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ON THE ELECTRON-MUON MASS RATIO*

D. DILLENBURG* and Th.A.J. MARIS*
Instituto de Física, Universidade Federal do Rio Grande do Sul, Porto Alegre, RS, Brasil

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The quantum electrodynamics (QED) of electrons is considered as a theory with a passive dilatation invariance which is perturbed by the electromagnetic coupling to hadrons and muons. A stability criterium is introduced and evaluated in lowest order of the perturbation. The resulting expression for the electron-muon mass ratio in terms of the vacuum polarization can be tested in $e^+ - e^-$ colliding beam experiments.

1. Introduction

Suppose one has a quantum field theory with a passive dilatation invariance. By this we mean that the theory allows for a continuous set of solutions (given as irreducible representations of the field operators in a Hilbert space) which are related to each other by dilatations, defined by the well-known algebraic transformations of the field operators. For that to occur it is necessary and sufficient that the basic theory does not contain any constant of non-vanishing mass dimension.

In order to avoid a continuous mass spectrum the passive dilatation symmetry is assumed to be spontaneously broken [1,2]. This is equivalent to the statement that one cannot obtain the other solutions of the theory by a unitary transformation of one of the solutions in its representation space (in contradistinction to the case of a "good" symmetry). Still another way of expressing the same is: the algebraic dilatation transformation of the field operators is not representable by a unitary transformation in the space of a solution [3].

For a more detailed discussion of the present, somewhat unconventional, point of view on spontaneously broken symmetries in which also the relation to the usual definitions and Goldstone's theorem is shown, see ref. [4].

To calculate physical quantities, one has to choose one member of the set of solutions (representations) and for a theory to be a useful one, each of the solutions should describe the whole of the physics of the system. Also each of these different

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* Senior Research Fellow, Conselho Nacional de Pesquisas (Brasil).

descriptions should give the same physical predictions as any of the other ones. Consequently the representations may differ from each other only with respect to quantities which are not actually measurable. In the case of dilatation invariance these are all quantities which have a non-vanishing mass dimension; only dimensionless quantities as mass ratios have a physical meaning. Such quantities are indeed independent of the arbitrary mass unit with characterizes any special solution belonging to the mentioned infinite set of the, only in a mathematical sense, non-equivalent representations.

Suppose in a system, described by a theory of the above type, a perturbation is introduced containing a small component which breaks the passive dilatation invariance, because of the occurrence of a numerical quantity of non-vanishing mass dimension. Now the various descriptions of the unperturbed system, viewed as zero order approximations to the solution of the perturbed system, are not anymore physically equivalent and will lead to physically quite different first and higher order corrections. In this sense the perturbation, how weak it may be, has introduced an absolute mass scale into the original system [5,6].

At this point there arise two different logical possibilities:

(a) The introduction of the perturbation renders the theory incomplete. One must explicitly give a number which characterizes the ratio between the mass scales of the original system and the one of the perturbation.

(b) The introduction of the perturbation does not change the property of the theory of being a complete one.

In the latter case the small perturbation "chooses" among the (now) physically quite different representations a particular one which for some reason is preferred. The basic question in this case is: by which criterium is this choice governed?

In this paper we shall apply the described idea to the QED of electrons perturbed by the coupling with muons and hadrons through the vacuum polarization. As will become clear, from the present point of view the mass renormalization praxis of introducing the experimental electron mass as a parameter in the theory amounts to the adoption of possibility (a); it effectively means that one resigns oneself to not understanding the experimental value of the electron mass. We shall investigate possibility (b).

In sects. 2–5 the model, the adopted stability criterium and the results of its application are described. The assumptions made in the present treatment, together with some observations, are collected in the last section.

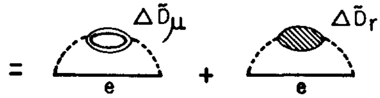
Throughout the paper we ignore the possibility that weak, gravitational or unknown interactions could be important for the problem; if this assumption should be incorrect the present model would need at least a modification.

2. Model and criterium

Consider the QED of electrons and photons in the absence of other types of particles. It is clear that in this case the absolute value of the electron mass has no physical importance and serves only to set the mass scale which is used. This is what one would expect in a theory with a spontaneously broken dilatation symmetry and in fact, both the usual formulation of the QED of electrons with infinite bare mass as well as the formulation with zero bare mass [7,8], have this property. We therefore shall consider the theory of the electron-photon system as having a spontaneously broken dilatation symmetry and possessing a continuous set of mathematically non-equivalent solutions of which each member can be characterized by its value m_e of the electron mass. Because the dilatation and γ_5 currents are not conserved, difficulties with Goldstone bosons do not arise [4].

We now break the passive symmetry of the electron-photon system by introducing the, in reality present, electromagnetic coupling with the muon and all the other remaining charged particles like hadrons, quarks, heavy leptons, etc. They will be denoted by the subscripts μ and r respectively. The lowest order perturbation of the electron selfenergy is graphically given by

$$-i\Delta\Sigma = (-i\Delta\Sigma_\mu) + (-i\Delta\Sigma_r)$$



(1)

Whereas for the uncoupled electron theory the values of all dimensionless quantities are independent of the representations, this is evidently not any more true after the coupling; now dependences on various mass ratios like m_μ/m_e, which vary with m_e, occur.

Assuming the dilatationally non-invariant part of the perturbation to be indeed small one has just the situation described in the introduction and the alternatives (a) and (b) mentioned there do arise. The usual mass renormalization procedure chooses possibility (a) by simply fixing the electron mass to be equal to the experimental mass. In the following, possibility (b) will be discussed.

In that case the question of the stability criterium, mentioned in sect. 1, comes up. The fact that in perturbation theory one has the freedom to arbitrarily choose the mass of the electron in relation to the ones of the perturbing fields without running into a contradiction, means that the stability criterium has to be explicitly imposed, at least in perturbation theory. Whether such a criterium can be derived from the complete theory, as known at present, or demands a generalization of the theory, is another question which will not be discussed here. In fact the general problem of the stability of spontaneously broken symmetry solutions in relativistic quantum field theory has not yet been resolved.

The present situation is reminiscent of the well-known one in quantum mechanics in which the Hamiltonian possesses a certain "good" symmetry which is broken by a small perturbation [5]. There one has the task of determining in a set of zero order eigenstates, connected by the symmetry transformations, the ones which are nearest to the new eigenstates, in particular to the one of lowest energy. This has a strong similarity with the present problem of selecting from the manifold of non-equivalent representations of the theory of electrons and uncoupled muons and hadrons, the stable one which is nearest to a representation of the coupled theory. The analogy suggests that the stability criterium should be a minimalization procedure of the type of the Ritz variation principle [9]. As in lowest order the inequivalent representations do not mix, we have the simplification that we need to minimize only with respect to one parameter, the zero order electron mass.

As the vacuum energy by relativistic invariance always vanishes, a generalized form of the variation principle will be considered, according to which one finds a stationary state with certain fixed values of some (also by the perturbation) conserved quantum numbers, by minimizing the energy, allowing for competition only states with the fixed values of the quantum numbers. Taking the value one for the electron number and zero for the momentum, the analogous procedure for our case would be to select as the stable representation the one which minimizes the electron self-mass, and this is the stability criterium which we adopt. The "physical" idea is that, if one switches on the perturbation in an electron world with the wrong electron mass, the electron will decay by γ emission until it has its lowest possible mass.

There still remains the question of the unit in which the electron mass, which is to be minimized, should be expressed; but fortunately the answer to this question is essentially unique. The stability criterium for which we look should be invariant under the addition to the perturbation of a small symmetric part (in which no other mass than the one of the electron occurs), because such a part could have been put in the zero order approximation without appreciably changing it. In particular should the criterium be identically fulfilled if the perturbation is symmetric or vanishes. Therefore the lowest order electron selfenergy, calculated for a certain zero-order representation, should be expressed in a quantity of mass dimension in that representation. The choice of this unit is indifferent. For simplicity taking the electron mass m_e (characterizing the representation) itself as unit and, multiplying by a factor m_e^2 for convenience, we obtain the criterium

$$m_e^2 \frac{d}{dm_e^2} \frac{\Delta\Sigma(m_e^2, m_i^2)}{m_e} = 0 , \qquad (2)$$

with m_e denoting the zero-order electron mass and m_i the masses occurring in the perturbation. Any symmetric part in the perturbation results in a contribution to $\Delta\Sigma$ which for dimensional reasons is proportional to m_e and consequently does not affect the criterium, as required.

The criterium constitutes an equation for the electron mass, assuming the muon mass and the hadron vacuum polarization to be given. Of course any stability criterium should be gauge independent. This can be explicitly verified for criterium (2), as is not surprising because of the gauge invariance of a self-mass.

3. The muon contribution

We start with the muonic perturbation which is given by $\Delta\Sigma_\mu$ of expression (1). One has in lowest order of α

$$\Delta\Sigma_\mu = \frac{-i\alpha}{4\pi^3}\int d^4k\, \gamma^\nu \frac{1}{\not p - \not k - m_e}\gamma_\nu\, \Delta\widetilde{D}_\mu(k^2)\,, \qquad (\not p = m_e)\,, \tag{3}$$

with

$$\Delta\widetilde{D}_\mu = \int d\kappa^2 \rho_\mu(\kappa^2)\, \frac{1}{k^2-\kappa^2}\,,$$

and $\rho_\mu(\kappa^2)$ being the spectral density of the photon propagator caused by the muon vacuum polarization, given by

$$\rho_\mu(\kappa^2) = \frac{\alpha}{3\pi}\,\theta\,(\kappa^2 - 4m_\mu^2)\,\frac{1}{\kappa^2}\left(1 + \frac{2m_\mu^2}{\kappa^2}\right)\sqrt{1 - \frac{4m_\mu^2}{\kappa^2}}\,. \tag{4}$$

First the approximate form

$$\rho'_\mu(\kappa^2) = \frac{\alpha}{3\pi}\,\theta\,(\kappa^2 - 4m_\mu^2)\,\frac{1}{\kappa^2} \tag{5}$$

will be used, because this simplifies the calculation and the result for the full expression (4) can be given in terms of the one following from the approximation (5). For the corresponding contribution to the photon propagator one finds

$$\Delta\widetilde{D}_\mu(k^2) = \int d\kappa^2\, \rho'_\mu(\kappa^2)\,\frac{1}{k^2-\kappa^2} = \frac{\alpha}{3\pi}\,\frac{1}{k^2}\,\ln\frac{-k^2+(2m_\mu)^2}{(2m_\mu)^2}\,. \tag{6}$$

This contribution has no pole at $k^2 = 0$ because that part of the muon vacuum polarization which leads to a change in the effective coupling constant at small k^2 values has been taken out and included in the (infinite) charge renormalization. This is for our considerations important, as the charge renormalization may be dependent on m_μ. The expression corresponding to (6) which directly follows from the calculation of the vacuum polarization graph, with the quadratic subtraction from gauge invariance included but before the charge renormalization has been applied, is of the form

$$\Delta D'_\mu(k^2) = \frac{\alpha}{3\pi}\,\frac{1}{k^2}\,\ln\frac{-k^2+(2m_\mu)^2}{\Lambda^2} \tag{7}$$

in which Λ is a large mass characterizing a general convergence factor or cut-off of the theory. The charge renormalization connecting the expressions (7) and (6) amounts to transferring the k^2 pole in $\Delta D'_\mu$ given by

$$\frac{\alpha}{3\pi}\frac{1}{k^2}\ln\frac{(2m_\mu)^2}{\Lambda^2} \tag{8}$$

to the free photon propagator, in effect decreasing its coupling constant. In a theory in which only the muon would occur, Λ would be proportional to the muon mass. In this case expression (8) would be independent of m_μ and the charge renormalization would not matter for the criterium. However if also a much lighter particle is contained in the theory in the same way as the muon, there is no reason anymore to take the cutoff proportional to the muon mass. In fact we shall assume that the non-dilatationally invariant part of the ultra-high momentum dependence of the photon propagator is to a good approximation governed by the mass of the lightest point-like charged particle, the electron. This assumption guarantees that the other particles really only cause a small perturbation of the passive dilatation invariance of a pure electron theory. It allows one to take all cutoffs proportional to the electron mass without introducing an unwanted dependence of renormalized coupling constant on any other mass.

With respect to the validity of the above assumption it should be remarked that perturbation theory has nothing to say on the involved finite part of the infinite subtraction term. One may argue that for Green functions symmetrically dependent on fields of particles which are approximately related as dilatational transforms (like the electron and muon), any large momentum will relative to the lightest mass be more asymptotic that relative to the heaviest one. Therefore both vertex-cut-off or increased propagator singularities will at very large momenta have the tendency to be dominated by the lightest mass, in particular if it is very small, as the electron mass is. But other possibilities are conceivable and it is an open question whether one can neglect the dependence on the heavier masses. We adopt this assumption because it leads to a simple and unambiguous result; even if it would turn out to be incorrect a modification of the present treatment might be possible.

The muon contribution to the criterium (2) may now be calculated by the insertion of expression (7), instead of ΔD, in eq. (3). We define the logarithmically divergent electron self-energy integral in eq. (3) by a suitable cutoff Nm_e, with sufficiently large N. Introducing new variables by taking m_e as the mass unit one obtains

$$\frac{\Delta\Sigma_\mu}{m_e} = \frac{-i\alpha^2}{12\pi^4}\int^{N} d^4k\,\gamma^\nu\frac{1}{\not p-\not k-1}\gamma_\nu\frac{1}{k^2}\ln\frac{-k^2+4m_\mu^2/m_e^2}{\Lambda^2/m_e^2}\,, \qquad (\not p=1)\,, \tag{9}$$

and

$$m_e^2\frac{d}{dm_e^2}\left(\frac{\Delta\Sigma_\mu}{m_e}\right) = \frac{-i\alpha^2}{12\pi^4}\int d^4k\,\gamma^\nu\frac{1}{\not p-\not k-1}\gamma_\nu\frac{4m_\mu^2/m_e^2}{k^2(k^2-4m_\mu^2/m_e^2)}\,, \qquad (\not p=1)\,, \tag{10}$$

where we have used the assumption that the cut-off is proportional to m_e.

Expression (10) is convergent and a standard Feynman parameter calculations gives

$$m_\mu^2 \frac{\mathrm{d}}{\mathrm{d}m_\mathrm{e}^2}\left(\frac{\Delta\Sigma_\mu}{m_\mathrm{e}}\right) = \frac{\alpha^2}{6\pi^2} F\left(\frac{4m_\mu^2}{m_\mathrm{e}^2}\right), \tag{11}$$

with

$$F(x) = \theta(4-x)\left[\tfrac{1}{2}x - \tfrac{1}{4}x^2 \ln x - 2\left(\tfrac{1}{2}x+1\right)\sqrt{x-\tfrac{1}{4}x^2}\,\mathrm{arctg}\,\frac{2\sqrt{x-\frac{1}{4}x^2}}{x}\right]$$
$$+\,\theta(x-4)\left[\tfrac{1}{2}x - \tfrac{1}{4}x^2 \ln x - \left(\tfrac{1}{2}x+1\right)\sqrt{\tfrac{1}{4}x^2-x}\,\ln\left(\tfrac{1}{2}x - 1 - \sqrt{\tfrac{1}{4}x^2-x}\right)\right], \tag{12}$$

and

$$\lim_{x\to\infty} F(x) = -\tfrac{3}{2}\ln x - \tfrac{3}{4}\,.$$

The use of the full expression (4) gives instead of eq. (7)

$$\Delta D_\mu'(k^2) = \int^{\Lambda^2} \mathrm{d}\kappa^2\, \rho_\mu(\kappa^2)\left(\frac{1}{k^2-\kappa^2} - \frac{1}{k^2}\right). \tag{13}$$

Inserting this perturbation in criterium (2), taking again m_e as mass unit and applying the m_e^2 differentiation under the integral, one obtains the same way as above

$$m_\mathrm{e}^2 \frac{\mathrm{d}}{\mathrm{d}m_\mathrm{e}^2}\left(\frac{\Delta\Sigma_\mu}{m_\mathrm{e}}\right) = \frac{\alpha^2}{6\pi^2} G\left(\frac{4m_\mu^2}{m_\mathrm{e}^2}\right), \tag{14}$$

with

$$G(x) = \frac{3}{4}\int_1^\infty \mathrm{d}y\, \frac{1}{y^3}\frac{1}{\sqrt{1-1/y}}\, F(xy)\,,$$
$$\lim_{x\to\infty} G(x) = -\tfrac{3}{2}\ln x - \tfrac{13}{4} + 3\ln 2\,. \tag{15}$$

4. The contribution of the other particles

To first order in α, the spectral density of the photon propagator can be written

$$\rho(\kappa^2) = \rho_\mathrm{e}(\kappa^2) + \rho_\mu(\kappa^2) + \rho_\mathrm{r}(\kappa^2)\,, \tag{16}$$

where ρ_r contains the contributions of the hadrons and of all other charged particles except the electron and muon. If some of these are pointlike fermions one will have

$$\rho_r(\kappa^2) \xrightarrow[\kappa^2\to\infty]{} \frac{\alpha}{3\pi}\frac{R(\infty)}{\kappa^2} . \tag{17}$$

with $R(\infty)$ a positive constant.

Suppose M is a mass which is sufficiently large (but otherwise arbitrary) so that $\rho_r(\kappa^2)$ for $\kappa^2 > M^2$ has already reached its asymptotic behaviour. We divide $\rho_r(\kappa^2)$ in two parts:

$$\rho_r(\kappa^2) = \rho_1(\kappa^2) + \rho_2(\kappa^2) ,$$

with

$$\rho_1(\kappa^2) = \rho_r(\kappa^2)\,\theta(M^2-\kappa^2) ,$$
$$\rho_2(\kappa^2) = \rho_r(\kappa^2)\,\theta(\kappa^2-M^2) \approx \frac{\alpha}{3\pi}\frac{R(\infty)}{\kappa^2}\,\theta(\kappa^2-M^2) . \tag{18}$$

The contribution of the second part can be evaluated as the expression (5) in the muon case, using the same assumption on the dominance of the lightest mass in the deviations from dilatation invariance at ultra-high momenta.

The result is analogous to eq. (11):

$$m_e^2 \frac{d}{dm_e^2}\;\frac{\Delta\Sigma_2}{m_e} = \frac{\alpha^2}{6\pi^2} R(\infty)\, F\!\left(\frac{M^2}{m_e^2}\right) . \tag{19}$$

For large values of M^2/m_e^2 one has from eq. (12):

$$m_e^2 \frac{d}{dm_e^2}\left(\frac{\Delta\Sigma_2}{m_e}\right) = \frac{\alpha^2}{6\pi^2}\, R(\infty) \left(-\tfrac{3}{2}\ln\frac{M^2}{m_e^2} - \frac{3}{4}\right) . \tag{20}$$

The entire contributions of all non-pointlike charged particles and the low energy parts of the pointlike ones to the spectral density of the photon propagator are contained in $\rho_1(\kappa^2)$. The corresponding self-energy, after taking m_e as the mass unit for the k integral, is

$$\frac{\Delta\Sigma_1}{m_e} = \frac{-i\alpha}{4\pi^3}\int^{M^2} d\kappa^2 \rho_r(\kappa^2) \int^{N} d^4k\, \gamma^\nu \frac{1}{\not p - \not k - 1}\gamma_\nu \frac{1}{k^2-\kappa^2/m_e^2} , \qquad (\not p = 1) ,$$

$$m_e^2 \frac{d}{dm_e^2}\left(\frac{\Delta\Sigma_1}{m_e}\right) = \frac{-i\alpha}{4\pi^3}\int^{M^2} d\kappa^2 \rho_r(\kappa^2) \int^{N} d^4k\, \gamma^\nu \frac{1}{\not p - \not k - 1}\gamma_\nu \frac{-\kappa^2}{m_e^2}$$
$$\frac{1}{(k^2-\kappa^2/m_e^2)^2} , \qquad (\not p = 1) . \tag{22}$$

The k integral in eq. (22) is convergent and one obtains:

$$m_e^2 \frac{d}{dm_e^2}\left(\frac{\Delta\Sigma_1}{m_e}\right) = \frac{\alpha}{2\pi}\int^{M^2} \rho_I(\kappa^2)\, H(\kappa^2/m_e^2)\, d\kappa^2 \,, \tag{23}$$

with

$$H(x) = \theta\,(4-x)x\left[1 - \tfrac{1}{2}x\ln x + \frac{x^2-2x-2}{2\sqrt{x-\frac{1}{4}x^2}}\operatorname{arctg}\frac{2\sqrt{x-\frac{1}{4}x^2}}{x}\right]$$
$$+\,\theta\,(x-4)x\left[1 - \tfrac{1}{2}x\ln x - \frac{x^2-2x-2}{4\sqrt{\frac{1}{4}x^2-x}}\ln\left(\tfrac{1}{2}x - 1 - \sqrt{\tfrac{1}{4}x^2-x}\right)\right],$$
$$\lim_{x\to\infty} H(x) = +\tfrac{3}{2}\,. \tag{24}$$

Taking into account that the electron is very much lighter than all other charged particles one has

$$m_e^2 \frac{d}{dm_e^2}\left(\frac{\Delta\Sigma_1}{m_e}\right) \approx \frac{\alpha}{2\pi}\frac{3}{2}\int^{M^2} \rho_I(\kappa^2)\, d\kappa^2 \,. \tag{25}$$

The total contribution of the charged particles besides the muon, for the actual case that all their masses are very large compared to the electron mass, is from eqs. (20) and (25)

$$m_e^2 \frac{d}{dm_e^2}\left(\frac{\Delta\Sigma_I}{m_e}\right) = \frac{\alpha^2}{4\pi^2}\left[\frac{3\pi}{\alpha}\int^{M^2} \rho_I(\kappa^2)\, d\kappa^2 - R(\infty)\left(\ln\frac{M^2}{m_e^2} + \tfrac{1}{2}\right)\right]. \tag{26}$$

The r.h.s. of this equation is independent of M, due to the asymptotic behaviour (17) of $\rho_I(\kappa^2)$. This independence is of course necessary for the consistency of our approach, becuase of the arbitrariness in the choice of M^2.

5. Result and comparison with experiments

Taking into account that the electron mass is very small compared to the masses of all other charged particles, we can to an excellent approximation use the asymptotic expressions (15) and (26) to obtain

$$m_e^2 \frac{d}{dm_e^2}\left(\frac{\Delta\Sigma}{m_e}\right) = \frac{\alpha^2}{4\pi^2}\left[-\ln\left(\frac{4m_\mu^2}{m_e^2}\right)\right.$$
$$\left. -\tfrac{13}{6} + 2\ln 2 + \frac{3\pi}{\alpha}\int^{M^2} \rho_I(\kappa^2)\, d\kappa^2 - R(\infty)\left(\ln\frac{M^2}{m_e^2} + \tfrac{1}{2}\right)\right] = 0\,. \tag{27}$$

The solution of this equation for the electron-muon mass ratio is

$$\frac{m_e}{m_\mu} = \exp\left\{\frac{(-3\pi/\alpha)\int^{M^2} \rho_r(\kappa^2)\,d\kappa^2 + R(\infty)\,(\ln(M^2/m_\mu^2) + \frac{1}{2}) + \frac{13}{6}}{2\,[1 + R(\infty)]}\right\}. \tag{28}$$

In the framework of our assumptions there can be no other mass of a charged lepton which is very light compared to all other charged particles.

From eq. (27) follows:

$$\frac{d^2}{(d\ln m_e^2)^2}\left(\frac{\Delta\Sigma}{m_e}\right) = m_e^2\frac{d}{dm_e^2}\,m_e^2\frac{d}{dm_e^2}\,\frac{\Delta\Sigma}{m_e} \approx \frac{\alpha^2}{4\pi^2}\,[1 + R(\infty)] > 0\,. \tag{29}$$

Consequently the electron mass has indeed a minimum as required by the variation principle.

Taking m_μ as the mass unit ($m_\mu = 1$) and $y = \ln(E/m_\mu)$, one may write eq. (28) also more compactly as

$$m_e = \exp\left\{\frac{-\int_{\frac{5}{6}}^{\infty}[R(y) - R(\infty)]\,dy}{1 + R(\infty)} + \frac{13}{12}\right\}, \tag{28'}$$

with $R = \rho_r/\rho_\mu$.

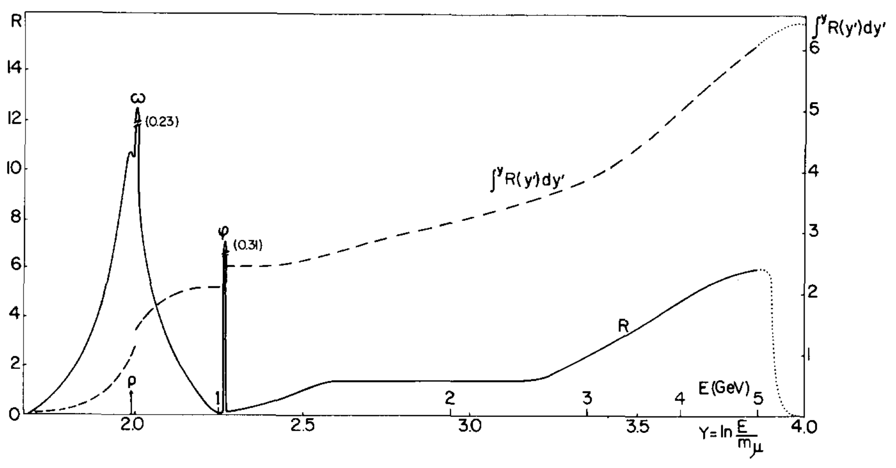


Fig. 1. The full curve is a sketch of the function $R(y)$ extracted from published $e^+ - e^-$ annihilation cross sections [12–16]; experimental uncertainties are of the order of 20%. For clarity, the narrow ω and ϕ peaks are cut and their areas indicated. The broken curve represents $\int^y R(y')\,dy'$. The dotted curves show the type of behaviour demanded by eq. (30) if $R(\infty)$ is zero.

From the experimental value $m_e = 0.00484$ follows:

$$[1 + R(\infty)]^{-1} \int_{\frac{5}{6}}^{\infty} [R(y) - R(\infty)] \, dy \approx 6.4 \, . \tag{30}$$

The function $R(y)$ is measurable in $e^+ - e^-$ annihilation. Results [12–16] of present experiments (see fig. 1) do not yet extend to sufficiently high energies to give a clear indication of the value of $R(\infty)$, but colliding beam measurements up to 9 GeV centre of mass energy ($y = 4.44$) are being planned [16]. The available experimental results which are subject to considerable systematical and statistical uncertainties are still compatible with $R(\infty) = 0$, i.e., with the electron and muon being the only pointlike charged particles. In this case the validity of eq. (30) would require a sharp drop to zero of $R(y)$ (see fig. 1) in the not yet measured domain $y > 3.86$. In any case for this equation to be true, $R(y)$ should first reverse it present rising behaviour before obtaining an asymptotic value.

6. Concluding remarks

For clarity we give a list of the main assumptions which have been made in the course of our treatment and add a few remarks:

(i) Only electromagnetic interactions have been taken into account. It is not clear that the introduction of weak [17], gravitational or other interactions is necessary for the present problem and these interactions would cause a great arbitrariness as they are not sufficiently known.

(ii) We assumed that the value of the electron mass in principle does not need to be given as an extra ingredient to the theory, but is determined by the existence of a stable solution. We took as stability criterium condition (2), which is based on an analogy and has not yet been derived from a general principle. It gives the correct result for a dilatational invariant perturbation and is gauge invariant.

The general question of the stability of spontaneously broken symmetry solutions in relativistic quantum field theory is open. As it seems not unlikely that the observed approximate symmetries of elementary particles could be described as spontaneously broken passive symmetries [2,11], the problem of the stability of a solution of such theories might be essential for a quantitative understanding of the observed symmetry breakings [2,11,18] which are usually incorporated as foreign elements into the theory.

(iii) We assumed that at ultra high momenta the photon propagator to a good approximation depends only on the momentum squared and on the mass of the lightest charged pointlike particle, if this is very light compared to all other charged particles. This last condition is actually fulfilled for the electron but not for the muon. The analogous discussion for the muon, in which the electron and muon are intercharged as compared to the present treatment, is therefore not valid, even if the basic theory is symmetric in respect to the electron and muon.

If future developments would show that one or more of the above assumptions would have to be modified one might hope that a treatment along the lines of the present paper would still be possible.

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