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Software Solutions for Antenna Design Exploration: A Comparison of Packages, Tools, Techniques and Algorithms for Various Design Challenges

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Overview— Numerous software packages exist for solving antenna design optimization problems with many of these employing a variety of approaches, leading in turn to variations in optimization performance. Antenna designers, often amateurish in optimization, can be confused as to which algorithm in which software package should be used. A wrong choice can cause the optimization to fail or expend considerable time on the computationally expensive 3-D electromagnetic (EM) simulations involved. Whilst it is true that the various algorithms, combined with the variety of complex challenges found in different real-world scenarios, make a direct comparison among tools difficult, a robust attempt at such an evaluation is overdue. In this paper, 17 major antenna optimization methods available, employed in design packages, are classified into 6 categories based on optimization theory. A simple yet clear evaluation of their performance is proposed. Three diverse, but typical and representative, design problems are used to perform the comparison. The performance, i.e., optimization ability and efficiency, of the various optimization methods is then evaluated through a comprehensive testing process. Finally, rules of thumb to select the most suitable optimizer, depending on the optimization problem to be solved, are provided for the first time.

Index Terms—Antenna Design Exploration, Antenna Optimization.

I. SUMMARY OF TOOLS COMPARED

In this research, several tools have been shown to be effective for antenna analysis and design, and are thus used by many researchers and engineers. A list of such tools is given here.

High-Frequency Structure Simulator (HFSS) — This is industry-standard software, that employs the Finite Element Method (FEM) solver for simulating high-frequency electromagnetic (EM) fields and structures. HFSS, now ANSYS EDT/HFSS, dates back to the late 1980s [1].

Computer Simulation Technology (CST) — Since the early 1990s, CST Microwave Studio (CST-MWS) has been a leader among highly specialized tools for 3-D EM simulation of high-frequency components and structures

using a variety of solver modules, such as time domain, frequency domain, eigenmode and asymptotic solvers [2].

Feldberechnung für Körpermitbeliebiger Oberfläche (FEKO) — Commercialized in 1997, FEKO is a computational electromagnetics software product for the simulation of EM structures, adopting the integral formulation of Maxwell's equations based on the Method of Moments (MoM) and asymptotic high-frequency techniques [3].

Matrix Laboratory (MATLAB) — MATLAB is a highly interactive, multi-paradigm numerical integrated development environment (IDE), dating back to the late 1970s. It contains toolboxes, such as that for antenna design, first featured in a 2015 release, which allows for the analysis and visualization of antennas by using MoM [4].

Antenna Design Explorer (ADE) — Released in 2017, ADE 1.0 is a GUI-based antenna design optimization tool, which features a few state-of-the-art antenna design exploration methods and runs in the MATLAB environment [5].

The purpose of this paper is to conduct performance evaluations across these tools, including optimization capacity, efficiency and reliability. Since all these packages offer comprehensive user guides and/or online support (often including video demonstrations), questions regarding relative usability characteristics are not considered in this paper.

II. OPTIMIZATION TECHNIQUES IN ANTENNA DESIGN

Antennas can be modeled based on design experience [6]. Very often, antennas based on design experience do not meet the desired specifications, but can be used as initial designs. For example, to model a patch antenna using a waveguide port, the width of the port can be 6 to 10 times the width of the microstrip feedline [6]. A random value or the mid-value from the range can be selected as an initial design for the antenna's port. This initial design is often not the optimum, resulting therefore in the need for design exploration. For some cases involving few design parameters, parameter sweeping can be used to improve the initial designs of antennas. However, this becomes very difficult when the number of parameters increases. An optimization procedure must then be performed to find near-

TABLE I
OPTIMIZATION METHODS AVAILABLE IN VARIOUS COMMERCIAL TOOLS AND
REFERENCE METHODS

| Optimization Methods | CST | HFSS | FEKO | MATLAB | ADE |
|--|-----|------|------|--------|-----|
| Classic Powell (CP) | ✓ | | | | |
| Covariance Matrix Adaptation Evolution Strategy (CMA-ES) | ✓ | | | | |
| Differential Evolution (DE) | | | | | ✓ |
| Genetic Algorithm (GA) | ✓ | | ✓ | ✓ | |
| Grid Search (GS) | | | ✓ | | |
| Integer-only Sequential Nonlinear Programming (ISNLP) | | ✓ | | | |
| Interpolated Quasi-Newton (IQN) | ✓ | | | | |
| Link to MATLAB Optimizers via Workbench | | ✓ | | | ✓ |
| Nelder-Mead Simplex Algorithm (N-M) | ✓ | | ✓ | ✓ | |
| Particle Swarm Optimization (PSO) | ✓ | | ✓ | ✓ | |
| Pattern Search (PS) | | | | ✓ | |
| Quasi-Newton Method (QN) | | ✓ | | ✓ | |
| Sequential Nonlinear Programming (SNLP) | | ✓ | | | |
| Sequential Quadratic Programming (SQP) | | | | ✓ | |
| Simulated Annealing (SA) | | | | ✓ | |
| Surrogate-model-assisted Differential Evolution for Antenna Optimization (SADEA) | | | | | ✓ |
| Trust Region Framework (TRF) | ✓ | | | | |

optimal values of all these design parameters.

Table I summarizes the main optimization approaches available in the tools to be compared. CAD/CEM platforms such as HFSS, CST, FEKO, MATLAB and ADE feature built-in toolboxes or external add-ons for antenna design optimization using a range of optimization methods. These methods can be broadly classified as follows:

- Local optimization with limited exploration ability, e.g. Classic Powell (CP) and Grid Search (GS) [7], [8], [9], [10], [11];
- Classical derivative-free standard local search, e.g. Nelder-Mead Simplex Algorithm (N-M) and Pattern Search (PS) [7], [8], [12], [13], [14];
- Standard derivative-based local search, e.g. Quasi-Newton (QN), Sequential Quadratic Programming (SQP), Sequential Non-linear Programming (SNLP) and Integer-only Sequential Non-linear Programming (ISNLP) [15], [16], [17], [18];
- Surrogate-assisted local optimization with enhanced exploration ability, e.g. Trust Region Framework (TRF) and Interpolated Quasi-Newton (IQN) [15], [19], [20];

TABLE II
ANTENNA EXAMPLES AND REFERENCE METHODS

| Reference Methods | YUA1 | DRA | YUA2 |
|--|------|-----|------|
| Covariance Matrix Adaptation Evolution Strategy (CMA-ES) | ✓ | ✓ | ✓ |
| Differential Evolution (DE) | ✓ | ✓ | ✓ |
| Interpolated Quasi-Newton (IQN) | | ✓ | ✓ |
| Nelder-Mead Simplex Algorithm (N-M) | ✓ | ✓ | ✓ |
| Particle Swarm Optimization (PSO) | ✓ | ✓ | ✓ |
| Pattern Search (PS) | ✓ | ✓ | ✓ |
| Sequential Quadratic Programming (SQP) | ✓ | | |
| Surrogate-model-assisted Differential Evolution for Antenna Optimization (SADEA) | ✓ | ✓ | ✓ |
| Trust Region Framework (TRF) | | ✓ | ✓ |

- Surrogate-assisted global optimization, e.g. Surrogate-model-assisted Differential Evolution for Antenna Optimization (SADEA) [21];
- Standard evolutionary algorithms, e.g. Particle Swarm Optimization (PSO) and Differential Evolution (DE), which are used popularly in the computational intelligence field, Covariance Matrix Adaptation Evolution Strategy (CMA-ES) which is fairly recent, Genetic Algorithms (GAs) and Simulated Annealing (SA), both of which are several decades old [22], [23], [24], [25], [26], [27], [28],[29].

In this study, the efficiency and optimization capacity of many of these methods are compared for single objective (constrained) optimization problems using three real-world antenna design examples. These design examples are carefully selected to provide a broad, yet representative, range of antenna optimization problems with essential, key antenna characterization parameters such as return loss, bandwidth, gain, directivity and radiation pattern. They are, in brief:

- A six-element Yagi-Uda antenna (YUA1) [30] producing a nine-variable problem for optimal directivity at a given design frequency.
- A hybrid dielectric resonator antenna (DRA) [31] giving rise to a seven-variable problem for optimizing the reflection coefficient (S_{11}) within a given operating frequency band.
- A planar Yagi-Uda antenna (YUA2) [32] that forms an eight-variable optimization problem for optimizing S_{11} within a given operating frequency band and under a constraint for a specified average gain over the bandwidth.

Each of the above design examples is fully described in the following sections. These examples and the respective

reference methods used in each example for demonstration are shown in Table II. Nevertheless, some methods listed in Table I are not used for comparison in the examples. The reasons are given below:

- SNLP and ISNLP are excluded due to their similarity to SQP, which consistently out-performs them [16], [17], [18];
- CP and GS are excluded because they have very limited exploration abilities [8], [9], [10], [11] and the corresponding low success rate;
- GAs and SA are excluded because they are relatively old techniques [28], [29], [33], [34], and have already been superseded by newer methods (as listed in Table II).

Also, to ensure model consistency in MATLAB and CST-MWS, TRF and IQN have been tested using only the CST-MWS antenna models and SQP has been tested using only the MATLAB antenna model.

III. HARDWARE AND SOFTWARE SETTINGS

All the examples in this study have been tested on a PC with an Intel 4-core i7-4770K CPU and 24 GB RAM. The total execution time is also recorded for every method and every example. To ensure a fair comparison, the algorithmic parameters adopted for all methods are based on default settings except where otherwise specified. It is assumed that these default settings are well investigated based on empirical studies, which show that they are appropriate for most cases. Furthermore, it is assumed that these settings are not going to be changed by most designers who are not experienced in optimization.

A population/swarm size of 50 individuals/particles is used for SADEA, CMA-ES, PSO and DE. The other default settings for SADEA and DE are according to [21] and [35]. As far as YUA1 is concerned, the default settings for CMA-ES are as recommended in [27] and [36], while PSO, PS, N-M and SQP are implemented using MATLAB default settings. As far as the hybrid DRA and YUA2 are concerned, PS is implemented using MATLAB default settings, while CMA-ES, PSO, N-M, TRF and IQN are implemented using CST-MWS Optimizer's default settings. However, a sigma value of 1.0 was chosen for TRF and 10 passes are selected for IQN in CST-MWS to enhance exploration and direct the search towards a near-global optimum in both cases. This is because a good initial design is rarely available for most antenna problems and the above setting helps TRF and IQN to avoid being trapped in local optima. For local optimization methods, the mid-values of the ranges of the design variables and four randomly generated candidate designs are used as initial points for design exploration in all test cases.

In each example, a reference objective value, R_r , is used to obtain the performance metrics of all methods. The quality of the results, R_q , is then determined using:

$$R_q = R_o / R_r \times 100\% \quad (1)$$

where R_o is either the average objective value obtained by each global optimization method at convergence, or the mid

objective value (after ranking all results from different initial designs) obtained by each local optimization method at termination. To have an optimal, R_q must be at least 100%. However; if R_q is greater than or equal to 90%, it is assumed that R_q approaches 100% for inclusiveness. That is, if $R_q \geq 90\%$, then $R_q \rightarrow 100\%$.

The optimization speed index, R_s , is determined using:

$$R_s = R_i / R_e \quad (2)$$

where R_i is the least number of EM simulations used to obtain R_r amongst all methods, and R_e is the number of EM simulations used by each method to obtain R_r .

IV. EXPERIMENTAL RESULTS AND COMPARISONS

A. Example 1 – (Six-Element Yagi-Uda Antenna (YUA1))

The first example is modeled using the MATLAB antenna toolbox with the design criteria in [30]. As shown in Fig. 1, YUA1 consists of a folded dipole as the driven element, a reflector and four directors. Each simulation takes about 0.09 s. This is the only test case with computationally inexpensive simulation, which is used to make 10 independent runs of standard global optimization methods affordable.

YUA1 can be used for terrestrial radio and television broadcasting and the structure is easy to implement. The sensitivity of its radiation pattern to its physical dimensions makes it very challenging to obtain the optimum dimensions for desired radiation characteristics using manual techniques [30], [37]. For design exploration, the lengths of the directors (dirL1, dirL2, dirL3 and dirL4), the respective spacing between the directors (dirS1, dirS2, dirS3 and dirS4) and the spacing between the driven element and reflector (refS) are adjusted for higher directivity at zenith (elevation equal to 90°) at a design frequency of 165 MHz (center of the 30 to 300 MHz band). This constitutes a goal optimization problem. The ranges of the design variables are shown in Table II and the design objective is defined as follows:

$$\max |D_{(\theta, \phi)}| \text{ at } 165 \text{ MHz} \quad (3)$$

where D is the directivity of the antenna, θ and ϕ are the azimuth and elevation angles, respectively. $\max |D_{(\theta, \phi)}|$ is the maximum value of the directivity for given values of θ and ϕ , which can vary from $(-180^\circ$ to $180^\circ)$ and $(-90^\circ$ to $90^\circ)$, respectively. The dimensions of the antenna are in λ_{YUA} , which is calculated as follows:

$$\lambda_{YUA} = c_0 / f_{YUA} = 1.81692 \text{ m} \quad (4)$$

where c_0 is the speed of light in vacuum ($2.998 \times 10^8 \text{ ms}^{-1}$) and f_{YUA} is the operating frequency ($165 \times 10^6 \text{ Hz}$).

To ensure all global optimization methods converge, the computing budget is as follows: 800 simulations for SADEA and 5000 simulations for each one of CMA-ES, PSO and DE, while 10 independent runs are performed for each one of the above four methods. The local optimization methods are applied with five independent runs per method

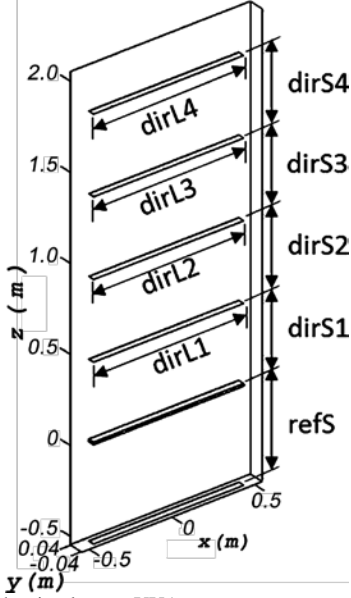


Fig. 1. Layout of the six-element YUA.

TABLE III

RANGES OF DESIGN VARIABLES (ALL IN λ_{YUA}) FOR DESIGN EXPLORATION (EXAMPLE 1)

| Variables | Lower Bound | Upper Bound |
|-----------|-------------|-------------|
| dirL1 | 0.20 | 0.70 |
| dirL2 | 0.20 | 0.70 |
| dirL3 | 0.20 | 0.70 |
| dirL4 | 0.20 | 0.70 |
| refS | 0.05 | 0.50 |
| dirS1 | 0.05 | 0.50 |
| dirS2 | 0.05 | 0.50 |
| dirS3 | 0.05 | 0.50 |
| dirS4 | 0.05 | 0.50 |

TABLE IV

INITIAL DESIGNS (ALL IN λ_{YUA}) FOR LOCAL OPTIMIZATION METHODS (EXAMPLE 1)

| Variables | 1st Initial Design | 2nd Initial Design | 3rd Initial Design | 4th Initial Design | 5th Initial Design |
|-----------|--------------------|--------------------|--------------------|--------------------|--------------------|
| dirL1 | 0.45 | 0.43 | 0.56 | 0.59 | 0.31 |
| dirL2 | 0.45 | 0.69 | 0.44 | 0.47 | 0.32 |
| dirL3 | 0.45 | 0.38 | 0.31 | 0.55 | 0.69 |
| dirL4 | 0.45 | 0.66 | 0.41 | 0.29 | 0.50 |
| refS | 0.28 | 0.42 | 0.17 | 0.06 | 0.37 |
| dirS1 | 0.28 | 0.10 | 0.32 | 0.25 | 0.48 |
| dirS2 | 0.28 | 0.36 | 0.25 | 0.42 | 0.13 |
| dirS3 | 0.28 | 0.27 | 0.31 | 0.45 | 0.08 |
| dirS4 | 0.28 | 0.15 | 0.50 | 0.22 | 0.29 |

using, respectively, the five initial designs in Table IV. These methods run in the MATLAB environment by calling the functions “patternsearch”, “fminsearch” and “fmincon ‘sqp’”), respectively for PS, N-M and SQP.

To evaluate all methods, R_r is set to 40.37, which corresponds to a value of $\max|D_{(0,90^\circ)}|$ equal to 10.19 dBi and a front-to-back ratio of 30.17 dBi at 165 MHz. Table V shows that SADEA, CMA-ES, PSO and DE obtain objective function values higher than the reference objective function value in all runs (even for their worst results). From Table VI, it can be seen that PS and N-M obtain objective function values higher than or approaching the reference objective function value for all initial designs. Table VI also shows that SQP obtains objective function values close to or

TABLE V
OPTIMIZED RESULTS FOR GLOBAL OPTIMIZATION METHODS (EXAMPLE 1)

| Method | Best | Worst | Average | Std. |
|--------|-------|-------|---------|------|
| SADEA | 66.01 | 57.50 | 61.76 | 3.17 |
| CMA-ES | 65.34 | 61.15 | 63.74 | 1.18 |
| PSO | 65.37 | 54.26 | 61.69 | 3.20 |
| DE | 61.57 | 49.71 | 54.68 | 4.29 |

TABLE VI
OPTIMIZED RESULTS FOR LOCAL OPTIMIZATION METHODS (EXAMPLE 1)

| Method | 1st Initial Design | 2nd Initial Design | 3rd Initial Design | 4th Initial Design | 5th Initial Design |
|------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| PS (Best) | 62.64 | 55.30 | 60.08 | 58.69 | 59.63 |
| N-M (Best) | 59.36 | 54.92 | 62.07 | 39.60 | 58.05 |
| SQP (Best) | 38.82 | 26.58 | 14.78 | 57.26 | 13.20 |

TABLE VII
PERFORMANCE METRICS OF ALL METHODS (EXAMPLE 1)

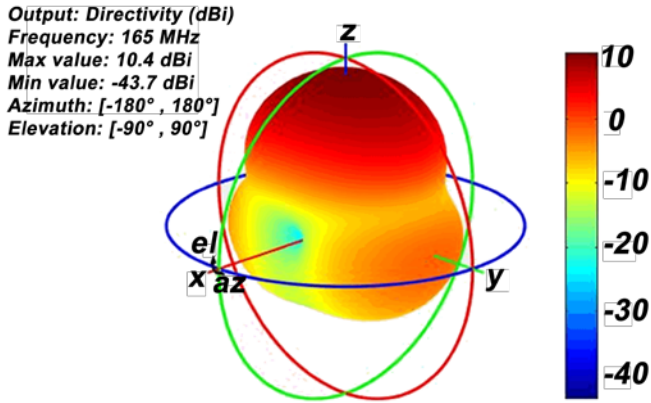
| Method | R_o | R_r | R_q | R_e | R_s |
|--------|-------|-------|---------|-------|-------|
| SADEA | 61.76 | 40.37 | 153.00% | 181 | 1.00 |
| CMA-ES | 63.76 | 40.37 | 157.90% | 477 | 0.38 |
| PSO | 61.69 | 40.37 | 152.83% | 677 | 0.27 |
| DE | 54.68 | 40.37 | 135.47% | 562 | 0.32 |
| PS | 59.63 | 40.37 | 147.73% | 352 | 0.51 |
| N-M | 58.05 | 40.37 | 143.82% | 229 | 0.79 |
| SQP | 26.58 | 40.37 | 65.59% | N/A | N/A |

higher than the reference function objective value for two initial designs (1st and 4th initial designs, respectively).

The performance metrics (R_q , R_e and R_s) of all methods are shown in Table VII. It can be seen that SADEA, CMA-ES, PSO, DE, PS and N-M all obtain results that meet the optimality criterion at varying optimization speed indices. From Fig. 3 and Table VII, SADEA uses the least number of EM simulations to obtain the reference objective (R_r) value compared to other methods. Then, R_i adopts the value of R_e that corresponds to SADEA, in order to calculate R_s according to (2), thus making the value of R_s equal to unity for SADEA. In this example, CMA-ES achieves the best performance, while SADEA and PSO achieve a little worse but similar performance; SADEA is several times faster than CMA-ES and PSO. Hence, the overall best performance belongs to SADEA. A typical SADEA result is shown in Fig. 2 for: dirL1 = 0.43 λ_{YUA} , dirL2 = 0.43 λ_{YUA} , dirL3 = 0.41 λ_{YUA} , dirL4 = 0.42 λ_{YUA} , refS = 0.15 λ_{YUA} , dirS1 = 0.14 λ_{YUA} , dirS2 = 0.17 λ_{YUA} , dirS3 = 0.10 λ_{YUA} and dirS4 = 0.14 λ_{YUA} .

B. Example 2 – (Hybrid Dielectric Resonator Antenna (DRA))

The second example is a hybrid DRA modeled in CST-MWS. The layout of the hybrid DRA is shown in Fig. 4 and the excitation is via aperture-coupling with $TE_{\delta 11}^x$ mode. The relative permittivity and loss tangent of the dielectric resonator (DR) are 10 and 0.0001, respectively. The substrate is RO4003C with a thickness of 0.5 mm, a relative permittivity of 3.38 and a loss tangent of 0.0027. The substrate is placed on a copper ground of 0.05 mm thickness. The mesh density is 12 cells per wavelength and the total number of mesh cells is approximately 22,000. Each EM simulation takes around 30 s. The hybrid DRA can be used in wireless access systems. It has a compact structure with complex implementation. The excitation



(a) 3-D radiation pattern.

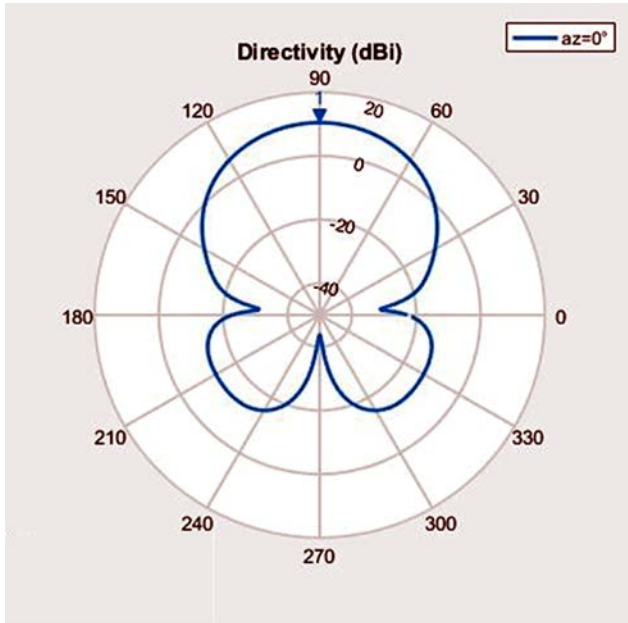
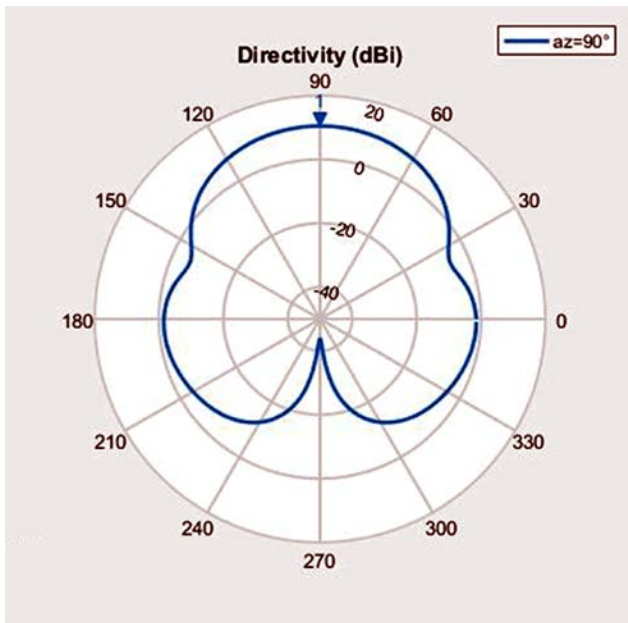
(b) 2-D Radiation pattern for a vertical cut plane ($az = 0^\circ$).(c) 2-D Radiation pattern for a vertical cut plane ($az = 90^\circ$).

Fig. 2. Radiation pattern of the optimized YUA1 derived from SADEA.

mode, coupling and resonances of the hybrid DRA are all

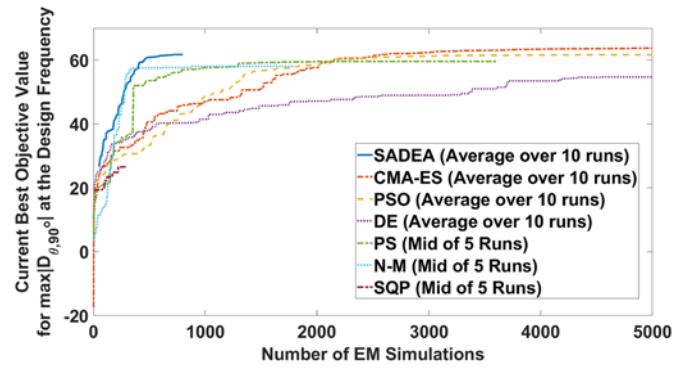


Fig. 3. Convergence graphs of all methods (Example 1).

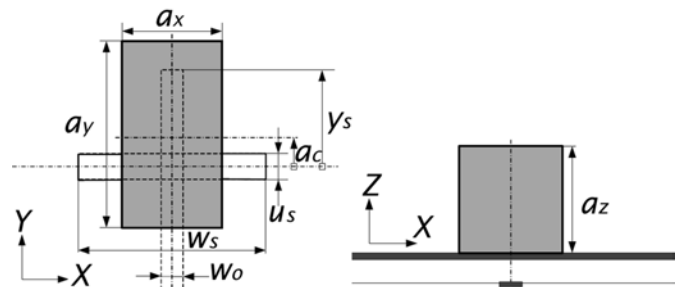


Fig. 4. Layout of the Hybrid DRA

controlled by its physical dimensions [38]. The estimation of the optimum dimensions using manual techniques is very challenging. An optimization procedure to find the optimum dimensions is also challenging because the optimal region of DRA is very narrow compared to most other antennas [39]. Therefore, the optimization algorithm is required to have strong exploration ability. The narrow optimal region also raises significant challenges for surrogate modeling methods [39].

The design exploration goal is to adjust the dimensions of the DR brick (a_x , a_y and a_z), the slot dimensions (u_s and w_s), the length of the microstrip slab (y_s) and the location of the DR relative to the slot (a_c), so that the bandwidth of the DRA is centered at 5.5 GHz in the operational band of 5.28 GHz to 5.72 GHz. There is thus a single objective function, which is to minimize the maximum reflection coefficient (S_{11}) from 5.28 GHz to 5.72 GHz as shown in the following expression:

$$\text{minimize } \max |S_{11}| \quad (5.28 \text{ GHz to } 5.72 \text{ GHz}) \quad (5)$$

This constitutes a goal optimization problem. The ranges of the design variables are shown in Table VIII. To keep the slot under the DRA in all possible cases, the geometric constraint used is $a_c \leq 0.5 \times a_y$.

To make all global optimization methods converge, the computing budget is as follows: 800 simulations for SADEA over ten independent runs, 5,000 simulations for each one of CMA-ES, PSO and DE. For CMA-ES, PSO and DE, three independent runs are carried out per method, since

TABLE VIII
 RANGES OF DESIGN VARIABLES (ALL IN MM) FOR DESIGN EXPLORATION
 (EXAMPLE 2)

| Variables | a_x | a_y | a_z | a_c | u_s | w_s | y_s |
|-------------|-------|-------|-------|-------|-------|-------|-------|
| Lower Bound | 6 | 12 | 6 | 6 | 0.5 | 4 | 2 |
| Upper Bound | 10 | 16 | 10 | 8 | 4 | 12 | 12 |

TABLE IX
INITIAL DESIGNS (ALL IN MM) FOR LOCAL OPTIMIZATION METHODS
(EXAMPLE 2)

| Variables | 1st Initial Design | 2nd Initial Design | 3rd Initial Design | 4th Initial Design | 5th Initial Design |
|-----------|--------------------|--------------------|--------------------|--------------------|--------------------|
| a_x | 8.00 | 8.79 | 7.10 | 6.20 | 9.77 |
| a_y | 14.00 | 14.58 | 12.91 | 14.79 | 15.77 |
| a_z | 8.00 | 7.23 | 9.18 | 9.65 | 8.47 |
| a_c | 7.00 | 0.15 | 0.36 | 4.76 | 0.80 |
| u_s | 2.25 | 2.56 | 3.37 | 0.68 | 3.68 |
| w_s | 8.00 | 5.19 | 11.52 | 11.62 | 8.22 |
| y_s | 7.00 | 6.19 | 6.49 | 7.64 | 5.92 |

TABLE X
OPTIMIZED RESULTS FOR GLOBAL OPTIMIZATION METHODS (EXAMPLE 2)

| Method | Best Objective Value | Worst Objective Value | Average Objective Value | Std. |
|--------|----------------------|-----------------------|-------------------------|------|
| SADEA | -26.5 dB | -22.2 dB | -24.5 dB | 1.3 |
| CMA-ES | -14.0 dB | -11.2 dB | -12.5 dB | 1.4 |
| PSO | -22.6 dB | -19.1 dB | -21.4 dB | 2.0 |
| DE | -24.7 dB | -23.1 dB | -24.1 dB | 0.9 |

additional runs are not affordable (a single run costs about two days). For the local optimization methods, five independent runs are implemented using, respectively, the initial designs shown in Table IX. PS runs in MATLAB environment by calling the “patternsearch” function, while TRF, N-M and IQN run in CST-MWS.

To evaluate all methods, R_r is set to be equal to -20 dB, which corresponds to a value of $\max|S_{11}|$ equal to -20 dB, a voltage standing wave ratio (VSWR) equal to 1.22 and an absolute value of reflection coefficient equal to 0.1 in the operating band (5.28 – 5.72 GHz). Table X shows that SADEA and DE obtain objective function values higher than the reference objective function value in all runs (even

TABLE XI
OPTIMIZED RESULTS FOR LOCAL OPTIMIZATION METHODS (EXAMPLE 2)

| Method | 1st Initial Design | 2nd Initial Design | 3rd Initial Design | 4th Initial Design | 5th Initial Design |
|--------|--------------------|--------------------|--------------------|--------------------|--------------------|
| PS | -20.7 dB | -17.8 dB | -9.7 dB | -11.2 dB | -11.5 dB |
| TRF | -19.1 dB | -18.4 dB | -19.0 dB | -20.5 dB | -21.2 dB |
| N-M | -14.8 dB | -11.9 dB | -11.8 dB | -14.1 dB | -11.9 dB |
| IQN | -12.1 dB | -12.2 dB | -11.5 dB | -11.3 dB | -12.2 dB |

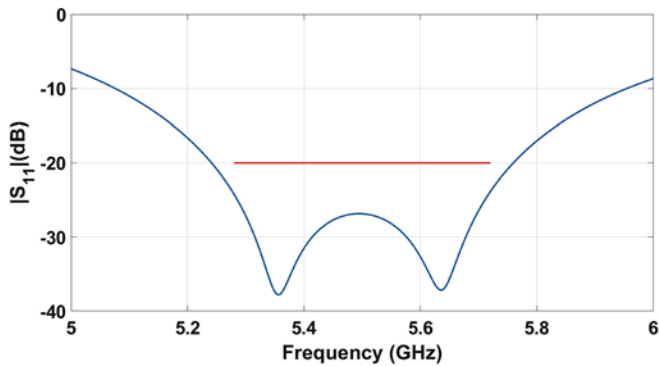


Fig. 5. Typical frequency response of the optimized hybrid DRA derived from SADEA.

in their worst cases). PSO satisfies the reference objective function value in two out of three runs, and the worst result is 95.3% of the reference objective value. For CMA-ES, in all the three runs, the best result is an objective function

TABLE XII
PERFORMANCE METRICS OF ALL METHODS (EXAMPLE 2)

| Method | R_o | R_r | R_q | R_e | R_s |
|--------|----------|----------|--------------------------------------|-------|-------|
| SADEA | -24.5 dB | -20.0 dB | 132.5% | 379 | 1.0 |
| CMA-ES | -12.5 dB | -20.0 dB | 62.3% | N/A | N/A |
| PSO | -21.3 dB | -20.0 dB | 106.7% | 1891 | 0.14 |
| DE | -24.1 dB | -20.0 dB | 120.4% | 4772 | 0.20 |
| PS | -11.5 dB | -20.0 dB | 57.7% | N/A | N/A |
| TRF | -19.1 dB | -20.0 dB | 95.3% ($R_q \rightarrow 100\%$) | 443 | 0.9 |
| N-M | -11.9 dB | -20.0 dB | 59.6% | N/A | N/A |
| IQN | -12.1 dB | -20.0 dB | 60.5% | N/A | N/A |

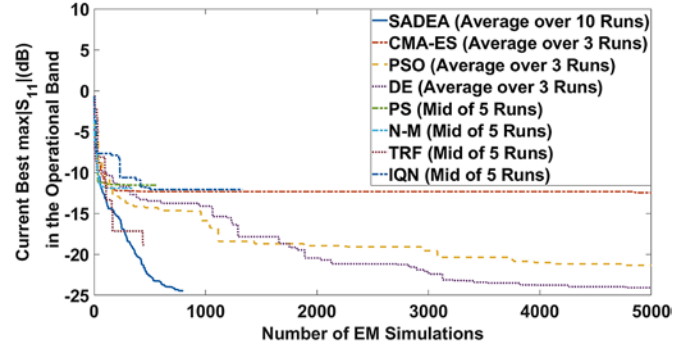


Fig. 6. Convergence graphs of all methods (Example 2).

value that is far from the reference objective value. As shown in Table XI, PS satisfies the reference objective function value for a single initial design (out of five); TRF satisfies or approaches the reference objective value for all initial designs; the results derived from N-M and IQN seem to be far from the reference objective function value for all respective initial designs.

The performance metrics (R_q , R_e and R_s) of all methods are shown in Table XII. As shown in this table, SADEA, PSO and DE obtain results that meet the optimality criterion at varying optimization speed indexes (R_s). Also, the result extracted for TRF is close to the optimality criterion. According to Fig. 6 and Table XII, SADEA uses the least number of EM simulations to obtain the desired value of R_r compared to other methods. Then, R_i adopts the value of R_e that corresponds to SADEA, in order to calculate R_s according to (2), thus making the value of R_s equal to unity for SADEA. In this example, SADEA performs best in both optimality and efficiency. A typical SADEA result is shown in Fig. 5 for: $a_x = 9.47$ mm, $a_y = 12.23$ mm, $a_z = 9.98$ mm, $a_c = 4.27$ mm, $u_s = 2.32$ mm, $w_s = 8.20$ mm and $y_s = 2.20$ mm.

C. Example 3 – (Planar Yagi-Uda Antenna (YUA2))

The third example is a planar YUA modeled in CST-MWS. As shown in Fig. 7, the structure of the planar YUA is made up of a director and a driven element fed by a 50 Ω microstrip-to-slot balun using a power divider. The planar YUA is implemented on a 0.635 mm thick RT6010 substrate with a relative permittivity of 10.2 and a loss tangent of 0.0023. The total number of mesh cells is over

TABLE XIII
RANGES OF DESIGN VARIABLES (ALL IN MM) FOR DESIGN EXPLORATION
(EXAMPLE 3)

| Variables | s_1 | s_2 | v_1 | v_2 | u_1 | u_2 | u_3 | u_4 |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Lower Bound | 3 | 1 | 5 | 2 | 2 | 2 | 1 | 1 |
| Upper Bound | 7 | 6 | 12 | 12 | 6 | 6 | 5 | 5 |

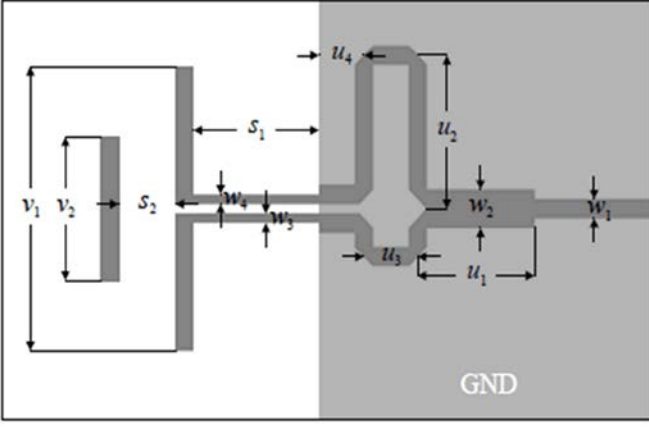


Fig. 7. Layout of the planar Yagi-Uda Antenna (YUA2)

86,000 at a mesh density of 15 cells per wavelength. Each simulation takes approximately 115 s.

The planar YUA can be used for radar communication applications. It has a compact structure with complex implementation. The configuration (quasi-Yagi) and excitation scheme (balun) of YUA2 make its response

TABLE XIV
INITIAL DESIGNS (ALL IN MM) FOR LOCAL OPTIMIZATION METHODS
(EXAMPLE 3)

| Variables | 1st Initial Design | 2nd Initial Design | 3rd Initial Design | 4th Initial Design | 5th Initial Design |
|-----------|--------------------|--------------------|--------------------|--------------------|--------------------|
| s_1 | 5.00 | 6.46 | 6.08 | 6.31 | 3.19 |
| s_2 | 3.50 | 3.08 | 2.82 | 5.67 | 2.17 |
| v_1 | 8.50 | 5.87 | 8.15 | 5.21 | 9.63 |
| v_2 | 7.00 | 6.66 | 11.92 | 3.04 | 9.20 |
| u_1 | 4.00 | 3.82 | 4.53 | 5.74 | 5.77 |
| u_2 | 4.00 | 2.99 | 5.91 | 5.30 | 2.38 |
| u_3 | 3.00 | 1.38 | 2.36 | 4.98 | 4.63 |
| u_4 | 3.00 | 3.17 | 2.22 | 1.97 | 3.62 |

characteristics (bandwidth and gain) highly sensitive to its physical dimensions [40]. The estimation of the optimum dimensions using manual techniques is very challenging. The design exploration goal is to adjust the dimensions (s_1 , s_2 , v_1 , v_2 , u_1 , u_2 , u_3 and u_4) for an operational bandwidth of 10 GHz to 11 GHz at an average gain (G) not smaller than 6 (7.8 dBi) over the bandwidth as shown in the following expression:

$$\begin{aligned} & \text{minimize } \max|S_{11}| (10 \text{ GHz to } 11 \text{ GHz}) \quad (6) \\ & \text{s.t. } \text{mean}(G) \geq 6 \end{aligned}$$

The above requirements constitute a constrained optimization problem. Note that the constraint is different from [32], which uses 6 dBi. The intention is to make the problem more challenging in order to test the optimization ability, while considering tight constraint(s) of the reference methods. To handle the constraint, a penalty coefficient of 50 is used for all applicable methods. The ranges of the design variables are shown in Table XIII. To make all global optimization methods converge, the computing budget is as follows: 600 simulations for SADEA over ten independent runs, 4,000 simulations for each one of CMA-ES, PSO and DE. For CMA-ES, PSO and DE, three independent runs are carried out per method, since, once again, additional runs are not affordable (a single run costs about five days). For the local optimization methods, five independent runs are

TABLE XV
OPTIMIZED RESULTS FOR GLOBAL OPTIMIZATION METHODS (EXAMPLE 3)

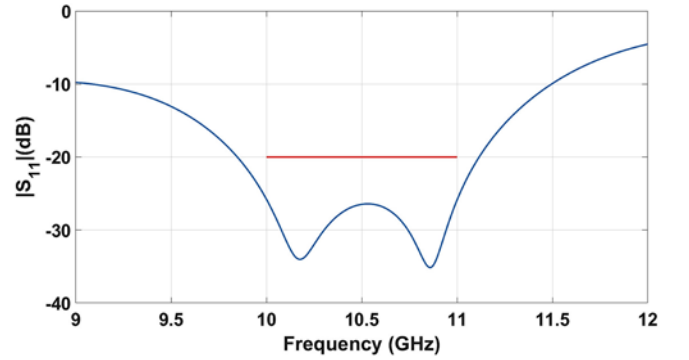
| Method | Best Objective Value | Worst Objective Value | Average Objective Value | Std. |
|--------|----------------------|-----------------------|-------------------------|------|
| SADEA | -25.7 dB | -21.9 dB | -23.4 dB | 1.2 |
| CMA-ES | -24.4 dB | -13.6 dB | -17.2 dB | 6.2 |
| PSO | -15.6 dB | -7.5 dB | -12.3 dB | 4.2 |
| DE | -6.27 dB | -4.9 dB | -5.5 dB | 0.7 |

TABLE XVI
OPTIMIZED RESULTS FOR LOCAL OPTIMIZATION METHODS (EXAMPLE 3)

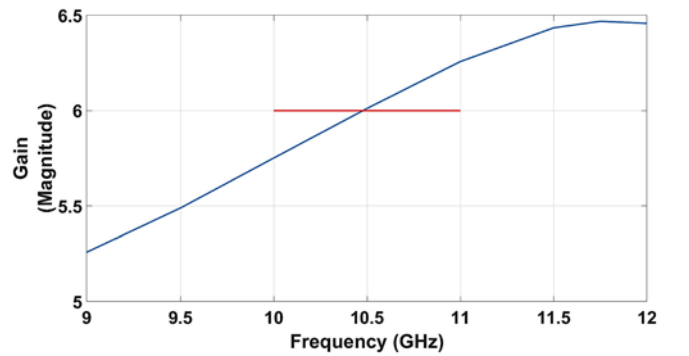
| Method | 1st Initial Design | 2nd Initial Design | 3rd Initial Design | 4th Initial Design | 5th Initial Design |
|--------|--------------------|--------------------|--------------------|--------------------|--------------------|
| PS | -15.6 dB | -23.1 dB | -18.6 dB | -15.2 dB | -8.9 dB |
| TRF | -18.2 dB | -18.6 dB | -18.6 dB | -18.2 dB | -18.6 dB |
| N-M | -22.7 dB | -19.7 dB | -17.2 dB | -17.3 dB | -13.4 dB |
| IQN | -17.5 dB | -14.0 dB | -19.1 dB | -16.9 dB | -13.3 dB |

carried out using, respectively, the five initial designs shown in Table XIV. PS runs in MATLAB environment by calling the “patternsearch” function, while TRF, N-M and IQN run in CST-MWS.

To evaluate all methods, R_r is set to be equal to -20 dB (i.e. $\max|S_{11}|$ of -20 dB, $VSWR = 1.22$, and absolute value of reflection coefficient equal to 0.1) with a mean gain of 6 in the operating band (10 – 11 GHz). All methods satisfy the constraint in (6) at convergence and/or termination. Table XV shows that SADEA obtains objective function values higher than the reference objective function value in all runs (even in the worst case). The results of PSO and DE are far from the reference objective function value for all runs, while CMA-ES satisfies the reference objective function



(a) S_{11} versus frequency.



(b) Gain versus frequency.

Fig. 8. Typical frequency response of the optimized YUA2 derived from SADEA.

TABLE XVII
PERFORMANCE METRICS OF ALL METHODS (EXAMPLE 3)

| Method | R_o | R_r | R_q | R_e | R_s |
|--------|----------|----------|--------------------------------------|-------|-------|
| SADEA | -23.4 dB | -20.0 dB | 128.7% | 212 | 1.0 |
| CMA-ES | -17.2 dB | -20.0 dB | 86.0% | N/A | N/A |
| PSO | -12.3 dB | -20.0 dB | 61.3% | N/A | N/A |
| DE | -5.5 dB | -20.0 dB | 27.5% | N/A | N/A |
| PS | -15.2 dB | -20.0 dB | 75.8% | N/A | N/A |
| TRF | -18.6 dB | -20.0 dB | 93.0% ($R_q \rightarrow 100\%$) | 469 | 0.5 |
| N-M | -17.3 dB | -20.0 dB | 86.5% | N/A | N/A |
| IQN | -16.9 dB | -20.0 dB | 84.4% | N/A | N/A |

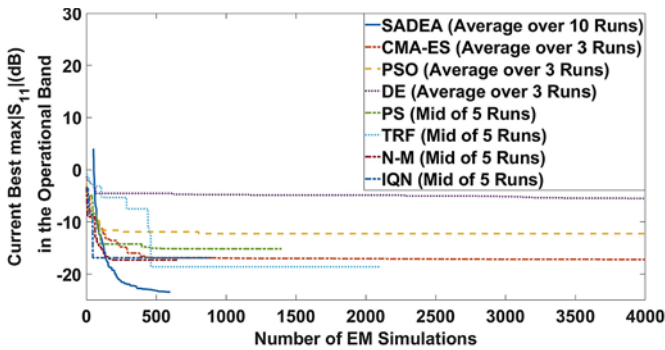


Fig. 9. Convergence graphs of all methods (Example 3)

value in one out of three runs. As shown in Table XVI, PS and N-M satisfy or approach the reference objective function value for two out of five initial designs. TRF approaches but does not satisfy the reference objective value for all initial designs; IQN approaches the reference objective function value only for a single initial design.

The performance metrics (R_q , R_e and R_s) of all methods are shown in Table XVII. Here it can be seen that SADEA meets the optimality criterion. Also, as shown in Table XVII, the result derived from TRF approaches the optimality criterion. According to Fig. 9 and Table XVII, SADEA uses the least number of EM simulations to obtain the desired value of R_r compared to other methods. R_i then adopts the value of R_e that corresponds to SADEA, in order to calculate R_s according to (2), thus making the value of R_s equal to unity for SADEA. In this example, SADEA performs best in both optimality and efficiency. A typical SADEA result is shown in Fig. 8 for: $s_1 = 5.06$ mm, $s_2 = 5.95$ mm, $v_1 = 7.51$ mm, $v_2 = 2.95$ mm, $u_1 = 4.22$ mm, $u_2 = 1.5$ mm, $u_3 = 3.80$ mm and $u_4 = 2.78$ mm.

V. OVERALL COMPARISONS, SUMMARY AND CONCLUSIONS

Three real-world antenna examples have been used to compare the efficiency and optimization capacity of various popular optimization methods available in software tools such as CST-MWS, ANSYS EDT/HFSS, FEKO, MATLAB and ADE 1.0. Note that for comparison purposes, the fidelities (i.e., levels of discretization) of the EM models in this paper are reasonably moderate. In real-world design, higher-fidelity and computationally more expensive EM models could be used. The quality of results and optimization speed for each method have been gathered using standard design criteria as a benchmark in each example. An overview of all reference methods based on the performance metrics (quality of results and optimization speed in terms of number of EM simulations) obtained in all examples is shown in Table XVIII and Fig. 10. In Fig. 10, the red line is a separator line between pass and fail

TABLE XVIII
OVERVIEW OF ALL REFERENCE METHODS BASED ON PERFORMANCE FOR ALL EXAMPLES

| Reference Methods | Example 1 (YUA1) | Example 2 (DRA) | Example 3 (YUA2) |
|-------------------|------------------|-----------------|------------------|
| SADEA | Pass | Pass | Pass |
| CMA-ES | Pass | Fail | Fail |
| PSO | Pass | Pass | Fail |
| DE | Pass | Pass | Fail |
| PS | Pass | Fail | Fail |
| N-M | Pass | Fail | Fail |
| SQP | Fail | N/A | N/A |
| TRF | N/A | Pass | Pass |
| IQN | N/A | Fail | Fail |

according to the benchmark for R_q stated earlier in Section III.

From Table XVIII, it can be seen that SADEA (in ADE 1.0) succeeds in all examples. TRF largely succeeds in all applicable test cases, although its performance is approaching but not satisfying the design criteria. PSO and DE succeed in the first two examples, while CMA-ES, PS and N-M succeed only in the first example. Finally, SQP and IQN fail in all applicable test cases. The performance metrics given in Fig. 10 show that: SADEA, CMA-ES, PSO, DE, PS and N-M obtain high-quality results for example 1 (YUA1) with SADEA and N-M using the least number of EM simulations. For example 2 (hybrid DRA), SADEA, PSO and DE obtain high-quality results; TRF obtains a reasonably good result; SADEA and TRF use the least number of EM simulations. For example 3 (YUA2), SADEA obtains a high-quality result and TRF obtains a reasonably good quality result; SADEA uses the least number of EM simulations.

Also, it has been shown that the performance of PS, SQP, TRF, N-M and IQN depends on the initial designs in all applicable examples. In other words, PS, SQP, TRF, N-M and IQN yield desirable results for good initial designs in all respective test cases. However, PS, SQP, N-M and IQN are very sensitive to the initial design, while TRF is less sensitive when it uses settings to promote global exploration. Using five initial designs, the mid objective value (after ranking all results from the initial designs) has been considered for performance evaluation of PS, SQP, TRF, N-M and IQN, respectively. Although an increase in the number of initial designs may alter the choice of the solutions adopted for performance evaluation, this approach is reasonable since good initial designs are often unavailable and unpredictable for local optimization techniques. Note that for methods whose result is sensitive to initial designs, their efficiency is difficult to be judged in general. Thus, "N/A" is set.

Using the details from Table XVIII and Fig. 10, Table XIX provides a summary of the optimization capacity, efficiency, reliability and comments for all reference methods. While, clearly, all antenna design problems have their own unique characteristics, possibly suiting different solution strategies, the three examples chosen in this paper are considered to cover typical and representative cases in antenna design area. The application of all the methods in the above examples have in summary shown the following:

- 1) SADEA offers significantly improved performance when compared to alternatives in all aspects. It is the suggested method, especially when there is no good initial design or the problem is challenging.

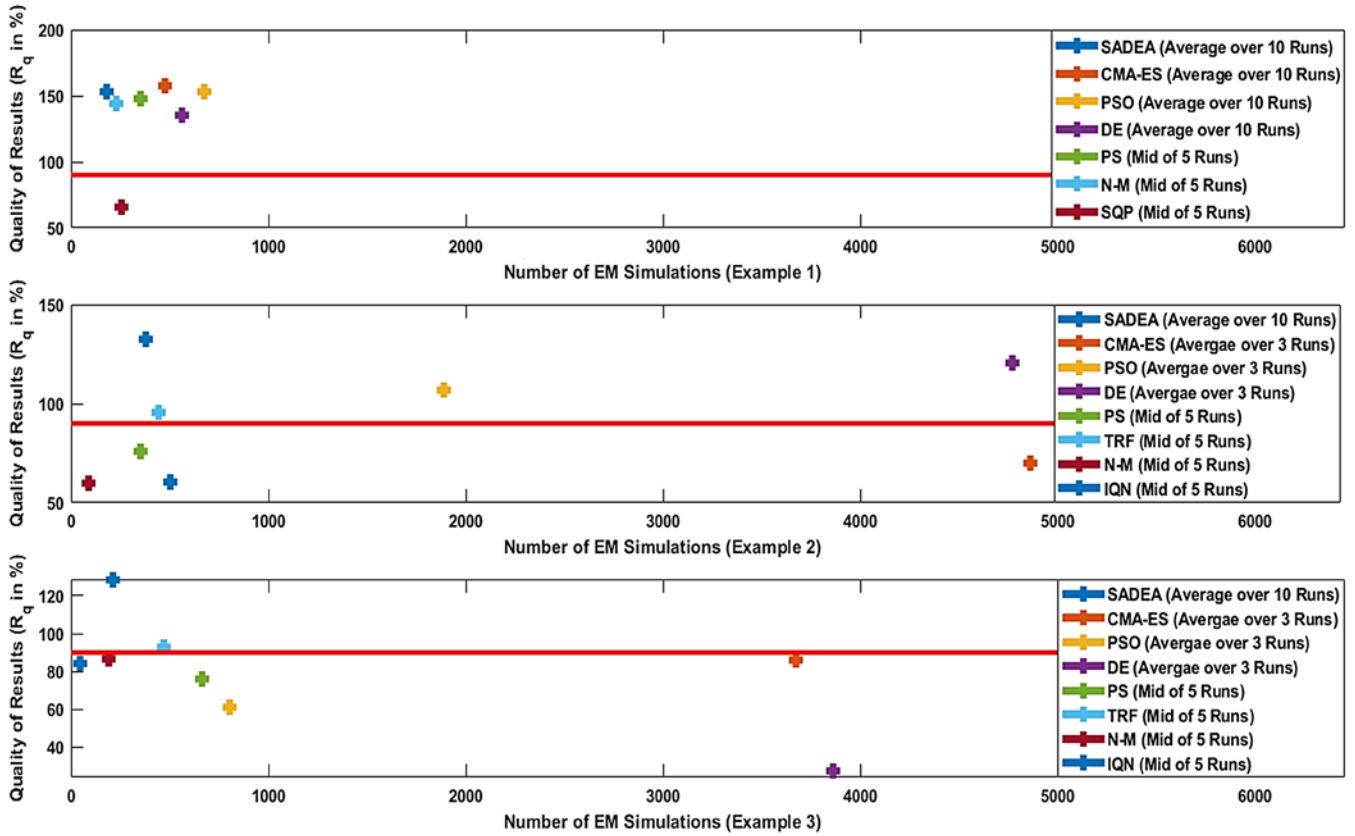


Fig. 10. Performance metrics of all reference methods for all examples (the red line is the separator between pass and fail).

TABLE XIX
OPTIMIZATION CAPACITY, EFFICIENCY, RELIABILITY AND CONDITION OF USE FOR ALL REFERENCE METHODS

| Reference Methods | Associated Tool(s) | Optimization Capacity | Efficiency | Reliability | Comments |
|-------------------|--------------------|-----------------------|------------|-------------|--|
| SADEA | ADE 1.0 | Very High | Very High | Very High | No specific conditions of use |
| CMA-ES | CST-MWS | Medium | Low | Medium | Effectiveness may reduce in the face of narrow optimal regions and or when handling tight constraints. |
| PSO | CST-MWS and MATLAB | High | Low | High | Effectiveness may reduce when handling tight constraints |
| DE | ADE 1.0 | High | Low | High | Effectiveness may reduce when handling tight constraints |
| PS | MATLAB | Medium | N/A | Low | Sensitive to initial designs |
| N-M | CST-MWS and MATLAB | Medium | N/A | Low | Sensitive to initial designs |
| SQP | MATLAB | Low | N/A | Low | Sensitive to initial designs |
| TRF | CST-MWS | High | High | High | Good quality results but not as good as SADEA; Less sensitive but is affected by initial designs |
| IQN | CST-MWS | Low | N/A | Low | Sensitive to initial designs |

- 2) CMA-ES, PSO and DE are not recommended because SADEA and these three methods conduct global optimization (without an initial design) and SADEA shows higher performance in optimization capacity, efficiency and reliability compared to them. In particular, the efficiency of these three methods is their essential weakness.
- 3) When there is already a good initial design, N-M, PS and TRF are recommended, because they can often obtain satisfactory results with fair efficiency.
- 4) When there is an initial design with a fair quality, TRF is a better choice than N-M and PS because it is not so sensitive to the initial designs and its optimization capacity is higher than them. When the problem is not challenging, TRF can obtain satisfactory results efficiently. However, discarding the fair-quality initial design and using SADEA may obtain higher-quality results with higher efficiency, especially when the problem is challenging.
- 5) SQP and IQN are not recommended because they fail most of the time.

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