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FERNANDO HENRIQUE DE PAULA E SILVA MENDES

MARKOV-SWITCHING MODELS: EMPIRICAL APPLICATIONS USING CLASSICAL AND BAYESIAN INFERENCE

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Tese submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como quesito parcial para obtenção do título de Doutor em Economia, com ênfase em Economia Aplicada.

Orientador: Prof. Dr. João Frois Caldeira

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BANCA EXAMINADORA:

Prof. Dr. João Frois Caldeira – Orientador
UFRGS

Prof. Dr. Flávio Augusto Ziegelmann
UFRGS

Prof. Dr. André Alves Portela Santos
UFSC

Prof. Dr. Osvaldo Candido da Silva Filho
UCB

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ABSTRACT

In this thesis, we present three empirical applications on finance and macroeconomics. The general modeling framework in all chapters is based on extensions of the Markov-switching model. And the statistical methodology is divided into two distinct areas; Classical and Bayesian inference.[1] In the first one, we test for the presence of duration dependence in the Brazilian business cycle. The main results indicated that as the recession ages, the probability of a transition into an expansion increases (positive duration dependence in recessions). On the other hand, as the expansions ages, the probability of a transition into a recession decreases (negative duration dependence in expansions). In the second paper, we extend the research concerned with the evaluation of alternative volatility modeling and forecasting methods for Bitcoin log-returns. The in-sample estimates suggest evidence of long memory in the data series. When performing one-day ahead Value-at-Risk (VaR), our results outperform all standard single-regime GARCH models considered in the study. Finally, in the third paper, we capture different regimes in Bitcoin volatility returns and test the mean-reversion hypothesis for multi-period returns. In general, we found evidence of mean-aversion for different holding returns. We also confirmed this result for alternative specifications and also carrying the analysis for sub-sample periods.

Keywords: Markov-switching, Duration dependence, Business cycle, Volatility, Mean-reversion.

[1]The Matlab codes developed for this thesis are available upon request.

Summary

1 Duration-dependent Markov-switching model: an empirical study for the Brazilian business cycle.

Abstract. This paper uses a duration-dependent Markov-switching model to identify business cycles in the Brazilian economy and to test for the presence of duration dependence in periods of expansion and contraction. The model is estimated using the growth rate of quarterly GDP from 1980:II to 2016:II. In the empirical application we found evidence of significant asymmetry in growth rates and duration dependence in the business cycle transition probabilities. The parameter estimates indicated that as the recession ages, the probability of a transition into an expansion increases (positive duration dependence in recessions). On the other hand, as the expansions ages, the probability of a transition into a recession decreases (negative duration dependence in expansions). The smoothed probabilities of the model captured several periods of contraction during the last three decades, matching the recession dates of the Business Cycle Dating Committee (CODACE) from the Getúlio Vargas Foundation.

Keywords: Business Cycles, Markov-switching model, Duration Dependence

1.1 Introduction

Measuring the state of the economy and understanding the transition between periods of recession and expansion has been an important topic in research regarding business cycles and has its foundations in the works of Fisher (1925) and Burns & Mitchell (1946). Based on these studies, various authors have sought to develop methodologies in order to capture regularities in economic activity that can define the phases of the business cycle and also the transition probabilities from an expansion to a recession and vice-versa.

The knowledge of the timing and duration of the business cycle is important for economic decision making, such as adopting anti-cyclical fiscal and monetary policies (see, for example, Castro 2010). However, the business cycle is characterized by nonlinearities (see, for example, Terasvirta & Anderson 1992, Beaudry & Koop 1993). More specifically, Keynes (1936) has argued that recession, although more aggressive, tend to be more short-lived than expansions; therefore, the research on duration dependence in business cycles attempts to answer the following question: "Are periods of expansion or contraction in economic activity more likely to end as they become older? More technically, do business cycles exhibit positive duration dependence?" Sichel (1991, p. 254).

Earlies studies about duration dependence analyzes the NBER chronology using non-parametric methods or hazard models (see, for example, Diebold & Rudebusch 1990, Sichel 1991, Diebold et al. 1993, among others). Based on the length of each phase, these studies found significant evidence of positive duration dependence for pre-WWII expansions and post-WWII contractions in U.S. economy. Another strand of the literature based on the Markov-Switching models, which defines the switches between expansions and recessions through a first-order Markov chain. Different to the existing studies, Durland & McCurdy (1994) extended Hamilton (1989) Markov-switching model to allow for duration dependence in recessions and in expansions. This methodology defines business cycle through an unobservable stochastic process, so that the business cycle chronology is not necessary.

Durland & McCurdy (1994) showed that as a contraction ages the probability of moving into an expansion increases, i.e., coming out of the recession is more plausible should the crisis be prolonged. In the opposite scenario, the model did not find significant results for duration dependence associated with the probability of a transition out of expansions, but could nicely match NBER business cycle dates. Lam (2004) generalized the model of Durland & McCurdy (1994) incorporating the duration dependence in the mean growth rate. The main conclusion of the author is that the probability of the expansion ending gradually decreases as the expansions ages, while the probability of the contraction ending increases as the contraction ages.

The duration dependence business cycle studies have generally focused on the developed countries, especially, U.S. economy, since their chronology are well documented by

NBER. Empirical research have remained limited for the developing countries (notable exceptions include: Ozun & Turk 2009, Castro 2015). Brazil is one of the emerging market economies that can constitute an important case of study. Markov-switching models and its variants have also been applied in the study of Brazilian business cycles, as shown in Chauvet (2002), Correa & Hillbrecht (2004), Cespedes et al. (2006), and Valls Pereira & Vieira (2014). However, none of the previous studies investigate the duration dependent feature of the Brazilian business fluctuations. Thus, our paper contribution try to fill the gap that can be observed in the empirical literature devoted to the Brazilian economy.

The purpose of this paper is to identify the timing, behavior and duration of business cycles in the Brazilian economy. In addition to identifying periods of recession and expansion, we test for the presence of duration dependence. More specifically, we implement a procedure to identify periods of recessions and expansions starting from the 1980s until 2016 and simultaneously test for the presence of duration dependence. We employ a duration-dependent Markov-switching model developed by Maheu & McCurdy (2000a) to study the U.S. bull and bear market and employed by Lam (2004) to study the U.S. business cycle.

The remainder of this paper is organized as follows: Section 2 presents the methodology; Section 3 describes the data and our empirical results; Section 4 concludes.

1.2 Methodology

1.2.1 Regime switching model with duration dependence

The Markov-switching model proposed by Hamilton (1989) is defined by:

$$y_t = \mu(S_t) + \sum_{i=1}^{p} \phi_i\{y_{t-i} - \mu(S_{t-i})\} + \varepsilon_t, \tag{1.1}$$

where y_t is the GDP growth rate, $\mu(S_t)$ is presented with the state variable S_t where $\mu(S_t) = \mu_0(1 - S_t) + \mu_1 S_t$ with μ_0 and μ_1 being parameters. If $S_t = 0$, then $\mu(S_t) = \mu_0$. If $S_t = 1$, then $\mu(S_t) = \mu_1$. The evolution of the unobserved state variable S_t follows a first-order Markov chain with transition probabilities and takes value 0 when the economy is in recession and 1 when the economy is in expansion, $\phi_1, \ldots, \phi_p$ are parameters, p the number of lags and ε_t is an error term at time t following an identically and independently normal distribution.

In an attempt to investigate the duration dependence in the business cycle, Durland & McCurdy (1994) extended the traditional Markov-switching model exploring high order Markov chains. In the duration-dependent Markov-switching specification, the probability of a regime change is a function of the previous state, as well as the duration of the previous state. Following Maheu & McCurdy (2000b), the length of occurrences of the state, S_t, can be characterized by:

$$D_t = min(D_{t-1}I(S_t, S_{t-1}) + 1, \tau) \tag{1.2}$$

where $I(S_t, S_{t-1}) = 1$ if $S_t = S_{t-1}$ and 0 otherwise. To make the estimation tractable is necessary to define a limiting parameter τ. The transition probabilities are parametrized using a logistic function. This ensures that the probabilities are between 0 and 1. Using i to index the state and d the duration (in quarters), where $\gamma_1(i)$ and $\gamma_2(i)$ are the parameters, the transition probability of staying in state i, given that we have been in state i for d_{t-1} periods is given by:

$$P\left(S_t = i | S_{t-1} = i, D_{t-1} = d_{t-1}\right) = \begin{cases} \dfrac{\exp(\gamma_1(i) + \gamma_2(i)d_{t-1})}{1 + \exp(\gamma_1(i) + \gamma_2(i)d_{t-1})}, & \text{if } d \leq \tau \\ \dfrac{\exp(\gamma_1(i) + \gamma_2(i)\tau)}{1 + \exp(\gamma_1(i) + \gamma_2(i)\tau)}, & \text{if } d > \tau \end{cases} \quad i = 0, 1. \tag{1.3}$$

The conditional probability of a state change, given that the state has achieved a duration d is described by the hazard function. Using the transition probabilities, it is given by:

$$1 - P\left(S_t = i | S_{t-1} = i, D_{t-1} = d_{t-1}\right) = \frac{1}{1 + \exp(\gamma_1(i) + \gamma_2(i)d_{t-1})}, \quad i = 0, 1. \tag{1.4}$$

A decreasing hazard function is referred to as negative duration dependence whereas an increasing hazard function characterizes the positive duration dependence. The parameter $\gamma_2(i)$ summarizes the duration effect on the hazard function. For example, $\gamma_2(i) < 0$ means that a long period in regime i implies a higher probability of state switching (positive duration dependence); $\gamma_2(i) = 0$ means that the transition probability is independent of the regime duration; and $\gamma_2(i) > 0$ implies that the longer the duration of regime i, the higher the chance of the process to remain at i (negative duration dependence).

The parameters of the duration-dependent Markov-switching model can be estimated using two different approaches: the maximum-likelihood method following Maheu & McCurdy (2000a), or using MCMC methods, such as in Pelagatti (2001). In this study, we employed the first method. For the model with two states, $S_t = i$ where $i = 0, 1$ and p is the number of lags of y_t, Maheu & McCurdy (2000a) defined a new latent variable $\mathcal{S}_t$, which covers all possible paths from $S_t = i$ to $S_{t-p} = i$ as well as the respective duration in the sequence of states up to τ. Using this approach, the duration-dependent model collapses into a first-order Markov model, where the transition matrix for $\mathcal{S}_t$, is given by:

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{N1} \\ p_{12} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & & \vdots \\ p_{1N} & p_{2N} & \cdots & p_{NN} \end{bmatrix}, \tag{1.5}$$

where $p_{ii} = P(\mathcal{S}_t = i|\mathcal{S}_{t-1} = i)$, $i, i = 1, \ldots, N$ is constructed using equations (1.3) and (1.4), and N represents the number of all states; $N = 2^{p+1} + 2(\tau - p - 1)$. Based on that, the authors show that estimation and smoothing can be performed with the usual techniques as seen in Hamilton (1989).

1.3 Data and Empirical Results

1.3.1 Data

We consider quarterly data from the Brazilian Gross Domestic Product (GDP) growth for the empirical analysis spanning the period from 1980:II to 2016:II, a total of 145 observations. Due to the fact that the current GDP series begins in 1996:1, we used two different bases to depict the data series for the entire period. The first data series refers to quarterly GDP with seasonal adjustment from 1996:I to 2016:II (base year 2010). The second data series refers to quarterly GDP with seasonal adjustment from 1980:I to 2014:III (base year 2000). In order to obtain the GDP from 1980 to 2016, the first data series was retropolated using the growth rate of the second data series. After the data treatment, the first difference of its logarithm was taken[1].

Before going to the estimates results, Table 1.1 presents an overview of the economic cycles in Brazil during the period considered in our data sample. The analysis is based on the Brazilian Business Cycle Dating Committee (CODACE) from the Getúlio Vargas Foundation.[2] During the last 36 years, the Brazilian economy went through 9 recessions. In this period, the economic growth was weak and volatile, excepted for some periods, for example, the most prosperous growth phase started at the end of 2003 lasting until the world crises of 2008-2009. The recessions were short and less severe in the two decades that followed the Real Plan in 1994, but it changed in 2014, the beginning of the last recession in our data sample. Further details summarizing the key events in recessionary (expansionary) phases, see, Weller (2019).

[1]The GDP data is obtained from Ipeadata (http://www.ipeadata.gov.br).

[2]The CODACE is a committee created in 2008 by the Getúlio Vargas Foundation to determine a chronology of reference for the Brazilian business cycles. Its form of organization and method of work follows the model adopted in many countries, notably the North American Data Committee, created in 1978 by the National Bureau of Economic Research (NBER).

Table 1.1: Quarterly chronology of business cycles in Brazil - CODACE

Recessions			Expansions		
Periods	Number of Quarters	Accumul. Growth %	Periods	Number of Quarters	Accumul. Growth %
1981:1 - 1983:1	9	-8.9	1983:2 - 1987:2	17	26.2
1987:3 - 1988:4	6	-4.2	1989:1 - 1989:2	2	8.1
1989:3 - 1992:1	11	-8.0	1992:2 - 1995:1	12	17.6
1995:2 - 1995:3	2	-2.8	1995:4 - 1997:4	9	8.5
1998:1 - 1999:1	5	-1.7	1999:2 - 2001:1	8	7.3
2001:2 - 2001:4	3	-0.9	2002:1 - 2002:4	4	5.1
2003:1 - 2003:2	2	-1.5	2003:3 - 2008:3	21	26.7
2008:4 - 2009:1	2	-6.0	2009:2 -2014:1	20	20.8
2014:2 - 2016:2	9	-7.6			

Note: The third (sixth) column of this table refers to the peak-trough (trough-peak) accumulated GDP growth rate in the period. Peak refers to the end of an expansion and is followed by the start of a recession in the next quarter. Trough refers to the final quarter of a recession, which is followed by the beginning of economic expansion in the next quarter.

1.3.2 Markov Model with Duration-Dependent Transition Probabilities

Table 1.2 reports the estimates for the duration-dependent Markov-switching model. For the quarterly frequency ranging from 1980:II to 2016:II, this specification capture a dichotomous pattern in the series associated with high and low economic growth phases. The recession is characterized by a negative mean value, $\mu_0 = -2.142\%$, while expansion is denoted by a positive mean value, $\mu_1 = 0.993\%$. These estimates are statistically significant and evidenced the asymmetry in the growth rates.[3] Regarding the duration dependence coefficients, our preliminary results indicated an asymmetry on the state temporal dependence in cyclical data. These estimates were also statistically significant implying a positive duration dependence in the recession, $\gamma_2(0) = -1.048$ and negative duration dependence in the expansion, $\gamma_2(1)= 0.289$.

Figure 1.1 shows the transition probability of equation 1.3. The dotted line represents the probability of remaining in the recession and the continuous line denotes the probability of remaining in the expansion. Note that the chance of remaining in the recession falls gradually over the quarters, that is, the probability of moving from recession to expansion increases as a function of time (increasing hazard function). On the contrary, the chance of remaining in the expansion increases gradually over the quarters, that is, the probability of moving from expansion to recession does not increase as a function of time (decreasing hazard function). After 7 quarters, the probability of switching the regime is duration independent and its value is represented by parameter τ, which was calibrated using grid search from $[5, 25]$ with the log-likelihood values as the criterion (see, Maheu & McCurdy 2000a).

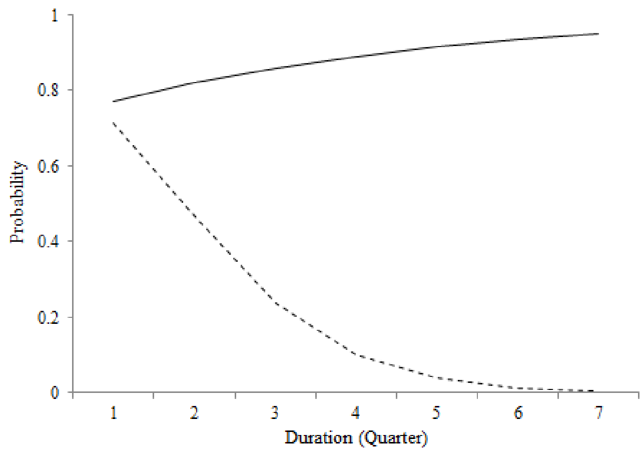
[3] We follow Chauvet (2002) setting $p = 0$ (see equation 1.1). In this case, we have $N = 2 + 2(\tau - 1)$.

Table 1.2: Markov Model with Duration-Dependent Transition Probabilities

Parameter	Estimate	Standard Error
μ_0	-2.142^{***}	0.536
μ_1	0.993^{***}	0.142
σ^2	1.993^{***}	0.452
$\gamma_1(0)$	1.968^{*}	1.582
$\gamma_2(0)$	-1.048^{*}	0.827
$\gamma_1(1)$	0.937	0.825
$\gamma_2(1)$	0.289^{**}	0.163
τ	7	
$\ln L$	-282.96	

Note: This table reports the estimates for the Markov model with duration-dependent transition probabilities for the quarterly Brazilian GDP growth rate from 1980:2 to 2016:2. $\ln L$ is the value of the log-likelihood. ***,**,* denote significance at the 1%, 5% and 10% levels, respectively.

Figure 1.1: Transition Probability: Recession (dotted line), Expansions (continuous line)



1.3.3 General Markov Model with Duration-Dependent Transition Probabilities

Despite the statistical evidence of negative duration dependence in expansions, much of the conventional wisdom state that a very long expansion is unstable and contraction is increasingly imminent (see, for example, Burns 1969 and Neftici 1982). To further investigate our preliminary results, we apply a general type of duration-dependent Markov-switching model in the same spirit of Lam (2004). In this specification, the duration variable enters both in the transition probabilities and in the mean process. The model is defined as:

$$y_t = \mu(S_t) + \psi(S_t)D_t + \sum_{i=1}^{p} \phi_i\{y_{t-i} - \mu(S_{t-i}) - \psi(S_{t-i})D_{t-i}\} + \varepsilon_t, \tag{1.6}$$

where $\psi(S_t) = \psi_0(1 - S_t) + \psi_1 S_t$ is the new component which ψ_0 and ψ_1 are parameters. This model allows duration to be a conditioning variable in the mean growth rates characterizing the dynamic behavior within each regime. The persistence in particular state implies that D_t increases and its effects are measured by the coefficient $\psi(S_t)$, which captures the relationship between the mean growth rate and the age of economic condition.[4]

The estimates of the general Markov model with duration-dependent transition probabilities are presented in Table 1.3. This specification also captured the asymmetry in the duration dependence dynamics. For the parameter ψ_1, the estimate value is negative (-0.244) and statistically significant, which implies that GDP growth rate declines as the expansion ages. For example, taking the value of τ, after 7 quarters, the GDP growth rate would be $\mu_1 = 0.52\%$. This result is related to the characterization of the business cycle, which suggest that expansion tends to be rapid in its early stages than its endings (Sichel 1994). On the other hand, Burns (1969) suggests that during recessions the rate of decline is usually fasted in the middle states than the early stages. This statement would be related to the estimates value of ψ_0, which is also negative (-0.27). However our result lacks statistical significance.

Table 1.3: General Markov Model with Duration-Dependent Transition Probabilities

Parameter	Estimate	Standard Error
μ_0	-1.454	1.416
μ_1	2.445^{***}	0.585
σ^2	1.927^{***}	0.253
$\gamma_1(0)$	2.549^{*}	1.758
$\gamma_2(0)$	-1.067^{*}	0.734
$\gamma_1(1)$	0.458	0.910
$\gamma_2(1)$	0.422^{**}	0.181
ψ_0	-0.278	0.536
ψ_1	-0.244^{**}	0.093
τ	7	
$\ln L$	-279.60	

Note: This table reports the estimates for the general Markov model with duration-dependent transition probabilities for the quarterly Brazilian GDP growth rate from 1980:2 to 2016:2. $\ln L$ is the value of the log-likelihood. ***,**,* denote significance at the 1%, 5% and 10% levels, respectively.

Overall results identified evidence of duration dependence in Brazilian business cycle. Our estimates are consistent with the findings of Lam (2004). However, the U.S. and Brazilian economy are very different, and there is not a unique justification for these kinds of features. Theoretically, the negative duration could be referred to the increasing

[4]We adopted the linear specification following Maheu & McCurdy (2000a). Other types of specifications can also be applied, for example, Lam (2004) assumes that the relationship between the mean growth rate and the age of current phase is quadratic.

probability of remaining in the expansion in the absence of external disturbances, while, the positive duration, could be related to the use of anti-cycle policies to mitigate the effect of the recessions. Nevertheless, there are several economic reasons why duration dependence might occur.

More recently, Rudebusch (2016) discuss several postwar changes in the U.S. economy that contributed to more robust and longer-lived expansions. For example, the increased share of services instead of tangible goods in the GDP would tend to diminish the importance of inventory fluctuations and moderate the business cycle. The author also highlights the postwar influence of the federal government actively focused on stabilizing the economy, which also included attempts to curtail recessions. Particularly, the Employment Act of 1946, applied broadly to the federal government, including to the Federal Reserve and in the conduct of monetary policy is a example of counter cyclical policy helped prolong business expansions and alter the pattern of business cycle age dependence (Diebold & Rudebusch 1999).

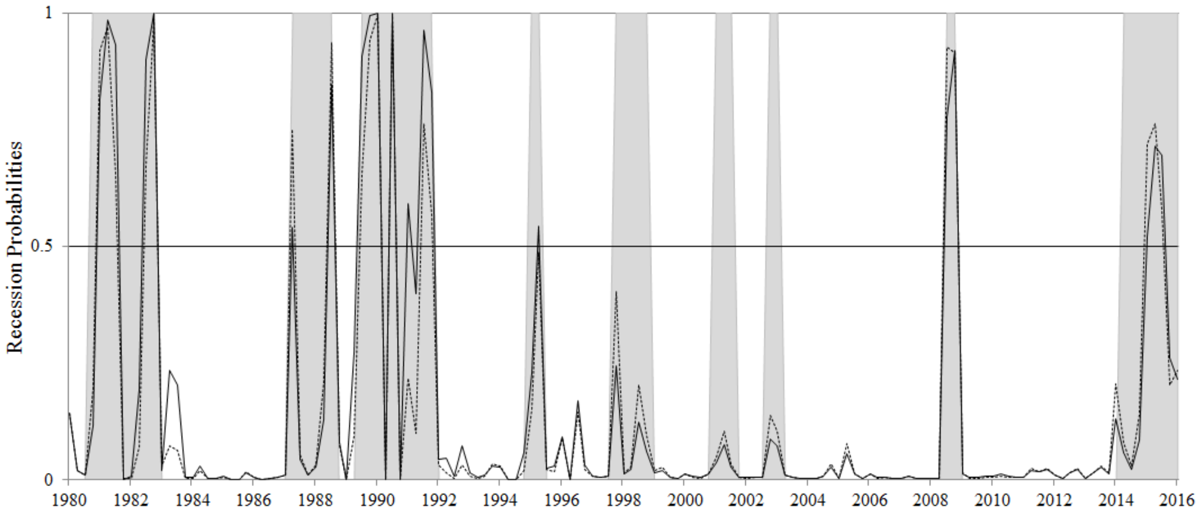
Notably, the Brazilian business cycle is characterized by volatility and stagnation. From 1980 to 2016, the overall growth results were mediocre compared to other developing economies. These stylized facts could be related to our general duration-dependent estimates since negative duration dependence could be associated with a growth rate that declines over the expansion. This results support what Brazilian economists refer to the stop-and-go process, whereas the country went through 9 recessions in the recent last thirty years. For the positive duration dependence, one possible narrative of our results could be referred to the fact that recessions were shorter after the Real Plan until 2014, as previously analyzed in Table 1.1. However, the economic forces behind the duration dependence effects could be investigated in future works.

1.3.4 Business Cycle Identification

Concerning the business cycle identification, Figure 1.2 plot the filtered probability of being in a recession and expansion phases. It is worth mentioning that the objective of this analysis is not to describe historical facts behind these results. Instead of this type of investigation, we compared our estimates to Table 1.1. In general, these models captured the main business cycle phases *vis-a-vis* to the CODACE. The general duration-dependent model seems to be slightly superior to the baseline duration specification. Using the Hamilton's 0.5-rule (recession probabilities higher or, equal to 0.5) the general model identified 6 out of 9 recessions from 1980 to 2016. For the expansion phases, both specifications match to the CODACE chronology.

It is important to point out that CODACE decisions are made on the basis of analyzing the most comprehensive set of variables, statistics and taking the point of view of its members. Moreover, the chronology is carried out many months after the facts have occurred, and therefore the duration-dependent model is not expected to be entirely

Figure 1.2: Duration-dependent Markov-switching model (dotted line), General duration-dependent Markov-switching model (continuous line) and Recession according to CODACE-FGV (shaded areas)



accurate to this chronology. However, the filtered probabilities are in line with previous studies in Brazil (see, for example, Chauvet 2002, Cespedes et al. 2006, Valls Pereira & Vieira 2014). Besides updating the data set, which takes account the latest recession in the Brazilian economy, the attractiveness of our empirical application stems for including the parameter that captures the duration-dependent, as the same time, we obtain the state probabilities. In both models, the hypothesis of duration dependence was confirmed suggesting this is a characteristic of the data.

1.3.5 Model Comparisons

Finally, we test the overall significance of the duration dependence comparing our models to the classic Markov-switching model, as seen in Durland & McCurdy (1994) and Lam (2004). For our first duration model, we were not able to reject the classic model in favor of the duration-dependent specification. The LR test statistic is 3.84, which has a p-value > 0.05 according to the χ^2 distribution with 2 degrees of freedom. On the order hand, we rejected the classic Markov-switching model in favor of the general duration-dependent model. The LR test statistic is 10.56, which has a p-value < 0.05 according to the χ^2 distribution with 4 degrees of freedom. Having these distinctive results (see, Table 1.4), we compare the duration-dependent models. The LR test statistic is 6.72, which also has a p-value < 0.05, according to the χ^2 distribution with 2 degrees of freedom. Overall results reject the null hypothesis of absence of duration dependence in mean growth rates and transition probabilities since the general model was statistically different from the classic Markov-switching and the baseline duration model.

Table 1.4: Likelihood-Ratio Test

Model	ln$L$	Test	d.f.	LR	p-value
$\mathcal{M}1$	- 284.88	$\mathcal{M}2$ x $\mathcal{M}1$	2	3.84	0.147
$\mathcal{M}2$	-282.96	$\mathcal{M}3$ x $\mathcal{M}1$	4	10.56	0.032
$\mathcal{M}3$	-279.60	$\mathcal{M}3$ x $\mathcal{M}2$	2	6.72	0.034

Note: $\mathcal{M}1$ refers to the Markov-switching model, $\mathcal{M}2$ is the duration-dependent Markov-switching model, and $\mathcal{M}3$ covers the General duration-dependent Markov-switching model. LR is the test statistics and d.f is the degrees of freedom.

1.4 Conclusion

In this article, we have identified business cycles in the Brazilian economy as well as evidence of duration dependence in the respective phases of expansion and recession. Using the duration-dependent Markov-switching model in growth rate of quarterly GDP from 1980:II to 2016:II, the estimates indicated that as the recession ages, the probability of a transition into an expansion increases (positive duration dependence in recessions). On the other hand, as the expansions ages, the probability of a transition into a recession decreases (negative duration dependence in expansions).

Regarding to the business cycles identification, the model probabilities proved to have a reasonable capacity of discerning periods of contraction and expansion. Instead of describing the historical facts behind these identifications, we compare the recession probabilities to the Business Cycle Dating Committee (CODACE) from the Getúlio Vargas Foundation, which is a reliable ex post reference for the Brazilian business cycle turning points. Using the Hamilton's 0.5-rule, the probabilities of the general duration-dependent Markov-switching model captured 6 out of the 9 recessions from 1980 to 2016.

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2 Duration-dependent Markov-switching in Bitcoin volatility with Value-at-Risk application.

Abstract. This paper extends research concerned with the evaluation of alternative volatility modeling and forecasting methods for Bitcoin log-returns by broadening the class of single-regime GARCH models to include duration-dependent Markov-switching models (DDMS). In addition to the in-sample statistical evaluation, we compare the DDMS model to single-regime GARCH-types predicting one-day-ahead Value-at-Risk (VaR). Despite using a two-state model, the DDMS specification used in this empirical application acts like a large N-state model capturing a broad range of volatility levels, since duration also affects the conditional variance. To access the economic significance of the Bitcoin volatility forecast to investors, we investigate the advantages of regime-switching models in the Bitcoin returns for the accurateness of the Value-at-Risk models. For one-day holding period, the best results were obtained for DDMS model at the 1% risk level.

Keywords: Bitcoin, Duration Dependence, Markov-switching, Value-at-Risk.

2.1 Introduction

In the last few years, the analysis of cryptocurrencies has attracted the interest of many investors, practitioners, and researchers. Since its creation in 2008, Bitcoin, the most popular cryptocurrency, exhibited an extreme increase in its market value. Despite its decentralized nature and reduction costs (Kim, 2017), the empirical literature suggests that Bitcoin is mainly used as an asset rather than a currency, mostly used for speculative purposes, occasioning high volatility and bubbles (Cheah and Fry, 2015; Hafner, 2018). However, the risk assessment of Bitcoin is crucial for financial agents and understanding its volatility is relevant to investment decisions.

The empirical literature on the behaviour of Bitcoin and other cryptocurrencies is expanding rapidally. Previous studies find that Bitcoin returns exhibit at the same time differences and similarities with other financial time series, e.g., foreign exchanges returns (Dyhrberg, 2016). The presence of volatility clustering and leverage effect justify the use of GARCH-type models in the empirical studies (see, for example, Baur et al., 2018; Katsiampa, 2017, among others). However, these applications also evidenced a high degree of persistence in the volatility dynamics. Tiwari et al. (2018) employed robust estimators and finds that Bitcoin returns and volatility exhibit long-memory characteristics. Phillip et al. (2018) examine the returns for 224 crypto-currencies and find that they exhibit long memory and stochastic volatility. Catania et al. (2018) used generalised autoregressive score (GAS) models and compares with GARCH specifications.

Another strand to study the volatility dynamics of Bitcoin returns includes the use of regime-switching models. This approach is useful for capturing shifts and turning points in the volatility process that is difficult to accommodate using GARCH-types models. In the abstention of regime changes, the estimation can be biased also affecting the precision of the volatility forecast. Earlier studies by Bariviera (2017) suggest the use of the regime-switching models in the Bitcoin volatility. Ardia et al. (2018) use Bayesian regime-switching models to show that the Bitcoin returns exhibit regime changes. Thies and Molnár (2018) also use a Bayesian approach and conclude that the structural breaks in average returns and volatility of Bitcoin occur frequently. However, most of the empirical literature have been focused on the in-sample framework and comparisons have been made based on information criteria.

This paper therefore seeks to extend previous research concerned with the evaluation of alternative methods to estimate and forecasting the Bitcoin log-returns volatility in two ways. First, we address the presence of regime switches using a duration-dependent Markov-switching model (DDMS). Second, in extending the scope of previous research through evaluative application, to assess the economic significance of Bitcoin volatility predictability to investors, we also compare the DDMS model to traditional single-regime GARCH-types models predicting one-day-ahead Value-at-Risk (VaR). In anticipation of the results to follow, unlike the traditional regime-switching models, the time-varying probability model used in this paper seems to improve the estimation of the intertemporal dependence in the volatility process. In general, our estimates evidenced a declining hazard function and positive persistence in volatility for both states; these results suggest evidences of long memory in the data series. Our results show that the DDMS specification outperformed the single-regime models both in-sample and out-of-sample.

The remainder of this paper is organized as follows: Section 2 presents the models do be estimated; Section 3 describes the data and our empirical findings; Section 4 concludes.

2.2 Methodology

In this paper, we follow the duration-dependent Markov-switching (DDMS) model proposed by Maheu McCurdy (2000). The nonlinear specification applied to the Bitcoin log-returns is a discrete-state Markov model, where the conditional transition probabilities, as well as the state-specific levels of volatility are functions of the duration.

Begin by assuming that the time series process is governed by a discrete mixture of distributions. The discrete mixing variable $S_t \in \{0, 1\}$ is unobserved and depends only on S_{t-1} and its duration D_{t-1}. The role of the duration variable is to depict the memory in the consecutive occurrence of a particular state given as:

$$D_t = \min\left(D_{t-1} I\left(S_t, S_{t-1}\right) + 1, \tau\right), \tag{2.1}$$

where $I(S_t, S_{t-1}) = 1$ if $S_t = S_{t-1}$ and otherwise 0. To make the estimation tractable, the limiting parameter τ is established to set the memory of the duration. The transition probability is parametrized using a logistic function. This ensures that the probabilities are between 0 and 1. Using i and d to index the state and duration, where $\gamma_1(i)$ e $\gamma_2(i)$ are the parameters, the transition probability for $i = 0, 1$ is given by:

$$P\left(S_t = i | S_{t-1} = i, D(S_{t-1}) = d\right) = \begin{cases} \dfrac{\exp(\gamma_1(i) + \gamma_2(i)d)}{1 + \exp(\gamma_1(i) + \gamma_2(i)d)}, & \text{if } d \leq \tau \\ \\ \dfrac{\exp(\gamma_1(i) + \gamma_2(i)\tau)}{1 + \exp(\gamma_1(i) + \gamma_2(i)\tau)}, & \text{if } d > \tau \end{cases} \qquad i = 0, 1. \tag{2.2}$$

The duration effect allows the transition probability to vary over time until it reaches τ, after that, the transition probabilities are constant. The conditional probability of a state change given that the state has achieved a duration d, is the hazard function. Using the transition probabilities, the hazard function is given by:

$$1 - P\left(S_t = i | S_{t-1} = i, D(S_{t-1}) = d\right) = \frac{1}{1 + \exp(\gamma_1(i) + \gamma_2(i)d)}, \quad i = 0, 1. \tag{2.3}$$

A decreasing hazard function is referred to as negative duration dependence whereas an increasing hazard function is positive duration dependence. The parameter $\gamma_2(i)$ summarize the duration effect on the hazard function. For example, $\gamma_2(i) < 0$ means that a long period in regime i implies a higher probability of state switching (positive duration dependence); $\gamma_2(i) = 0$ means that the transition probability is independent of the regime duration; and $\gamma_2(i) > 0$ implies that the longer the duration of regime i, the higher the chance of the process to remain at i (negative duration dependence).

Given the transition probabilities in equations (2.1)-(2.3), the model equations follow:

$$r_t = c + \sum_{i=1}^{s} \phi_i r_{t-i} + \epsilon_t, \tag{2.4}$$

$$\epsilon_t = \sigma(S_t, D_t) z_t, \quad z_t \sim N(0,1), \quad S_t = 0,1 \tag{2.5}$$

$$\sigma(S_t, D_t) = (\omega(S_t) + \zeta(S_t) D_t)^2, \tag{2.6}$$

where r_t is the Bitcoin log-returns, $\phi_1, \ldots, \phi_s$ are the autoregressive parameters, and ε_t is the error term at time t. In this model, the latent state variable affects the level of volatility by $\omega(S_t) = \omega_0(1 - S_t) + \omega_1 S_t$, while the duration of the states, D_t, affect the dynamics of volatility through $\zeta(S_t)D_t$, where, $\zeta(S_t) = \zeta_0(1 - S_t) + \zeta_1 S_t$. z_t is is assumed to follow an identically and independently normal distribution.

The parameters of the duration-dependent Markov-switching model can be estimated using two different approaches: the maximum-likelihood method following Maheu & McCurdy (2000), or using MCMC methods, such as in Pelagatti (2001). In this study, we employed the first approach, and the reader is referred to Maheu & McCurdy (2000) for further details of the estimation procedure. However, it is important to highlight that is possible to determinate τ using a grid search with the log-likelihood values as the criterion. Although, we use a different strategy setting τ as a large number in the attempt to capture all the duration effect on the time series process.

2.3 Data and Empirical Results

2.3.1 Data

The dataset analyzed in this paper is the daily closing prices for the Bitcoin Coindesk Index[1], the same data as in Katsiampa (2017). In contrast to the previous work, we update the sample period from 18th July 2010 to 1st May 2018, producing a total of 2845 observations. The descriptive statistics for the Bitcoin log-returns are reported in Table 2.1. The mean is 0.4051 with a standard deviation of 5.8721. The largest price

[1] The data series are available online at http://www.coindesk.com/price.

increase is 42.4579, while the largest price decrease is -49.1528. The skewness is small and negative, and the kurtosis is higher than the normal distribution value. The Jarque-Bera (JB) statistics also indicated the departure from normality, while the value of the ARCH (5) suggest the presence of ARCH eects in the log-returns.

Table 2.1: Descriptive Statistics

Obs.	Mean (%)	Std. Dev. (%)	Min (%)	Max (%)	Skewness	Kurtosis	JB	ARCH (5)
2,844	0.405	5.872	−49.153	42.458	−0.352	14.617	16,051.3***	73.443***

Note: This table presents summary statistics of the log-returns on Bitcoin. The sample period is daily from 07/18/2010 – 05/01/2018. Bitcoin log-returns trade 7 days a week and have 2,845 observations during the sample period. *** indicates the rejection of the null hypotheses at the 1% level. Log-returns are calculated by $100 \times (lnP_t - lnP_{t-1})$.

2.3.2 Duration dependent model estimates

Full-sample estimates for the duration-dependent Markov-switching models are reported in Table 2.2. To analyze the overall statistical significance of the duration dependence effect, we also consider two nested specifications, MS-2 and DDMS-2, respectively. In the MS-2 model, the transition probabilities and the volatility are both independent of the duration, $\gamma_2(i) = 0$, $\zeta(i) = 0$. For the DDMS-2 model, the transition probability are unrestricted, $\gamma_2(i) \neq 0$, while volatility is absent of duration, $\zeta(i) = 0$. The most general specification is the DDMS-DD.

The last two columns of Table 2.2 present the estimates of the DDMS-DD model. The coefficients that capture the duration dependence of the transition probabilities, $\gamma_2(1)$ and $\gamma_2(2)$, are significantly positives. This result evidence that both states become more persistent over time, which implies a declining hazard function. For the duration effect associated with conditional standard deviation, the coefficients estimates are also significantly positives suggesting the increase of volatility as duration increase. These results were observed using two distinct values for the duration, 25 and 40, respectively.

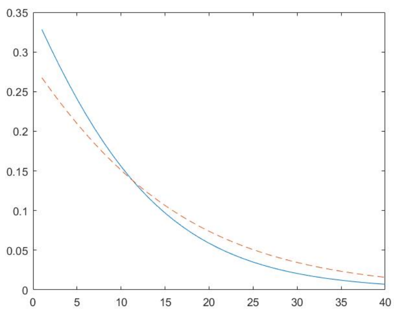
Figure 2.1 summarizes these evidence for $\tau = 40$. Panel (a) presents the hazard function and Panel (b) the standard deviation function. The continuous line is the low volatility state ($S_t = 0$), where the dotted line is the high volatility state ($S_t = 1$). The hazard functions are similar, but the volatilities levels are distinct as the length of duration. On average, the Bitcoin returns spend 49% of the time in the high volatility state and 51% in the low volatility state. The unconditional probability for each state can be computed as $P(S = i) = \sum_{d=1}^{\tau} P(S = i, D = d)$ for $i = 0, 1$, where $P(S, D)$ are the joint unconditional probabilities. Using $\tau = 25$, we observed the same values for both states, 50% of the time, respectively. These results suggest the long run characteristic of the market and reflect the recurring switch in the volatility states dynamics.

Table 2.2: Maximum Likelihood Estimates of Duration-Dependent Markov-Switching Models

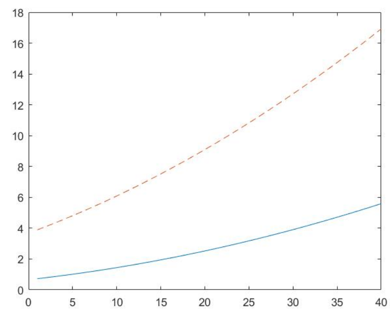
Parameter	MS-2	DDMS-2 ($\tau = 25$)	DDMS-DD ($\tau = 25$)	DDMS-DD ($\tau = 40$)
c	0.285***	0.282***	0.108***	0.119***
	(0.052)	(0.050)	(0.038)	(0.038)
ϕ_1	0.007	−0.011	−0.014	−0.019
	(0.018)	(0.017)	(0.017	(0.017)
$\omega(0)$	1.984***	1.839***	0.760***	0.814***
	(0.061)	(0.065)	(0.027)	(0.030)
$\zeta(0)$	–	–	0.062***	0.039***
			(0.002)	(0.001)
$\omega(1)$	9.556***	9.610***	1.760***	1.919***
	(0.275)	(0.277)	(0.076)	(0.086)
$\zeta(1)$	–	–	0.093***	0.055***
			(0.007)	(0.004)
$\gamma_1(0)$	2.508***	0.740***	0.193	0.607***
	(0.132)	(0.148)	(0.181)	(0.164)
$\gamma_2(0)$	–	0.123***	0.165***	0.108***
		(0.014)	(0.021)	(0.015)
$\gamma_1(1)$	1.814***	0.160	0.701***	0.927***
	(0.154)	(0.165)	(0.143)	(0.165)
$\gamma_2(1)$	-	0.151***	0.104***	0.080***
		(0.021)	(0.016)	(0.013)
$Log(L)$	−8,013.2	−7951.6	−7,897.8	−7,896.3
$Q(10)$	33.311	38.350	31.331	37.195
	[0.000]	[0.000]	[0.000]	[0.000]
$Q^2(10)$	42.301	66.879	12.995	11.996
	[0.000]	[0.000]	[0.224]	[0.285]

Note: This table reports the estimates for the Markov model with duration-dependent transition probabilities for the daily log-returns of the Bitcoin price index from 18th July 2010 to 1st May 2018. $Log(L)$ is the value of the log-likelihood. Standard errors are in parentheses. The p-values associated with the statistical tests are presented in brackets. ***,** ,* denote significance at the 1, 5 and 10% levels, respectively.

Figure 2.1: Duration Effects



(a) Hazard function



(b) Standard Deviation function

Table 2.2 also reports, for each model, the log-likelihood value and the Ljung-Box test statistics for autocorrelation in the squared standardized residuals. According to this diagnostics, the general model presents the best fit in-sample and also captured serial correlation in the squared standardized residuals. However, we test the significance of the duration dependence by applying the likelihood ratio test. For the MS-2 model, the LR statistic for the null hypothesis of absence of duration in transition probability and volatility is 230.8, which has a p-value $< 1\%$ according to the asymptotically $\chi^2(4)$ distribution. For the DDMS-2 model, the LR statistic for the volatility restriction is 107.6, which has a p-value $< 1\%$ according to the asymptotically $\chi^2(2)$ distribution.[2]

Overall, results not only points out the duration dependence in modeling perspective, but highlight theoretical arguments of its existence. For example, the persistence associated with declining hazard function indicated that if the low (high) volatility state was observed in the last period, the probability to continue in the same low (high) state increase. These results suggest evidence of long memory in the data series.On the other hand, the duration effect associated with conditional standard deviation is related to Bitcoin high volatility levels and speculation factors.

Since its creation, the Bitcoin prices increase with the dramatically growing interest in cryptocurrency with led to higher speculation levels. The Bitcoin prices rise as demand increases and the supply are limited. Furthermore, the Bitcoin is not issued by any legal support or government. In this context, Bitcoin is a high-risk asset with high volatility. The low US interest rate policy after the financial crisis in 2008 and 2009 lasted for several years could be contributed to make the investors looking for another source of value found Bitcoin, whose prices escalated accordingly. Tanking this fact, we have found distinct volatilities levels, the increase in volatility as duration increase in both of cases is consistent with the speculative behavior of the Bitcoin.

2.3.3 Models comparison

In the following subsection, we compare the DDMS-DD model to the traditional volatility models using in-sample statistics and predicting one-day-ahead Value-at-Risk. Particularly, we follow some of the Garch-types models explored by Katsiampa (2017), since we use the same data series extended until May 2018.[3] In general, we obtained the same results as in the previews work, which indicates that the optimal model in terms of goodness-of-fit is the AR-CGARCH, see Table 2.3.

Comparing single regime models with Markov-switching specifications is a well-known issue in the empirical literature. Standard econometrics tests for model specification are not valid since some parameters are unidentified under the null. Taking this fact, Table 2.4 presents some in-sample goodness-of-fit statistics. The largest log-likelihood value among

[2]These results were obtained setting $\tau = 25$.

[3]We follow Katsiampa (2017) setting low-orders of Garch-type models.

the Garch-type models is given by the AR-CGARCH, while for the Markov-switching models, and overall, the best result is reached with the DDMS-DD. The Akaike Information Criterion (AIC), Schwarz Criterion (BIC) and Hannan–Quinn Criterion (HQC) also indicated these results. Table 2.4 also shows that according to MSE and QLIKE loss functions the DDMS-DD outperform the Garch-type models.

Table 2.3: Maximum Likelihood Estimates of Standard AR(1)-GARCH(1,1)-type models

Parameter	AR-GARCH	AR-EGARCH	AR-GJR-GARCH	AR-APARCH	AR-CGARCH	AR-FIGARCH	AR-HYGARCH
c	0.2340*** (0.0614)	0.5018*** (0.0239)	0.2516*** (0.0639)	0.3075*** (0.0546)	0.2101*** (0.0665)	0.2106*** (0.0632)	0.2156*** (0.0655)
ϕ_1	0.0643*** (0.0220)	0.0613*** (0.0212)	0.0626*** (0.0220)	0.0588*** (0.0218)	0.0539** (0.0223)	0.0519** (0.0278)	0.0498** (0.0257)
ω	0.8410*** (0.0287)	−0.0569*** (0.0076)	0.8401*** (0.0295)	0.4952*** (0.0360)	1,522.78** (707.58)	0.9378 (0.7368)	0.5338 (0.5873)
α	0.2317*** (0.0098)	0.3972*** (0.0129)	0.2477*** (0.0157)	0.2353*** (0.0091)	0.1706*** (0.0090)	0.1318 (0.2609)	0.0834 (0.2824)
β	0.7784*** (0.0061)	0.9319*** (0.0026)	0.7793*** (0.0065)	0.7903*** (0.0058)	0.7942*** (0.0087)	0.3935 (0.3143)	0.3475 (0.3306)
γ	–	0.0131 (0.0083)	–	1.5338*** (0.0616)	–	0.5193*** (0.1082)	0.5077*** (0.1148)
δ, $log(\delta)$	–	–	−0.0346** (0.0163)	−0.0257 (0.0188)	–	–	0.0950 (0.0646)
ρ	–	–	–	–	0.9999*** (0.0001)	–	–
θ	–	–	–	–	0.0603*** (0.0042)	–	–
$Log(L)$	−8,251.7	−8,253.3	−8,250.7	−8,246.0	−8,213.0	−8,218.9	−8,214.1
$Q(10)$	45.010 [0.000]	43.370 [0.000]	44.203 [0.000]	44.478 [0.000]	46.898 [0.000]	45.968 [0.000]	47.525 [0.000]
$Q^2(10)$	5.851 [0.828]	3.844 [0.954]	5.922 [0.822]	7.610 [0.667]	4.723 [0.909]	4.135 [0.845]	4.863 [0.772]

Note: This table reports the estimates for the AR(1)-GARCH(1,1)-type models for the daily log-returns of the Bitcoin price index from 18th July 2010 to 1st May 2018. $Log(L)$ is the value of the log-likelihood. We estimated the HYGARCH using the G@RCH module in OxMetrics. The program reports $log(\delta)$ instead of δ. Standard errors are in parentheses. The p-values associated with the statistical tests are presented in brackets. ***,** ,* denote significance at the 1, 5 and 10% levels, respectively.

Figure 2.2 plots the estimates of volatility for the DDMS-DD model.[4] Comparing the DDMS-DD with the AR-CGARCH suggests that using duration as an instrument in the conditional variance and transition matrix is a substitute for Garch-type models. Unlike the standard Markov-switching models, the DDMS-DD is particularly suited to capture the persistence associated with volatility clustering because the duration variable provides a parsimonious parameterization of potential high-order dependence in the data series. Furthermore, persistence in volatility levels is time varying in this specification.

On the other hand, the AR-CGARCH suggests the importance of distinguish short-run and a long-run component in conditional variance which implies the varying level in volatility process. Furthermore, empirical studies (Maheu, 2005) demonstrated that the component model is potentially able to reproduce long memory properties in the autocorrelation of squared returns. Although different specifications, this can bright some

[4]Further information to calculate the conditional standard deviations implied by the DDMS-DD, see, Maheu and McCurdy (2000).

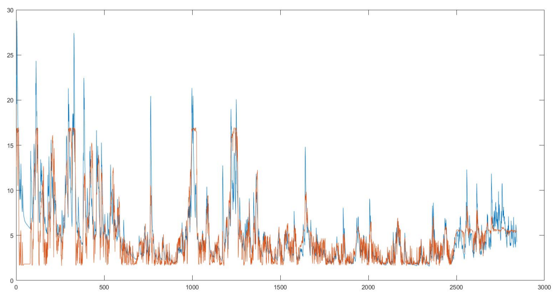
explanation for the similar pattern in volatility dynamics.

2.4: In-sample goodness-of-fit statistics

Model	AIC	Rank	BIC	Rank	HQ	Rank	*Log*(*L*)	Rank	MSE	Rank	QLIKE	Rank
AR-GARCH	5.8084	7	5.8189	6	5.8122	7	−8,251.7	8	15,142	9	3.9789	9
AR-EGARCH	5.8103	9	5.8229	9	5.8148	9	−8,253.3	9	14,032	3	3.9746	7
AR-GJR-GARCH	5.8085	8	5.8210	8	5.8130	8	−8,250.7	7	15,062	8	3.9775	8
AR-APARCH	5.8059	6	5.8205	7	5.8111	6	−8,246.0	6	14,765	5	3.9735	6
AR-CGARCH	5.7826	3	5.7973	3	5.7879	3	−8,213.0	3	14,774	6	3.9525	3
AR-FIGARCH	5.7860	4	5.7986	5	5.7905	5	−8,218.9	5	14,594	4	3.9563	5
AR-HYGARCH	5.7883	5	5.7980	4	5.7886	4	−8,214.1	4	15,023	7	3.9527	4
DDMS-DD (25)	5.5611	2	5.5820	2	5.5686	2	−7,897.8	2	12,900	1	3.4356	1
DDMS-DD (40)	5.5599	1	5.5809	1	5.5675	1	−7,896.3	1	13,064	2	3.4562	2

Note: MSE is the Mean Squared Error loss function calculated as $n^{-1}\sum_{t=1}^{n}\left(\hat{\sigma}_{t/t-1} - \hat{h}^{1/2}_{t/t-1}\right)^2$ and QLIKE is the Quasi–Like loss function calculated as $n^{-1}\sum_{t=1}^{n}\left(log\hat{h}_{t/t-1} + \hat{\sigma}^2_{t/t-1}\hat{h}^{-1}_{t/t-1}\right)$. Since volatility is unobserved and realized volatility measures are not available, we proxy volatility with the square of the realized log-returns, $r_t^2 = \hat{h}_{t/t-1}$. Further discussion about loss functions choice, see, Patton (2011).

Figure 2.2: Conditional Standard Error: AR-CGARCH X DDMS(40)



We now turn to an out-of-sample analysis reporting the one-day-ahead Value-at-Risk (VaR) for each model. Value-at-risk (VaR) measures risk in terms of returns at a given probability: for example, a VaR of −1% at the 5% confidence level indicates that there is a probability of 5% of a return that is lower or equal to −1%. Intuitively, VaR^i_ϑ corresponds to the losses above a certain threshold. In this empirical application, the recursive forecasting estimation ranges from May 1st, 2015 to May 1st, 2018, producing a total of 1,097 out-of-sample observations. We backtest the accuracy of the VaR following Christoffersen (1998). Table 2.5 reports the hit rate and the p-values of the independence, unconditional coverage and conditional coverage tests for the 5%, 2.5%, and 1% risk levels.

Our results show that DDMS-DD model presents the lowest hit rate for the three risk levels. For instance, the best result is obtained at the 1% risk level, since p-values indicated the non-rejection of the null hypothesis in all backtests. Among the Garch-types models, the best result is obtained with the AR-EGARCH, especially for the 2.5%

and 5.0% risk levels. However, our results follow the empirical literature which state that Markov-switching models tend to give accurate predictions, especially for VaR at 1% level (Guidolin and Timmermann, 2006).

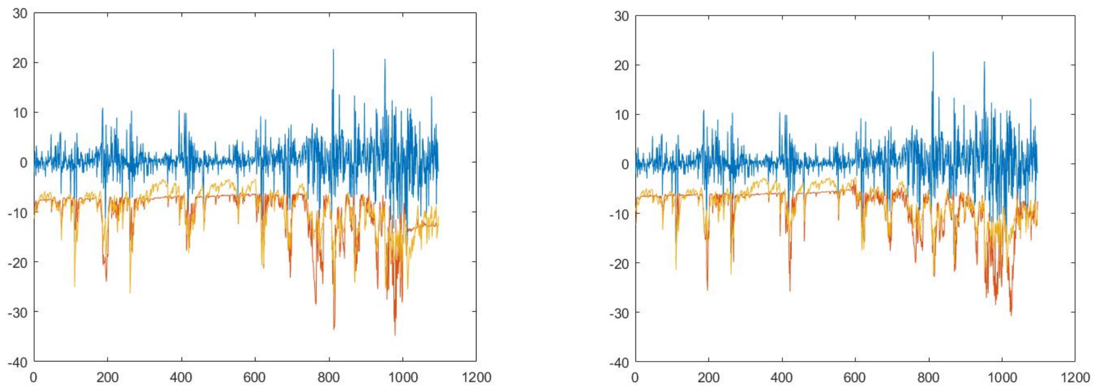
2.5: Forecasting - Backtesting Results

	Hit Rate	U.C	Indep.	C.C	Hit Rate	U.C	Indep.	C.C	Hit Rate	U.C	Indep.	C.C
	$\vartheta = 1\%$				$\vartheta = 2.5\%$				$\vartheta = 5\%$			
AR-GARCH	2.5%	0.1522	0.4911	0.2830	2.6%	0.9079	0.2256	0.4767	4.1%	0.1616	0.4141	0.2690
AR-EGARCH	1.4%	0.2455	0.5188	0.4137	2.5%	0.9382	0.2428	0.5041	3.8%	0.0457	0.2723	0.0744
AR-GJR-GARCH	1.4%	0.2455	0.5188	0.4137	2.8%	0.4950	0.8946	0.7854	4.0%	0.1217	0.3756	0.2039
AR-APARCH	1.4%	0.2455	0.5188	0.4137	3.0%	0.2936	0.0898	0.1366	4.5%	0.4134	0.0055	0.0153
AR-CGARCH	1.5%	0.1522	0.4911	0.2830	2.6%	0.9079	0.2256	0.4767	4.3%	0.2683	0.4968	0.4302
AR-FIGARCH	1.8%	0.0139	0.3885	0.0334	3.2%	0.1583	0.4348	0.2725	4.3%	0.2683	0.1928	0.2321
AR-HYGARCH	1.6%	0.0505	0.4381	0.1093	2.7%	0.6202	0.8442	0.8675	4.3%	0.2683	0.1928	0.2321
AR-GAS	1.5%	0.1522	0.4911	0.2830	2.6%	0.9079	0.2256	0.4767	4.0%	0.1217	0.3756	0.2039
AR-EGAS	1.6%	0.0898	0.4642	0.1814	2.7%	0.6202	0.8442	0.8675	4.5%	0.4134	0.0236	0.0552
AR-AEGAS	1.7%	0.0271	0.4129	0.0621	3.0%	0.2936	0.0168	0.0330	4.6%	0.4998	0.0071	0.0212
AR-DDMS-DD (25)	1.2%	0.5474	0.5464	0.7139	2.2%	0.5017	0.0139	0.0388	3.3%	0.0056	0.0056	0.0005
AR-DDMS-DD (40)	1.1%	0.7558	0.1173	0.2795	2.5%	0.9079	0.0353	0.1084	3.4%	0.0089	0.0011	0.0002

Note: This table reports the backtesting results for the DDMS-DD and Garch-types estimates at the $\vartheta = 1\%$, $\vartheta = 2.5\%$ and $\vartheta = 5\%$ levels. The hit rate and the p-values of the independence ("Indep."), unconditional coverage ("U.C.") and conditional coverage("C.C.") tests are also reported, respectively.

To illustrate the results presented in Table 2.5, we plot in Figure 2.3 the Bitcoin returns (blue line) over the out-of-sample period. For the 1% risk level, panel (a) presents the VaR estimates for the DDMS-DD(40) versus AR-EGARCH. For the 2.5% risk level, panel (b) presents the VaR estimates for the DDMS-DD(25) versus AR-EGARCH. In general, we observe that the VaR estimates obtained with the AR-EGARCH model (orange line) tend to be more noisy than those obtained with DDMS-DD specifications (red line).

Figure 2.3: Value-at-Risk



(a) VaR 1%: DDMS-DD(40) X AR-EGARCH (b) VaR 2.5%: DDMS-DD(25) X AR-EGARCH

Finally, the forecasting accurateness of the competing models is also evaluated with statistical loss functions. Our analysis is conducted using the MSE and the QLIKE losses, and the duration dependence specification is the benchmark model in the Diebold Mariano test of equal predictive ability. Since the squared returns represent a very noisy estimate of the true volatility (see, Andersen and Bollerslev, 1998), we calculated the realized volatility as the sum of one and five-minutes squared returns aggregated over the relevant forecast horizon of one day.[5]

2.6 One-day ahead volatility forecast: May 1st, 2015 - May 1st, 2018

	Squared Returns		Realized Volatility -1 Min.		Realized Volatility - 5 Min	
	MSE	QLIKE	MSE	QLIKE	MSE	QLIKE
DDMS-DD (25)	2524.3	3.6250	2.7780	**3.6483**	2670.4	3.6381
DDMS-DD (40)	**2135.7**	**3.6073**	**2.6011**	3.6566	**2436.7**	**3.6361**
AR-GARCH	1591.2*	3.4535*	2195.1*	3.5413*	2078.6*	3.5247*
AR-EGARCH	1632.3*	3.4850*	2177.7*	3.5484*	2051.7*	3.5303*
AR-GJR-GARCH	1598.7*	3.4540*	2204.8*	3.5394*	2087.0*	3.5226*
AR-APARCH	1564.4*	3.4653*	2201.2*	3.5500*	2081.0*	3.5387*
AR-CGARCH	1574.5*	3.4456*	2173.2*	3.5339*	2056.1*	3.5163*
AR-FIGARCH	1567.9*	3.4447*	2154.6*	3.5060*	2041.6*	3.4776*
AR-HYGARCH	1630.2*	3.4441*	2115.8*	3.4857*	1999.6*	3.4484*
GAS (GARCH)	1604.4*	3.4691*	2196.3*	3.5516*	2080.4*	3.5400*
EGAS	1525.9*	3.5024*	2390.8	3.6626	2285.7	3.6655
AEGAS	1520.3*	3.4878*	2380.4	3.6386	2268.6	3.6377

Note: The realized volatility was calculated as the sum of one and five-minutes squared returns aggregated over the relevant forecast horizon of 1 and 5-days ahead. * refers to the p-value less than 1% in the Diebold Mariano test. We perform this test using the highlighted duration dependence model with the lowest loss function value as the benchmark.

Analyzing Table 2.6, the duration models did not outperform the Garch-type models. However, empirical studies have highlighted that volatility forecasting and VaR forecasting are two different objectives. The results of the earlier to not imply in the former, and vice-versa. In general, our results goes directly with Dacco and Satchell's (1999) which advert that the choice of the correct loss function is crucial to evaluate the accuracy of volatility forecasts of non-linear models. Using two different loss function, we observed that the duration dependence specifications are relatively different when compared to the Garch-type models. Another point that should be stressed refers to the volatility proxy. Our measure is larger than others realized volatility measures, since it is not robust to jumps and microstructure noise (see, Barndor-Nielsen Shephard, 2004). In this case,

[5]Intra-daily returns on the Bitcoin index are obtained from https://www.kaggle.com/mczielinski/bitcoin-historical-data

further research is needed to construct out-of-sample evaluation procedures to identify the best models in terms of statistical forecasting.

2.4 Conclusion

We have used duration-dependent Markov-switching models (DDMS) to test for the presence of duration dependence in the volatility dynamics of Bitcoin returns. Additionally, we compare the DDMS model to traditional single-regime GARCH models predicting one-day-ahead Value-at-Risk (VaR), since regime-switching models tend to give accurate predictions. In general, the coefficients of the transition probabilities are significantly positive, which suggest that both states become more persistent over time. For the duration effect associated with conditional standard deviation, the coefficients estimates are also significantly positives indicating the increase of volatility as duration increase. Our results suggest evidence of long memory in the data series as observed by other authors. When performing one-day ahead VaR, the DDMS-DD model presents the lowest hit rate among the models. The best result is obtained at the 1% risk level, since p-values indicated the non-rejection of the null hypothesis in all backtests.

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GARCH-type models

AR(1)-GARCH(1,1)-type models

	$r_t = c + \phi r_{t-1} + u_t, \quad u_t = \sigma_t z_t, \quad z_t \sim N(0,1)$	
GARCH	$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2$	Bollerslev, 1986
EGARCH	$\log(\sigma_t^2) = \omega + \alpha \left[\left\lvert \frac{u_{t-1}}{\sigma_{t-1}} \right\rvert - \sqrt{2\pi} \right] + \beta \log(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sigma_{t-1}}$	Nelson, 1991
GJR-GARCH	$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$	Glosten et al., 1993
APARCH	$\sigma_t^\delta = \omega + \alpha(\lvert u_{t-1}\rvert - \gamma u_{t-1})^\delta + \beta \sigma_{t-1}^\delta$	Ding et al., 1993
FIGARCH	$\sigma_t^2 = \omega + (1 - \beta L - (1 - \alpha L)(1 - L)^\gamma) u_t^2 + \beta \sigma_{t-1}^2$	Baillie et al., 1996
HYGARCH	$\sigma_t^2 = \omega + (1 - \beta L - (1 - \alpha L)(1 + \delta((1 - L)^\gamma - 1))) u_t^2 + \beta \sigma_{t-1}^2$	Davidson, 2004
CGARCH	$\sigma_t^2 = q_t + \alpha \left(u_{t-1}^2 - q_{t-1}\right) + \beta \left(\sigma_{t-1}^2 - q_{t-1}\right)$	Engle & Lee, 1993
	$q_t = \omega + \rho \left(q_{t-1} - \omega\right) + \theta \left(u_{t-1}^2 - \sigma_{t-1}^2\right)$	

GAS-type models

The Generalised Autoregressive Score (GAS) models were proposed by Harvey & Chakravarty (2008) and Creal et al. (2012) to deal with large returns in a GARCH approach. The general framework is as follows. Let $p\left(y_t|f_t\right)$ denote a conditional observation density for observations y_t and f_t a time varying parameter. Assume the parameter f_t follows the updating equation

$$f_{t+1} = \omega + \beta f_t + \alpha \kappa_{t-1},$$

where $\kappa_t = S_t \triangledown_t$. $\triangledown_t$ is the score with respect to the parameter f_t and S_t is a time dependent scaling matrix.[6] In the GAS approach, the evolution of the volatility equation $(f_t = \sigma_t^2)$ depends on the past values of the score of the conditional distribution.

The specification of the GAS (Garch) is given by:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1} \sigma_{t-1}^2 + f_1 \sigma'_{t-1}$$

where $u_t = z_t^2 - 1$ and $z_t \sim N\left(0,1\right)$.

The Exponential GAS (EGAS) is defined as:

$$\log \sigma_t^2 = \omega + \alpha_1 u_{t-1} + f_1 \log \sigma_{t-1}^2.$$

The Asymmetric Exponential GAS (AEGAS) which considered the leverage effect is defined as:

$$\log \sigma_t^2 = \omega + \alpha_1 u_{t-1} + \gamma_1 l_{t-1} + f_1 \log \sigma_{t-1}^2,$$

where $l_t = sgn\left(-z_t\right)\left(u_t + 1\right)$ for the symmetric distributions (i.e. Normal, Student-t and GED) and $l_t = sgn\left(-z_t^*\right)\left(u_t + 1\right)$ for SKST (skewed-Student-t distribution). Therefore $E(l_t) = 0$ for symmetric and $E(l_t) = \frac{1-\zeta^2}{1+\zeta^2}$ for SKST.

[6] We set S_t following Harvey & Chakravarty (2008).

3 Testing for Mean-Reversion in Bitcoin returns with Gibbs-sampling-augmented randomization

Abstract. In the present paper, we attempt to verify whether the Bitcoin log-returns are mean-reverted in the presence of heteroskedastic disturbances driven by a mixture distribution. To tackle this problem, we use the autoregression test of mean-reversion based on the Gibbs-sampling-augmented randomization methodology. In general, our results indicated that Bitcoin is mean-averting for different returns horizons, model specifications and for sub-sample periods, which show the explosive characteristic of the Bitcoin in the period of analysis from 2010 to 2019.

Keywords: Autoregression tests; Mean-reversion in Bitcoin market, Markov-switching models, Gibbs-sampling-augmented randomization.

3.1 Introduction

The analysis of mean-reversion in financial assets attracts the interest of many investors, market practitioners, and researchers. It has fundamental implications for investment decisions from portfolio selection to pricing of options. For example, a mean-reverting stock market suggests that assets are less risky in the long run. The research in mean-reversion for stock prices has shown evidence that stock returns are to some extent predictable (see, for example, Nelson, 1976; Fama, 1981, among others). The presence of mean-reverting components in asset prices is directly at odds with the Efficient Market Hypothesis, and it may imply pricing irregularities or market manipulation. In a seminal work Fama and French (1988) analyzed autocorrelations of stock returns for increasing holding periods. A mean-reverting component of prices can induce strong negative autocorrelation in long-horizon returns. For periods of length between 3 and 5-years, long-term mean-reversion was empirically observed using stock returns between 1926 and 1985. More specifically, 3-year returns showed a negative correlation of 25%, while 5-year returns showed a negative correlation of 40%.

However, the autoregression testing of mean-reversion that ignores patterns of heteroscedascity is biased towards rejecting the null hypothesis of mean-aversion too often (Kim and Nelson, 1998). Kim and Nelson (1998) question the often used assumption of homoscedastic volatility and argue that the significant divergences in mean-reversion results are explained by the volatility's time-variation. They used a Bayesian Markov-swtiching model capturing shifts and turning points in the volatility process, and propose a Gibbs-sampling-augmented randomization, which is able to obtain the distribution of the null hypothesis of mean-aversion in the presence of shifts in the volatility. Their findings indicate that mean-aversion in stock prices are more common than previously found in the literature.

Hence, it is not surprising that mean-reversion has been tested extensively for many traditional financial assets as well as commodities and over a variety of time periods, using different methods. However, Bitcoin has so far been unexplored, despite representing an important case study, since it is the most traded in the world among cryptocurrencies. We add to the literature on cryptocurrencies by studying the mean-reversion of Bitcoin returns.

Cryptocurrencies have recently received a great deal of attention in the literature. In addition, many investors and market practitioners are interested in the behavior of Bitcoin for trading purposes. Studies have observed that Bitcoin returns exhibit at the same time differences and similarities with other financial time series (Dyhrberg, 2016). Particularly, the presence of volatility clustering justifies the use of heteroscedastic models in the empirical studies (see, for example, Katsiampa, 2017; Baur et al., 2018; Chaim and Laurini, 2018, among others). In this regard, the Markov-switching models are useful for capturing shifts and turning points in the volatility process that is difficult to accommodate using GARCH-types models. Several applications (see, for example, Bariviera, 2017; Ardia et al., 2018) suggest the use of regime-switching models and show that the Bitcoin returns exhibit regime changes. Nonetheless, no previous study has utilized this methodology for the mean-reversion analysis.

To the best of our knowledge, testing of mean-reversion in the Bitcoin returns is not currently explored in the literature (notable exceptions include: Corbet and Katsiampa, 2018). Thus, this paper aims to extend the Bitcoin mean-reversion analysis in two different ways. Firstly, Corbet and Katsiampa (2018) find evidence of asymmetric reverting patterns for positive and negative returns. In contrast, we focus on latent regime switching properties of Bitcoin returns. This approach allows for jumps in the unconditional volatility, and it is interesting when analyzing mean-reversion of returns in the long-run. Using a flexible 3-state Markov-switching variance model, our paper is based on the autoregression test of mean-reversion following Kim and Nelson (1998). Secondly, we analyze multi-period Bitcoin returns, since this is relevant as investors may change their trading strategies according to the Bitcoin's long-term behavior. In anticipation of the results to follow, we report that the 3-state Markov-switching model is identifiable and well-adjusted to the data. Differently, to the existing literature, the methodology is suitable to capture long-run dynamics of volatility changes while the Bayesian approach captured the uncertainty in volatility and parameters. In general, the Bitcoin is strongly mean-avert for all returns horizons, using alternative models specifications and taking account for sub-sample periods. This result indicates that the Bitcoin returns are unpredictable, especially considering their explosive characteristic from 2010 to 2019.

In this recent literature, a closely related topic to our work focus on testing the Efficient Market Hypothesis (EMH) for cryptocurrencies (Urquhart, 2016; Nadarajah and Chu, 2017; Vidal-Tomás and Ibañez, 2018). Urquhart (2016) was the first author to analyze this form of efficiency in the Bitcoin market, rejecting the null hypothesis of informational efficiency regardless of the test. Nadarajah and Chu (2017) test the EMH not on the Bitcoin returns but using power transformations of daily returns, whose results showed that the transformed Bitcoin returns are actually market efficient. Vidal-Tomás and Ibañez (2018) examine the semi-strong efficiency of Bitcoin in the Bitstamp and Mt.Gox markets, showing how the digital currency responds to monetary policy and Bitcoin events. Recently, Demir et al. (2018) indicate that Economic Policy Uncertainty (EPU) can be used to predict Bitcoin returns. And also those papers related to the explosive behavior like Cheah and Fry (2015), Corbet et al. (2018), among others. Therefore, our analysis contributes not only to the mean-reverting literature but also to those papers in Bitcoin related to the Efficient Market Hypothesis and explosive behavior of Bitcoin given the connection of the topics.

The remainder of this paper is organized as follows: Section 2 presents the methodology; Section 3 describes the data and our empirical results; Section 4 concludes.

3.2 Methodology

3.2.1 The autoregression test

Among the different approaches to study the mean-reversion hypotheses, Fama and French (1988), simply estimate a regression model using multi-period returns, since the estimated slope coefficient is a measure of the serial autocorrelation of the multi-period returns.[1] Denoting p_t the log of stock price in period t, the multi-period return from t to $t+k$ is given by $R_{t,t+K} = p_{t+k} - p_t$. Therefore, the mean-reverting process can be characterized by negative estimated values of the slope coefficient in the following regression model:

$$R_{t,t+K} = \alpha_K + \beta_K R_{t-K,t} + \varepsilon_{t,t+k}. \tag{3.1}$$

Despite multi-period stock returns seen to be characterized by negative autocorrelation over long intervals, many authors have challenged the reliability of Fama and French's results. Testing whether the sample autocorrelation is significantly below zero demands the sampling distribution and the standard error of β_K under the null hypothesis of no mean-reversion. Taking this fact, statistical issues such as the autocorrelations downward bias in finite samples or volatility changes over the sample period have been appointed by empirical studies. For example, Kim et al. (1991) re-examine Fama and French's work estimating the unknown distribution of the autoregressive statistics. Using randomization methods, the authors relaxed the normality assumption for stock returns and suggested that significance levels are much lower than the previously reported.

3.2.2 Bayesian approach to autoregression test

In the same spirit of Kim et al. (1991), Kim and Nelson (1998) offer an alternative approach to address the issues of heteroscedasticity. Based on previous results in the literature (see, Kim et al., 1998) the authors apply the Gibbs-sampling-augmented-randomization method in the autoregressive test of mean-reversion. In their application, a three-state Markov-switching variance model[2] is used to approximate the data generating process under the null hypothesis, such as:

$$y_t \sim N(0, \sigma_t^2), \tag{3.2}$$

$$\sigma_t^2 = \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t} + \sigma_3^2 S_{3t}, \tag{3.3}$$

[1]For further details, see Cochrane (2001).
[2]See, Hamilton (1989).

$$S_{kt} = \begin{cases} 1 & if \quad S_t = k \\ 0, & otherwise \end{cases}, \qquad k = 1, 2, 3 \tag{3.4}$$

$$\sigma_1^2 < \sigma_2^2 < \sigma_{3,}^2, \tag{3.5}$$

where y_t is the one-period stock log-return subtracted by the mean (i.e., demeaned log-return), and S_t is an latent state variable following an first-order Markov process with transition probabilities given by:

$$Pr[S_t = j | S_{t-1} = i] = p_{ij}, \quad i, j = 1, 2, 3 \tag{3.6}$$

$$\sum_{j=1}^{3} p_{ij} = 1. \tag{3.7}$$

Using Bayesian methods, the authors estimated the above model applying the Gibbs-sampling technique, which enable us to access the posterior distributions of the parameters σ_1^2, σ_2^2, σ_3^2, p_{ij}; $i = 1, 2, 3$; $j = 1, 2, 3$, and the latent states S_t, $t = 1, 2, ..., T$. Theoretically, the estimation procedure provides the uncertainty associated with the parameters and states through the posterior distributions. For example, the r-th run of the Gibbs Sampling produce the simulated parameters, $\sigma_1^2(r)$, $\sigma_2^2(r)$ $\sigma_3^2(r)$, and the states $S_t(r)$, $t = 1, 2, ...T$ that allow us to generate $\sigma_t^2(r)$, $t = 1, 2, ..., T$ following equation (3.3). Based on Kim and Nelson (1998), the subsequent steps describe the Gibbs-sampling-augmented randomization method in the autoregressive tests of mean-reversion:

- **Step 1** Standardize the demeaned log-return y_t, $t = 1, 2, ..., T$, with the simulated $\sigma_t(r)$, $t = 1, 2, ..., T$ at the r-th run of Gibbs-sampling.

- **Step 2** Calculate the autoregression coefficients $\hat{\beta}_K^*(r)$ using the standardized returns.
- **Step 3** Randomize the standardized returns from Step 1 and calculate the autoregression coefficients $\hat{\beta}_K^{**}(r)$ that are both standardized and randomized.

- **Step 4** Compare $\hat{\beta}_K^*(r)$ and $\hat{\beta}_K^{**}(r)$.

The steps 1 to 4 are repeated 10,000 times. Steps 1 and 2 refers to the posterior distribution of the autoregression coefficients for standardized returns. Step 3 refers to the posterior distribution of the autoregression coefficients for standardized under the null hypothesis, i.e., no mean-reversion. We count how many times $\hat{\beta}_K^*(r)$ falls below $\hat{\beta}_K^{**}(r)$ to estimate the significance level of the tests.

3.3 Data and Empirical Results

3.3.1 Data

The dataset analyzed in this paper is the weekly closing prices for the Bitcoin (in U.S. dollar) from October 2010 to January 2019 (443 observations) obtained from Bloomberg. The choice of using weekly data takes into consideration a trade-off between the number of observations available for the analysis and the presence of noise regarding the regime switching identification. The use of daily data ultimately confuses the detection of distinguishable regimes. However, monthly data would considerably reduce the number of observations to run the autoregressive test for multi-periods returns. The descriptive statistics for the Bitcoin log-returns are reported in Table 3.1. The mean is 0.0247 with a standard deviation of 0.1574. The skewness is positive, and the kurtosis is higher than 3. The Jarque-Bera (JB) statistics also indicated the departure from normality, while the value of the ARCH (4) suggests the presence of ARCH effects in the log-returns.

Table 3.1: Summary statistics of the Bitcoin original returns

Obs.	Mean	Std. Dev.	Min	Max	Skewness	Kurtosis	JB	ARCH (4)
443	0.0247	0.1574	−0.7223	0.8658	0.6758	8.5413	600.5 (0.000)	116.8 (0.000)

Note: This table presents summary statistics of the weekly original Bitcoin returns from 08/06/2010 to 02/01/2019. Parentheses refer to the p-values of Jarque-Bera test and ARCH test, respectively.

3.3.2 Estimation Results

We first evaluate if the three-state Markov-switching model captures most of the dynamics in Bitcoin returns variance. Our analyses based on the Kim et al. (1998) approach. For example, Figure 3.1 plots the average of 10,000 realizations of the standardized returns described in the Gibbs sampling-augmented randomization method.[3] Analyzing the summary statistics of the simulated standardized returns (see Table 3.2), we observe no ARCH effect at 5% significance level, the kurtosis value is close to the normal distribution and the Jarque-Bera test does not reject the null at 1% significance level.

3.2: Summary statistics of the simulated standardized returns: 3 Regimes Model

Obs.	Mean	Std. Dev.	Min	Max	Skewness	Kurtosis	JB	ARCH (4)
443	-0.1044	0.9438	−2.4336	2.7398	0.1673	2.3789	9.1882 (0.0161)	8.5028 (0.0748)

Note: This table presents summary statistics of the standardized returns (average of 10,000 realizations from Gibbs-sampling). Parentheses refer to the p-values of Jarque-Bera test and ARCH test, respectively.

[3]We also average the variance series when all Gibbs-sampling interactions over, see, Figure I in the Appendix.

Figure 3.1: Standardized returns (average of 10,000 realizations from Gibbs-sampling): 3 Regimes Model

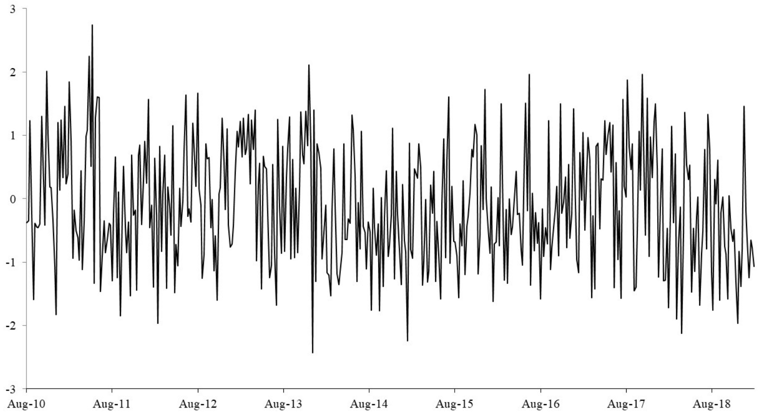

Table 3.3 lists the estimates of variance and transition probability parameters for the Markov-switching model. In our analysis, the Gibbs-sampling iterates on sampling from the following conditional densities: $S|\Sigma, P$; $\Sigma|P, S$, and $P|\Sigma, S$, where, $\Sigma = \{\sigma_1^2,\ \sigma_2^2,\ \sigma_3^2\}$, $P = \{p_{ij};\ i, j = 1, 2, 3;\}$, and $S = \{S_t,\ t = 1, 2, ..., T\}$ are the blocks of parameters. Draws of the variance and transition probability can be sampled from the conditional Inverse-gamma and Dirichlet posteriors distributions, respectively. The state variables are sampled using the so-called forward and backward smoother (see, Chib, 1996). Using non-informative priors for all model's parameters, we identify distinct variance values, characterizing low, medium and high volatility states in Bitcoin returns. We also computed the model's smoothed probabilities; the regimes are persistent and distincts for which level of volatility, this result is in line with the transition probabilities values, see Figure II in the Appendix.

Table 3.3: Bayesian inferences on parameters estimates: 3 Regimes Model

Parameter	Posterior			
	Mean	Median	SD	95 % interval
σ_1^2	0.0018	0.0017	0.0006	(0.0011, 0.0029)
σ_2^2	0.0124	0.0122	0.0023	(0.0093, 0.0163)
σ_3^2	0.0990	0.0955	0.0238	(0.0683, 0.1426)
p_{11}	0.8296	0.8331	0.0503	(0.7418, 0.9059)
p_{12}	0.1345	0.1307	0.0474	(0.0625, 0.2172)
p_{21}	0.0747	0.0709	0.0299	(0.0332, 0.1299)
p_{22}	0.8843	0.8880	0.0332	(0.8237, 0.9321)
p_{31}	0.0500	0.0442	0.0313	(0.0103, 0.1104)
p_{32}	0.0985	0.0926	0.0464	(0.0333, 0.1828)

Note: Non-informative priors were given for all parameters of the model. SD refers to standard deviation.

The autoregressive test based on the Gibbs-sampling-augmented randomization method is reported in Table 3.4. In our analysis, we considered several multi-period lag returns ranging from 4 weeks (one month) to 52 weeks, approximately one year. In general, our results indicated very strong evidence of mean-aversion, since the slope coefficients for the standardized returns are positive. The lowest p-value for the test of mean-aversion is 0.026 at lag 24. For long periods, 44, 48 and 52 weeks, the slope coefficients for the standardized returns are negative, but not statistically significant for mean-reversion.

Table 3.4: Autoregression tests based on standardized returns: 3 Regimes Model

Lag K (weeks)												
4	8	12	16	20	24	28	32	36	40	44	48	52
Posterior distribution of for standardized Bitcoin returns (Mean, Median, SD)												
0.1388	0.1493	0.1413	0.1762	0.2695	0.3100	0.2839	0.1893	0.0986	0.0084	-0.0620	-0.1045	-0.1252
0.1388	0.1501	0.1420	0.1776	0.2708	0.3114	0.2847	0.1902	0.1006	0.0110	-0.0585	-0.1017	-0.1218
0.0316	0.0461	0.0592	0.0661	0.0582	0.0546	0.0580	0.0685	0.0764	0.0816	0.0855	0.0900	0.0926
Sampling distribution. of based on standardized and randomized returns (Mean, Median, SD)												
-0.0115	-0.0245	-0.0374	-0.0520	-0.0662	-0.0771	-0.0901	-0.1065	-0.1222	-0.1366	-0.1585	-0.1684	-0.1841
-0.0104	-0.0255	-0.0387	-0.0523	-0.0674	-0.0803	-0.0942	-0.1144	-0.1270	-0.1448	-0.1717	-0.1820	-0.1964
0.0782	0.1104	0.1309	0.1552	0.1726	0.1880	0.2021	0.2140	0.2243	0.2387	0.2473	0.2568	0.2674
p-values												
0.9621	0.9242	0.8934	0.9105	0.9653	0.9740	0.9592	0.9050	0.8170	0.7185	0.6525	0.6049	0.5912

We also investigated our preliminary results using two different approaches. First, we re-estimate the Markov-switching model and the autoregressive test using the sub-sample period from 2010 to 2016. Second, we applied the autoregressive test using a two-state Markov-switching model and tested if the model specification would considerably affect its results. Recently, empirical studies have suggested that Bitcoin entered in a consistent bubble-phase since 2017. Before 2017, Bitcoin prices remained bounded around U$ 1,000, but after this period, prices sharply increased breaking the value of U$ 17,000 in December of 2017, and so far has remained well above the values registered in 2010-2016.

Table 3.5: Autoregression tests based on standardized returns, 2010–2016: 3 Regimes Model

Lag K (weeks)												
4	8	12	16	20	24	28	32	36	40	44	48	52
Posterior distribution of for standardized Bitcoin returns (Mean, Median, SD)												
0.1619	0.0787	0.0391	0.0941	0.2331	0.3311	0.3544	0.3356	0.2846	0.2179	0.1498	0.1069	0.0788
0.1625	0.0799	0.0388	0.0945	0.2353	0.3342	0.3578	0.3384	0.2876	0.2203	0.1513	0.1079	0.0802
0.0413	0.0528	0.0688	0.0833	0.0745	0.0683	0.0715	0.0783	0.0840	0.0891	0.0921	0.0973	0.1023
Sampling distribution. of based on standardized and randomized returns (Mean, Median, SD)												
-0.0156	-0.0328	-0.0485	-0.0692	-0.0886	-0.1091	-0.1295	-0.1500	-0.1684	-0.1852	-0.2075	-0.2381	-0.2571
-0.0156	-0.0334	-0.0494	-0.0707	-0.0893	-0.1155	-0.1354	-0.1577	-0.1808	-0.1979	-0.2198	-0.2525	-0.2779
0.0911	0.1268	0.1535	0.1781	0.1968	0.2150	0.2294	0.2438	0.2575	0.2662	0.2812	0.2907	0.3043
p-values												
0.9635	0.7950	0.6983	0.7973	0.9326	0.9712	0.9754	0.9674	0.9484	0.9181	0.8814	0.8700	0.8493

For the sub-sample period 2010-2016, (see, Table 3.5) we also obtained similar results indicating mean-aversion for the Bitcoin returns. In this case, the slope coefficients for the standardized returns are positive for all lags returns. The lowest p-value for the test of mean-aversion is 0.024 at lag 28 and 0.205 for the highest at lag 8. However, most of lags presented p-value inferior of 10%. For the two-state Markov-switching model, we also obtained similar results to the previous autoregressive test, both for full sample and sub-sample periods (see Tables I and II in the Appendix section)

Despite the extensive research devoted to the Bitcoin market, the number of papers examining the mean-reversion hypotheses is relatively small. For the best of our knowledge, Corbet and Katsiampa (2018) found evidence of asymmetric reverting patterns in the Bitcoin price returns using ANAR models. Differently to our approach, the authors concluded that negative price returns present stronger reverting behaviors compared to positive returns. However, this reverting pattern becomes more symmetrical for lower data frequencies. Further extension of the present paper could incorporate alternative models of mean-reversion, since our results contrast to the existing in the empirical literature. Taking this fact, one readily extension of this work is the implementation of the variance-ratio test using the Markov-switching model as seen in Kim et al. (1998). Another further interesting extension is to analyze mean-reversion of different cryptocurrencies, given the increasing literature of the efficient market hypothesis using different cryptocurrencies (Wei, 2018; Vidal-Tomás et al., 2019; Bouri et al., 2019).

3.4 Conclusion

In this paper, we captured different states in Bitcoin volatility returns using a 3-state Bayesian Markov-switching model, and use the Gibbs-sampling-augmented randomization method in the autoregressive test of mean-reversion for multi-period returns. For the sample period from 2010 to 2019, we found strong evidence of mean-aversion using weekly data. We also confirmed our preliminary results for different models specifications and also carrying the analysis for sub-sample periods.

In general, our results indicate that Bitcoin returns do not display evidence of mean-reverting components either in the medium or long-run. This implies that the they are unpredictable, and their movement could be explained by random innovations, supporting the hypothesis of market efficiency. In this context winning Bitcoin portfolios are expected to outperform losing portfolios in the long-run.

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Appendix

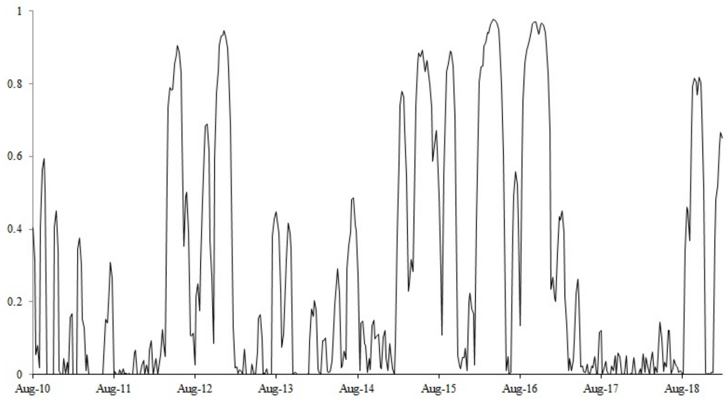
Table.I: Autoregression tests based on standardized returns: 2 Regimes Model

Lag K (weeks)												
4	8	12	16	20	24	28	32	36	40	44	48	52
Posterior distribution of for standardized Bitcoin returns (Mean, Median, SD)												
0.1761	0.1421	0.1043	0.1392	0.2558	0.3072	0.2873	0.1967	0.1096	0.0213	-0.0481	-0.0879	-0.1066
0.1763	0.1436	0.1055	0.1418	0.2591	0.3090	0.2890	0.1985	0.1112	0.0225	-0.0470	-0.0868	-0.1055
0.0334	0.0373	0.0443	0.0491	0.0418	0.0369	0.0393	0.0419	0.0431	0.0439	0.0450	0.0461	0.0469
Sampling distribution. of based on standardized and randomized returns (Mean, Median, SD)												
-0.0122	-0.0249	-0.0348	-0.0486	-0.0624	-0.0759	-0.0899	-0.1086	-0.1232	-0.1352	-0.1573	-0.1677	-0.1888
-0.0114	-0.0242	-0.0353	-0.0492	-0.0641	-0.0786	-0.0931	-0.1113	-0.1292	-0.1414	-0.1639	-0.1807	-0.1994
0.0780	0.1102	0.1348	0.1542	0.1725	0.1871	0.2012	0.2157	0.2258	0.2392	0.2491	0.2596	0.2651
p-values												
0.9866	0.9233	0.8377	0.8760	0.9606	0.9772	0.9645	0.9115	0.8360	0.7377	0.6698	0.6273	0.6314

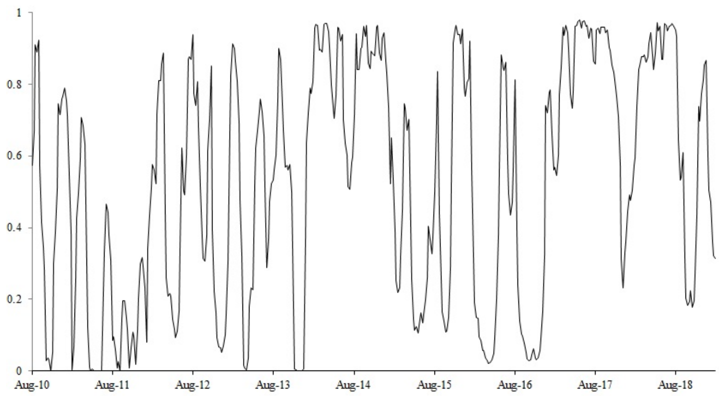
Table.II: Autoregression tests based on standardized returns, 2010–2016: 2 Regimes Model

Lag K (weeks)												
4	8	12	16	20	24	28	32	36	40	44	48	52
Posterior distribution of for standardized Bitcoin returns (Mean, Median, SD)												
0.2178	0.1044	0.0146	0.0663	0.2308	0.3409	0.3588	0.3199	0.2583	0.1864	0.1137	0.0646	0.0323
0.2194	0.1067	0.0162	0.0705	0.2361	0.3448	0.3623	0.3227	0.2605	0.1876	0.1140	0.0643	0.0312
0.0375	0.0444	0.0523	0.0618	0.0547	0.0481	0.0493	0.0472	0.0451	0.0443	0.0444	0.0488	0.0554
Sampling distribution. of based on standardized and randomized returns (Mean, Median, SD)												
-0.0147	-0.0320	-0.0484	-0.0676	-0.0874	-0.1076	-0.1264	-0.1469	-0.1695	-0.1882	-0.2145	-0.2367	-0.2569
-0.0149	-0.0316	-0.0512	-0.0685	-0.0901	-0.1120	-0.1304	-0.1548	-0.1791	-0.2050	-0.2278	-0.2551	-0.2770
0.0904	0.1276	0.1551	0.1776	0.1955	0.2115	0.2312	0.2435	0.2555	0.2699	0.2779	0.2923	0.3032
p-values												
0.9896	0.8426	0.6471	0.7603	0.9389	0.9777	0.9786	0.9647	0.9443	0.9052	0.8689	0.8411	0.8307

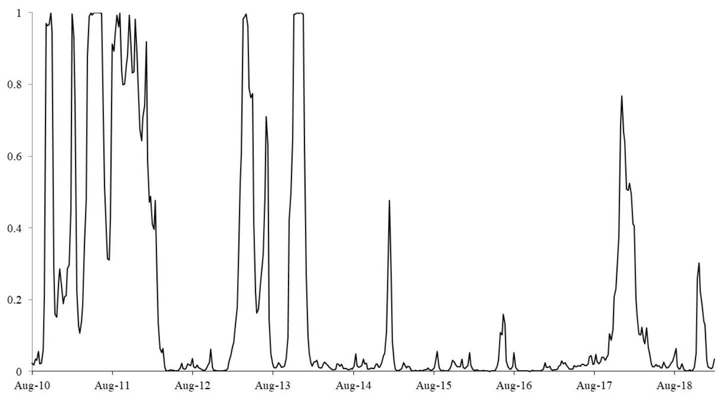
Figure.II: Probabilities for the three-regime model



(a) Probability low-variance state



(b) Probability medium-variance state



(c) Probability high-variance state

Figure.I: Estimated variance of Bitcoin returns: 3 Regimes Model