

Dielectric tensor for inhomogeneous plasmas in inhomogeneous magnetic field

R. Gaelzer, L. F. Ziebell, and O. J. G. Silveira

Citation: *Physics of Plasmas* **6**, 4533 (1999); doi: 10.1063/1.873740

View online: <http://dx.doi.org/10.1063/1.873740>

View Table of Contents: <http://scitation.aip.org/content/aip/journal/pop/6/12?ver=pdfcov>

Published by the [AIP Publishing](#)

Articles you may be interested in

[Dielectric permittivity tensor and low frequency instabilities of a magnetoactive current-driven plasma with nonextensive distribution](#)

Phys. Plasmas **22**, 122102 (2015); 10.1063/1.4936825

[Effect of magnetic field on the wave dispersion relation in three-dimensional dusty plasma crystals](#)

Phys. Plasmas **19**, 073704 (2012); 10.1063/1.4737600

[The dispersion relation and excitation of transverse magnetic mode electromagnetic waves in rippled-wall waveguide with a plasma rod and an annular dielectric](#)

Phys. Plasmas **19**, 013109 (2012); 10.1063/1.3676154

[Relativistic plasma dielectric tensor evaluation based on the exact plasma dispersion functions concept](#)

Phys. Plasmas **13**, 072105 (2006); 10.1063/1.2217309

[Effects of magnetic curvature on the lower-hybrid-drift instability](#)

Phys. Plasmas **8**, 1159 (2001); 10.1063/1.1354646



PFEIFFER VACUUM

VACUUM SOLUTIONS FROM A SINGLE SOURCE

Pfeiffer Vacuum stands for innovative and custom vacuum solutions worldwide, technological perfection, competent advice and reliable service.

Dielectric tensor for inhomogeneous plasmas in inhomogeneous magnetic field

R. Gaelzer^{a)}

Instituto de Física e Matemática, Universidade Federal de Pelotas, Caixa Postal 354, 96010-900 Pelotas, RS, Brazil

L. F. Ziebell^{b)} and O. J. G. Silveira

Instituto de Física, Universidade Federal do Rio Grande do Sul, Caixa Postal 15051, 91501-970 Porto Alegre, RS, Brazil

(Received 22 June 1999; accepted 14 September 1999)

The derivation of explicit expressions for the effective dielectric tensor to be utilized in the dispersion relation for weakly inhomogeneous plasmas is discussed. The general expressions obtained are useful for situations with simultaneous existence of weak inhomogeneities in density and magnetic field. The particular case of a Maxwellian distribution in velocity space for the electron population is discussed, and relatively compact expressions for the dielectric tensor are obtained, which depend on the inhomogeneous plasma dispersion function introduced by [Gaelzer *et al.*, Phys. Rev. E **55**, 5859 (1997)] and ultimately on the well-known Fried–Conte function and its derivatives. © 1999 American Institute of Physics. [S1070-664X(99)04512-7]

I. INTRODUCTION

The dielectric properties of weakly inhomogeneous plasmas may be well described by a dispersion relation that is formally the same as that utilized for an homogeneous plasma, with the dielectric tensor replaced by an *effective dielectric tensor*.¹ The derivation of the effective dielectric tensor is based on an iterative procedure, due to Beskin, Gurevich, and Istomin, denoted in what follows as the “BGI procedure,” which is aimed to assure that the absorption coefficient obtained from the solution of the dispersion relation is effectively connected with energy exchange between waves and particles.¹

We have adopted the BGI procedure in our studies on wave propagation and absorption. In some of these studies, we considered the situation in which the magnetic field is homogeneous and other plasma parameters can be inhomogeneous,^{2–6} obtaining explicit expressions for the effective dielectric tensor components, which were shown to satisfy the correct symmetry properties, as does the tensor for an homogeneous plasma,² for any direction of propagation relative to the inhomogeneity. In another approach, we considered the situation in which the magnetic field is inhomogeneous and other plasma parameters are homogeneous, also deriving explicit expressions for the components of the effective dielectric tensor, with the correct symmetry properties.^{7–9} In several of these publications, as well as in a recent commentary upon the symmetry featured by the effective dielectric tensor,¹⁰ we compared our formulation with other approaches found in the literature, including approaches which do not preserve Onsager symmetry. The interested reader is therefore directed to these previous publications for a list of references on the subject of wave propagation and absorption in inhomogeneous plasmas.

In the present paper we present a generalization of the formalism, taking into account the simultaneous presence of inhomogeneities in the magnetic field and in other plasma parameters. The motivation for this generalization is the following. As is known, the so-called lower-hybrid-drift instability (LHDI) has been considered as a possible trigger mechanism for current disruptions in the Earth magnetotail.^{11–16} Some early studies of this instability considered it as approximately electrostatic,^{11,12} but the electromagnetic character was soon recognized as important for the analysis of the instability, which occurs due to electron inhomogeneities. However, in the ensuing treatments of the instability including fully electromagnetic features, a conventional approach was utilized for the derivation of the dielectric tensor, and Onsager symmetry was not preserved for the elements of this tensor.^{15,16} This feature was noticed later on and an *ad hoc* solution was proposed in order to restore the desired symmetry.¹⁷ However, some important shortcomings of the *ad hoc* solution are noted in Ref. 17 itself, which also recognizes the need for a satisfactory and consistent approach to the problem.

This is the framework where we believe that the formalism developed in the present paper can be useful. We arrive at quite general expressions, based on the previous approaches, and particularize these general expressions in the nonrelativistic limit for the case of a Maxwellian distribution function of velocities, with a linear density profile. The final expressions obtained incorporate the relevant effects of inhomogeneities, and are written in terms of the inhomogeneous plasma dispersion function (PDF) and in terms of the Fried–Conte function and its derivatives.⁹ As demonstrated previously, in the nonrelativistic limit the inhomogeneous PDF may also be written as a function of the Fried–Conte function and its derivatives, which makes the final expressions quite practical to be utilized. In Sec. II we describe these more recent developments. Some final remarks and com-

^{a)}Electronic mail: rudi@ufpel.tche.br

^{b)}Electronic mail: ziebell@if.ufrgs.br

ments on the perspective for future work are added in Sec. III.

II. OUTLINE OF THE DERIVATION OF THE EFFECTIVE DIELECTRIC TENSOR FOR INHOMOGENEOUS MEDIA

Let us consider a magnetized plasma, with magnetic field pointing along the z direction, and featuring inhomoge-

neities in magnetic field and other plasma parameters along the x direction. Some particular cases have already been considered in recent years, and the main results obtained will be reproduced here.

For instance, in the case of homogeneous magnetic field and other plasma parameters inhomogeneous, the components of the effective dielectric tensor may be written as follows:²

$$\begin{aligned} \epsilon_{ij} = & \delta_{ij} - \delta_{iz}\delta_{jz} \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega^2} \int d^3u \frac{u_{\perp} L(f_{\alpha})}{\gamma_{\alpha}} \frac{u_{\parallel}}{u_{\perp}} - i \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega} \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u e^{iD_{n\alpha}(0)\tau} u_{\perp} \mathcal{L}(f_{\alpha}) \left(\frac{u_{\parallel}}{u_{\perp}} \right)^{\delta_{iz} + \delta_{jz}} (R_{ij}^R + iR_{ij}^I) \\ & + (\delta_{iy}\delta_{jz} + \delta_{iz}\delta_{jy}) \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega^2} \frac{c}{\Omega_{\alpha}} \int d^3u \frac{u_{\parallel} f'_{\alpha}}{\gamma_{\alpha}} - i \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega} \frac{c}{\omega} \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u e^{iD_{n\alpha}(0)\tau} u_{\perp} \\ & \times \frac{f'_{\alpha} b_{\alpha} \sin \psi}{\gamma_{\alpha}} \left(\frac{u_{\parallel}}{u_{\perp}} \right)^{\delta_{iz} + \delta_{jz}} (R_{ij}^R + iR_{ij}^I) - i \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega} \frac{c}{\Omega_{\alpha}} \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u e^{iD_{n\alpha}(0)\tau} u_{\perp}^2 \mathcal{L}(f'_{\alpha}) \left(\frac{u_{\parallel}}{u_{\perp}} \right)^{\delta_{iz} + \delta_{jz}} \\ & \times \frac{1}{2} [(S_{ij}^R + iS_{ij}^I) + (S_{ji}^R + iS_{ji}^I)^*], \end{aligned} \tag{1}$$

where $f'_{\alpha} \equiv \partial f_{\alpha} / \partial x$, $b_{\alpha} = k_{\perp} u_{\perp} c / \Omega_{\alpha}$, $\mathbf{u} = \mathbf{p} / (m_{\alpha} c)$, $\Omega_{\alpha} = q_{\alpha} B_0 / (m_{\alpha} c)$, and ψ denotes the angle between the vector \mathbf{k}_{\perp} and the direction of the inhomogeneity. We have also used the following definitions:

$$D_{n\alpha}(\epsilon_B) = \gamma\omega - ck_{\parallel}u_{\parallel} - n\Omega_{\alpha}(1 + \epsilon_B x) - \epsilon_B \frac{k_{\perp} u_{\perp}^2 c^2}{2\Omega_{\alpha}} \sin \psi$$

$$\mathcal{L}(f_{\alpha}) = \partial_{u_{\perp}} f_{\alpha} - \frac{N_{\parallel} u_{\perp}}{\gamma} L(f_{\alpha}), \quad L(f_{\alpha}) = \frac{u_{\parallel}}{u_{\perp}} \partial_{u_{\perp}} f_{\alpha} - \partial_{u_{\parallel}} f_{\alpha}.$$

It is important to remark that in Eq. (1) we have considered the particular case of $D_{n\alpha}(0)$. The quantity ϵ_B which appears in $D_{n\alpha}(\epsilon_B)$ will be defined below, when considering inhomogeneous magnetic field.

Definitions of R_{ij} , S_{ij} , and other basic quantities appearing in Eq. (1) can be found in Ref. 2 and will not be repeated here for the sake of space economy. Equation (1) is exactly the same as Eq. (9) of Ref. 2, although it is written in a slightly modified form in order to be more adequate for our present purposes. The original form can be easily recovered, however, by some simple procedures, as explained in the following. First of all, the integral in the variable τ can be easily performed,

$$\frac{1}{D_{n\alpha}} = -i \int_0^{\infty} d\tau e^{iD_{n\alpha}\tau}.$$

In addition to this, the resonant contribution can be separated from the principal part,

$$\frac{1}{D_{n\alpha}} = P \frac{1}{D_{n\alpha}} - i\pi \delta(D_{n\alpha}),$$

where P means the principal part of an integral. After these simple steps, it is very easy to convert the operators L and \mathcal{L} , as well as the quantity $D_{n\alpha}$, to the definitions utilized in Ref. 2.

Equation (1) can be cast in an even more suitable and compact form by the definitions of the following auxiliary quantities:

$$\begin{aligned} \pi_{n\alpha} = & \left(\frac{nJ_n(b_{\alpha})}{b_{\alpha}} \cos \psi - iJ'_n(b_{\alpha}) \sin \psi \right) \hat{\mathbf{e}}_x \\ & + \left(\frac{nJ_n(b_{\alpha})}{b_{\alpha}} \sin \psi + iJ'_n(b_{\alpha}) \cos \psi \right) \hat{\mathbf{e}}_y \\ & + \frac{v_{\parallel}}{v_{\perp}} J_n(b_{\alpha}) \hat{\mathbf{e}}_z, \end{aligned} \tag{2}$$

$$\begin{aligned} \Phi_{n\alpha} = & \left\{ \left[\left(\frac{n^2}{b_{\alpha}^2} - \frac{1}{2} \right) J_n(b_{\alpha}) - \frac{J'_n(b_{\alpha})}{b_{\alpha}} \right] \sin(2\psi) \right. \\ & - i \left[\frac{n}{b_{\alpha}^2} J_n(b_{\alpha}) - \frac{nJ'_n(b_{\alpha})}{b_{\alpha}} \right] \cos(2\psi) \left. \right\} \hat{\mathbf{e}}_x - \left\{ \left[\left(\frac{n^2}{b_{\alpha}^2} \right. \right. \right. \\ & - \frac{1}{2} \left. \left. \left. J_n(b_{\alpha}) - \frac{J'_n(b_{\alpha})}{b_{\alpha}} \right] \cos(2\psi) + i \left[\frac{n}{b_{\alpha}^2} J_n(b_{\alpha}) \right. \right. \right. \\ & \left. \left. \left. - \frac{nJ'_n(b_{\alpha})}{b_{\alpha}} \right] \sin(2\psi) \right\} \hat{\mathbf{e}}_y + \frac{v_{\parallel}}{v_{\perp}} \left[\frac{n}{b_{\alpha}} J_n(b_{\alpha}) \sin \psi \right. \right. \\ & \left. \left. + iJ'_n(b_{\alpha}) \cos \psi \right] \hat{\mathbf{e}}_z. \end{aligned} \tag{3}$$

As a result of using these definitions, we may write

$$\begin{aligned}
 \vec{\epsilon} = & \vec{1} - i \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega} \sum_{n \rightarrow -\infty}^{\infty} \int_0^{\infty} d\tau \int d^3u u_{\perp} \mathcal{L}(f_{\alpha 0}) \\
 & \times e^{iD_{n\alpha}(0)\tau} \pi_{n\alpha}^* \pi_{n\alpha} - \hat{\mathbf{e}}_z \hat{\mathbf{e}}_z \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega^2} \int d^3u \frac{u_{\parallel}}{\gamma_{\alpha}} L f_{\alpha 0} \\
 & - i \sum_{\alpha} \frac{4\pi q_{\alpha}^2 c}{m_{\alpha}\omega} \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u e^{iD_{n\alpha}(0)\tau} u_{\perp} \\
 & \times \frac{f'_{\alpha} b_{\alpha} \sin \psi}{\gamma_{\alpha}} \pi_{n\alpha}^* \pi_{n\alpha} - i \sum_{\alpha} \frac{4\pi q_{\alpha}^2 c}{m_{\alpha}\omega} \frac{c}{\Omega_{\alpha}} \\
 & \times \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u e^{iD_{n\alpha}(0)\tau} u_{\perp}^2 \\
 & \times \mathcal{L}(f'_{\alpha}) \left[\frac{J_{n\alpha}}{2} \hat{\mathbf{e}}_y \pi_{n\alpha} + \Phi_{n\alpha}^* \pi_{n\alpha} \right]^H \\
 & + (\hat{\mathbf{e}}_y \hat{\mathbf{e}}_z + \hat{\mathbf{e}}_z \hat{\mathbf{e}}_y) \sum_{\alpha} \frac{4\pi q_{\alpha}^2 c}{m_{\alpha}\omega^2} \frac{c}{\Omega_{\alpha}} \int d^3u \frac{u_{\parallel} f'_{\alpha}}{\gamma_{\alpha}}, \quad (4)
 \end{aligned}$$

where $[\dots]^H$ means the Hermitian part of the dyadic, and $\hat{\mathbf{e}}_x$, $\hat{\mathbf{e}}_y$, and $\hat{\mathbf{e}}_z$ are the unit vectors.

For inhomogeneous plasmas where the magnetic field itself is not uniform, a fundamental difference exists relative to the case of the homogeneous field. The unperturbed orbits of the plasma particles are not affected by inhomogeneities in plasma parameters like density and temperature, but they are affected by the magnetic field inhomogeneity. As a consequence, the resonance condition in momentum space is affected by the inhomogeneity, and it is necessary to add an infinite number of corrections in order to build up the effective dielectric tensor.¹ The procedure has been carried out previously, by considering inhomogeneous magnetic field

and other plasma parameters as being homogeneous, assuming that the magnetic field is pointing along the z axis and is inhomogeneous along the x direction, $\mathbf{B}_0(x) = B_0(1 + \epsilon_B x) \hat{\mathbf{e}}_z$. The waves were assumed to be propagating in arbitrary direction, making an angle θ with the magnetic field, with the perpendicular component of the wave vector, \mathbf{k}_{\perp} , making an angle ψ with the inhomogeneity. Details of the derivation may be found in Ref. 9. For our present purposes, it is sufficient to reproduce here the components of the effective dielectric tensor, as they appear in Eq. (3) of Ref. 9,

$$\begin{aligned}
 \vec{\epsilon} = & \vec{1} - i \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega} \sum_{n \rightarrow -\infty}^{\infty} \int_0^{\infty} d\tau \int d^3u u_{\perp} \mathcal{L}(f_{\alpha 0}) \\
 & \times e^{iD_{n\alpha}(\epsilon_B)\tau} [F_{n\alpha}(\tau)]^{(|n|-1)} \frac{\Pi_{n\alpha}^- \Pi_{n\alpha}^+}{(W_n^- W_n^+)^{|n|}} \\
 & - \hat{\mathbf{e}}_z \hat{\mathbf{e}}_z \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega^2} \int d^3u \frac{u_{\parallel}}{\gamma} L(f_{\alpha 0})(u_{\perp}^2, u_{\parallel}), \quad (5)
 \end{aligned}$$

where

$$\begin{aligned}
 \Pi_{n\alpha}^{\pm} = & \pm \left[n J_{|n|}(W_n^{\pm}) G_{n\alpha}^{\pm}(\tau) + i \frac{J_{|n|+1}(W_n^{\pm})}{W_n^{\pm}} F_{n\alpha}^{1/2}(\tau) b_{\alpha} \sin \psi \right] \\
 & \times \hat{\mathbf{e}}_x + i \left[|n| J_{|n|}(W_n^{\pm}) G_{n\alpha}^{\pm}(\tau) \mp \frac{J_{|n|+1}(W_n^{\pm})}{W_n^{\pm}} \right. \\
 & \left. \times F_{n\alpha}^{1/2}(\tau) (b_{\alpha} \cos \psi \pm \mathcal{K}_n \tau) \right] \hat{\mathbf{e}}_y \\
 & + \frac{u_{\parallel}}{u_{\perp}} J_{|n|}(W_n^{\pm}) F_{n\alpha}^{1/2}(\tau) \hat{\mathbf{e}}_z,
 \end{aligned}$$

$$G_{n\alpha}^{\pm}(\tau) = i \sqrt{\frac{\mathcal{K}_n \tau \mp b_{\alpha} \cos \psi + i S_n b_{\alpha} \sin \psi}{\mathcal{K}_n \tau \pm b_{\alpha} \cos \psi + i S_n b_{\alpha} \sin \psi}}, \quad G_{n\alpha}^+ G_{n\alpha}^- = -1, \\
 F_{n\alpha}^{1/2} G_{n\alpha}^{\pm} = -(\mathcal{K}_n \tau \mp b_{\alpha} \cos \psi + i S_n b_{\alpha} \sin \psi),$$

$$W_n^{\pm} = [b_{\alpha}^2 \pm 2b_{\alpha} \cos \psi \mathcal{K}_n \tau + \mathcal{K}_n^2 \tau^2]^{1/2}, \quad \mathcal{K}_n = n \epsilon_B c u_{\perp} / 2,$$

$S_n = \text{sign}(n)$, and $D_{n\alpha}(\epsilon_B)$ is as defined along with Eq. (1).

For the more general case with the simultaneous presence of weak inhomogeneities both in the magnetic field and in other plasma parameters which appear in the distribution function, the same general approach applied to these previous cases may be followed. Since the inhomogeneity is weak, gradient effects are kept only up to first order, except those necessary to avoid secular terms in the unperturbed orbits.⁸ The outcome is such that we may collect together Eqs. (1) and (5), and write the effective dielectric tensor $\vec{\epsilon}$ as follows:

$$\vec{\epsilon} = \vec{1} + \vec{Q}_B + \vec{Q}_P, \quad (6)$$

where

$$\begin{aligned}
 \vec{Q}_B = & -i \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega} \sum_{n \rightarrow -\infty}^{\infty} \int_0^{\infty} d\tau \int d^3u u_{\perp} \mathcal{L}(f_{\alpha}) \\
 & \times e^{iD_{n\alpha}(\epsilon_B)\tau} [F_{n\alpha}(\tau)]^{(|n|-1)} \frac{\Pi_{n\alpha}^- \Pi_{n\alpha}^+}{(W_n^- W_n^+)^{|n|}} \\
 & - \hat{\mathbf{e}}_z \hat{\mathbf{e}}_z \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega^2} \int d^3u \frac{u_{\parallel}}{\gamma} L(f_{\alpha})(u_{\perp}^2, u_{\parallel}), \quad (7)
 \end{aligned}$$

$$\begin{aligned} \vec{Q}_P = & -i \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega} \frac{c}{\omega} \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u e^{iD_{n\alpha}(0)\tau} u_{\perp} \\ & \times \frac{f'_{\alpha} b_{\alpha} \sin \psi}{\gamma_{\alpha}} \pi_{n\alpha}^* \pi_{n\alpha} - i \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega} \frac{c}{\Omega_{\alpha}} \\ & \times \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u e^{iD_{n\alpha}(0)\tau} u_{\perp}^2 \\ & \times \mathcal{L}(f'_{\alpha}) \left[\frac{J_{n\alpha}}{2} \hat{\mathbf{e}}_y \pi_{n\alpha} + \Phi_{n\alpha}^* \pi_{n\alpha} \right]^H \\ & + (\hat{\mathbf{e}}_y \hat{\mathbf{e}}_z + \hat{\mathbf{e}}_z \hat{\mathbf{e}}_y) \sum_{\alpha} \frac{4\pi q_{\alpha}^2}{m_{\alpha}\omega^2} \frac{c}{\Omega_{\alpha}} \int d^3u \frac{u_{\parallel} f'_{\alpha}}{\gamma_{\alpha}}. \end{aligned} \quad (8)$$

Let us now assume a model distribution function which is Maxwellian in velocity space and features a linear spatial profile, which is a suitable approximation for the case of weak density inhomogeneity. For simplicity, we consider uniform and isotropic temperature,

$$\begin{aligned} f_{\alpha}(u_{\perp}^2, u_{\parallel}, x) = & (1 - \epsilon_{\alpha} x) n_0 \frac{\mu_{\alpha}^{3/2}}{(2\pi)^{3/2}} e^{-\mu_{\alpha} u^2/2} \\ \equiv & (1 - \epsilon_{\alpha} x) n_0 g_{\alpha}(u_{\perp}^2, u_{\parallel}), \end{aligned} \quad (9)$$

where $\mu_{\alpha} = m_{\alpha} c^2 / T_{\alpha}$.

Using this distribution function, after some calculations, Eqs. (7) and (8) can be written as

$$\begin{aligned} \vec{Q}_B = & i \sum_{\alpha} (1 - \epsilon_{\alpha} x) X_{\alpha} \omega \mu_{\alpha} \sum_{n \rightarrow -\infty}^{\infty} \int_0^{\infty} d\tau \int d^3u u_{\perp}^2 g_{\alpha} \\ & \times e^{iD_{n\alpha}(\epsilon_B)\tau} [F_{n\alpha}(\tau)]^{(|n|-1)} \frac{\Pi_{n\alpha}^- \Pi_{n\alpha}^+}{(W_n^- W_n^+)^{|n|}}, \end{aligned} \quad (10)$$

$$\begin{aligned} \vec{Q}_P = & i \sum_{\alpha} \epsilon_{\alpha} X_{\alpha} \omega \frac{c}{\omega} \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u e^{iD_{n\alpha}(0)\tau} u_{\perp} \\ & \times \frac{g_{\alpha} b_{\alpha} \sin \psi}{\gamma_{\alpha}} \pi_{n\alpha}^* \pi_{n\alpha} - i \sum_{\alpha} \epsilon_{\alpha} \mu_{\alpha} X_{\alpha} \omega \frac{c}{\Omega_{\alpha}} \\ & \times \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u e^{iD_{n\alpha}(0)\tau} u_{\perp}^3 g_{\alpha} \left[\frac{J_{n\alpha}}{2} \hat{\mathbf{e}}_y \pi_{n\alpha} \right. \\ & \left. + \Phi_{n\alpha}^* \pi_{n\alpha} \right]^H - (\hat{\mathbf{e}}_y \hat{\mathbf{e}}_z + \hat{\mathbf{e}}_z \hat{\mathbf{e}}_y) \\ & \times \sum_{\alpha} \epsilon_{\alpha} X_{\alpha} \frac{c}{\Omega_{\alpha}} \int d^3u \frac{u_{\parallel} g_{\alpha}}{\gamma_{\alpha}}, \end{aligned} \quad (11)$$

where $X_{\alpha} = \omega_p^2 / \omega^2$.

The last term in Eq. (11) is indeed null with the symmetrical distribution g_{α} defined in Eq. (9), but appears here only for completeness. From Eq. (10), it is seen that tensor \vec{Q}_B , which incorporates the magnetic field inhomogeneities, may be separated into two parts, one of order (ϵ_{α}^0) , and another of order (ϵ_{α}^1) . In order to avoid quadratic terms with

simultaneous effects of magnetic field and density inhomogeneity, we make $\epsilon_B \rightarrow 0$ in the term of order (ϵ_{α}^1) . What remains is the following:

$$\begin{aligned} \vec{Q}_B = & i \sum_{\alpha} X_{\alpha} \omega \mu_{\alpha} \sum_{n \rightarrow -\infty}^{\infty} \int_0^{\infty} d\tau \int d^3u u_{\perp}^2 g_{\alpha} \\ & \times e^{iD_{n\alpha}(\epsilon_B)\tau} [F_{n\alpha}(\tau)]^{(|n|-1)} \frac{\Pi_{n\alpha}^- \Pi_{n\alpha}^+}{(W_n^- W_n^+)^{|n|}} \\ & - ix \sum_{\alpha} \epsilon_{\alpha} X_{\alpha} \omega \mu_{\alpha} \sum_{n \rightarrow -\infty}^{\infty} \int_0^{\infty} d\tau \int d^3u u_{\perp}^2 g_{\alpha} \\ & \times e^{iD_{n\alpha}(0)\tau} \pi_{n\alpha} \pi_{n\alpha}^*. \end{aligned} \quad (12)$$

We can at this point rewrite the dielectric tensor, adding together all the terms containing ϵ_{α} into a quantity which will be denoted by $\vec{\chi}_P$, while the term which contains the magnetic field inhomogeneity will be called $\vec{\chi}_B$,

$$\vec{\epsilon} = \vec{I} + \vec{\chi}_B + \vec{\chi}_P, \quad (13)$$

where

$$\begin{aligned} \vec{\chi}_P = & i \sum_{\alpha} \epsilon_{\alpha} X_{\alpha} \omega \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u e^{iD_{n\alpha}(0)\tau} u_{\perp} g_{\alpha} \\ & \times \left[\frac{c}{\omega} \frac{b_{\alpha} \sin \psi}{\gamma_{\alpha}} - \mu_{\alpha} u_{\perp} x \right] \pi_{n\alpha}^* \pi_{n\alpha} \\ & - i \sum_{\alpha} \epsilon_{\alpha} \mu_{\alpha} X_{\alpha} \omega \frac{c}{\Omega_{\alpha}} \sum_{n \rightarrow -\infty}^{+\infty} \int_0^{\infty} d\tau \int d^3u \\ & \times e^{iD_{n\alpha}(0)\tau} u_{\perp}^3 g_{\alpha} \left[\frac{J_{n\alpha}}{2} \hat{\mathbf{e}}_y \pi_{n\alpha} + \Phi_{n\alpha}^* \pi_{n\alpha} \right]^H, \end{aligned} \quad (14)$$

$$\begin{aligned} \vec{\chi}_B = & i \sum_{\alpha} X_{\alpha} \omega \mu_{\alpha} \sum_{n \rightarrow -\infty}^{\infty} \int_0^{\infty} d\tau \int d^3u u_{\perp}^2 g_{\alpha} e^{iD_{n\alpha}(\epsilon_B)\tau} \\ & \times [F_{n\alpha}(\tau)]^{(|n|-1)} \frac{\Pi_{n\alpha}^- \Pi_{n\alpha}^+}{(W_n^- W_n^+)^{|n|}}. \end{aligned} \quad (15)$$

A. Evaluation of the velocity and time integrals appearing in the $\vec{\chi}_B$ tensor

The evaluation of the velocity integrals which appear in the $\vec{\chi}_B$ tensor, Eq. (15), can be analytically performed both in the weakly relativistic and in the nonrelativistic limit. After these integrals, the components of $\vec{\chi}_B$ can be written in terms of the so-called inhomogeneous plasma dispersion function PDF, as demonstrated elsewhere.^{7,9} This inhomogeneous PDF can be related to other well-known plasma dispersion functions appearing in the case of homogeneous plasmas, like Fried–Conte or Shkarofskii functions.^{7,9} For the present application, we will consider the nonrelativistic limit. For an isotropic distribution such as Eq. (9), the nonrelativistic inhomogeneous PDF may be written as follows:

$$\mathcal{G}_{r,p,m,l} = -i \int_0^\infty dt \frac{(it)^r e^{izt}}{(1+i\zeta_\alpha t)^p} e^{-\beta t^2} e^{-(\nu_\alpha^2 + \chi_{n\alpha}^2 t^2)/(1+i\zeta_\alpha t)}$$

$$\times \frac{[H_{n\alpha}(t)]^m}{[S_{n\alpha}(t)]^l} I_l \left(\frac{S_{n\alpha}(t)}{1+i\zeta_\alpha t} \right), \tag{16}$$

where

$$H_{n\alpha}(t) = \nu_\alpha^2 - i2S_n \nu_\alpha \sin \psi \chi_{n\alpha} t - \chi_{n\alpha}^2 t^2,$$

$$S_{n\alpha}(t) = \sqrt{\nu_\alpha^4 - 2\nu_\alpha^2 \cos 2\psi \chi_{n\alpha}^2 t^2 + \chi_{n\alpha}^4 t^4},$$

and where we have introduced the following definitions: $t = (\omega/\mu_\alpha) \tau$, $z = \mu_\alpha \delta_{n\alpha}$, $\delta_{n\alpha} = 1 - nY_\alpha(1 + \epsilon_B x)$, $\beta = \mu_\alpha N_\parallel^2/2$, $\chi_{n\alpha} = \mu_\alpha^{1/2} \rho_n$, $\zeta_\alpha = N_B N_\perp \sin \psi/Y_\alpha$, $Y_\alpha = \Omega_\alpha/\omega$, $\rho_n = nN_B/2$, and $N_B = \epsilon_B c/\omega$. Moreover, $\nu_\alpha = N_\perp / (\mu_\alpha^{1/2} Y_\alpha) = ck_\perp / (\mu_\alpha^{1/2} \Omega_\alpha)$, and $I_n(z)$ is the modified Bessel function of the first kind. The relationship between the inhomogeneous PDF and well-known functions from the literature is particularly useful for computational reasons, since the analytic properties of these functions are well studied, and many useful mathematical relationships can be found in the literature. These features can be illustrated here by the consideration of an example. For instance, let us consider the case of $\psi = \pi/2$, and harmonic $n = 0$. In that case the inhomogeneous PDF becomes

$$\mathcal{G}_{r,p,m,l} = -i \nu_\alpha^{2(m-l)} \int_0^\infty dt \frac{(it)^r e^{i\mu_\alpha t}}{(1+i\zeta_\alpha t)^p} e^{-\beta t^2}$$

$$\times e^{-\nu_\alpha^2/(1+i\zeta_\alpha t)} I_l \left(\frac{\nu_\alpha^2}{1+i\zeta_\alpha t} \right)$$

$$= -i \nu_\alpha^{2(m-l)} \int_0^\infty dt \frac{(it)^r e^{i\mu_\alpha t}}{(1+i\zeta_\alpha t)^p} e^{-\beta t^2} \mathcal{H}_n \left(\frac{\nu_\alpha^2}{1+i\zeta_\alpha t} \right), \tag{17}$$

where we have introduced the auxiliary function \mathcal{H}_n ,

$$\mathcal{H}_n(x) \equiv e^{-x} I_n(x). \tag{18}$$

For the particular case of $p = 1$, we may use Eq. (69) of Ref. 18, obtaining

$$\mathcal{G}_{r,p,m,l} = -i2\nu_\alpha^{2(m-l)} \int_0^\infty du u e^{-u^2} J_l^2(\sqrt{2}\nu_\alpha u)$$

$$\times \int_0^\infty dt (it)^r e^{i(\mu_\alpha - \zeta_\alpha u^2)t} e^{-\beta t^2}. \tag{19}$$

Using now the following definition of the Fried–Conte function and its derivative,

$$Z(z) = i \int_0^\infty dy e^{(izy - y^2/4)}, \tag{20a}$$

$$Z^{(n)}(z) = i \int_0^\infty dy (iy)^n e^{(izy - y^2/4)}, \tag{20b}$$

one may express the inhomogeneous PDF for $\psi = \pi/2$ in terms of the derivative of order r of the Fried–Conte function,

$$\mathcal{G}_{r,p,m,l} = -\frac{2\nu_\alpha^{2(m-l)}}{(\sqrt{2}\mu_\alpha N_\parallel)^{(r+1)}} \int_0^\infty du u e^{-u^2} J_l^2(\sqrt{2}\nu_\alpha u) Z^{(r)}$$

$$\times \left(\frac{\mu_\alpha - \zeta_\alpha u^2}{\sqrt{2}\mu_\alpha N_\parallel} \right). \tag{21}$$

For other values of p similar but more complicated expressions may be found.

Complete expressions for the components of $\vec{\chi}_B$ in terms of the inhomogeneous PDF, for any value of ψ , can be found in Ref. 9. As an example, for completeness, we reproduce here the expression for the components xy and yx of that tensor:

$$(\chi_B)_{yx}^{xy} = -\sum_\alpha \mu_\alpha X_\alpha \sum_{n \rightarrow -\infty}^{\infty} \{ \pm i2\nu_\alpha \chi_{n\alpha} \sin \psi [\mathcal{G}_{1,1/2,3,|n|,|n|} - \nu_\alpha^2 \mathcal{G}_{1,1/2,3,|n|,|n|+1} + \chi_{n\alpha}^2 \mathcal{G}_{3,1/2,3,|n|,|n|+1}] \pm in [|n| \mathcal{G}_{0,1/2,1,|n|-1,|n|}$$

$$- \nu_\alpha^2 (\mathcal{G}_{0,1/2,2,|n|-1,|n|} - \mathcal{G}_{0,1/2,2,|n|,|n|+1}) + \chi_{n\alpha}^2 (\mathcal{G}_{2,1/2,2,|n|-1,|n|} + \mathcal{G}_{2,1/2,2,|n|,|n|+1})] - \nu_\alpha^2 \sin 2\psi$$

$$\times [\mathcal{G}_{0,1/2,3,|n|,|n|} - |n| \mathcal{G}_{0,1/2,2,|n|-1,|n|} - \nu_\alpha^2 \mathcal{G}_{0,1/2,3,|n|,|n|+1} + \chi_{n\alpha}^2 \mathcal{G}_{2,1/2,3,|n|,|n|+1}] - 2n\nu_\alpha \chi_{n\alpha} \cos \psi \mathcal{G}_{1,1/2,2,|n|-1,|n|} \}. \tag{22}$$

B. Evaluation of the velocity integrals appearing in the $\vec{\chi}_P$ tensor

In the nonrelativistic limit, the components of $\vec{\chi}_P$ may be written as

$$\vec{\chi}_P = i \sum_\alpha \epsilon_\alpha X_\alpha \omega \sum_{n \rightarrow -\infty}^{+\infty} \int_0^\infty d\tau e^{i(\omega - n\Omega_\alpha(0))\tau} \left[\left(\frac{c}{\omega} \frac{ck_\perp}{\Omega_\alpha} \sin \psi - 2\mu_\alpha x \right) \vec{I}^{(1)} - 2\mu_\alpha \frac{c}{\Omega_\alpha} (\vec{I}^{(2)} + \vec{I}^{(3)}) \right], \tag{23}$$

where

$$\vec{I}^{(1)} = \int d^3u e^{-ick_{\parallel}u_{\parallel}\tau} u_{\perp}^2 g_{\alpha} \vec{\pi}_{n\alpha}^* \vec{\pi}_{n\alpha},$$

$$\vec{I}^{(2)} = \int d^3u e^{-ick_{\parallel}u_{\parallel}\tau} u_{\perp}^3 g_{\alpha} \left[\frac{J_n \hat{e}_y}{2} \vec{\pi}_{n\alpha} \right]^H,$$

and

$$\vec{I}^{(3)} = \int d^3u e^{-ick_{\parallel}u_{\parallel}\tau} u_{\perp}^3 g_{\alpha} [\Phi_{n\alpha}^* \vec{\pi}_{n\alpha}]^H.$$

The evaluation of these integrals requires a large amount of algebra, and will not be shown here. The final outcome of the velocity integrals is presented in what follows.

For the $\vec{I}^{(1)}$ components,

$$I_{xx}^{(1)} = \frac{1}{\mu_{\alpha}} \left\{ \left[\frac{n^2}{v_{\alpha}^2} \mathcal{H}_n(v_{\alpha}^2) - 2v_{\alpha}^2 \mathcal{H}'_n(v_{\alpha}^2) \right] (1 - \cos^2 \psi) + \cos^2 \psi \frac{n^2}{v_{\alpha}^2} \mathcal{H}_n(v_{\alpha}^2) \right\} e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}, \quad (24a)$$

$$I_{(xy)}^{(1)} = \frac{1}{\mu_{\alpha}} [2v_{\alpha}^2 \sin \psi \cos \psi \pm in] \mathcal{H}'_n(v_{\alpha}^2) e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}, \quad (24b)$$

$$I_{(xz)}^{(1)} = -i \frac{ck_{\parallel}\tau}{\mu_{\alpha}} \left(\frac{n\Omega_{\alpha}}{ck_{\perp}} \cos \psi \mathcal{H}_n(v_{\alpha}^2) \pm i \sin \psi \frac{ck_{\perp}}{\mu_{\alpha}\Omega_{\alpha}} \mathcal{H}'_n(v_{\alpha}^2) \right) e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}, \quad (24c)$$

$$I_{yy}^{(1)} = \frac{1}{\mu_{\alpha}} \left\{ \left[\frac{n^2}{v_{\alpha}^2} \mathcal{H}_n(v_{\alpha}^2) - 2v_{\alpha}^2 \mathcal{H}'_n(v_{\alpha}^2) \right] (1 - \sin^2 \psi) + \sin^2 \psi \frac{n^2}{v_{\alpha}^2} \mathcal{H}_n(v_{\alpha}^2) \right\} e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}, \quad (24d)$$

$$I_{(yz)}^{(1)} = -i \frac{ck_{\parallel}\tau}{\mu_{\alpha}} \left(\frac{n\Omega_{\alpha}}{ck_{\perp}} \sin \psi \mathcal{H}_n(v_{\alpha}^2) \mp i \cos \psi \frac{ck_{\perp}}{\mu_{\alpha}\Omega_{\alpha}} \mathcal{H}'_n(v_{\alpha}^2) \right) e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}, \quad (24e)$$

$$I_{zz}^{(1)} = \frac{1}{\mu_{\alpha}} \left(1 - \frac{c^2 k_{\parallel}^2 \tau^2}{\mu_{\alpha}} \right) \mathcal{H}_n(v_{\alpha}^2) e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}. \quad (24f)$$

In Eqs. (24b), (24c), and (24e), the upper (lower) component of tensor $\vec{I}^{(1)}$, is related to the upper (lower) sign of the imaginary number.

For the $\vec{I}^{(2)}$ components,

$$I_{xx}^{(2)} = I_{xz}^{(2)} = I_{zx}^{(2)} = I_{zz}^{(2)} = 0, \quad (25a)$$

$$I_{(xy)}^{(2)} = \frac{1}{2\mu_{\alpha}} \left\{ \frac{n\Omega_{\alpha}}{ck_{\perp}} \cos \psi [\mathcal{H}_n(v_{\alpha}^2) + v_{\alpha}^2 \mathcal{H}'_n(v_{\alpha}^2)] \pm i \sin \psi \frac{ck_{\perp}}{\mu_{\alpha}\Omega_{\alpha}} \left[(1 - 2v_{\alpha}^2) \mathcal{H}'_n(v_{\alpha}^2) + \left(\frac{n^2}{v_{\alpha}^2} - 1 \right) \mathcal{H}_n(v_{\alpha}^2) \right] \right\} e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}, \quad (25b)$$

$$I_{yy}^{(2)} = \frac{n\Omega_{\alpha}}{ck_{\perp}} \sin \psi \frac{1}{\mu_{\alpha}} [\mathcal{H}_n(v_{\alpha}^2) + v_{\alpha}^2 \mathcal{H}'_n(v_{\alpha}^2)] e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}, \quad (25c)$$

$$I_{yz}^{(2)} = I_{zy}^{(2)} = -i \frac{1}{2\mu_{\alpha}} \frac{ck_{\parallel}\tau}{\mu_{\alpha}} [\mathcal{H}_n(v_{\alpha}^2) + v_{\alpha}^2 \mathcal{H}'_n(v_{\alpha}^2)] e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}. \quad (25d)$$

For the $\vec{I}^{(3)}$ components,

$$I_{xx}^{(3)} = \sin \psi \frac{n\Omega_{\alpha}}{\mu_{\alpha}ck_{\perp}} \left\{ \cos^2 \psi [-2\mathcal{H}_n(v_{\alpha}^2) + 2v_{\alpha}^2 \mathcal{H}'_n(v_{\alpha}^2)] + \frac{n^2}{v_{\alpha}^2} \mathcal{H}_n(v_{\alpha}^2) - (1 + 2v_{\alpha}^2) \mathcal{H}'_n(v_{\alpha}^2) \right\} \times e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}, \quad (26a)$$

$$I_{(xy)}^{(3)} = \frac{1}{2} \frac{\Omega_{\alpha}}{ck_{\perp}} \frac{1}{\mu_{\alpha}} \left\{ n \cos \psi (4 \cos^2 \psi - 3) [\mathcal{H}_n(v_{\alpha}^2) - v_{\alpha}^2 \mathcal{H}'_n(v_{\alpha}^2)] \pm i \sin \psi \left[\left(-2 \frac{n^2}{v_{\alpha}^2} - n^2 + v_{\alpha}^2 \right) \mathcal{H}_n(v_{\alpha}^2) + (2n^2 + v_{\alpha}^2) + 2v_{\alpha}^4 \mathcal{H}'_n(v_{\alpha}^2) \right] \right\} e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}, \quad (26b)$$

$$I_{(xz)}^{(3)} = -i \frac{1}{2\mu_{\alpha}} \frac{ck_{\parallel}\tau}{\mu_{\alpha}} \left\{ 2 \left[\left(\frac{n^2}{v_{\alpha}^2} - 1 \right) \mathcal{H}_n(v_{\alpha}^2) - \mathcal{H}'_n(v_{\alpha}^2) \right] \sin \psi \cos \psi \pm in \left[2 \cos^2 \psi \left(\frac{1}{v_{\alpha}^2} \mathcal{H}_n(v_{\alpha}^2) - \mathcal{H}'_n(v_{\alpha}^2) \right) - \frac{1}{v_{\alpha}^2} \mathcal{H}_n(v_{\alpha}^2) + 2 \mathcal{H}'_n(v_{\alpha}^2) \right] \right\} \times e^{-c^2 k^{(2\parallel)} \tau^2 / (2\mu_{\alpha})}, \quad (26c)$$

$$I_{yy}^{(3)} = \sin \psi \frac{n}{\mu_\alpha} \frac{\Omega_\alpha}{ck_\perp} \left\{ \cos^2 \psi [2\mathcal{H}_n(v_\alpha^2) - 2v_\alpha^2 \mathcal{H}'_n(v_\alpha^2)] + \left(\frac{n^2}{v_\alpha^2} - 1 \right) \mathcal{H}_n(v_\alpha^2) - (v_\alpha^2 + 1) \mathcal{H}'_n(v_\alpha^2) \right\} e^{-c^2 k_{\parallel}^2 \tau^2 / (2\mu_\alpha)}, \quad (26d)$$

$$I_{zy}^{(3)} = i \frac{1}{2\mu_\alpha} \frac{ck_\parallel \tau}{\mu_\alpha} \left\{ \left[\left(-2\frac{n^2}{v_\alpha^2} + 1 \right) \mathcal{H}_n(v_\alpha^2) + (v_\alpha^2 + 1) \mathcal{H}'_n(v_\alpha^2) + 2 \cos^2 \psi \left(\left[\frac{n^2}{v_\alpha^2} - 1 \right] \mathcal{H}_n(v_\alpha^2) - \mathcal{H}'_n(v_\alpha^2) \right) \right] \mp i \left[\frac{n}{v_\alpha^2} \mathcal{H}_n(v_\alpha^2) - n \mathcal{H}'_n(v_\alpha^2) \right] 2 \sin \psi \cos \psi \right\} e^{-c^2 k_{\parallel}^2 \tau^2 / (2\mu_\alpha)}, \quad (26e)$$

$$I_{zz}^{(3)} = n \frac{\Omega_\alpha}{ck_\perp} \frac{1}{\mu_\alpha} \sin \psi \left[1 - \frac{c^2 k_{\parallel}^2 \tau^2}{\mu_\alpha} \right] \mathcal{H}_n(v_\alpha^2) e^{-c^2 k_{\parallel}^2 \tau^2 / (2\mu_\alpha)}. \quad (26f)$$

C. Final form for the $\tilde{\chi}_P$ tensor and for the effective dielectric tensor

One must now apply the time integrals that appear in (23) to the results presented in Eqs. (24), (25), and (26). In doing so, the integrations are again identified with the Fried-Conte function (20), and we arrive at the final and compact expressions for the components of the tensor $\tilde{\chi}_P$. Since the calculations are also lengthy and tedious, but otherwise straightforward, details are again omitted, and we show in what follows the final expressions, evaluated at position $x = 0$:

$$(\chi_P)_{xx} = \sin \psi \frac{1}{k_\parallel} \frac{ck_\perp}{\omega} \sum_\alpha \left(\frac{1}{2\mu_\alpha} \right)^{1/2} \epsilon_\alpha \frac{X_\alpha}{Y_\alpha} \sum_{n \rightarrow -\infty}^{+\infty} \times \left[\frac{n^2}{v_\alpha^2} \mathcal{H}_n(v_\alpha^2) - 2v_\alpha^2 \mathcal{H}'_n(v_\alpha^2) \sin^2 \psi - \frac{1}{Y_\alpha} \frac{n}{v_\alpha^2} \left\{ 2 \sin^2 \psi [\mathcal{H}_n(v_\alpha^2) - v_\alpha^2 \mathcal{H}'_n(v_\alpha^2)] + \frac{n^2}{v_\alpha^2} \mathcal{H}_n(v_\alpha^2) - \mathcal{H}'_n(v_\alpha^2) - 2\mathcal{H}_n(v_\alpha^2) \right\} \right] Z(z_n), \quad (27a)$$

$$(\chi_P)_{yx}^{(xy)} = \frac{1}{k_\parallel} \frac{ck_\perp}{\omega} \sum_\alpha \left(\frac{1}{2\mu_\alpha} \right)^{1/2} \epsilon_\alpha \frac{X_\alpha}{Y_\alpha} \sum_{n \rightarrow -\infty}^{+\infty} \times \left[\sin \psi [2v_\alpha^2 \sin \psi \cos \psi + in] \mathcal{H}'_n(v_\alpha^2) - \frac{1}{Y_\alpha} \frac{1}{v_\alpha^2} \left\{ n \cos \psi [\mathcal{H}_n(v_\alpha^2) - 2 \sin^2 \psi [\mathcal{H}_n(v_\alpha^2) - v_\alpha^2 \mathcal{H}'_n(v_\alpha^2)]] \pm i \sin \psi \left[-\frac{n^2}{v_\alpha^2} \mathcal{H}_n(v_\alpha^2) + (n^2 + v_\alpha^2) \mathcal{H}'_n(v_\alpha^2) \right] \right\} \right] Z(z_n), \quad (27b)$$

$$(\chi_P)_{zx}^{(xz)} = \frac{1}{2} \frac{1}{k_\parallel} \sum_\alpha \epsilon_\alpha \frac{X_\alpha}{Y_\alpha} \sum_{n \rightarrow -\infty}^{+\infty} \left\{ \sin \psi Y_\alpha \times [-n \cos \psi \mathcal{H}_n(v_\alpha^2) \mp i v_\alpha^2 \sin \psi \mathcal{H}'_n(v_\alpha^2)] \pm in \left[\cos^2 \psi \left(\frac{1}{v_\alpha^2} \mathcal{H}_n(v_\alpha^2) - \mathcal{H}'_n(v_\alpha^2) \right) - \frac{1}{2v_\alpha^2} \mathcal{H}_n(v_\alpha^2) + \mathcal{H}'_n(v_\alpha^2) \right] + \left[\left(\frac{n^2}{v_\alpha^2} - 1 \right) \times \mathcal{H}_n(v_\alpha^2) - \mathcal{H}'_n(v_\alpha^2) \right] \sin \psi \cos \psi \right\} Z^{(1)}(z_n), \quad (27c)$$

$$(\chi_P)_{yy} = \sin \psi \frac{1}{k_\parallel} \frac{ck_\perp}{\omega} \sum_\alpha \left(\frac{1}{2\mu_\alpha} \right)^{1/2} \epsilon_\alpha \frac{X_\alpha}{Y_\alpha} \sum_{n \rightarrow -\infty}^{+\infty} \times \left[\frac{n^2}{v_\alpha^2} \mathcal{H}_n(v_\alpha^2) - 2v_\alpha^2 \mathcal{H}'_n(v_\alpha^2) \cos^2 \psi - \frac{1}{Y_\alpha} \frac{n}{v_\alpha^2} \left\{ 2 \cos^2 \psi [\mathcal{H}_n(v_\alpha^2) - v_\alpha^2 \mathcal{H}'_n(v_\alpha^2)] + \frac{n^2}{v_\alpha^2} \mathcal{H}_n(v_\alpha^2) - \mathcal{H}'_n(v_\alpha^2) \right\} \right] Z(z_n), \quad (27d)$$

$$(\chi_P)_{zy}^{(yz)} = \frac{1}{2} \frac{1}{k_\parallel} \sum_\alpha \epsilon_\alpha \frac{X_\alpha}{Y_\alpha} \sum_{n \rightarrow -\infty}^{+\infty} \left[Y_\alpha \sin \psi \times [-n \sin \psi \mathcal{H}_n(v_\alpha^2) \pm i v_\alpha^2 \cos \psi \mathcal{H}'_n(v_\alpha^2)] + \frac{n^2}{v_\alpha^2} \mathcal{H}_n(v_\alpha^2) - \frac{1}{2} \mathcal{H}'_n(v_\alpha^2) - \cos^2 \psi \left[\left(\frac{n^2}{v_\alpha^2} - 1 \right) \mathcal{H}_n(v_\alpha^2) - \mathcal{H}'_n(v_\alpha^2) \right] \pm i \left[\frac{1}{v_\alpha^2} \mathcal{H}_n(v_\alpha^2) - \mathcal{H}'_n(v_\alpha^2) \right] n \sin \psi \cos \psi \right] Z^{(1)}(z_n), \quad (27e)$$

$$(\chi_P)_{zz} = \sin \psi \frac{1}{k_{\parallel}} \frac{ck_{\perp}}{\omega} \sum_{\alpha} \left(\frac{1}{2\mu_{\alpha}} \right)^{1/2} \epsilon_{\alpha} \frac{X_{\alpha}}{Y_{\alpha}} \sum_{n=-\infty}^{+\infty} \left[1 - \frac{1}{Y_{\alpha}} \frac{n}{\nu_{\alpha}^2} \right] \mathcal{H}_n(\nu_{\alpha}^2) \left[Z(z_n) + \frac{1}{2} Z^{(2)}(z_n) \right], \quad (27f)$$

where the argument of the Fried–Conte function is

$$z_n = \left(\frac{\mu_{\alpha}}{2} \right)^{1/2} \frac{(\omega - n\Omega_{\alpha}(0))}{ck_{\parallel}}.$$

These expressions, together with the components of $\vec{\chi}_B$ as given by Eq. (22), must be introduced in Eq. (13) in order to have the complete expressions for the components of the effective dielectric tensor. They may appear to be very impressive. However, as we have already demonstrated, for practical use it is quite straightforward to proceed with the approximations that are convenient in each case, like selecting the most important harmonics, or using the small Larmor radius approximation. These expressions incorporate inhomogeneity effects and preserve the correct symmetry expected from the dielectric tensor, and the fact that they are ultimately written in terms of a well-known plasma dispersion function adds to their simplicity and utility for use in the study of plasma waves in situations where inhomogeneity effects are expected to be important.

In order to demonstrate the use of approximations of these complete expressions in practical use, we consider in what follows the case of propagation in the y – z plane, which means $\psi = \pi/2$, using as an example the component $(\chi_P)_{xx}$ for the electron population. For waves with frequency in the vicinity of the lower hybrid frequency, we can keep only harmonics $n=0, \pm 1$, and neglect the effect of higher harmonics. From Eq. (27), we obtain

$$\begin{aligned} (\chi_P)_{xx}^e &= \frac{1}{k_{\parallel}} \frac{ck_{\perp}}{\omega} \left(\frac{1}{2\mu_e} \right)^{1/2} \epsilon_e \frac{X_e}{Y_e} \left\{ -2\nu_e^2 \mathcal{H}'_0(\nu_e^2) Z(z_0) \right. \\ &+ \sum_{n=\pm 1} \left[\frac{1}{\nu_e^2} \mathcal{H}_n(\nu_e^2) - 2\nu_e^2 \mathcal{H}'_n(\nu_e^2) \right. \\ &- \frac{n}{Y_e} \frac{1}{\nu_e^2} \left. \left. \left[\frac{1}{\nu_e^2} \mathcal{H}_n(\nu_e^2) - [1 + 2\nu_e^2] \mathcal{H}'_n(\nu_e^2) \right] \right] Z(z_n) \right\}. \end{aligned}$$

The absolute values of the arguments of the Fried–Conte functions are

$$|z_{\pm}| = \left| \frac{\omega \pm \Omega_e}{k_{\parallel} v_e} \right| \approx \left| \frac{\Omega_e}{k_{\parallel} v_e} \right| = \frac{\lambda_{\parallel}}{r_{Le}},$$

where r_{Le} is the electron Larmor radius, and $\nu_e = (2T_e/m_e)^{1/2}$. For $\lambda_{\parallel} \gg r_{Le}$, it is possible to further neglect the contribution from the $n = \pm 1$ terms compared to the $n = 0$ contribution, and what remains is simply the following:

$$\begin{aligned} (\chi_P)_{xx}^e &= -\frac{1}{k_{\parallel}} \frac{ck_{\perp}}{\omega} \left(\frac{1}{2\mu_e} \right)^{1/2} \epsilon_e \frac{X_e}{Y_e} 2\nu_e^2 \mathcal{H}'_0(\nu_e^2) Z(z_0) \\ &= -\epsilon_e \frac{\sqrt{2}}{k_{\parallel}} X_e \nu_e^3 \mathcal{H}'_0(\nu_e^2) Z(z_0), \end{aligned} \quad (28)$$

where $z_0 = (\mu_e/2)^{1/2} \omega / (ck_{\parallel}) = \omega / (k_{\parallel} v_e)$. It is important to remember that ν_e and Y_e are negative quantities, according to the definitions presented along with Eq. (16).

III. CONCLUSIONS AND FUTURE DEVELOPMENTS

In the present study we have presented an effective dielectric tensor to be utilized in the dispersion relation for electromagnetic waves in weakly inhomogeneous plasmas with inhomogeneities both in the magnetic field and in other plasma parameters. This new development follows and unifies previous approaches developed for the case of magnetic field inhomogeneity with uniform density and plasma temperature,^{7–9} and for the case in which the magnetic field is homogeneous and other plasma parameters can be inhomogeneous.^{2–6} The development of this unified approach, incorporating both gradients of the magnetic field and of other plasma parameters, will make possible a considerable increase in the range of phenomena to be investigated.

We are presently working on the use of the formulation reported in the present paper in the investigation of phenomena that occur in regions where the field gradient is predominantly perpendicular, like the magnetic equator of planetary magnetospheres or the central region of coronal loops in the solar corona and chromosphere. Particularly, we are interested in the cross-field drift instabilities, which are due to plasma inhomogeneities and are assumed to be important in the energization phase of magnetotail substorms.^{17,19,20} The lack of Onsager symmetry of the conventional approach has already been recognized and corrected in an *ad hoc* manner,¹⁷ and it may be expected that the new expressions developed here may bring some interesting contribution to the understanding of the physical picture of the situation. We intend to report our findings in a forthcoming publication.

ACKNOWLEDGMENTS

This work has been partially supported by the Brazilian agencies Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Fundação de Amparo à Pesquisa do Estado do Rio Grande do Sul (FAPERGS), and Financiadora de Estudos e Projetos (FINEP).

¹V. S. Beskin, A. V. Gurevich, and Y. I. Istomin, *Sov. Phys. JETP* **65**, 715 (1987).

²R. A. Caldela Filho, R. S. Schneider, and L. F. Ziebell, *J. Plasma Phys.* **42**, 165 (1989).

³R. A. Caldela Filho, Ph.D. thesis, Universidade Federal do Rio Grande do Sul, Curso de Pós-Graduação em Física, 1990.

⁴R. A. Caldela Filho, R. S. Schneider, and L. F. Ziebell, *J. Plasma Phys.* **43**, 335 (1990).

⁵C. J. de H. Cavalcanti, M.Sc. Dissertation, Universidade Federal do Rio Grande do Sul, Curso de Pós-Graduação em Física, 1994.

- ⁶C. J. H. Cavalcanti, R. S. Schneider, and L. F. Ziebell, *J. Plasma Phys.* **52**, 195 (1994).
- ⁷R. Gaelzer, Ph.D. thesis, Universidade Federal do Rio Grande do Sul, Curso de Pós-Graduação em Física, 1995.
- ⁸R. Gaelzer, R. S. Schneider, and L. F. Ziebell, *Phys. Rev. E* **51**, 2407 (1995).
- ⁹R. Gaelzer, R. S. Schneider, and L. F. Ziebell, *Phys. Rev. E* **55**, 5859 (1997).
- ¹⁰R. S. Schneider, L. F. Ziebell, and R. Gaelzer, *Phys. Plasmas* **4**, 3091 (1997).
- ¹¹N. A. Krall and P. C. Liewer, *Phys. Rev. A* **4**, 2094 (1971).
- ¹²R. C. Davidson and N. T. Gladd, *Phys. Fluids* **18**, 1327 (1975).
- ¹³N. T. Gladd, *Phys. Fluids* **19**, 27 (1976).
- ¹⁴R. C. Davidson, N. T. Gladd, C. S. Wu, and J. D. Huba, *Phys. Fluids* **20**, 301 (1977).
- ¹⁵J. B. Hsia, S. M. Chiu, M. F. Hsia *et al.*, *Phys. Fluids* **22**, 1737 (1979).
- ¹⁶Y. M. Zhou, H. K. Wong, C. S. Wu, and D. Winske, *J. Geophys. Res.* **88**, 3026 (1983).
- ¹⁷P. H. Yoon, A. T. Y. Lui, and C.-L. Chang, *Phys. Plasmas* **1**, 3033 (1994).
- ¹⁸P. A. Robinson, *J. Math. Phys.* **27**, 1206 (1986).
- ¹⁹C. S. Wu, P. H. Yoon, L. F. Ziebell *et al.*, *J. Geophys. Res.* **97**, 141 (1992).
- ²⁰P. H. Yoon and A. T. Y. Lui, *Phys. Fluids B* **5**, 836 (1993).