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Transition from thermal to turbulent equilibrium with a resulting electromagnetic spectrum

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A recent paper [Ziebell *et al.*, Phys. Plasmas **21**, 010701 (2014)] discusses a new type of radiation emission process for plasmas in a state of quasi-equilibrium between the particles and enhanced Langmuir turbulence. Such a system may be an example of the so-called "turbulent quasi-equilibrium." In the present paper, it is shown on the basis of electromagnetic weak turbulence theory that an initial thermal equilibrium state (i.e., only electrostatic fluctuations and Maxwellian particle distributions) transitions toward the turbulent quasi-equilibrium state with enhanced electromagnetic radiation spectrum, thus demonstrating that the turbulent quasi-equilibrium discussed in the above paper correctly describes the weakly turbulent plasma dynamically interacting with electromagnetic fluctuations, while maintaining a dynamical steady-state in the average sense. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4863453]

I. INTRODUCTION

Nonlinear processes associated with the beam-plasma instability are widely regarded as being responsible for the emission of electromagnetic (EM) waves in a plasma, according to a model proposed in the decade of 1950s.[1] In this paradigm, electrons in a beam excite Langmuir (L) waves via the bump-on-tail instability, followed by nonlinear coupling processes of wave decay and scattering, which include the generation of backward-propagating L waves and nonlinear interactions between L waves and ion-sound (S) waves. The eventual generation of EM radiation results from partial conversion of electrostatic wave energy into transverse (T) radiation. Such an elaborate process intimately associated with the existence of electron beams is known as *plasma emission*, and has become the bedrock based upon which standard interpretation of bursty radio emission phenomenon, such as the solar type II and type III emissions, is customarily made.[2–4]

In a field-free and collisionless plasma, plasma emission is the only known radio emission mechanism that can generate radio-frequency EM waves. However, when electron-ion collisions are present in the plasma, classical Bremsstrahlung (or free-free) emission is also operative. In magnetized plasmas, gyro-synchrotron radiation and the electron cyclotron maser mechanisms may, under various conditions, also be at work.[5] Čerenkov emission in plasmas only involves longitudinal modes. As such, it does not lead to direct radiation emission.

In a recently published paper,[6] the present authors discussed a novel radiation mechanism in unmagnetized plasmas containing a thermal population of electrons only. In the proposed scenario of Ref. 6, the EM wave spectrum results from nonlinear interactions involving L and S wave spectra and plasma particles, with the L and S spectra being in equilibrium with the particles.

The system of particles and fluctuations described in Ref. 6 can be described as a turbulent quasi-equilibrium state. Treumann[7,8] first discussed the notion of turbulent equilibrium in plasmas, which involves plasma particles constantly exchanging momentum and energy with enhanced EM fluctuations, but on average the system is in dynamical equilibrium. Later work by Treumann and Jaroschek[9] showed that non-Maxwellian power-law velocity distributions may be expected to occur in collisionless plasmas in a quasi-stationary turbulent state. More recently, the first concrete solution of the turbulent quasi-equilibrium was obtained in Refs. 10–12, where electron kappa distribution function and the concomitant Langmuir turbulence spectrum were shown to form a set of self-consistent asymptotically steady-state solutions of the electrostatic weak turbulence equation.

In the present paper, we extend the analysis of Refs. 10–12 to the fully electromagnetic formalism. The main focus is on the transition from a thermal to the turbulent equilibrium spectrum of EM waves described in Ref. 6. The time evolution of wave and particle spectra is followed, starting from a thermal equilibrium (Maxwell-Boltzmann) electron distribution and the spontaneously generated L and S spectral equilibria. It is shown that the final state of EM radiation corresponds to the turbulent equilibrium discussed in Ref. 6.

The essential nonlinear processes are the same as in the standard plasma emission. They are described by weak EM plasma turbulence theory as found in the standard literature.[13] The complete set of equations was recently reviewed and extended in Ref. 14, describing the time evolution and interactions among L, S, and transverse (*T*) waves, as well as among plasma particles. Here, we solve the full set of EM weak turbulence equations, and verify that an initial null spectrum of T waves evolves toward the asymptotic solution

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discussed in Ref. 6, thus demonstrating that once we include weakly turbulent nonlinear processes into account, the isotropic distribution of electrons spontaneously emits a spectrum of EM waves.

The structure of the paper is as follows: In Sec. II, we briefly describe the theoretical formulation and the setup for the numerical analysis. Section III presents the results of numerical analysis. In Sec. IV, we discuss the physical mechanism responsible for the emission of radiation. Finally, we summarize the findings of the present paper in Sec. V.

II. THEORETICAL FORMULATION AND NUMERICAL SETUP

For the purpose of the present paper, we start from a general self-consistent set of equations that comprise the electromagnetic weak turbulence theory, as they have appeared in Ref. 14. In the Appendix, we reproduce the entire set of equations in un-normalized form, but for the sake of convenience here we present only the non-dimensional forms, which are more suitable for numerical analysis. The equation for L wave is as follows:

$$
\begin{aligned}
\frac{\partial \mathcal{E}^{\sigma L}_{\mathbf{q}}}{\partial \tau} &= \left\{ \mu^L_{\mathbf{q}} \frac{\pi}{q^2} \int d\mathbf{u}\, \delta(\sigma z^L_{\mathbf{q}} - \mathbf{q}\cdot\mathbf{u}) \left[g\, \Phi_e(\mathbf{u}) + (\sigma z^L_{\mathbf{q}})\, \mathbf{q}\cdot \frac{\partial \Phi_e(\mathbf{u})}{\partial \mathbf{u}}\, \mathcal{E}^{\sigma L}_{\mathbf{q}} \right] \right\}_{Lql} + \left\{ 2\sigma \mu^L_{\mathbf{q}} z^L_{\mathbf{q}} \sum_{\sigma',\sigma''=\pm 1} \int d\mathbf{q}'\, \frac{\mu^L_{\mathbf{q}'}\, \mu^S_{\mathbf{q}-\mathbf{q}'}\, (\mathbf{q}\cdot\mathbf{q}')^2}{q^2\, q'^2\, |\mathbf{q}-\mathbf{q}'|^2} \right.\\
&\quad \times \left[\sigma z^L_{\mathbf{q}}\, \mathcal{E}^{\sigma' L}_{\mathbf{q}'} \mathcal{E}^{\sigma'' S}_{\mathbf{q}-\mathbf{q}'} - \left(\sigma' z^L_{\mathbf{q}'}\, \mathcal{E}^{\sigma'' S}_{\mathbf{q}-\mathbf{q}'} + \sigma'' z^L_{\mathbf{q}-\mathbf{q}'}\, \mathcal{E}^{\sigma' L}_{\mathbf{q}'} \right) \mathcal{E}^{\sigma L}_{\mathbf{q}} \right] \delta(\sigma z^L_{\mathbf{q}} - \sigma' z^L_{\mathbf{q}'} - \sigma'' z^S_{\mathbf{q}-\mathbf{q}'}) \Big\}_{LdLS}\\
&\quad + \left\{ \frac{\sigma \mu^L_{\mathbf{q}} z^L_{\mathbf{q}}}{16} \sum_{\sigma',\sigma''} \int d\mathbf{q}'\, \frac{\mu^L_{\mathbf{q}'}\, \mu^T_{\mathbf{q}-\mathbf{q}'}\, (\mathbf{q}\times\mathbf{q}')^2}{q^2\, q'^2\, |\mathbf{q}-\mathbf{q}'|^2} \left(\frac{q'^2}{\sigma' z^L_{\mathbf{q}'}} + \frac{q^2}{\sigma z^L_{\mathbf{q}}} \right)^2 \left[\sigma z^L_{\mathbf{q}}\, \mathcal{E}^{\sigma'' T}_{\mathbf{q}-\mathbf{q}'} \mathcal{E}^{\sigma' L}_{\mathbf{q}'} - \left(\sigma' z^L_{\mathbf{q}'}\, \mathcal{E}^{\sigma'' T}_{\mathbf{q}-\mathbf{q}'} + \sigma'' 2 z^T_{\mathbf{q}-\mathbf{q}'}\, \mathcal{E}^{\sigma' L}_{\mathbf{q}'} \right) \mathcal{E}^{\sigma L}_{\mathbf{q}} \right] \right.\\
&\quad \times\, \delta\left(\sigma z^L_{\mathbf{q}} - \sigma' z^L_{\mathbf{q}'} - \sigma'' z^T_{\mathbf{q}-\mathbf{q}'} \right) \Big\}_{LdLT} + \left\{ \sigma \mu^L_{\mathbf{q}} z^L_{\mathbf{q}} \sum_{\sigma',\sigma''} \int d\mathbf{q}'\, \frac{\mu^S_{\mathbf{q}'}\, \mu^T_{\mathbf{q}-\mathbf{q}'}\, (\mathbf{q}\times\mathbf{q}')^2}{q^2\, q'^2\, |\mathbf{q}-\mathbf{q}'|^2} \right.\\
&\quad \times \left[\sigma z^L_{\mathbf{q}}\, \mathcal{E}^{\sigma'' T}_{\mathbf{q}-\mathbf{q}'} \mathcal{E}^{\sigma' S}_{\mathbf{q}'} - \left(\sigma' z^L_{\mathbf{q}'}\, \mathcal{E}^{\sigma'' T}_{\mathbf{q}-\mathbf{q}'} + \sigma'' 2 z^T_{\mathbf{q}-\mathbf{q}'}\, \mathcal{E}^{\sigma' S}_{\mathbf{q}'} \right) \mathcal{E}^{\sigma L}_{\mathbf{q}} \right] \delta\left(\sigma z^L_{\mathbf{q}} - \sigma' z^S_{\mathbf{q}'} - \sigma'' z^T_{\mathbf{q}-\mathbf{q}'} \right) \Big\}_{LdST}\\
&\quad + \left\{ \frac{\sigma z^L_{\mathbf{q}}}{16} \sum_{\sigma',\sigma''} \int d\mathbf{q}'\, \frac{q^2}{(z^T_{\mathbf{q}'})^2\, (z^T_{\mathbf{q}-\mathbf{q}'})^2} \left(1 + \frac{[\mathbf{q}'\cdot(\mathbf{q}-\mathbf{q}')]^2}{q^2\, |\mathbf{q}-\mathbf{q}'|^2} \right) \left[\sigma z^L_{\mathbf{q}}\, \mathcal{E}^{\sigma' T}_{\mathbf{q}'} \mathcal{E}^{\sigma'' T}_{\mathbf{q}-\mathbf{q}'} - \left(\sigma' 2 z^T_{\mathbf{q}'}\, \mathcal{E}^{\sigma'' T}_{\mathbf{q}-\mathbf{q}'} + \sigma'' 2 z^T_{\mathbf{q}-\mathbf{q}'}\, \mathcal{E}^{\sigma' T}_{\mathbf{q}'} \right) \mathcal{E}^{\sigma L}_{\mathbf{q}} \right] \right.\\
&\quad \times\, \delta\left(\sigma z^L_{\mathbf{q}} - \sigma' z^T_{\mathbf{q}'} - \sigma'' z^T_{\mathbf{q}-\mathbf{q}'} \right) \Big\}_{LdTT} + \left\{ \sigma z^L_{\mathbf{q}} \sum_{\sigma'} \int d\mathbf{q}' \int d\mathbf{u}\, \frac{\mu^L_{\mathbf{q}}\, \mu^L_{\mathbf{q}'}\, (\mathbf{q}\cdot\mathbf{q}')^2}{q^2\, q'^2}\, \delta\left[\sigma z^L_{\mathbf{q}} - \sigma' z^L_{\mathbf{q}'} - (\mathbf{q}-\mathbf{q}')\cdot\mathbf{u} \right] \right.\\
&\quad \times \left[g\left(\sigma z^L_{\mathbf{q}}\, \mathcal{E}^{\sigma' L}_{\mathbf{q}'} - \sigma' z^L_{\mathbf{q}'}\, \mathcal{E}^{\sigma L}_{\mathbf{q}} \right) [\Phi_e(\mathbf{u}) + \Phi_i(\mathbf{u})] + \frac{m_e}{m_i}\, \mathcal{E}^{\sigma' L}_{\mathbf{q}'} \mathcal{E}^{\sigma L}_{\mathbf{q}}\, (\mathbf{q}-\mathbf{q}')\cdot \frac{\partial \Phi_i(\mathbf{u})}{\partial \mathbf{u}} \right] \Big\}_{LsLL}\\
&\quad + \left\{ \sigma z^L_{\mathbf{q}} \sum_{\sigma'} \int d\mathbf{q}' \int d\mathbf{u}\, \frac{\mu^L_{\mathbf{q}}\, \mu^T_{\mathbf{q}'}\, (\mathbf{q}\times\mathbf{q}')^2}{q^2\, q'^2}\, \delta[\sigma z^L_{\mathbf{q}} - \sigma' z^T_{\mathbf{q}'} - (\mathbf{q}-\mathbf{q}')\cdot\mathbf{u}] \left[g\left(\sigma z^L_{\mathbf{q}}\, \frac{\mathcal{E}^{\sigma' T}_{\mathbf{q}'}}{2} - \sigma' z^T_{\mathbf{q}'} \mathcal{E}^{\sigma L}_{\mathbf{q}} \right) [\Phi_e(\mathbf{u}) + \Phi_i(\mathbf{u})] \right. \right.\\
&\quad + \left. \left. \frac{m_e}{m_i}\, \frac{\mathcal{E}^{\sigma' T}_{\mathbf{q}'}}{2}\, \mathcal{E}^{\sigma L}_{\mathbf{q}}\, (\mathbf{q}-\mathbf{q}')\cdot \frac{\partial \Phi_i(\mathbf{u})}{\partial \mathbf{u}} \right] \right\}_{LsLT}. \qquad (1)
\end{aligned}
$$

For S mode, the dynamical equation is the following:

$$
\begin{aligned}
\frac{\partial \mathcal{E}^{\sigma S}_{\mathbf{q}}}{\partial \tau} &= \left\{ \mu^S_{\mathbf{q}} \frac{\pi}{q^2} \int d\mathbf{u}\, \delta(\sigma z^S_{\mathbf{q}} - \mathbf{q}\cdot\mathbf{u}) \left[g[\Phi_e(\mathbf{u}) + \Phi_i(\mathbf{u})] + (\sigma z^L_{\mathbf{q}}) \left(\mathbf{q}\cdot \frac{\partial \Phi_e(\mathbf{u})}{\partial \mathbf{u}} + \frac{m_e}{m_i}\, \mathbf{q}\cdot \frac{\partial \Phi_i(\mathbf{u})}{\partial \mathbf{u}} \right) \mathcal{E}^{\sigma S}_{\mathbf{q}} \right] \right\}_{Sql}\\
&\quad + \left\{ \sigma z^L_{\mathbf{q}} \sum_{\sigma',\sigma''} \int d\mathbf{q}'\, \frac{\mu^S_{\mathbf{q}}\, \mu^L_{\mathbf{q}'}\, \mu^L_{\mathbf{q}-\mathbf{q}'}\, [\mathbf{q}'\cdot(\mathbf{q}-\mathbf{q}')]^2}{q^2\, q'^2\, |\mathbf{q}-\mathbf{q}'|^2} \left[\sigma z^L_{\mathbf{q}}\, \mathcal{E}^{\sigma' L}_{\mathbf{q}'} \mathcal{E}^{\sigma'' L}_{\mathbf{q}-\mathbf{q}'} - \left(\sigma' z^L_{\mathbf{q}'}\, \mathcal{E}^{\sigma'' L}_{\mathbf{q}-\mathbf{q}'} + \sigma'' z^L_{\mathbf{q}-\mathbf{q}'}\, \mathcal{E}^{\sigma' L}_{\mathbf{q}'} \right) \mathcal{E}^{\sigma S}_{\mathbf{q}} \right] \right.\\
&\quad \times\, \delta\left(\sigma z^S_{\mathbf{q}} - \sigma' z^L_{\mathbf{q}'} - \sigma'' z^L_{\mathbf{q}-\mathbf{q}'} \right) \Big\}_{SdLL} + \left\{ \sigma z^L_{\mathbf{q}} \sum_{\sigma',\sigma''} \int d\mathbf{q}'\, \frac{\mu^S_{\mathbf{q}}\, \mu^L_{\mathbf{q}'}\, \mu^T_{\mathbf{q}-\mathbf{q}'}\, (\mathbf{q}\times\mathbf{q}')^2}{q^2\, q'^2\, |\mathbf{q}-\mathbf{q}'|^2} \right.\\
&\quad \times \left[\sigma z^L_{\mathbf{q}}\, \mathcal{E}^{\sigma'' T}_{\mathbf{q}-\mathbf{q}'} \mathcal{E}^{\sigma' L}_{\mathbf{q}'} - \left(\sigma' z^L_{\mathbf{q}'}\, \mathcal{E}^{\sigma'' T}_{\mathbf{q}-\mathbf{q}'} + \sigma'' 2 z^T_{\mathbf{q}-\mathbf{q}'}\, \mathcal{E}^{\sigma' L}_{\mathbf{q}'} \right) \mathcal{E}^{\sigma S}_{\mathbf{q}} \right] \delta\left(\sigma z^S_{\mathbf{q}} - \sigma' z^L_{\mathbf{q}'} - \sigma'' z^T_{\mathbf{q}-\mathbf{q}'} \right) \Big\}_{SdLT} \qquad (2)
\end{aligned}
$$

and for T mode, it is as follows:

$$\frac{\partial \mathcal{E}_{\mathbf{q}}^{\sigma T}}{\partial \tau} = \Bigg\{ \frac{\sigma \mu_{\mathbf{q}}^T z_{\mathbf{q}}^T}{32} \sum_{\sigma',\sigma''} \int d\mathbf{q}' \, \frac{\mu_{\mathbf{q}'}^L \mu_{\mathbf{q}-\mathbf{q}'}^L (\mathbf{q}\times\mathbf{q}')^2}{q^2 q'^2 |\mathbf{q}-\mathbf{q}'|^2} \left(\frac{q'^2}{\sigma' z_{\mathbf{q}'}^L} - \frac{|\mathbf{q}-\mathbf{q}'|^2}{\sigma'' z_{\mathbf{q}-\mathbf{q}'}^L} \right)^2 \Big[\sigma 2 z_{\mathbf{q}}^T \mathcal{E}_{\mathbf{q}'}^{\sigma' L} \mathcal{E}_{\mathbf{q}-\mathbf{q}'}^{\sigma'' L} - \Big(\sigma' z_{\mathbf{q}'}^L \mathcal{E}_{\mathbf{q}-\mathbf{q}'}^{\sigma'' L} + \sigma'' z_{\mathbf{q}-\mathbf{q}'}^L \mathcal{E}_{\mathbf{q}'}^{\sigma' L} \Big) \mathcal{E}_{\mathbf{q}}^{\sigma T} \Big]$$

$$\times \, \delta\Big(\sigma z_{\mathbf{q}}^T - \sigma' z_{\mathbf{q}'}^L - \sigma'' z_{\mathbf{q}-\mathbf{q}'}^L \Big) \Bigg\}_{TdLL} + \Bigg\{ \sigma \mu_{\mathbf{q}}^T z_{\mathbf{q}}^T \sum_{\sigma',\sigma''} \int d\mathbf{q}' \, \frac{\mu_{\mathbf{q}'}^L \mu_{\mathbf{q}-\mathbf{q}'}^S (\mathbf{q}\times\mathbf{q}')^2}{q^2 q'^2 |\mathbf{q}-\mathbf{q}'|^2}$$

$$\times \Big[\sigma 2 z_{\mathbf{q}}^T \mathcal{E}_{\mathbf{q}'}^{\sigma' L} \mathcal{E}_{\mathbf{q}-\mathbf{q}'}^{\sigma'' S} - \Big(\sigma' z_{\mathbf{q}'}^L \mathcal{E}_{\mathbf{q}-\mathbf{q}'}^{\sigma'' S} + \sigma'' z_{\mathbf{q}-\mathbf{q}'}^L \mathcal{E}_{\mathbf{q}'}^{\sigma' L} \Big) \mathcal{E}_{\mathbf{q}}^{\sigma T} \Big] \, \delta\Big(\sigma z_{\mathbf{q}}^T - \sigma' z_{\mathbf{q}'}^L - \sigma'' z_{\mathbf{q}-\mathbf{q}'}^S \Big) \Bigg\}_{TdLS}$$

$$+ \Bigg\{ \frac{\sigma z_{\mathbf{q}}^T}{8} \sum_{\sigma',\sigma''} \int d\mathbf{q}' \, \frac{|\mathbf{q}-\mathbf{q}'|^2}{(z_{\mathbf{q}}^T)^2 (z_{\mathbf{q}'}^T)^2} \left(1 + \frac{(\mathbf{q}\cdot\mathbf{q}')^2}{q^2 q'^2} \right) \Big[\sigma 2 z_{\mathbf{q}}^T \mathcal{E}_{\mathbf{q}'}^{\sigma' T} \mathcal{E}_{\mathbf{q}-\mathbf{q}'}^{\sigma'' L}$$

$$- \Big(\sigma' 2 z_{\mathbf{q}'}^T \mathcal{E}_{\mathbf{q}-\mathbf{q}'}^{\sigma'' L} + \sigma'' z_{\mathbf{q}-\mathbf{q}'}^L \mathcal{E}_{\mathbf{q}'}^{\sigma' T} \Big) \mathcal{E}_{\mathbf{q}}^{\sigma T} \Big] \, \delta\Big(\sigma z_{\mathbf{q}}^T - \sigma' z_{\mathbf{q}'}^T - \sigma'' z_{\mathbf{q}-\mathbf{q}'}^L \Big) \Bigg\}_{TdTL}$$

$$+ \Bigg\{ \sigma z_{\mathbf{q}}^T \sum_{\sigma'} \int d\mathbf{q}' \int d\mathbf{u} \, \frac{\mu_{\mathbf{q}}^T \mu_{\mathbf{q}'}^L (\mathbf{q}\times\mathbf{q}')^2}{q^2 q'^2} \, \delta[\sigma z_{\mathbf{q}}^T - \sigma' z_{\mathbf{q}'}^L - (\mathbf{q}-\mathbf{q}')\cdot\mathbf{u}] \Bigg[g \Bigg(\sigma z_{\mathbf{q}}^T \mathcal{E}_{\mathbf{q}'}^{\sigma' L} - \sigma' z_{\mathbf{q}'}^L \frac{\mathcal{E}_{\mathbf{q}}^{\sigma T}}{2} \Bigg) [\Phi_e(\mathbf{u}) + \Phi_i(\mathbf{u})]$$

$$+ \frac{m_e}{m_i} \mathcal{E}_{\mathbf{q}'}^{\sigma' L} \frac{\mathcal{E}_{\mathbf{q}}^{\sigma T}}{2} (\mathbf{q}-\mathbf{q}') \cdot \frac{\partial \Phi_i(\mathbf{u})}{\partial \mathbf{u}} \Bigg] \Bigg\}_{TsTL}. \tag{3}$$

The above set of equations for the three basic normal modes of an unmagnetized plasma, namely, L, S, and T modes, are to be solved together with the dynamical equations for the particle distribution functions,

$$\frac{\partial \Phi_a(\mathbf{u})}{\partial \tau} = \frac{e_a^2}{e^2} \frac{m_e^2}{m_a^2} \sum_{\sigma} \sum_{\alpha=L,S} \int d\mathbf{q} \left(\frac{\mathbf{q}}{q} \cdot \frac{\partial}{\partial \mathbf{u}} \right) \times \mu_{\mathbf{q}}^{\alpha} \, \delta(\sigma z_{\mathbf{q}}^{\alpha} - \mathbf{q}\cdot\mathbf{u}) \left(g \frac{m_a}{m_e} \frac{\sigma z_{\mathbf{q}}^L}{q} \Phi_a(\mathbf{u}) + \mathcal{E}_{\mathbf{q}}^{\sigma\alpha} \frac{\mathbf{q}}{q} \cdot \frac{\partial \Phi_a(\mathbf{u})}{\partial \mathbf{u}} \right). \tag{4}$$

In the above $a = e$ denotes the electrons, and $a = i$ stands for the ions. For these equations, we have used the following normalized quantities and definitions:

$$z \equiv \frac{\omega}{\omega_{pe}}, \quad \tau \equiv t\omega_{pe}, \quad \mathbf{q} \equiv \frac{\mathbf{k} v_{te}}{\omega_{pe}}, \quad \mathbf{u} \equiv \frac{\mathbf{v}}{v_{te}},$$

$$\mu_{\mathbf{q}}^L = 1, \quad \mu_{\mathbf{q}}^T = 1, \quad \mu_{\mathbf{q}}^S = \frac{q^3}{2^{3/2}} \sqrt{\frac{m_e}{m_i}} \left(1 + \frac{3T_i}{T_e} \right)^{1/2},$$

$$\lambda_{De}^2 = \frac{T_e}{4\pi \hat{n} e^2} = \frac{v_{te}^2}{2\omega_{pe}^2}, \quad g = \frac{1}{2^{3/2} (4\pi)^2 \hat{n} \lambda_{De}^3},$$

as well as the normalized distribution functions and wave spectra,

$$\Phi_a(\mathbf{u}) = v_{te}^3 F_a(\mathbf{v}), \quad \mathcal{E}_{\mathbf{q}}^{\sigma\alpha} = \frac{(2\pi)^2 g}{m_e v_{te}^2} \frac{I_{\mathbf{k}}^{\sigma\alpha}}{\mu_{\mathbf{k}}^{\alpha}}.$$

The dispersion relations for plasma normal modes are given in dimensionless form by

$$z_{\mathbf{q}}^L = \left(1 + \frac{3}{2} q^2 \right)^{1/2}, \quad z_{\mathbf{q}}^S = \frac{qA}{(1+q^2/2)^{1/2}}, \quad z_{\mathbf{q}}^T = \left(1 + \frac{c^2}{v_{te}^2} q^2 \right)^{1/2},$$

where

$$A = \frac{1}{\sqrt{2}} \left(\frac{m_e}{m_i} \right)^{1/2} \left(1 + \frac{3T_i}{T_e} \right)^{1/2}.$$

Here, $v_{te} = (2T_e/m_e)^{1/2}$ is the electron thermal speed and λ_{De} is the electron Debye length, T_e being the electron temperature.

Let us first comment on the terms appearing in Eq. (1), which describe the evolution of L mode. The first term on the right-hand side, enclosed within the large curly brackets, is denoted by subscript *Lql*. It describes the spontaneous emission and quasilinear (i.e., induced emission) effects for L mode. The second term describes the effects of three-wave decay involving L and S mode waves, and is denoted as *LdLS*. The third term depicts three-wave decay processes involving L and and T mode waves, and is labeled as *LdLT*. The fourth term describes three-wave decay processes involving S and T modes, and is designated as *LdST*. The fifth term enclosed within the large curly brackets denotes three-wave decay processes that involve one L mode wave and two T waves, and its designation is *LdTT*. The sixth term stands for the scattering process involving L waves, and its designation is *LsLL*. Finally, the seventh term describes the effects of scattering involving L and T waves, and is denoted as *LsLT*. For alternative discussions on various linear and nonlinear wave-particle and wave-wave interaction processes, see the Appendix.

The equation for S wave, Eq. (2), contains on the right-hand side the first term enclosed within the large curly brackets that describes the spontaneous emission and quasilinear effects, and is denoted as *Sql*. The second and third terms describe the three-wave decay terms. Specifically, the second term involving L waves are designated by *SdLL*, while the third terms that involves L and T waves are labeled as *SdLT*.

The evolution of T mode is described by Eq. (3). It is seen that there is no contribution of terms related to spontaneous emission and quasilinear effects. This is because the linear wave-particle resonance between T mode and the particles is impossible since no particles can have speeds greater than the speed of light *in vacuo*. From such a physical ground, the linear wave-particle resonance terms are ignored at the outset (see the Appendix for further discussion). The first term on the right-hand side describes the effects of three-wave decay involving L waves, and is denoted as *TdLL*. The second term describes the influence of three-wave decay involving L and and S waves, and is denoted as *TdLS*. The third term is denoted as *TdTL*, and it characterizes three-wave decay interactions involving one L wave and a T mode. The fourth term describes the nonlinear scattering process involving L and T waves, and is labeled as *TsTL*.

In the equation for the particle distribution functions, the term with g describes the effects of spontaneous fluctuations, and the term with the velocity derivative describes the quasilinear diffusion process. For details on the derivation of the above equations, the reader is referred to Refs. 14 and 15. See also, the Appendix for un-normalized forms of the entire set of equations.

III. NUMERICAL ANALYSIS

The objective of the present paper is to investigate the possibility of T wave emission in thermal plasmas by nonlinear processes, when the initial configuration corresponds to a quiescent thermal equilibrium state. For numerical analysis, we consider a two dimensional approximation. We assume an equilibrium situation, and therefore we write the initial electron and ion distribution functions as Maxwellian distributions, which are given as follows:

$$F_a(\mathbf{v},0) = \frac{1}{\pi v_{ta}^2} \exp\left(-\frac{v^2}{v_{ta}^2}\right), \tag{5}$$

where $a = e$ for electrons and $a = i$ for ions, and where $v_{ta} = (2T_a/m_a)^{1/2}$ is the thermal speed for particles of species a. The spectrum of T waves is assumed to be nonexistent at initial time, since according to the standard theory of plasmas in thermal equilibrium, no EM waves can be emitted and re-absorbed. However, thermal distribution of particles can spontaneously emit and re-absorb longitudinal electrostatic modes. The intensities of L and S modes are thus initialized by assuming that these waves are in equilibrium with the particle distribution functions. That is, the intensities for these modes are calculated from the balance requirement of spontaneous emission and re-absorption in Eqs. (1) and (2), or equivalently, by balancing spontaneous and induced emissions.[16–19] The resulting thermal intensities can, of course, be computed on the basis of the fluctuation-dissipation theorem. The initial spectra thus computed are given by

$$\begin{aligned} \mathcal{E}_{\mathbf{q}}^{\sigma L}(0) &= \frac{g}{2\,(z_{\mathbf{q}}^L)^2}, \\ \mathcal{E}_{\mathbf{q}}^{\sigma S}(0) &= \frac{g}{2z_{\mathbf{q}}^L\, z_{\mathbf{q}}^S} \frac{\exp(-\xi_{\mathbf{q}}^2) + \eta^{1/2}\exp(-\eta\,\xi_{\mathbf{q}}^2)}{\exp(-\xi_{\mathbf{q}}^2) + (T_e/T_i)\,\eta^{1/2}\exp(-\eta\,\xi_{\mathbf{q}}^2)}, \\ \xi_{\mathbf{q}}^2 &= \frac{(z_{\mathbf{q}}^S)^2}{q^2}, \quad \eta = \frac{m_i}{m_e}\frac{T_e}{T_i}. \end{aligned} \tag{6}$$

The set of equations (1)–(3) for the waves and Eq. (4) for the electrons are solved in 2D wave number space and 2D velocity space, by employing a splitting method with fixed time step for the evolution of the distribution and a Runge-Kutta method with the same fixed time step for the wave equations. The ion distribution is assumed to be fixed along all the time evolution of the system. As noted, the T waves are initially absent.

For all the numerical examples to be discussed subsequently, we use the normalized time interval $\Delta\tau = 0.1$. We employ 51×51 grids for $q_\perp$ and $q_\parallel$, with $0 < q_\perp = k_\perp v_{te}/\omega_p < 0.6$, and $0 < q_\parallel = k_\parallel v_{te}/\omega_p < 0.6$. For the velocities, we use a 51×101 grid for the $(u_\perp, u_\parallel) = (v_\perp/v_{te}, v_\parallel/v_{te})$ space, covering the velocity range $0 < u_\perp = v_\perp/v_{te} < 12$ and $-12 < u_\parallel = v_\parallel/v_{te} < 12$. For subsequent numerical solutions, we assume different values of the plasma parameter $(\hat{n}\lambda_D^3)^{-1}$, and different values of the parameter v_{te}^2/c^2, related to electron temperature.

The first case to be considered is with the plasma parameter given by $(\hat{n}\lambda_D^3)^{-1} = 5.0 \times 10^{-3}$ and $v_{te}^2/c^2 = 4.0 \times 10^{-3}$. Right at the onset of the time evolution, the nonlinear processes that are incorporated into Eq. (3) immediately generate a finite spectrum of T waves. Figure 1 shows the normalized spectrum of T waves, which is obtained from the numerical solution, as a function of the components of normalized wave number, $q_\parallel$ and $q_\perp$, at normalized times $\tau = 100$, 1000, 5000, and 10 000. The spectrum at $\tau = 100$ is shown in Fig. 1(a). It is seen that at this time an isotropic spectrum has developed, with maximum normalized intensity around 1.0×10^{-7}, and a minimum at very small q. For comparison, the maximum of the spectrum of L waves occurs for $q \to 0$, and is about 4.0×10^{-5}. In Fig. 1(b), the case of $\tau = 1000$ is depicted, which shows that the intensity of the T wave has grown in general, and that the minimum at $q = 0$ is not so pronounced in comparison with the surrounding values of q as it was in panel (a). The general tendency continues along time evolution. In Fig. 1(c), where the case of $\tau = 5000$ is shown, it can be seen that the spectrum at $q = 0$ is almost at the same level as the spectrum at surrounding values of q. The spectrum continues to grow, albeit at weaker pace, indicating a tendency to saturation. In Fig. 1(d), we show the spectrum at $\tau = 10\,000$, with the prominent intensity at very long wavelengths ($q \simeq 0$).

It is important to note that wave kinetic equations for both L and S mode wave spectral intensities are numerically solved together with that of T wave. The electron particle kinetic equation is also numerically solved in a self-consistent manner. However, the dynamical changes in L and S mode intensities as well as that of the electron velocity distribution

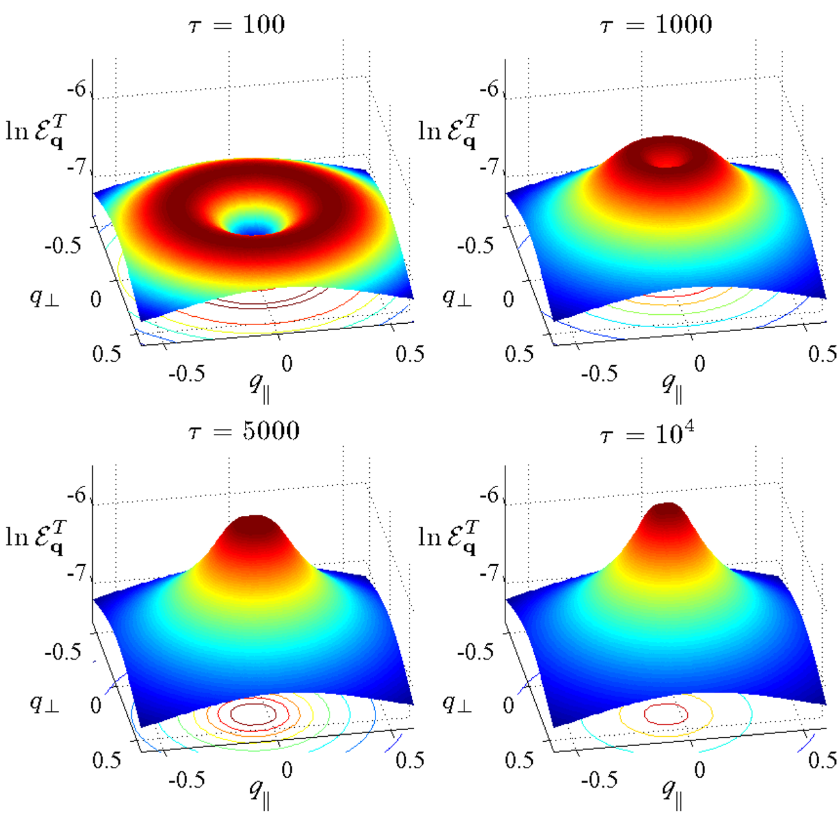

FIG. 1. Normalized T wave intensity, vs $q_\perp = k_\perp v_{te}/\omega_p$ and $q_\parallel = k_\parallel v_{te}/\omega_p$, in vertical logarithmic scale. (a) $\tau = 100$; (b) $\tau = 1000$; (c) $\tau = 5000$; (d) $\tau = 10000$. Input parameters are $v_f/v_{te} = 5.0$, $T_f/T_e = T_b/T_e = 1.0$, $T_e/T_i = 7.0$, $\left(\hat{n}\lambda_D^3\right)^{-1} = 5.0 \times 10^{-3}$, and $v_e^2/c^2 = 4.0 \times 10^{-3}$.

function are almost non-existent, which shows that over the range of dimensionless temporal variable considered in the present numerical analysis, the system remains largely in equilibrium, except for the EM wave spectrum.

Figure 2 shows a reduced view of the spectrum of T waves, obtained after integration along component $q_\perp$. It shows the reduced T wave normalized intensity vs $q_\parallel$, for $\tau = 100$, 200, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, and 10 000. A tendency for saturation and the reduction of the wave growth is clearly noticeable, indicating that the situation at $\tau = 10\,000$ can be considered as approaching the asymptotic state. Figure 2 also shows the curve corresponding to the approximated analytical solution that has been reported in Ref. 6.

In order to investigate the influence of the electron temperature on the nonlinear emission of transverse waves, we consider a case with the same plasma parameter as the previous case, namely, $\left(\hat{n}\lambda_D^3\right)^{-1} = 5.0 \times 10^{-3}$, but with a different value of the parameter v_{te}^2/c^2, by choosing $v_{te}^2/c^2 = 1.0 \times 10^{-3}$. Figure 3 shows the reduced view of the spectrum obtained for the T waves, i.e., the spectrum integrated along component $q_\perp$, vs $q_\parallel$, for the same values of τ as in Fig. 2, except that we now consider $v_{te}^2/c^2 = 1.0 \times 10^{-3}$.

We also consider a case with the same electron temperature as the original case, but with a different plasma density, by

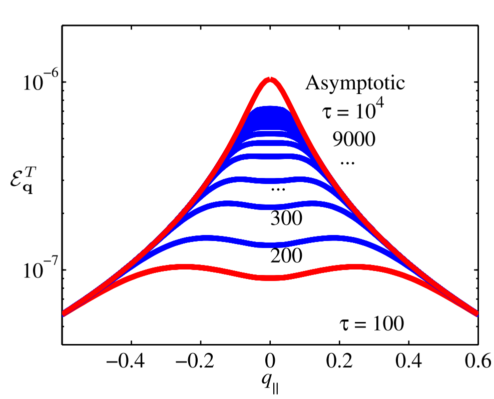

FIG. 2. Reduced T wave intensity vs $q_\parallel = k_\parallel v_{te}/\omega_p$, in vertical logarithmic scale, for several values of the reduced time τ corresponding to $\tau = 100$ (bottom-most curve shown in red), 200, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, and $\tau = 10\,000$ (the rest in blue). The figure also shows the approximate analytical solution (13) (topmost red curve) for the asymptotically steady-state (i.e., the turbulent equilibrium EM spectral intensity). Parameters are as in Fig. 1.

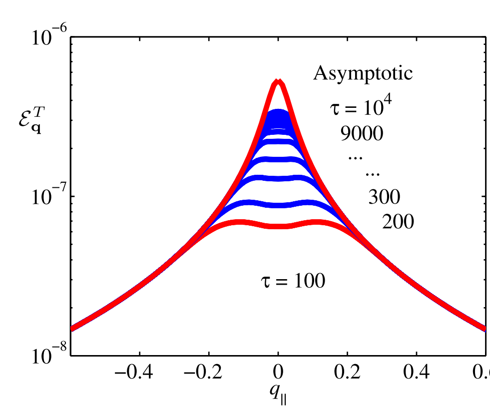

FIG. 3. Reduced T wave intensity vs $q_\parallel = k_\parallel v_{te}/\omega_p$, in vertical logarithmic scale, for several values of the reduced time τ: $\tau = 100$ (bottom-most red curve), 200, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, and $\tau = 10\,000$ (the rest in blue). The figure also shows the asymptotic solution (13) (topmost red curve). The plasma parameter is given by $\left(\hat{n}\lambda_D^3\right)^{-1} = 5.0 \times 10^{-3}$, and the electron temperature corresponds to $v_{te}^2/c^2 = 1.0 \times 10^{-3}$. Other parameters as in Fig. 1.

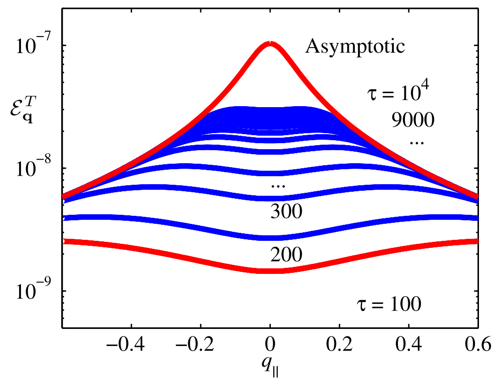

FIG. 4. Reduced T wave intensity vs $q_\parallel = k_\parallel v_{te}/\omega_p$, in vertical logarithmic scale, for several values of the reduced time τ: $\tau = 100$ (bottom-most red curve), 200, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, and $\tau = 10\,000$ (the rest in blue). The asymptotic solution (13) is also plotted (top-most red curve). The plasma parameter is $(\hat{n}\lambda_D^3)^{-1} = 5.0 \times 10^{-4}$, and the temperature is specified by $v_{te}^2/c^2 = 4.0 \times 10^{-3}$. Other parameters as in Fig. 1.

assuming the plasma parameter to be $(\hat{n}\lambda_D^3)^{-1} = 5.0 \times 10^{-4}$, and with $v_{te}^2/c^2 = 4.0 \times 10^{-3}$. The spectrum obtained for the T waves, integrated along component $q_\perp$, is shown in Fig. 4, vs $q_\parallel$, for the same values of τ as in Fig. 2 or Fig. 3.

On the basis of results shown in Figs. 1–3, we may thus conclude that the system of thermal electron distribution and longitudinal electrostatic fluctuations, when subject to nonlinear interactions, will induce nonlinear response to the spontaneously emitted thermal electrostatic fluctuations, with the net result being the emission of EM fluctuations. This finding can be interpreted in the following two alternative ways: The first is that such a process constitutes a novel radiation mechanism for a thermal plasma, since this implies that a thermal distribution of electrons not only emits and reabsorbs longitudinal fluctuations spontaneously, but they also must generate transverse EM fluctuations. Such an emission mechanism has not been discussed in the literature, and Ref. 6 emphasizes the ramification in such a light. The second interpretation, which is the focus of the present paper, is that for a collision-free plasma interacting through collective force, the thermal equilibrium is not a true steady-state solution, but rather, the system must make a transition to a quasi-equilibrium state dictated by the fundamental equations of weak turbulence theory, where there must exist a finite level of EM radiation. Such a new asymptotic state can be termed the *turbulent* equilibrium. In what follows, we further elucidate on the primary physical process that leads to the emission of EM fluctuation.

IV. DISCUSSION: PHYSICAL MECHANISM OF EM EMISSION

The primary physical mechanism responsible for the generation of the T mode can be investigated by considering each term among the weak turbulence equations, and determine the highest relevance. By turning each term on and off artificially and comparing the outcome against the full solution, we concluded that the most relevant term is the scattering term denoted as *TsLT* in Eq. (3). We have verified this by keeping the term *TsLT*, while turning other terms off in Eq. (3). The spectrum of T mode generated thusly turned out to be virtually indistinguishable from that obtained by considering the full equation. Similar tests performed by keeping either the *TdLL* term only or the *TdLT* term only led to the development of a spectrum with much lower intensity. When keeping only the effect of the *TdTL* term in Eq. (3), the T wave spectrum does not evolve at all, since the three wave decay process in this context requires the presence of T mode at the outset to be operative. From this, we may conclude that the scattering process is the most effective term in the generation of EM wave spectrum, starting from an equilibrium situation with zero T mode intensity. This finding corroborates the assumption that has been employed in an approximated analytical approach in Ref. 6.

In what follows, we discuss some details concerning the scattering term in Eq. (3), specifically, the term designated by *TsTL*, and show that, after suitable approximations, it can be written in a very simple form, which can ultimately lead to an approximate asymptotic solution as has been reported in Ref. 6. The approximate treatment of the scattering term starts by assuming that the electron and ion distributions do not evolve appreciably over the temporal range of the computation. We thus assume that they are described by the initial Maxwellian distribution. In the case of two dimensions in wave number and velocity space, the resonance condition can be used to eliminate one of the velocity integrals, and the remaining velocity integral can be performed analytically considering the Maxwellian distributions. We then use the symmetry properties of the integrand in order to write the wave-number integrals only in terms of positive wave numbers. By introducing s_1 and s_2 as the signs of q_x' and q_z', respectively, it can be shown that the scattering term *TsTL* simplifies as follows:

$$\begin{aligned}\{\text{scattering term in Eq. (3)}\}_{TsTL} = {} & \frac{n_*}{n_e}\frac{1}{\sqrt{\pi}}\frac{\sigma z_{\mathbf{q}}^T}{\mathrm{q}^2}\sum_{\sigma',s_1,s_2=\pm 1}\int_0^\infty dq_x'\int_0^\infty dq_z'\, M(\mathbf{q},\mathbf{q}') \\ & \times\Bigg\{g_*\left(\sigma\, z_{\mathbf{q}}^T\,\mathcal{E}_{\mathbf{q}'}^{\sigma' L} - s_2\sigma' z_{\mathbf{q}'}^L\,\frac{\mathcal{E}_{\mathbf{q}}^{\sigma T}}{2}\right)\left(\frac{T_*}{T_e}\right)^{1/2}\exp\left(-\frac{T_*}{T_e}\,\zeta^2(\mathbf{q},\mathbf{q}')\right) \\ & + \left[g_*\left(\sigma\, z_{\mathbf{q}}^T\,\mathcal{E}_{\mathbf{q}'}^{\sigma' L} - s_2\sigma' z_{\mathbf{q}'}^L\,\frac{\mathcal{E}_{\mathbf{q}}^{\sigma T}}{2}\right) - 2\,\frac{T_*}{T_i}\,(\sigma z_{\mathbf{q}}^T - s_2\sigma' z_{\mathbf{q}'}^L)\,\mathcal{E}_{\mathbf{q}'}^{\sigma' L}\,\frac{\mathcal{E}_{\mathbf{q}}^{\sigma T}}{2}\right] \\ & \times\left(\frac{m_i}{m_e}\frac{T_*}{T_i}\right)^{1/2}\exp\left(-\frac{m_i}{m_e}\frac{T_*}{T_i}\,\zeta^2(\mathbf{q},\mathbf{q}')\right)\Bigg\},\end{aligned} \tag{7}$$

where

$$M(\mathbf{q},\mathbf{q}') = \frac{1}{{q_x'}^2 + {q_z'}^2} \times \frac{q^2({q_x'}^2 + {q_z'}^2) - (s_1 q_x q_x' + s_2 q_z q_z')^2}{\sqrt{(q_x - s_1 q_x')^2 + (q_z - s_2 q_z')^2}},$$

$$\zeta^2(\mathbf{q},\mathbf{q}') = \frac{(\sigma z_{\mathbf{q}}^T - s_2\sigma' z_{\mathbf{q}'}^L)^2}{(q_x - s_1 q_x')^2 + (q_z - s_2 q_z')^2}.$$

Note that the integrand within the integrals over wave number components is dominated by the exponential functions, which contain in the argument the quantity $(\sigma z_{\mathbf{q}}^T - s_2\sigma' z_{\mathbf{q}'}^L)^2$. Owing to the dependence on the exponential functions, the most significant contributions to the integrand are from the terms in which $s_2\sigma' = \sigma$. Moreover, the most significant region of integration is around $q' = q_*$, where $z_{q_*}^L = z_q^T$. That is,

$$q_* = \left(\frac{2}{3}\frac{c^2}{v_e^2}\right)^{1/2} q. \tag{8}$$

For q near the middle of the numerical grid in our numerical analysis, for instance, $q \simeq 0.2$, say, the interaction due to scattering will involve an L mode with $q' = q_* > 1$. Owing to the fact that $c/v_e \gg 1$, the resonant wave number will lie outside the computing grid. In the numerical code, however, we make use of the analytical L mode spectrum (6), which is assumed to be non-evolving.

Taking into account the condition $s_2\sigma' = \sigma$, and the fact that the dispersion relations depend on the absolute value of the wave number, we write Eq. (7) in polar coordinates,

$$\text{Eq. (7)} = \frac{n_*}{n_e}\frac{z_q^T}{\sqrt{\pi}} \sum_{\sigma' s_1 = \pm 1} \int_0^{\pi/2} \mathrm{d}\phi' \int_0^\infty \mathrm{d}q'\, M \left\{ g_* \left(z_q^T\, \mathcal{E}_{\mathbf{q}'}^{\sigma' L} - z_{q'}^L \frac{\mathcal{E}_{\mathbf{q}}^{\sigma T}}{2} \right) \left(\frac{T_*}{T_e}\right)^{1/2} \exp\left(-\frac{T_*}{T_e}\zeta^2 \right) \right.$$
$$\left. + \left[g_* \left(z_q^T\, \mathcal{E}_{\mathbf{q}'}^{\sigma' L} - z_{q'}^L \frac{\mathcal{E}_{\mathbf{q}}^{\sigma T}}{2} \right) - 2\frac{T_*}{T_i}(z_q^T - z_{q'}^L)\, \mathcal{E}_{\mathbf{q}'}^{\sigma' L} \frac{\mathcal{E}_{\mathbf{q}}^{\sigma T}}{2} \right] \left(\frac{m_i}{m_e}\frac{T_*}{T_i}\right)^{1/2} \exp\left(-\frac{m_i}{m_e}\frac{T_*}{T_i}\zeta^2 \right) \right\}, \tag{9}$$

where

$$M = \frac{q'\left[1 - (s_1 \sin\phi \sin\phi' + \sigma\sigma' \cos\phi\cos\phi')^2\right]}{R},$$
$$\zeta^2 = \frac{(z_q^T - z_{q'}^L)^2}{R^2},$$
$$R^2 = (q\sin\phi - s_1 q'\sin\phi')^2 + (q\cos\phi - \sigma'\sigma q'\cos\phi')^2. \tag{10}$$

Further approximations can be made. We expand in Taylor series the expression in the argument of the exponential function,

$$\zeta^2 \simeq -\frac{3}{2}\frac{q_*}{(z_q^T)}\frac{(q' - q_*)}{R^2}.$$

Using this expression, we can write

$$\left(\frac{m_\alpha}{m_e}\frac{T_*}{T_\alpha}\right)^{1/2} \exp\left(-\frac{m_\alpha}{m_e}\frac{T_*}{T_\alpha}\zeta^2 \right)$$
$$\simeq \left(\frac{m_\alpha}{m_e}\frac{T_*}{T_\alpha}\right)^{1/2} \exp\left(-\frac{m_\alpha}{m_e}\frac{T_*}{T_\alpha}\frac{9}{4}\frac{q_*^2}{(z_q^T)^2}\frac{(q'-q_*)^2}{R^2} \right)$$
$$\Rightarrow \frac{2\sqrt{\pi}}{3}\frac{(z_q^T)}{q_*} R\,\delta(q' - q_*).$$

Making use of the above delta-function approximation, it is now trivial to perform the integral over q'. It turns out that the electron and ion contributions are the same so that their respective contributions in the scattering term $TsTL$ become the following:

$$\frac{n_*}{n_e}\frac{4}{3}(z_q^T)^3 g_* \sum_{\sigma' s_1 = \pm 1} \int_0^{\pi/2} d\phi'$$
$$\times \left[1 - (s_1\sin\phi\sin\phi' + \sigma\sigma'\cos\phi\cos\phi')^2\right]$$
$$\times \left(\mathcal{E}^{\sigma' L}_{q_*\sin\phi', q_*\cos\phi'} - \frac{\mathcal{E}_{\mathbf{q}}^{\sigma T}}{2} \right). \tag{11}$$

The integrand in the expression above can be rewritten by explicitly considering the signs of σ' and s_1, for given σ. The different terms can be added, making use of trigonometric identities, and finally the term corresponding to the spontaneous scattering in the equation for T mode may be written as follows:

$$\frac{n_*}{n_e}\frac{4}{3}(z_q^T)^3 g_* \int_0^{\pi/2} d\phi' \left[1 - \cos(2\phi)\cos(2\phi')\right]$$
$$\times \left(\mathcal{E}^{+L}_{q_*\sin\phi', q_*\cos\phi'} + \mathcal{E}^{-L}_{q_*\sin\phi', q_*\cos\phi'} - \mathcal{E}_{\mathbf{q}}^{\sigma T} \right). \tag{12}$$

One may carry out a similar analysis with the term that corresponds to the contribution from the induced scattering. However, the induced scattering term is much less significant than the spontaneous scattering terms, and we will not write it here explicitly, for the sake of economy of space. Assuming that in the asymptotic state the spectra of forward and backward propagating waves are the same and independent of ϕ, and considering that the evolution of T mode is approximately governed by the scattering term, the approximate form given by Eq. (12) shows that the asymptotic state is given by

$$\mathcal{E}_{\mathbf{q}}^T \simeq 2\mathcal{E}_{\mathbf{q}_*}^L = \frac{g}{1 + 3q_*^2}, \tag{13}$$

as recently shown in Ref. 6.

V. SUMMARY

In the present paper, we have utilized the equations of fully electromagnetic weak turbulence theory, which include quasilinear and spontaneous effects, three-wave decay, and scattering, to address the problem of transition from thermal equilibrium to an alternative equilibrium state characterized by the presence of finite level of electromagnetic radiation spectrum. We first demonstrated this by a numerical analysis in which we started from a thermal equilibrium situation, in which ions and electrons are described by Maxwellian velocity distribution functions and the spectra of L and S waves are specified by the balance condition between the quasilinear effects (induced emission) and the spontaneous emission effects. It is important to note that in such a classical thermal equilibrium, the spectrum of transverse radiation (T mode) is absent at the initial time.

The numerical analysis showed that nonlinear processes, in particular, scattering involving particles and L waves, generate a finite level of T mode waves over the entire range of wavelengths. At the initial stage, the maximum of this spectrum occurs for wavelengths that are larger than the electron Debye length, but as time progresses, the region of very long wavelengths continues to evolve, until for sufficiently long time scale, the spectrum of T mode radiation attains a quasi-asymptotic state with maximum intensity at very long wavelengths (normalized wave number $q \simeq 0$). The characteristics of this quasi-asymptotic state conform to the approximate analytical solution discussed in Ref. 6. We also performed an approximate analytical treatment of the most important nonlinear process, namely the scattering term involving particles and L mode wave, reproducing the approximate asymptotic T mode radiation spectrum.

The present finding implies that for a collision-free plasma interacting through collective force, the thermal equilibrium is not a true steady-state solution, but rather, the system must make a transition to an alternative quasi-equilibrium state where there must exist a finite level of EM radiation. Such a new asymptotic state can be termed the *turbulent* equilibrium.[6–12]

ACKNOWLEDGMENTS

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APPENDIX: EQUATIONS OF EM WEAK TURBULENCE THEORY

This section presents the basic equations of EM weak turbulence theory in un-normalized form. The detailed derivations can be found in Refs. 14 and 15. The wave intensities for the plasma normal modes are defined by their electric field energy. For longitudinal modes, the intensities $I_{\mathbf{k}}^{\sigma\alpha}$ for $\alpha = L, S$, are defined by $\langle \delta E_{\parallel}^2 \rangle_{\mathbf{k},\omega} = \sum_{\sigma=\pm 1} \sum_{\alpha=L,S} I_{\mathbf{k}}^{\sigma\alpha} \delta(\omega - \sigma\omega_{\mathbf{k}}^{\alpha})$. The transverse mode has both electric and magnetic fields, but it is sufficient to define the spectral wave intensity in terms of electric field, $\langle \delta E_{\perp}^2 \rangle_{\mathbf{k}\omega} = \sum_{\sigma=\pm 1} I_{\mathbf{k}}^{\sigma T}\ \delta(\omega - \sigma\omega_{\mathbf{k}}^{T})$, as the magnetic field intensity is trivially given by $\langle \delta B^2 \rangle_{\mathbf{k}\omega} = |ck/\omega|^2 \langle \delta E_{\perp}^2 \rangle_{\mathbf{k}\omega}$. The L mode wave kinetic equation is given by

$$
\begin{aligned}
\frac{\partial I_{\mathbf{k}}^{\sigma L}}{\partial t} &= \frac{4\pi e^2}{m_e k^2} \int d\mathbf{v}\, \delta(\sigma\omega_{\mathbf{k}}^L - \mathbf{k}\cdot\mathbf{v}) \left(\hat{n} e^2 f_e(\mathbf{v}) + \pi(\sigma\omega_{\mathbf{k}}^L)\, \mathbf{k}\cdot \frac{\partial f_e(\mathbf{v})}{\partial \mathbf{v}}\, I_{\mathbf{k}}^{\sigma L} \right) + \pi\sigma\omega_{\mathbf{k}}^L \frac{e^2}{2T_e^2} \sum_{\sigma',\sigma''=\pm 1} \int d\mathbf{k}'\, \frac{\mu_{\mathbf{k}-\mathbf{k}'}^S (\mathbf{k}\cdot\mathbf{k}')^2}{k^2 k'^2 |\mathbf{k}-\mathbf{k}'|^2} \\
&\times \left(\sigma\omega_{\mathbf{k}}^L I_{\mathbf{k}'}^{\sigma' L} \frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' S}}{\mu_{\mathbf{k}-\mathbf{k}'}^S} - \sigma'\omega_{\mathbf{k}'}^L \frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' S}}{\mu_{\mathbf{k}-\mathbf{k}'}^S} I_{\mathbf{k}}^{\sigma L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^L I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma L} \right) \delta(\sigma\omega_{\mathbf{k}}^L - \sigma'\omega_{\mathbf{k}'}^L - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^S) \\
&+ \pi\sigma\omega_{\mathbf{k}}^L \frac{e^2}{8 m_e^2 \omega_{pe}^2} \sum_{\sigma',\sigma''} \int d\mathbf{k}'\, \frac{(\mathbf{k}\times\mathbf{k}')^2}{k^2 k'^2 |\mathbf{k}-\mathbf{k}'|^2} \left(\frac{k'^2}{\sigma'\omega_{\mathbf{k}'}^L} + \frac{k^2}{\sigma\omega_{\mathbf{k}}^L} \right)^2 \left(\sigma\omega_{\mathbf{k}}^L \frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' T}}{2} I_{\mathbf{k}'}^{\sigma' L} - \sigma'\omega_{\mathbf{k}'}^L \frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' T}}{2} I_{\mathbf{k}}^{\sigma L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^T I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma L} \right) \\
&\times \delta(\sigma\omega_{\mathbf{k}}^L - \sigma'\omega_{\mathbf{k}'}^L - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^T) + \pi\sigma\omega_{\mathbf{k}}^L \frac{e^2}{4m_e^2} \sum_{\sigma',\sigma''} \int d\mathbf{k}'\, \frac{k^2}{(\omega_{\mathbf{k}'}^T)^2 (\omega_{\mathbf{k}-\mathbf{k}'}^T)^2} \left(1 + \frac{[\mathbf{k}'\cdot(\mathbf{k}-\mathbf{k}')]^2}{k^2 |\mathbf{k}-\mathbf{k}'|^2} \right) \\
&\times \left(\sigma\omega_{\mathbf{k}}^L \frac{I_{\mathbf{k}'}^{\sigma' T}}{2} \frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' T}}{2} - \sigma'\omega_{\mathbf{k}'}^T \frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' T}}{2} I_{\mathbf{k}}^{\sigma L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^T \frac{I_{\mathbf{k}'}^{\sigma' T}}{2} I_{\mathbf{k}}^{\sigma L} \right) \delta(\sigma\omega_{\mathbf{k}}^L - \sigma'\omega_{\mathbf{k}'}^T - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^T) \\
&+ \pi\sigma\,\omega_{\mathbf{k}}^L \frac{e^2}{2T_e^2} \sum_{\sigma',\sigma''} \int d\mathbf{k}'\, \frac{\mu_{\mathbf{k}'}^S (\mathbf{k}\times\mathbf{k}')^2}{k^2 k'^2 |\mathbf{k}-\mathbf{k}'|^2} \left(\sigma\omega_{\mathbf{k}}^L \frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' T}}{2} \frac{I_{\mathbf{k}'}^{\sigma' S}}{\mu_{\mathbf{k}'}^S} - \sigma'\omega_{\mathbf{k}'}^L \frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' T}}{2} I_{\mathbf{k}}^{\sigma L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^T \frac{I_{\mathbf{k}'}^{\sigma' S}}{\mu_{\mathbf{k}'}^S} I_{\mathbf{k}}^{\sigma L} \right) \\
&\times \delta(\sigma\omega_{\mathbf{k}}^L - \sigma'\omega_{\mathbf{k}'}^S - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^T) - \sigma\omega_{\mathbf{k}}^L \frac{e^2}{\hat{n}\, m_e^2\, \omega_{pe}^2} \sum_{\sigma'} \int d\mathbf{k}' \int d\mathbf{v}\, \frac{(\mathbf{k}\cdot\mathbf{k}')^2}{k^2 k'^2}\, \delta[\sigma\omega_{\mathbf{k}}^L - \sigma'\omega_{\mathbf{k}'}^L - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}] \\
&\times \left[\frac{\hat{n}\, e^2}{\omega_{pe}^2} \left(\sigma'\omega_{\mathbf{k}'}^L I_{\mathbf{k}}^{\sigma L} - \sigma\omega_{\mathbf{k}}^L I_{\mathbf{k}'}^{\sigma' L} \right) [f_e(\mathbf{v}) + f_i(\mathbf{v})] - \pi \frac{m_e}{m_i} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma L} (\mathbf{k}-\mathbf{k}')\cdot \frac{\partial f_i(\mathbf{v})}{\partial \mathbf{v}} \right] \\
&- \sigma\omega_{\mathbf{k}}^L \frac{e^2}{\hat{n}\, m_e^2\, \omega_{pe}^2} \sum_{\sigma'} \int d\mathbf{k}' \int d\mathbf{v}\, \frac{(\mathbf{k}\times\mathbf{k}')^2}{k^2 k'^2}\, \delta[\sigma\omega_{\mathbf{k}}^L - \sigma'\omega_{\mathbf{k}'}^T - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}] \\
&\times \left[\frac{\hat{n} e^2}{\omega_{pe}^2} \left(\sigma'\omega_{\mathbf{k}'}^T I_{\mathbf{k}}^{\sigma L} - \sigma\omega_{\mathbf{k}}^L \frac{I_{\mathbf{k}'}^{\sigma' T}}{2} \right) [f_e(\mathbf{v}) + f_i(\mathbf{v})] - \pi \frac{m_e}{m_i} \frac{I_{\mathbf{k}'}^{\sigma' T}}{2} I_{\mathbf{k}}^{\sigma L} (\mathbf{k}-\mathbf{k}')\cdot \frac{\partial f_i(\mathbf{v})}{\partial \mathbf{v}} \right],
\end{aligned} \tag{A1}
$$

where

$$\mu_{\mathbf{k}}^{S}=|k|^{3}\lambda_{De}^{3}\left(\frac{m_e}{m_i}\right)^{1/2}\left(1+\frac{3T_i}{T_e}\right)^{1/2}. \qquad \text{(A2)}$$

Note that the first velocity integral term on the right-hand side of Eq. (A1) that contains the resonance factor $\delta(\sigma\omega_{\mathbf{k}}^{L}-\mathbf{k}\cdot\mathbf{v})$ (i.e., linear wave-particle interaction) represents the spontaneous and induced emissions of L waves; the second $\mathbf{k}'$-integral term dictated by the three-wave resonance condition $\delta(\sigma\omega_{\mathbf{k}}^{L}-\sigma'\omega_{\mathbf{k}'}^{L}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{S})$ (nonlinear three-wave interaction) represents the decay/coalescence involving L mode with another L mode and an S mode; the next $\mathbf{k}'$-integral associated with $\delta(\sigma\omega_{\mathbf{k}}^{L}-\sigma'\omega_{\mathbf{k}'}^{L}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{T})$ represents the decay/coalescence of two L modes into a T mode at twice the plasma frequency; the third $\mathbf{k}'$-integral associated with $\delta(\sigma\omega_{\mathbf{k}}^{L}-\sigma'\omega_{\mathbf{k}'}^{T}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{T})$ represents the decay/coalescence of two T modes into an L mode.; the fourth $\mathbf{k}'$-integral with the factor $\delta(\sigma\omega_{\mathbf{k}}^{L}-\sigma'\omega_{\mathbf{k}'}^{S}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{T})$ corresponds to the decay/coalescence process involving L, S, and a transverse mode T at the plasma frequency; the double integral term $\int d\mathbf{v}\int d\mathbf{k}'\cdots$ dictated by the nonlinear wave-particle resonance condition $\delta[\sigma\omega_{\mathbf{k}}^{L}-\sigma'\omega_{\mathbf{k}'}^{L}-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]$ represents the spontaneous and induced scattering processes involving two Langmuir waves and the particles, and the similar term which contains $\delta[\sigma\omega_{\mathbf{k}}^{L}-\sigma'\omega_{\mathbf{k}'}^{T}-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]$ corresponds to spontaneous and induced scattering processes involving L and T modes and the particles.

For the ion-sound mode, $\alpha=S$, the wave kinetic equation is given by

$$\begin{aligned}
\frac{\partial}{\partial t}\frac{I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}^{S}} &= \mu_{\mathbf{k}}^{S}\frac{4\pi e^2}{m_e k^2}\int d\mathbf{v}\,\delta(\sigma\omega_{\mathbf{k}}^{S}-\mathbf{k}\cdot\mathbf{v})\left[\hat{n}\,e^2\,[f_e(\mathbf{v})+f_i(\mathbf{v})]+\pi\,(\sigma\omega_{\mathbf{k}}^{L})\left(\mathbf{k}\cdot\frac{\partial f_e(\mathbf{v})}{\partial\mathbf{v}}+\frac{m_e}{m_i}\mathbf{k}\cdot\frac{\partial f_i(\mathbf{v})}{\partial\mathbf{v}}\right)\frac{I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}^{S}}\right]\\
&+\pi\sigma\omega_{\mathbf{k}}^{L}\frac{e^2}{4T_e^2}\sum_{\sigma',\sigma''}\int d\mathbf{k}'\,\frac{\mu_{\mathbf{k}}^{S}[\mathbf{k}'\cdot(\mathbf{k}-\mathbf{k}')]^2}{k^2k'^2|\mathbf{k}-\mathbf{k}'|^2}\left(\sigma\omega_{\mathbf{k}}^{L}I_{\mathbf{k}'}^{\sigma'L}I_{\mathbf{k}-\mathbf{k}'}^{\sigma''L}-\sigma'\omega_{\mathbf{k}'}^{L}I_{\mathbf{k}-\mathbf{k}'}^{\sigma''L}\frac{I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}^{S}}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L}I_{\mathbf{k}'}^{\sigma'L}\frac{I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}^{S}}\right)\\
&\times\delta(\sigma\omega_{\mathbf{k}}^{S}-\sigma'\omega_{\mathbf{k}'}^{L}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L})+\pi\sigma\omega_{\mathbf{k}}^{L}\frac{e^2}{2T_e^2}\sum_{\sigma',\sigma''}\int d\mathbf{k}'\,\frac{\mu_{\mathbf{k}}^{S}(\mathbf{k}\times\mathbf{k}')^2}{k^2k'^2|\mathbf{k}-\mathbf{k}'|^2}\\
&\times\left(\sigma\omega_{\mathbf{k}}^{L}\frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma''T}}{2}I_{\mathbf{k}'}^{\sigma'L}-\sigma'\omega_{\mathbf{k}'}^{L}\frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma''T}}{2}\frac{I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}^{S}}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{T}I_{\mathbf{k}'}^{\sigma'L}\frac{I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}^{S}}\right)\delta(\sigma\omega_{\mathbf{k}}^{S}-\sigma'\omega_{\mathbf{k}'}^{L}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{T}).
\end{aligned} \qquad \text{(A3)}$$

The first velocity integral term on the right-hand side of Eq. (A3) that contains the resonance factor $\delta(\sigma\omega_{\mathbf{k}}^{S}-\mathbf{k}\cdot\mathbf{v})$ represents the spontaneous and induced emissions of S waves; the first $\mathbf{k}'$-integral term dictated by the three-wave resonance condition $\delta(\sigma\omega_{\mathbf{k}}^{S}-\sigma'\omega_{\mathbf{k}'}^{L}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{S})$ corresponds to the decay/coalescence involving S mode with two L modes; the next $\mathbf{k}'$-integral associated with $\delta(\sigma\omega_{\mathbf{k}}^{S}-\sigma'\omega_{\mathbf{k}'}^{L}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{T})$ represents the decay/coalescence involving an S and L modes and a T mode.

For the transverse mode T, the wave kinetic equation is given by

$$\begin{aligned}
\frac{\partial}{\partial t}\frac{I_{\mathbf{k}}^{\sigma T}}{2} &= \frac{\pi^2e^2}{2m_e}\int d\mathbf{v}\,\frac{(\mathbf{k}\times\mathbf{v})^2}{k^2}\,\delta(\sigma\omega_{\mathbf{k}}^{T}-\mathbf{k}\cdot\mathbf{v})\left(\frac{m_e}{4\pi^2}f_e(\mathbf{v})+\frac{1}{\sigma\omega_{\mathbf{k}}^{T}}\mathbf{k}\cdot\frac{\partial f_e(\mathbf{v})}{\partial\mathbf{v}}\frac{I_{\mathbf{k}}^{\sigma T}}{2}\right)+\pi\,\sigma\,\omega_{\mathbf{k}}^{T}\frac{e^2}{32\,m_e^2\,\omega_{pe}^2}\sum_{\sigma',\sigma''}\int d\mathbf{k}'\,\frac{(\mathbf{k}\times\mathbf{k}')^2}{k^2k'^2|\mathbf{k}-\mathbf{k}'|^2}\\
&\times\left(\frac{k'^2}{\sigma'\omega_{\mathbf{k}'}^{L}}-\frac{|\mathbf{k}-\mathbf{k}'|^2}{\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L}}\right)^2\left(\sigma\omega_{\mathbf{k}}^{T}I_{\mathbf{k}'}^{\sigma'L}I_{\mathbf{k}-\mathbf{k}'}^{\sigma''L}-\sigma'\omega_{\mathbf{k}'}^{L}I_{\mathbf{k}-\mathbf{k}'}^{\sigma''L}\frac{I_{\mathbf{k}}^{\sigma T}}{2}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L}I_{\mathbf{k}'}^{\sigma'L}\frac{I_{\mathbf{k}}^{\sigma T}}{2}\right)\delta(\sigma\omega_{\mathbf{k}}^{T}-\sigma'\omega_{\mathbf{k}'}^{L}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L})\\
&+\pi\,\sigma\,\omega_{\mathbf{k}}^{T}\frac{e^2}{4T_e^2}\sum_{\sigma',\sigma''}\int d\mathbf{k}'\,\frac{\mu_{\mathbf{k}-\mathbf{k}'}^{S}(\mathbf{k}\times\mathbf{k}')^2}{k^2k'^2|\mathbf{k}-\mathbf{k}'|^2}\left(\sigma\omega_{\mathbf{k}}^{T}I_{\mathbf{k}'}^{\sigma'L}\frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma''S}}{\mu_{\mathbf{k}-\mathbf{k}'}^{S}}-\sigma'\omega_{\mathbf{k}'}^{L}\frac{I_{\mathbf{k}-\mathbf{k}'}^{\sigma''S}}{\mu_{\mathbf{k}-\mathbf{k}'}^{S}}\frac{I_{\mathbf{k}}^{\sigma T}}{2}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L}I_{\mathbf{k}'}^{\sigma'L}\frac{I_{\mathbf{k}}^{\sigma T}}{2}\right)\\
&\times\delta(\sigma\omega_{\mathbf{k}}^{T}-\sigma'\omega_{\mathbf{k}'}^{L}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{S})+\pi\sigma\omega_{\mathbf{k}}^{T}\frac{e^2}{4m_e^2}\sum_{\sigma',\sigma''}\int d\mathbf{k}'\,\frac{|\mathbf{k}-\mathbf{k}'|^2}{(\omega_{\mathbf{k}}^{T})^2(\omega_{\mathbf{k}'}^{T})^2}\left(1+\frac{(\mathbf{k}\cdot\mathbf{k}')^2}{k^2k'^2}\right)\\
&\times\left(\sigma\omega_{\mathbf{k}}^{T}\frac{I_{\mathbf{k}'}^{\sigma'T}}{2}I_{\mathbf{k}-\mathbf{k}'}^{\sigma''L}-\sigma'\omega_{\mathbf{k}'}^{T}I_{\mathbf{k}-\mathbf{k}'}^{\sigma''L}\frac{I_{\mathbf{k}}^{\sigma T}}{2}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L}\frac{I_{\mathbf{k}'}^{\sigma'T}}{2}\frac{I_{\mathbf{k}}^{\sigma T}}{2}\right)\delta(\sigma\omega_{\mathbf{k}}^{T}-\sigma'\omega_{\mathbf{k}'}^{T}-\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L})\\
&-\sigma\omega_{\mathbf{k}}^{T}\frac{e^2}{2\hat{n}\,m_e^2\,\omega_{pe}^2}\sum_{\sigma'}\int d\mathbf{k}'\int d\mathbf{v}\,\frac{(\mathbf{k}\times\mathbf{k}')^2}{k^2k'^2}\,\delta[\sigma\omega_{\mathbf{k}}^{T}-\sigma'\omega_{\mathbf{k}'}^{L}-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]\\
&\times\left[\frac{\hat{n}\,e^2}{\omega_{pe}^2}\left(\sigma'\omega_{\mathbf{k}'}^{L}\frac{I_{\mathbf{k}}^{\sigma T}}{2}-\sigma\omega_{\mathbf{k}}^{T}I_{\mathbf{k}'}^{\sigma'L}\right)[f_e(\mathbf{v})+f_i(\mathbf{v})]-\pi\frac{m_e}{m_i}I_{\mathbf{k}'}^{\sigma'L}\frac{I_{\mathbf{k}}^{\sigma T}}{2}(\mathbf{k}-\mathbf{k}')\cdot\frac{\partial f_i(\mathbf{v})}{\partial\mathbf{v}}\right].
\end{aligned} \qquad \text{(A4)}$$

The first velocity integral term on the right-hand side of Eq. (A4) that contains the resonance factor $\delta(\sigma\omega_{\mathbf{k}}^T - \mathbf{k}\cdot\mathbf{v})$ corresponds to the spontaneous and induced emissions of T waves. However, since the particles cannot have velocity higher than the speed of light *in vacuo*, while the transverse waves are superluminal modes, the wave-particle resonance cannot be satisfied. For this reason, this term can be ignored at the outset. However, this term is included here only for the sake of completeness. The second $\mathbf{k}'$-integral terms dictated by the three-wave resonance condition $\delta(\sigma\omega_{\mathbf{k}}^T - \sigma'\omega_{\mathbf{k}'}^L - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^L)$ represents the coalescence of two L modes into a T mode at the second harmonic plasma frequency; the next $\mathbf{k}'$-integrals associated with the factor $\delta(\sigma\omega_{\mathbf{k}}^T - \sigma'\omega_{\mathbf{k}'}^L - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^S)$ describe the merging of L and S modes into a T mode at the fundamental plasma frequency. The third $\mathbf{k}'$-integrals with $\delta(\sigma\omega_{\mathbf{k}}^T - \sigma'\omega_{\mathbf{k}'}^T - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^L)$ depict the merging of a T mode and an L mode into the next higher harmonic T mode. The double integral term $\int d\mathbf{v}\int d\mathbf{k}'\cdots$ dictated by the nonlinear wave-particle resonance condition $\delta[\sigma\omega_{\mathbf{k}}^T - \sigma'\omega_{\mathbf{k}'}^L - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]$ represents spontaneous and induced scattering processes involving T and L modes and the particles.

The linear dispersion relations for electrostatic (Langmuir L and ion-sound S) and transverse electromagnetic (T) modes are given by

$$\begin{aligned}
\omega_{\mathbf{k}}^L &= \omega_{pe}\,(1 + 3k^2\lambda_D^2/2),\\
\omega_{\mathbf{k}}^S &= kc_S\,(1 + 3T_e/T_i)^{1/2}\,(1 + k^2\lambda_D^2)^{-1/2},\\
\omega_{\mathbf{k}}^T &= (\omega_{pe}^2 + c^2k^2)^{1/2},
\end{aligned} \tag{A5}$$

where $\omega_{pe} = (4\pi\hat{n}e^2/m_e)^{1/2}$ is the plasma frequency, $\lambda_D = [T_e/(4\pi\hat{n}e^2)]^{1/2}$ is the Debye length, T_e and T_i are electron and ion temperatures, respectively, and $c_S = (T_e/m_i)^{1/2}$ represents the ion-sound speed.

[1]V. L. Ginzburg and V. V. Zheleznyakov, Sov. Astron. **2**, 653 (1958).
[2]*Solar Radiophysics*, edited by D. J. McLean and N. R. Labrum (Cambridge University Press, New York, 1985).
[3]M. V. Goldman, Sol. Phys. **89**, 403 (1983).
[4]D. B. Melrose, *Instabilities in Space and Laboratory Plasmas* (Cambridge University Press, New York, 1986).
[5]V. V. Zheleznyakov, *Radiation in Astrophysical Plasmas* (Kluwer, Dordrecht, 1996).
[6]L. F. Ziebell, P. H. Yoon, F. J. R. Simões, Jr., R. Gaelzer, and J. Pavan, Phys. Plasmas **21**, 010701 (2014).
[7]R. A. Treumann, Phys. Scr. **59**, 19 (1999).
[8]R. A. Treumann, Phys. Scr. **59**, 204 (1999).
[9]R. A. Treumann and C. H. Jaroschek, Phys. Rev. Lett. **100**, 155005 (2008).
[10]P. H. Yoon, Phys. Plasmas **19**, 052301 (2012).
[11]P. H. Yoon, Phys. Plasmas **19**, 012304 (2012).
[12]P. H. Yoon, L. F. Ziebell, R. Gaelzer, R. P. Lin, and L. Wang, Space Sci. Rev. **173**, 459 (2012).
[13]R. C. Davidson, *Methods in Nonlinear Plasma Theory* (Academic, New York, 1972).
[14]P. H. Yoon, L. F. Ziebell, R. Gaelzer, and J. Pavan, Phys. Plasmas **19**, 102303 (2012).
[15]P. H. Yoon, Phys. Plasmas **13**, 022302 (2006).
[16]A. G. Sitenko, *Electromagnetic Fluctuations in Plasma* (Academic, New York, 1967).
[17]N. Meyer-Vernet, J. Geophys. Res. **84**, 5373, doi:10.1029/JA084iA09p05373 (1979).
[18]P. H. Yoon, Phys. Plasmas **14**, 064504 (2007).
[19]R. Schlickeiser and P. H. Yoon, Phys. Plasmas **19**, 022105 (2012).