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Comment on ''Onsager symmetry for inhomogeneous magnetized plasmas'' [Phys. Plasmas 3, 4325 (1996)]

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In his recent paper, Nambu addressed the problem of finding the response tensor for inhomogeneous plasmas and discussed its symmetry properties.[1] Early on in the Introduction of Ref. 1, it is recognized that the dielectric tensor for inhomogeneous plasmas, if derived using the ''localized mode approximation,'' does not feature Onsager symmetry, and therefore contains an unphysical imaginary part that is not connected to wave–particle resonance.[1] At this point, reference is made to a previous paper by Caldela Filho *et al.*, published in 1989, where these undesirable features of the dielectric tensor in the plane wave approximation were pointed out.[2]

However, in the arguments that appear at the end of the Introduction of Ref. 1, one finds the statement that the paper is devoted to ''propose a modified local mode approximation to recover the full symmetry properties for electromagnetic waves.'' Therefore, Ref. 1 fails to recognize that an effective dielectric tensor satisfying Onsager and Hermitian symmetry was already derived by Caldela Filho *et al.* with the use of the modified local mode approximation, and was available in the formerly mentioned paper by Caldela Filho *et al.*[2] In what follows we will give a brief account of basic features of the derivation of the effective dielectric tensor as obtained in Ref. 2, in order to restate the priority of their achievement.

In the paper by Caldela Filho *et al.*, it was assumed that the plasma parameters can be weakly inhomogeneous along a direction that is perpendicular to the direction of an external and uniform magnetic field.[2] The magnetic field was assumed to be along the z direction, $\mathbf{B}_0 = B_0\mathbf{e}_z$, while the inhomogeneity was considered to be along the x direction. This geometry is the same as the geometry utilized in Ref. 1, with the irrelevant difference that the latter assumes inhomogeneity along the y direction. Due to the weak inhomogeneity, in both cases the distribution function was expanded in Taylor series and only the first two terms were retained [see Eq. (9) of Ref. 1 and Eq. (3) of Ref. 2]. Using this expanded distribution function, a plane wave approximation for the dielectric tensor was obtained. The tensor obtained from this plane wave approximation, which does not possess Onsager symmetry, was called $\boldsymbol{\varepsilon}^0$ and was used to derive the effective dielectric tensor, as proposed by Beskin, Gurevich, and Istomin,[3]

$$\varepsilon_{ij} = \varepsilon_{ij}^0 + \frac{i}{2}\sum_l \frac{\partial^2 \varepsilon_{ij}^0}{\partial k_l\,\partial r_l}. \tag{1}$$

This is the same as Eq. (5) of Ref. 2 and can be recognized as the same as Eq. (23) of Ref. 1, which uses the same procedure to derive the modified dielectric tensor. After a lenghty calculation, Caldela Filho *et al.* arrived to the explicit expressions for the effective dielectric tensor components, which are reproduced in what follows:[2]

$$\begin{aligned}\varepsilon_{ij}' = {}& \delta_{ij} - \delta_{iz}\delta_{jz}\sum_\alpha \frac{X_\alpha}{n_\alpha}\int d\mathbf{p}\,\frac{\mathscr{L}(f_\alpha)}{\gamma_\alpha}\frac{p_\parallel}{p_\perp} + \sum_\alpha \frac{X_\alpha}{n_\alpha}\sum_{n=-\infty}^{+\infty}\int d\mathbf{p}\,\frac{p_\perp \varphi_0(f_\alpha)}{D_{n\alpha}}\left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{iz}+\delta_{jz}}(R_{ij}^R + iR_{ij}^I)\\ &+ (\delta_{iy}\delta_{jz} + \delta_{iz}\delta_{jy})\sum_\alpha \frac{X_\alpha}{n_\alpha}\frac{1}{m_\alpha\Omega_\alpha}\int d\mathbf{p}\,\frac{p_\parallel f_\alpha'}{\gamma_\alpha} + \sum_\alpha \frac{X_\alpha}{n_\alpha}\frac{1}{m_\alpha\omega}\sum_{n=-\infty}^{+\infty}\int d\mathbf{p}\,p_\perp\frac{f_\alpha' b_\alpha \sin\psi}{\gamma_\alpha D_{n_\alpha}}\left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{iz}+\delta_{jz}}\\ &\times (R_{ij}^R + iR_{ij}^I) + \sum_\alpha \frac{X_\alpha}{n_\alpha}\frac{1}{m_\alpha\Omega_\alpha}\sum_{n=-\infty}^{+\infty}\int d\mathbf{p}\,p_\perp^2\frac{\varphi_0(f_\alpha')}{D_{n_\alpha}}\left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{iz}+\delta_{jz}}\frac{1}{2}[(S_{ij}^R + iS_{ij}^I) + (S_{ji}^R + iS_{ji}^I)^*],\end{aligned} \tag{2}$$

where[2] (see also corrigendum appended at the end of Ref. 4)

$$f'_\alpha \equiv \frac{\partial f_\alpha}{\partial x}, \quad \mathscr{L}(f_\alpha) \equiv p_\parallel \frac{\partial f_\alpha}{\partial p_\perp} - p_\perp \frac{\partial f_\alpha}{\partial p_\parallel},$$

$$\varphi_0(g) \equiv \frac{\partial g}{\partial p_\perp} + \frac{k_\parallel}{m_\alpha \gamma_\alpha \omega}\left(p_\perp \frac{\partial g}{\partial p_\parallel} - p_\parallel \frac{\partial g}{\partial p_\perp}\right),$$

$$D_{n\alpha} \equiv \gamma_\alpha - k_\parallel p_\parallel/(m_\alpha \omega) - n\Omega_\alpha/\omega,$$

$$R^R_{xx} = J'^2_n + \cos^2\psi\left(\frac{n^2}{b^2_\alpha} J^2_n - J'^2_n\right), \quad R^I_{xx} = 0,$$

$$R^R_{xy} = \left(\frac{n^2}{b^2_\alpha} J^2_n - J'^2_n\right)\sin\psi\cos\psi, \quad R^I_{xy} = \frac{n}{b_\alpha} J_n J'_n,$$

$$R^R_{xz} = \frac{n}{b_\alpha} J^2_n \cos\psi, \quad R^I_{xz} = J_n J'_n \sin\psi,$$

$$R^R_{yy} = \frac{n^2}{b^2_\alpha} J^2_n + \cos^2\psi\left(J'^2_n - \frac{n^2}{b^2_\alpha} J^2_n\right), \quad R^I_{yy} = 0,$$

$$R^R_{yz} = \frac{n}{b_\alpha} J^2_n \sin\psi, \quad R^I_{yz} = -J_n J'_n \cos\psi,$$

$$R^R_{zz} = J^2_n, \quad R^I_{zz} = 0, \quad R^R_{ij} = R^R_{ji}, \quad R^I_{ij} = -R^I_{ji},$$

$$S^R_{xx} = \frac{n}{b_\alpha}\sin\psi\left[J'_n\left(J'_n - \frac{J_n}{b_\alpha}\right) - \xi\right],$$

$$S^I_{xx} = \cos\psi\left[\left(-\frac{n^2}{b^3_\alpha} J^2_n + \left(1 - \frac{n^2}{b^2_\alpha}\right) J_n J'_n + \frac{2}{b_\alpha} J'^2_n\right) + \eta\right],$$

$$S^R_{xy} = \frac{n}{b_\alpha}\cos\psi\left[\left(2\frac{n^2}{b^2_\alpha} J^2_n - \frac{J_n J'_n}{b_\alpha} - J^2_n - J'^2_n\right) + \xi\right],$$

$$S^I_{xy} = \sin\psi\left[\left(\frac{n^2}{b^2_\alpha} J_n J'_n - \frac{n^2}{b^3_\alpha} J^2_n\right) + \eta\right],$$

$$S^R_{xz} = \sin\psi\cos\psi\left(2\frac{n^2}{b^2_\alpha} J^2_n - J^2_n - 2\frac{J_n J'_n}{b_\alpha}\right),$$

$$S^I_{xz} = \left(\frac{n}{b_\alpha} J_n J'_n - \frac{n}{b^2_\alpha} J^2_n\right) + \cos^2\psi\left(2\frac{n}{b^2_\alpha} J^2_n - 2\frac{n}{b_\alpha} J_n J'_n\right),$$

$$S^R_{yx} = \frac{n}{b_\alpha}\cos\psi\left[\left(\frac{n^2}{b^2_\alpha} J^2_n + \frac{J_n J'_n}{b_\alpha} - 2J'^2_n\right) + \xi\right],$$

$$S^I_{yx} = \sin\psi\left[\left(\frac{J'^2_n}{b_\alpha} - \frac{n^2}{b^2_\alpha} J_n J'_n\right) + \eta\right],$$

$$S^R_{yy} = \frac{n}{b_\alpha}\sin\psi\left[\left(\frac{n^2}{b^2_\alpha} J^2_n - \frac{J_n J'_n}{b_\alpha}\right) + \xi\right],$$

$$S^I_{yy} = \cos\psi\left[\left(2\frac{n^2}{b^3_\alpha} J^2_n - \frac{n^2}{b^2_\alpha} J_n J'_n - \frac{J'^2_n}{b_\alpha}\right) - \eta\right],$$

$$S^R_{yz} = \left(\frac{n^2}{b^2_\alpha} J^2_n - \frac{J_n J'_n}{b_\alpha}\right) + \cos^2\psi\left(J^2_n - 2\frac{n^2}{b^2_\alpha} J^2_n + 2\frac{J_n J'_n}{b_\alpha}\right),$$

$$S^I_{yz} = \sin\psi\cos\psi\left(2\frac{n}{b^2_\alpha} J^2_n - 2\frac{n}{b_\alpha} J_n J'_n\right),$$

$$S^R_{zx} = \sin\psi\cos\psi\left(\frac{n}{b^2_\alpha} J^2_n - J'^2_n\right), \quad S^I_{zx} = -\frac{n}{b_\alpha} J_n J'_n,$$

$$S^R_{zy} = \frac{n^2}{b^2_\alpha} J^2_n + \cos^2\psi\left(J'^2_n - \frac{n^2}{b^2_\alpha} J^2_n\right), \quad S^I_{zy} = 0,$$

$$S^R_{zz} = \frac{n}{b_\alpha} J^2_n \sin\psi, \quad S^I_{zz} = -J_n J'_n \cos\psi,$$

$$\eta = \cos^2\psi\left(2\frac{n^2}{b^3_\alpha} J^2_n - J_n J'_n - 2\frac{J'^2_n}{b_\alpha}\right),$$

$$\xi = -\cos^2\psi\left(2\frac{n^2}{b^2_\alpha} J^2_n - J^2_n - 2J'^2_n\right).$$

The Bessel functions and its derivatives depend on the argument $b_\alpha = (k_\perp p_\perp / m_\alpha \Omega_\alpha)$, $\psi = \tan^{-1}(k_y/k_x)$ is the angle between the vector $k_\perp$ and the direction of the inhomogeneity ($k_\perp$ and $k_\parallel$ are, respectively, the perpendicular and parallel components of the wave vector), $X_\alpha \equiv \omega^2_{p\alpha}/\omega^2$, $\omega^2_{p\alpha} = 4\pi q^2_\alpha n_\alpha / m_\alpha$, n_α is the density of particles of species α, and γ_α is the relativistic factor. It is easy to verify that in the homogeneous limit the effective dielectric tensor $\boldsymbol{\varepsilon}$ reduces to the dielectric tensor of a homogeneous magnetized plasma. It is also easy to verify that $\varepsilon^h_{ij}(-\mathbf{B}_0) = \varepsilon^h_{ji}(\mathbf{B}_0)$ and $\varepsilon^{nh}_{ij}(-\mathbf{B}_0) = -\varepsilon^{nh}_{ji}(\mathbf{B}_0)$ [not $\varepsilon^{nh}_{ij}(-\mathbf{B}_0) = -\varepsilon^{nh}_{ij}(\mathbf{B}_0)$, as it is wrongly typed in Ref. 2], where ε^h_{ij} and ε^{nh}_{ij} are, respectively, the homogeneous and the nonhomogeneous parts of the effective dielectric tensor ε_{ij}.

The expressions for the effective dielectric tensor obtained in Ref. 2 and reproduced here are valid for electromagnetic waves propagating in arbitrary directions relative to the magnetic field and to the inhomogeneity, and are fully relativistic, in contrast to those obtained in Ref. 1, which are nonrelativistic. It is straightforward to verify that these expressions for the dielectric tensor satisfy both Hermitian and Onsager symmetries,

$$\varepsilon_{ij}[\mathbf{B}_0, \mathbf{k}; F_0(p^2_\perp, p_\parallel)] = \varepsilon_{ji}[-\mathbf{B}_0, -\mathbf{k}; F_0(p^2_\perp, -p_\parallel)], \tag{3}$$

where $F_0(p^2_\perp, p_\parallel)$ is the equilibrium distribution function, which is explicitly written in order to emphasize the necessary change in the sign of the parallel momentum, required in order to obtain the time-reversed distribution function. Equation (3) expresses the generalized Onsager symmetry, valid for fluctuations around metastable equilibrium.[5,6]

The expressions for the effective dielectric tensor as given by Eq. (2) have been used for numerical investigations on electron cyclotron absorption.[4,7] Another version of the effective dielectric tensor, valid for the case of inhomogeneous magnetic field, where the resonant denominator itself is modified by the inhomogeneity, was also obtained using the same general approach.[6,8]

The fact that these preceding developments were missed in Ref. 1 is remarkable, especially those appearing in Ref. 2. We hope to have contributed to shedding some light on the development of the research of the subject of dielectric properties of inhomogeneous plasmas.

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