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Risk parity in the brazilian market

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Abstract

Using sectorial indices of the Brazilian market, we compare the portfolio optimization approach known as risk parity with minimum variance and equally weighted approaches. We apply various estimators for the covariance matrix to each portfolio strategy, since portfolio variance is considered as risk measure. Empirical results demonstrate that the risk parity approach provides more diversified portfolios and stable weights in the out-of-sample than the other two approaches, thereby avoiding the dangers of excessive concentration and reducing transaction costs. Furthermore, the results demonstrate that different estimators of the covariance matrix had little influence on the results obtained through the risk parity approach.

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1. Introduction

The search for efficiency in resource allocation is a key topic in finance and economics. The mean-variance approach, introduced by Markovitz (1952), is still commonly applied, being used as a reference for new approaches in several recent works, such as Fliege and Werner (2014), Kolm et al. (2014), Nguyen and Lo (2012) and Behr et al. (2013). However, this perspective has some limitations, especially regarding estimation errors of the covariance matrix and the expectation of future returns (Muirhead, 1987).

Considering those negative aspects, alternative approaches based on novel covariance matrix structuring methodologies, such as principal components and Bayesian shrinkage, have been studied (Clarke et al., 2006; Engle and Sheppard, 2008; DeMiguel, Garlappi, and Uppal, 2009; Behr et al., 2013; Fan et al., 2012). However, these methodologies present similar out-of-sample performance as more traditional methods (Scherer, 2007).

To minimize the above shortcomings, a novel approach called risk parity has been developed, focusing on allocation of risk rather than allocation of capital (see Maillard et al., 2008). The main idea of this approach is to select assets so that they contribute the same level of risk. Risk parity uses only the estimation of the covariance matrix to obtain the optimal portfolio. It does not seek to minimize volatility, but rather seeks to maximize risk diversification among the assets. The amount invested in each asset and the expected return are not taken into account, but the volatility and correlations are. Thus, both total and marginal contributions of each asset are evaluated. Although the theoretical background for this investment portfolio management approach was developed in the 50's and 60's, it was only after the late 2000 financial crisis that risk parity gained wide interest, given its good performance compared with more traditional strategies (Allen, 2010). Since then, several studies have focused on this approach (Maillard et al., 2008; Schachter and Thiagarajan, 2011; Clarke et al., 2013; Qian, 2011; Roncalli and Weisang, 2016; Bai et al., 2016, among others). An important advantage of using a risk parity approach is that return expectations are not necessary, since setting those expectations is often a point of criticism. Furthermore, historical returns are not put into any relationship to risk or used as an approximation for future returns.

This study has two main objectives as follows: (i) to analyze the risk parity portfolio in the Brazilian market, a context in which risk parity has not been studied, making it possible to investigate whether the conclusions of Maillard et al. (2008), Griveau-Billion (2013) and Lee (2011) for European and American markets remain valid in the Brazilian market; and (ii) to search for the most effective way of estimating the covariance matrix for the risk parity approach, an undertaking that was also not found in the current literature. Most studies on risk parity use a comparative analysis, in which the portfolio obtained is compared with those obtained through the minimum variance and the equally weighted approaches. We use six different estimators to obtain a covariance matrix, from the simplest one, the sample covariance matrix, to more complex models, such as multivariate GARCH (Generalized Autoregressive Conditional Heteroscedasticity) models, which are presented in Bauwens et al. (2006).

Using an experimental framework to exploit risk parity in the Brazilian market, we consider hypothetical investment portfolios in which the sectorial indices of the Brazilian stock market are considered as assets. The obtained results allow the conclusion that the risk parity approach provides more diversified portfolios and stable weights in out-of-sample, thus avoiding the dangers of excessive concentration and reducing transaction costs when compared to minimum variance and equally weighted portfolios. The results point in the

same direction as those obtained by Maillard et al. (2008), Griveau-Billion (2013) and Lee (2011) for European and American markets. The results also show that different estimators of the covariance matrix had little influence on the results obtained through the risk parity approach.

Therefore, this paper provides two major contributions to the empirical finance literature. First, it implements, in a manner unprecedented to the Brazilian market, the portfolio selection approach based on risk parity and contrasts this approach with two approaches widely used in the literature, minimum variance portfolios and equally weighted portfolios. Second, unlike previous studies that have analyzed the risk parity approach in other markets, this study considers different estimators for the covariance matrix, including multivariate GARCH models. The empirical results show that the goal of creating a portfolio in which each asset (asset class) equally contributes to the risk of the portfolio was achieved. As shown by the average turnover calculation, the optimal portfolios obtained by the risk parity approach demonstrated much more stable weight over time than the investment policy of minimum variance did, and this stability contributes to a reduction in transaction costs. Furthermore, the goal of reducing excessive concentration in a few assets was clearly achieved. Additionally, the portfolios obtained by the risk parity approach showed levels of return, volatility and Sharpe ratios better than those of many portfolios obtained by the benchmarking models considered.

The paper is organized as follows. In the next section, a background on risk parity is presented. The experimental design is explained in detail in Section 3. Section 4 presents and discusses the obtained results. Finally, Section 5 presents the study's conclusions, both summarizing the results obtained and recommending further research.

2. Background

The risk parity approach we consider in this paper is based on (Maillard et al., 2008), and involves the calculation of the covariance matrix, represented by Σ . Furthermore, the volatility of the portfolio is given by $\sigma(x) = (x^T \Sigma x)^{\frac{1}{2}}$, where $x = [x_1, x_2, \dots, x_n]^T$ is the vector of asset weights. Therefore, considering that the variance of asset i is σ_i^2 and the covariance between assets i and j is σ_{ij} , it follows that the marginal and total contribution to risk is given by:

$$\frac{\partial \sigma(x)}{\partial x_i} = \frac{x_i \sigma_i^2 + \sum_{j \neq i} x_j \sigma_{ij}}{\sigma(x)} \quad \text{and} \quad \sigma_i(x) = x_i \partial_{x_i} \sigma(x).$$

The quest for equal contributions to the risk of each asset is $\sigma_1(x) = \sigma_2(x) = \dots = \sigma_n(x)$. Therefore, the portfolio obtained with risk parity can be formulated as follows:

$$x^* = \left\{ x \in [0,1]^n: \sum x_i = 1; x_i \partial_{x_i} \sigma(x) = x_j \partial_{x_j} \sigma(x), \forall i, j \right\}$$

Maillard et al. (2008) propose three different strategies: risk parity (RP), equal weights ($1/n$) and minimum variance (mv), with the following properties, respectively: (i) $x_i \partial_{x_i} \sigma(x) = x_j \partial_{x_j} \sigma(x)$; (ii) $x_i = x_j$; and (iii) $\partial_{x_i} \sigma(x) = \partial_{x_j} \sigma(x)$. The minimum variance portfolio only seeks for the marginal contributions to risk to be equal. It is expected that the relationship among the portfolios obtained by these approaches can be represented by $\sigma_{mv} < \sigma_{PR} < \sigma_{1/n}$.

3. Experimental Study

To apply the risk parity approach in the Brazilian market, we conducted extensive experimentation using data from the Brazilian stock market, BM&F-BOVESPA. We consider portfolios created from the seven sectoral indices of BM&F-BOVESPA, as follows: IEE (Electrical Energy Index), INDX (Industrial Index), ICON (Consumption Index), IMOB (Real Estate Index), IFNC (Financial Index), IMAT (Basic Materials Index) and UTIL (Utility Index). This methodology has already been applied by Lee (2011), who uses sectorial indices to analyze the US stock market.

Since the IMOB index only contains historical data from January 2008 onwards, data were collected from January 2008 to July 2014 using Economatica, a recognized Brazilian database. For the optimization approaches, data from the last two years were used for each portfolio created, and the portfolio was rebalanced monthly. As an example, for the first portfolio obtained, data between January 2008 and December 2009 were used, the portfolio was kept the same during the month of January 2010, then it was rebalanced with data between February 2008 and January 2010, and finally it was evaluated in February 2010.

We consider six different estimators for the covariance matrix. The simplest method merely involves use of the sample covariance. The second method is the exponentially weighted moving average (EWMA), with a decay factor equal to 0.94, which is indicated as most appropriate when daily data are used (Bauwens et al., 2006).

The third method is the RiskMetrics™ 2006 model, which is a combination of two exponential decay functions. This strategy is described by Zumbach (2007), and we use the parameters proposed by this author. The RiskMetrics 2006 Covariance is computed as follows:

$$H_t = \sum_{i=1}^N w_i h_{i,t}$$

$$h_{i,t} = (1 - \lambda_i) R_{t-1} R'_{t-1} + \lambda_i h_{i,t-1}$$

$$w_i = \frac{1}{C} \left(1 - \frac{\ln(\tau_i)}{\ln(\tau_0)} \right)$$

$$\lambda_i = e^{\left(-\frac{1}{\tau_i} \right)}$$

$$\tau_i = \tau_1 \rho^{i-1}, \quad i = 1, 2, \dots, N$$

where C is a normalization constant that ensures that $\sum_{i=1}^N w_i = 1$. The 2006 methodology uses a 3-parameter model that includes a logarithmic decay factor, τ_0 (1560), a lower cut-off, τ_1 (4), and an upper cutoff τ_{max} (512), [suggested values in parentheses] as described by Zumbach (2007). One additional parameter, ρ , is required to operationalize the model, and RiskMetrics suggests $\sqrt{2}$.

We also consider two multivariate GARCH methods, VECH and MGARCH (Bauwens et al., 2006). VECH is an estimation method that uses variance targeting, aiming to overcome difficulties encountered in the application of the quasi-maximum likelihood (QML) in the GARCH models. VECH is basically a two-step procedure (Francq et al., 2011). First, the volatility equation becomes a parameter in which the intercept is replaced by the unconditional variance of the returns. Second, the unconditional variance is estimated, and, conditionally to this measure, the remaining parameters are estimated by QML. In this case,

the assumptions for the conditional covariance matrix to be positive definite are simplified. According to Engle and Sheppard (2008), the specification is defined by:

$$H_t = C + \alpha R_t R_t' + \beta H_{t-1},$$

where α and β are scalars, H_t is the conditional variance matrix of the returns, R_t . Assuming stationary covariance with the aim of overcoming the curse of dimensionality and $\bar{H} = E[R_t R_t']$ being an unconditional covariance of the returns, we have:

$$E[H_t] = C + \alpha E[R_t R_t'] + \beta E[H_{t-1}],$$

$$\bar{H} = C + \alpha \bar{H} + \beta \bar{H}$$

$$C = (u' - \alpha - \beta) \bar{H}$$

where C can be concentrated out of the evolution of H_t and replaced with a consistent estimate $(u' - \alpha - \beta) \bar{H}$ where u is a $K \times 1$ vector of ones. The fifth model used in this work is MGARCH, which is also a generalization of the univariate GARCH method. MGARCH has the same parameters as the VEC method.

Finally, the last methodology considered is a non-linear combination of univariate GARCH methods. This method treats the conditional correlation as a constant and it is referred by Bauwens et al. (2006) simply as ‘‘CCC’’ and is defined as follows:

$$H_t = D_t \psi_t D_t,$$

where D_t is a diagonal matrix of the conditional standard deviations and ψ is a correlation matrix. Under certain conditions, this decomposition allows for separate estimation of the volatility and correlation parameters (Engle and Sheppard, 2008). The CCC model can be consistently estimated in two steps (Bollerslev, 1990). The first step specifies univariate GARCH processes for the conditional variance of each asset series, $R_{i,t}$:

$$h_{i,t} = \omega_i + \alpha R_{i,t-1}^2 + \beta h_{i,t-1}.$$

The CCC model assumes that $\psi_t = \bar{\psi}, \forall t$. The constant conditional correlation is estimated

using the standardized residuals $\left(\hat{\epsilon}_{i,t} = \frac{R_{i,t}}{\sqrt{\hat{h}_{i,t}}} \right)$ and the usual correlation estimator:

$$\hat{R}_{ij} = \frac{\sqrt{\sum_{t=1}^T \hat{\epsilon}_{i,t} \hat{\epsilon}_{j,t}}}{\sqrt{\sum_{t=1}^T \hat{\epsilon}_{i,t}^2 \sum_{t=1}^T \hat{\epsilon}_{j,t}^2}}.$$

To compare the performance of different strategies, we explore the results of different metrics: return, volatility, Sharpe Ratio, turnover, weight distribution, allocation concentration and maximum drawdown. The explanation for each is provided in the next section.

4. Computational Results

Figures 1 and 2 illustrate a portfolio obtained during the evaluation period (with data between January 2008 and December 2009), using the sampling and CCC methods, respectively, to estimate the covariance matrix. The trend of the risk parity approach in providing portfolios with well distributed weights is quite clear in both figures.

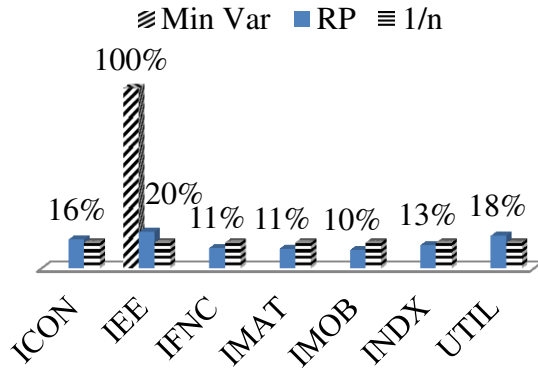


Figure 1. Weight distribution in the portfolios obtained by the sample method estimating the covariance matrix with data between January 2008 and December 2009.

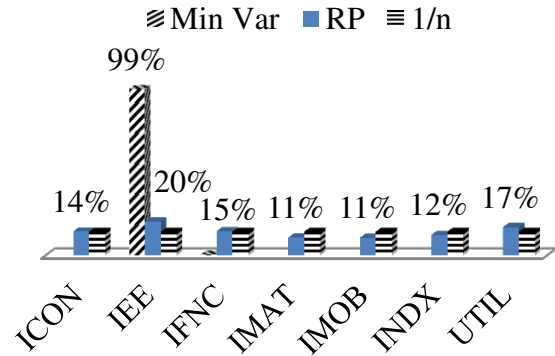


Figure 2. Weight distribution in the portfolios obtained by the CCC method estimating the covariance matrix with data between January 2008 and December 2009.

Table 1 presents the return, volatility, and Sharpe ratio results obtained from the risk parity approach for the entire assessment period, using the six different estimators of the covariance matrix presented in the previous section. The results seem to indicate that for the risk parity approach, the estimator used to obtain the covariance matrix does not make much difference in the three metrics. It is noteworthy that, in this case, the Sharpe ratio was calculated by simply dividing the annual return by the risk, without taking into account the risk of a risk-free asset. Because it was established that the risk parity strategy proved to have low sensitivity to the method of estimating the covariance matrix, the results obtained by sample estimation of the covariance matrix shall be used to compare this approach with the other investment portfolio optimization strategies.

Table 1. Return, volatility and Sharpe ratio of the risk parity approach.

Evaluation Parameters	Sample	EWMA	RiskMetrics™	CCC	VEC	MGARCH
Annual return	4.71%	4.52%	4.54%	4.52%	4.62%	4.40%
Annual risk	16.19%	16.06%	16.08%	16.10%	16.16%	16.18%
Sharpe ratio	0.291	0.282	0.282	0.281	0.286	0.272

We focus now on comparison of the risk parity approach using the sample covariance estimator with the equally weighted ($1/n$) and minimum variance strategies, the latter using the six different estimators for the covariance matrix. Regarding the return, the minimum

variance approach presents a slight advantage with some of the covariance matrix estimators, outperforming the two other approaches considered in some cases. For better visualization, Figure 3 presents the results of the risk parity approach using the sample covariance method, and compares them with those of the equally weighted approach, and the average results obtained with the six different estimators in the minimum variance strategy. There is a considerable mismatch between the minimum variance approach and the others in 2011, but the three approaches' returns graphs appear to behave similarly in 2012.

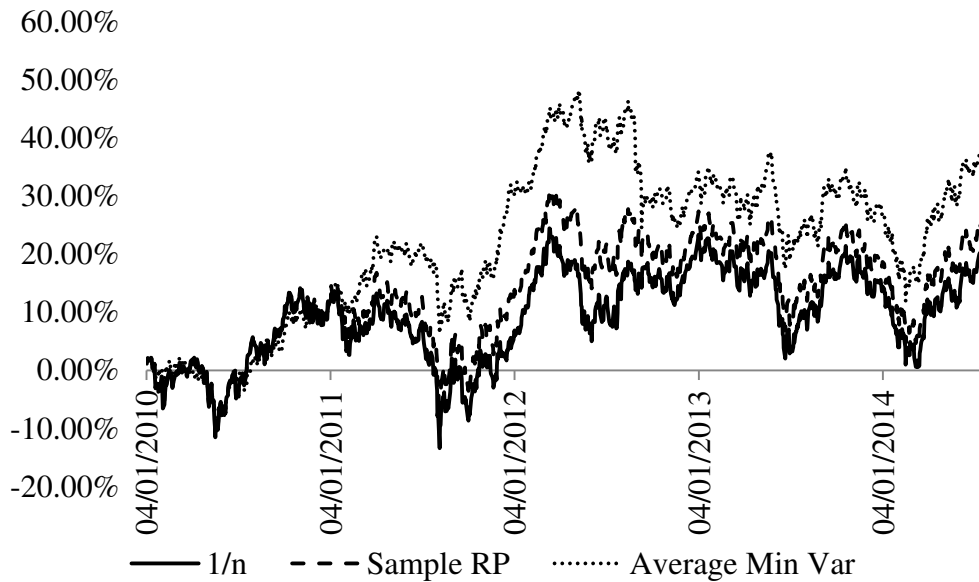


Figure 3. Return of the approaches studied for the whole period.

Comparative analysis was also conducted for turnover, volatility, Sharpe ratio, maximum drawdown and concentration. All the results consider transaction costs that may be relevant to differences between risk parity and other approaches (Anderson et al., 2012). A variable cost of 0.06% was defined as suggested by French (2008). In order to present the differences in transaction costs among the different approaches, the average turnover, which represents the percentage of the portfolio that was rebalanced after each evaluation period, was calculated (see Table 2).

Table 2. Average turnover of the equally weighted, RP and Min Var. portfolios

Estimator	Average Turnover		
	1/n	RP	Min Var
Sample	1.35%	1.40%	2.02%
EWMA	1.35%	5.47%	25.59%
RiskMetrics™ 2006	1.35%	4.61%	21.12%
CCC	1.35%	5.03%	22.70%
VEC	1.35%	2.29%	9.00%
MGARCH	1.35%	2.51%	10.63%

It is clear in Table 2 that the minimum variance approach has higher average turnover than the other approaches. Furthermore, the sample estimation method provides lower

turnover than the other methods, since more recent data are not given greater weight when this estimator is used. After the application of the transaction costs resulting from the turnovers in each rebalance, the annual returns for each strategy are obtained and shown in Table 3. It should be noted that the results for risk parity presented in Tables 3 to 6 reflect those obtained using the sample covariance matrix. The same results were observed with the other covariance estimation methods, so they were omitted in order to give clarity to the relevant results. Table 3 shows that the minimum variance approach obtained superior return performance in 2011 and early 2014. On other hand, the equal weight and RP approaches achieved higher absolute returns in 2012.

Table 3. Average return comparison.

Annual return	1/n	RP	Sample	EWMA	Minimum Variance			
					RiskMetrics™ 2006	CCC	VEC	MGARCH
2010	11.0%	12.2%	11.9%	9.0%	10.4%	10.6%	12.4%	12.2%
2011	-5.9%	-0.3%	18.4%	15.7%	17.6%	18.9%	17.9%	17.7%
2012	15.0%	11.2%	-4.3%	5.6%	5.8%	0.9%	-0.0%	-4.2%
2013	-3.9%	-3.7%	-4.9%	-0.9%	-1.6%	-2.7%	-4.4%	-5.2%
2014	3.5%	5.2%	12.1%	0.8%	2.9%	10.7%	10.1%	9.2%

Table 4 shows the strategies' annual volatilities. The risk parity approach shows an intermediate level of volatility, between the volatility levels of the other two approaches, in all cases. This result is observed regardless of the estimation method of the covariance matrix used in the minimum variance approach. Thus, in all cases the results for risk parity are consistent with those found by Maillard et al. (2008).

Table 4. Volatility comparison.

Annual risk	1/n	RP	Sample	EWMA	Minimum variance			
					RiskMetrics™ 2006	CCC	VEC	MGARCH
2010	16.9%	15.5%	12.3%	12.50%	12.3%	12.2%	12.2%	12.2%
2011	20.3%	18.6%	14.2%	14.4%	14.4%	14.1%	14.1%	14.1%
2012	15.6%	14.6%	14.5%	13.4%	13.0%	12.7%	14.2%	14.5%
2013	15.1%	14.8%	14.0%	13.5%	13.3%	13.3%	13.7%	14.1%
2014	16.9%	16.7%	15.4%	15.8%	15.5%	16.1%	15.3%	15.7%

Table 5 presents the Sharpe ratios for the evaluated approaches. The Sharpe ratio was calculated solely by observing the relationship between return and volatility, without considering a risk-free asset. This methodology is used by De Miguel et al. (2014), whose intention is solely to compare different approaches. On this metric, the minimum variance strategy again significantly outperformed the other strategies in 2011. Moreover, in this strategy again underperformed other strategies in 2012. In the analysis of the year 2014: note that even with higher returns, such yields are only achieved with higher volatility, resulting in a Sharpe ratio without the large differences observed in Table 3.

Table 5. Sharpe ratio comparison.

Sharpe	1/n	RP	Sample	EWMA	Minimum Variance	CCC	VEC	MGARCH
					RiskMetrics™ 2006			
2010	0.65	0.79	0.97	0.72	0.84	0.87	1.02	1.00
2011	-0.29	-0.02	1.29	1.09	1.23	1.33	1.26	1.25
2012	0.96	0.77	-0.30	0.42	0.44	0.07	0.00	-0.29
2013	-0.26	-0.25	-0.35	-0.06	-0.12	-0.20	-0.32	-0.37
2014	0.21	0.31	0.78	0.05	0.19	0.66	0.66	0.59

Another important tool for assessing the portfolio optimization approaches is the maximum drawdown, which is presented by Chekhlov et al. (2005). This indicator is a reference regarding the maximum loss observed in a certain evaluation period. Table 6 shows the results of maximum drawdown for each year studied and also for the whole period (between 2010 and 2014).

Table 6. Annual maximum drawdown comparison.

	2010	2011	2012	2013	2014	2010- 2014
1/n	-13.4%	-23.8%	-15.6%	-17.3%	-12.2%	-24.1%
Risk Parity	-12.6%	-21.5%	-13.1%	-16.1%	-11.8%	-21.5%
Sample	-11.7%	-13.9%	-20.4%	-15.0%	-11.8%	-29.3%
EWMA	-12.3%	-14.7%	-16.0%	-14.5%	-12.1%	-18.9%
Min. RiskMetrics™ 2006	-12.0%	-13.8%	-14.9%	-14.3%	-11.8%	-19.2%
Var. VEC	-11.9%	-13.5%	-17.7%	-14.7%	-11.8%	-25.9%
CCC	-11.7%	-12.3%	-16.5%	-14.0%	-12.1%	-24.0%
MGARCH	-11.7%	-13.4%	-20.6%	-14.7%	-12.7%	-29.8%

Table 6 reflects mixed results in the maximum drawdown among the different approaches. The risk parity strategy showed lower maximum drawdowns than the equal weight strategy did. Between the risk parity and minimum variance approaches, the results are similar, and the maximum drawdown is greater for risk parity only in 2011.

A positive aspect of the risk parity approach described in the literature (Maillard et al., 2008) is its low tendency to concentrate most of the resources in only one or a few assets, which reflects its great advantage of providing protection against concentration risk, or the risk of sudden high volatility from any asset. Otherwise, if a great percentage of the resources were allocated to just one asset, any sharp drop in that asset's value in a short time period will negatively affect the portfolio. In order to verify whether this result would be reproduced with the Brazilian market data, the trend of risk concentration was evaluated in a comparative manner. As the equal weights strategy cannot have weight concentration in one asset, the comparison in this case is performed between the risk parity and minimum variance strategies. The first step is to evaluate the portfolios as a whole; then, the standard deviation of the weights of each portfolio was calculated, making it possible to see how diverse each portfolio is. Such data are presented in Figure 4, which shows clearly that the risk parity

approach is again an intermediate strategy, between the equal weight and minimum variance strategies, in terms of diversification.

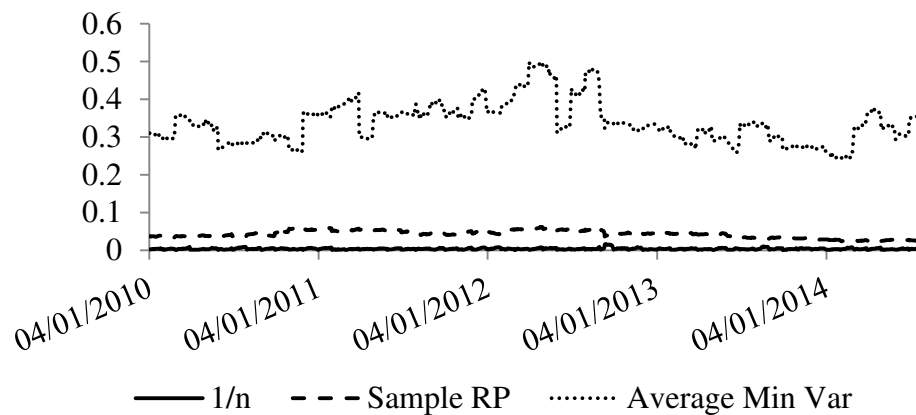


Figure 4. Concentration of resources (standard deviation of the weights).

As the standard deviation of the weights is equal to zero when the portfolio is well diversified in weights (all weights are equal), the results show a clear difference in diversification level between the risk parity and minimum variance approaches. Another possible method to examine the level of concentration of resources is to analyze the number of times the most heavily weighted asset in the portfolio holds a certain percentage of resources available. Figure 5 shows the percentage of times a certain level of concentration was found in the portfolios of both approaches.

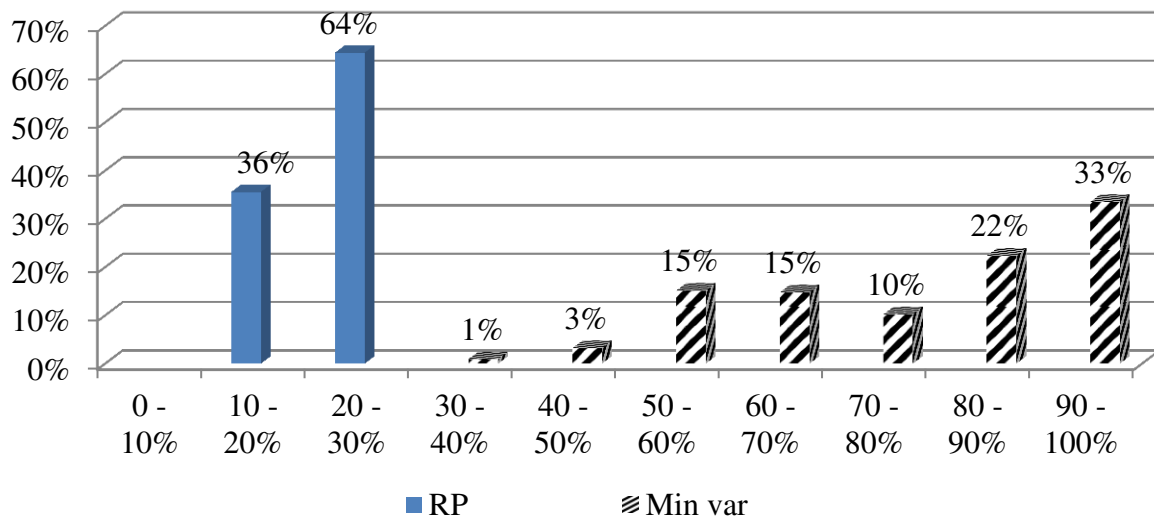


Figure 5. Level of concentration of the most present asset weight.

On one hand, 33% of the portfolios formed from the minimum variance approach had between 90% and 100% of its resources concentrated in a single asset. On the other hand, 64% of the portfolios formed from the risk parity approach had between 20% and 30% of its resources concentrated in only one asset. These results assure that the RP approach performs better than the minimum variance approach in terms of concentration risk.

5. Conclusions

First, it is worth highlighting that this research is the first study on risk parity applied to the Brazilian market, and it uses as assets a specific group of indices. Thus, the results obtained, especially in relation to the absolute return from the portfolios formed, cannot be considered as an absolute truth pertaining to the entire Brazilian stock market. However, certain findings were consistent with the results based on other markets, which is extremely positive and opens doors to new research.

In this work, we examine whether the positive aspects of the risk parity approach, as described by authors who have studied this approach in the context of the American and European markets, are valid in Brazil. Researchers of this portfolio optimization strategy argue that its benefits include a trend toward portfolio diversification, an intermediate volatility between the volatilities of the minimum variance and equal weights approaches, and stability in the portfolio rebalance. These points were affirmed in this study, as during the entire study period, the portfolio formed from the risk parity strategy generated an intermediate level of volatility, as well as average turnover results that were lower than those from the minimum variance approach. These are the reasons why implementation of the analyses of the European and US markets can be assumed, at least in part, in the Brazilian financial context. In addition to generating an intermediate level of volatility, the portfolio formed from risk parity also generated an intermediate level of diversification in asset weights, reducing concentration risks usually observed in portfolios formed from the minimum variance approach.

The use of different estimators for the covariance matrix had little influence on risk parity results, but this was not true for the minimum variance results, as already reported in the literature (Moreno et al., 2005; Francq et al., 2011). This result is an initial insight that can spur further research on risk parity. For instance, a larger number of assets with different estimators for the covariance matrix could be explored.

Clearly, more studies on the subject should be conducted, filling gaps that still remain on the subject by searching mainly for embodiments of choice on which assets should be included in the risk parity portfolio so that this strategy can be as efficient as possible. Other risk measures (Value at Risk, Expected Shortfall) and covariance estimators (e.g., Dynamic Conditional Correlation GARCH and Copula methods) can also be incorporated into the analysis (Chen et al., 2011).

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