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## SOME DYNAMIC MODELS OF RIGID MEMORY MECHANISMS

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### ABSTRACT

Rigid memory mechanisms have played an important role in the history of mankind, contributing greatly to the industrial, economic, social changes in society, thus leading to a real evolution of mankind. Used in automated tissue wars, in cars as distribution mechanisms, automated machines, mechanical transmissions, robots and mechatronics, precision devices, and medical devices, these mechanisms have been real support for mankind along the time. For this reason, it considered being useful this paper, which presents some dynamic models that played an essential role in designing rigid memory mechanisms.

**Keywords:** Distribution mechanism; Rigid memory mechanisms; Variable internal damping; Dynamic model; Angular speed variation; Dynamic coefficient.



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## 1. INTRODUCTION

Mechanisms with rigid memory or as are commonly known cam and punch mechanisms have played an essential role in technology and industry, managing at least two industrial revolutions followed by major changes across society. They were the first mechanical transmissions used in the wars of tissue that changed the face of the industry when they were introduced into massive work.

All rigid memory devices have also played an essential role in transport since the introduction of Otto's internal combustion engine. Rigid memory mechanisms have been indispensable in clocks, clocks, fine mechanics, small device mechanisms, and the medical industry, which today plays an essential role in engineering medicine. Today rigid memory mechanisms are widely used in the machine building industry, in robotics and mechatronics, power engineering, as mechanical transmissions.

The development and diversification of road vehicles and vehicles, especially of cars, together with thermal engines, especially internal combustion engines (being more compact, robust, more independent, more reliable, stronger, more dynamic etc.), has also forced the development of devices, mechanisms, and component assemblies at an alert pace. The most studied are power and transmission trains.

The four-stroke internal combustion engine (four-stroke, Otto or Diesel) comprises in most cases (with the exception of rotary motors) and one or more camshafts, valves, valves, and so on.

The classical distribution mechanisms are robust, reliable, dynamic, fast-response, and although they functioned with very low mechanical efficiency, taking much of the engine power and effectively causing additional pollution and increased fuel consumption, they could not be abandoned until the present. Another problem was the low speed from which these mechanisms begin to produce vibrations and very high noises.

Regarding the situation realistically, the mechanisms of cam casting and sticking are those that could have produced more industrial, economic, social revolutions in the development of mankind. They have contributed substantially to the development of internal combustion engines and their spreading to the detriment of external combustion (Steam or Stirling) combustion engines.

In 1680, Dutch physicist Christian Huygens designs the first internal combustion engine.



In 1807, the Swiss Francois Isaac de Rivaz invented an internal combustion engine that uses a liquid mixture of hydrogen and oxygen as fuel. However, Rivaz's engine for its new engine has been a major failure, so its engine has passed to the deadline, with no immediate application.

In 1824, English engineer Samuel Brown adapted a steam engine to make it work with gasoline.

In 1858, Belgian engineer Jean Joseph Etienne Lenoir invented and patented two years later, the first real-life internal combustion engine with spark-ignition, liquid gas (extracted from coal), a two-stroke engine . In 1863, all Belgian Lenoir is adapting a carburetor to his engine by making it work with oil (or gasoline).

In 1862, the French engineer Alphonse Beau de Rochas first patented the four-stroke internal combustion engine (but without building it).

It is the merit of German engineers Eugen Langen and Nikolaus August Otto to build (physically, practically the theoretical model of the French Rochas), the first four-stroke internal combustion engine in 1866, with electric ignition, charging and distribution in a form Advanced.

Ten years later (in 1876), Nikolaus August Otto patented his engine.

In the same year (1876), Sir Dougald Clerk, arranges the two-time engine of the Belgian Lenoir, (bringing it to the shape known today).

In 1885, Gottlieb Daimler arranges a four-stroke internal combustion engine with a single vertical cylinder and an improved carburetor.

A year later, his compatriot Karl Benz brings some improvements to the four-stroke gasoline engine. Both Daimler and Benz were working new engines for their new cars (so famous).

In 1889, Daimler improves the four-stroke internal combustion engine, building a "two cylinder in V", and bringing the distribution to today's classic form, "with mushroom-shaped valves."

In 1890, Wilhelm Maybach, builds the first four-cylinder four-stroke internal combustion.

In 1892, German engineer Rudolf Christian Karl Diesel invented the compression-ignition engine, in short the diesel engine.

In 2010, more than 800 million vehicles circulated across the planet (ANTONESCU, 2000; ANTONESCU; PETRESCU, 1985; ANTONESCU; PETRESCU, 1989; ANTONESCU et al., 1985a; ANTONESCU et al., 1985b; ANTONESCU et al., 1986; ANTONESCU et al., 1987; ANTONESCU et al., 1988; ANTONESCU et al., 1994; ANTONESCU et al., 1997; ANTONESCU et al., 2000 a; ANTONESCU et al. 2000b; ANTONESCU et al., 2001; AVERSA et al., 2017a; AVERSA et al., 2017b; AVERSA et al., 2017c; AVERSA et al., 2017d; AVERSA et al., 2017e; MIRSAYAR et al., 2017; PETRESCU et al., 2017a; PETRESCU et al., 2017b; PETRESCU et al., 2017c; PETRESCU et al., 2017d; PETRESCU et al., 2017e; PETRESCU et al., 2017f; PETRESCU et al., 2017g; PETRESCU et al., 2017h; PETRESCU et al., 2017i; PETRESCU et al., 2015; PETRESCU; PETRESCU, 2016; PETRESCU; PETRESCU, 2014; PETRESCU; PETRESCU, 2013a; PETRESCU; PETRESCU, 2013b; PETRESCU; PETRESCU, 2013c; PETRESCU; PETRESCU, 2013d; PETRESCU; PETRESCU, 2011; PETRESCU; PETRESCU, 2005a; PETRESCU; PETRESCU, 2005b; PETRESCU, 2015a; PETRESCU, 2015b; PETRESCU, 2012a; PETRESCU, 2012b; HAIN, 1971; GIORDANA et al., 1979; ANGELES; LOPEZ-CAJUN, 1988; TARAZA et al., 2001; WIEDERRICH; ROTH, 1974; FAWCETT; FAWCETT, 1974; JONES; REEVE, 1974; TESAR; MATTHEW, 1974; SAVA, 1970; KOSTER, 1974).

## **2. THE STATE OF THE ART**

The Peugeot Citroën Group in 2006 built a 4-valve hybrid engine with 4 cylinders the first cam opens the normal valve and the second with the phase shift. Almost all current models have stabilized at four valves per cylinder to achieve a variable distribution. Hain (1971) proposes a method of optimizing the cam mechanism to obtain an optimal (maximum) transmission angle and a minimum acceleration at the output. Giordano (1979) investigates the influence of measurement errors in the kinematic analysis of the camel.

In 1985, P. Antonescu presented an analytical method for the synthesis of the cam mechanism and the flat barbed wire, and the rocker mechanism. Angeles and Lopez-Cajun (1988) presented the optimal synthesis of the cam mechanism and oscillating plate stick. Taraza (2001) analyzes the influence of the cam profile, the variation of the angular speed of the distribution shaft and the power, load, consumption and emission parameters of the internal combustion engine.

Petrescu and Petrescu (2005), present a method of synthesis of the rotating camshaft profile with rotary or rotatable tappet, flat or roller, in order to obtain high yields at the exit.

Wiederrich and Roth, (1974), there is presented a basic, single-degree, dual-spring model with double internal damping for simulating the motion of the cam and punch mechanism. In the paper (FAWCETT; FAWCETT, 1974) is presented the basic dynamic model of a cam mechanism, stick and valve, with two degrees of freedom, without internal damping.

A dynamic model with both damping in the system, external (valve spring) and internal one is the one presented in the paper (JONES; REEVE, 1974). A dynamic model with a degree of freedom, generalized, is presented in the paper (Tesar and Matthew, 1974), in which there is also presented a two-degree model with double damping.

In the paper (SAVA, 1970) is proposed a dynamic model with 4 degrees of freedom, obtained as follows: the model has two moving masses these by vertical vibration each impose a degree of freedom one mass is thought to vibrate and transverse, generating yet another degree of freedom and the last degree of freedom is generated by the torsion of the camshaft.

Also in the paper (SAVA, 1970) is presented a simplified dynamic model, amortized. In (SAVA, 1970) there is also showed a dynamic model, which takes into account the torsional vibrations of the camshaft. In the paper (KOSTER, 1974) a four-degree dynamic model with a single oscillating motion mass is presented, representing one of four degrees of freedom. The other three freedoms result from a torsional deformation of the camshaft, a vertical bending (z), camshaft and a bending strain of the same shaft, horizontally (y), all three deformations, in a plane perpendicular to the axis of rotation. The sum of the momentary efficiency and the momentary losing coefficient is 1.

The work is especially interesting in how it manages to transform the four degrees of freedom into one, ultimately using a single equation of motion along the main axis. The dynamic model presented can be used wholly or only partially, so that on another classical or new dynamic model, the idea of using deformations on different axes with their cumulative effect on a single axis is inserted. In works (ANTONESCU et al., 1987; PETRESCU; PETRESCU, 2005a) there is presented a dynamic model with a degree of freedom, considering the internal damping of the system (c), the damping for which is considered a special function. More precisely, the damping coefficient of the system (c) is defined as a variable parameter depending on the reduced mass of the mechanism ( $m^*$  or  $J_{\text{reduced}}$ ) and time, i.e, c, depends on

the time derivative of  $m_{\text{reduced}}$ . The equation of differential movement of the mechanism is written as the movement of the valve as a dynamic response.

### 3. MATERIALS AND METHODS

#### 3.1. Dynamic model with a degree of freedom with double internal damping

Wiederrich and Roth (1974) presented a basic single-degree model with two springs and double internal damping to simulate the movement of the cam and punch mechanism (see Figure 1) and the relationships (1-2).

$$\ddot{x} + 2\xi_2\omega_2\dot{x} + \omega_2^2x = \omega_1^2y + 2\xi_1\omega_1\dot{y} \tag{1}$$

$$\omega_1 = \frac{K_1}{M}; \quad \omega_2 = \frac{(K_1 + K_2)}{M}; \quad 2\xi_1\omega_1 = \frac{c_1}{M}; \quad 2\xi_2\omega_2 = \frac{(c_1 + c_2)}{M} \tag{2}$$

The motion equation of the proposed system (1) uses the notations (relations) in the system (2);  $\omega_1$  and  $\omega_2$  represents the system's own pulses and is calculated from the relationship system (2), depending on the elasticities  $K_1$  and  $K_2$  of the system in Figure 1 and the reduced mass  $M$  of the system.

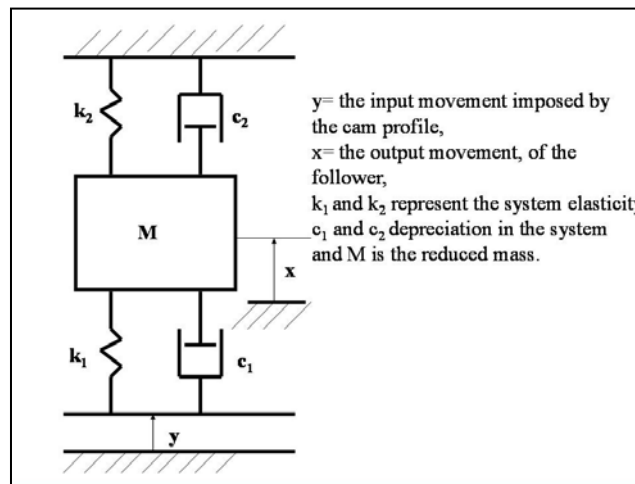


Figure 1: Dynamic model with a degree of freedom with double internal damping

#### 3.2. Dynamic model with two degrees of freedom without internal damping

Fawcett and Fawcett (1974) presented the basic dynamic model of a mechanism with cam, barrel and valve, with two degrees of freedom, without internal damping (see Figure 2, eq. 3-5).

$$y = x + z \tag{3}$$

$$m \frac{d^2 y}{dt^2} + (K_1 + K)y = K_1 x - s_0 \quad (4)$$

$$F_n = m_1 \ddot{x} - K_1(y - x) = m_1 \ddot{x} - k_1 z \quad (5)$$

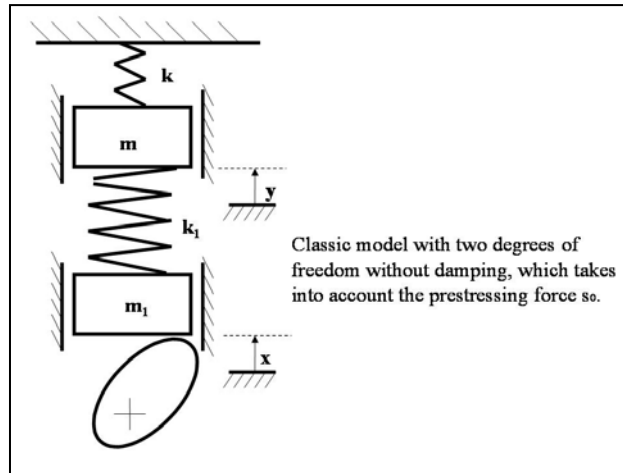


Figure 2: Dynamic model with two degrees of freedom without internal damping

### 3.3. Dynamic model with a degree of freedom with internal and external damping

A dynamic model with both system damping, external (spring valve) and internal damping has been presented in the paper (JONES; REEVE, 1974), (see Figure 3).

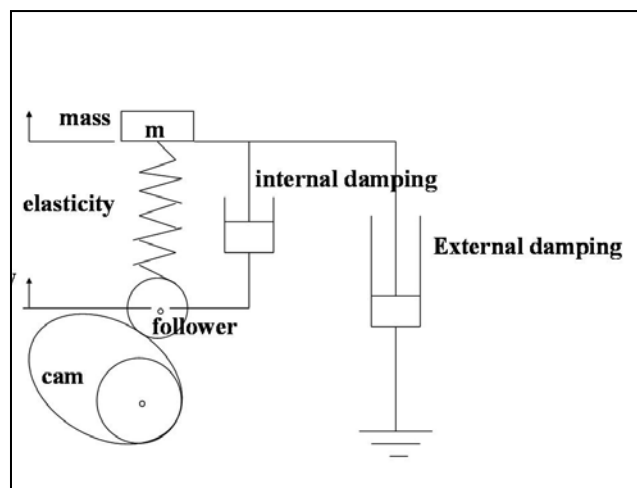


Figure 3: Dynamic model with a degree of freedom with internal and external damping

### 3.4. Dynamic model with a degree of freedom, taking into account the internal damping of the valve spring

A dynamic model with a generalized degree of freedom is presented by Tesar and Matthew (1974) (see Figure 4):

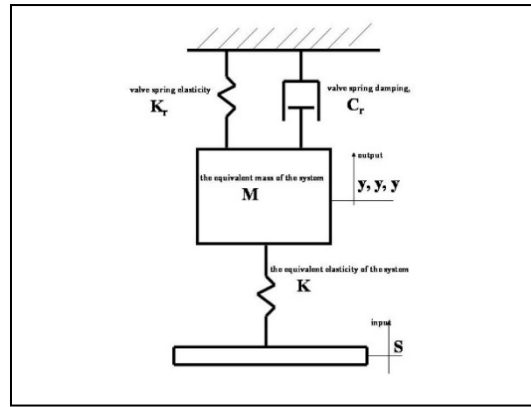


Figure 4: Dynamic model with a degree of freedom, taking into account the internal damping of the valve spring

The motion equation is written as (6):

$$\frac{M}{K} \frac{d^2 y}{dt^2} + \frac{C_r}{K} \frac{dy}{dt} + \frac{(K + K_r)}{K} y = S \quad (6)$$

Using the known relation (7), equation (6) takes the form (8):

$$\frac{d^K y}{dt^K} = y^{(K)} \omega^K \quad (7)$$

$$S = \mu_M y'' + \mu_C y' + \mu_K y \quad (8)$$

where the coefficients  $\mu$  have the form (9):

$$\mu_M = \frac{M}{K} \omega^2; \mu_C = \frac{C_r}{K} \omega; \mu_K = \frac{(K + K_r)}{K} \cong 1, \text{ with } K_r \ll K \quad (9)$$

The vertical reaction has the form (10):

$$F_K = K(S - y) + P = M\omega^2 y'' + C_r \omega y' + K_r y + P \quad (10)$$

### 3.5. Dynamic two-degree, dual damping model

Tesar and Matthew (1974) presented also the model with two degrees of freedom (see Fig. 5) with double damping:



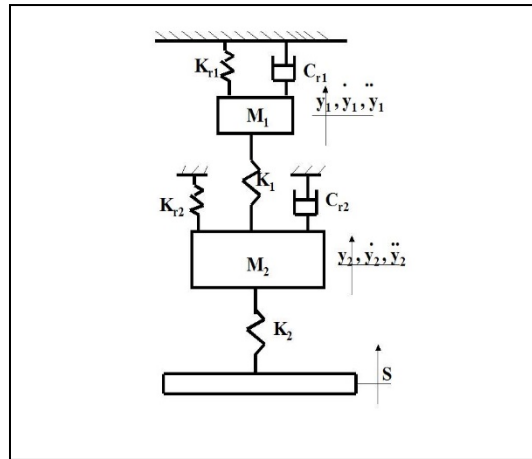


Figure 5: Dynamic two-degree, dual damping model

The calculation relationships used are (11-16):

$$S = P_4 y_1'''' + P_3 y_1''' + P_2 y_1'' + P_1 y_1' + P_0 y_1 \quad (11)$$

$$P_4 = \frac{M_1 M_2}{K_1 K_2} \omega^4 \quad (12)$$

$$P_3 = \frac{(M_2 C_{r1} + M_1 C_{r2})}{K_1 K_2} \omega^3 \quad (13)$$

$$P_2 = \frac{[M_2 (K_1 + K_{r1}) + M_1 (K_1 + K_2 + K_{r2}) + C_{r1} C_{r2}]}{K_1 K_2} \omega^2 \quad (14)$$

$$P_1 = \frac{[C_{r2} (K_1 + K_{r1}) + C_{r1} (K_1 + K_2 + K_{r2})]}{K_1 K_2} \omega \quad (15)$$

$$P_0 = \frac{(K_1 K_{r1} + K_1 K_2 + K_2 K_{r1} + K_1 K_{r2} + K_{r1} K_{r2})}{K_1 K_2} \quad (16)$$

### 3.6. Dynamic model with four degrees of freedom, with torsional vibrations

In the paper Sava (1970) a dynamic model with 4 degrees of freedom is proposed, obtained as follows:

The model has two moving masses; these by vertical vibration each impose a degree of freedom; one mass is thought to vibrate and transverse, generating yet another degree of freedom; and the last degree of freedom is generated by torsional torsion of the camshaft (see Figure 6).

The calculation relationships are (17-20).

The first two equations resolve normal vertical vibrations, the third equation takes into account the camshaft torsional vibration, and the last equation (independent of the others), the fourth, deals only with the transverse vibration of the system.

$$M\ddot{x}_1 + 2c\dot{x}_1 + (k + K)x_1 - c\dot{x}_2 - Kx_2 = -P(t) \tag{17}$$

$$m\ddot{x}_2 + 2c\dot{x}_2 + (K + k_{ac})x_2 - c\dot{x}_1 - Kx_1 = F_v + c\dot{s} + k_{ac}s \tag{18}$$

$$J\ddot{q} + c_r\dot{q} + k_r q - s'k_{ac}x_2 - cs'\dot{x}_2 = -s'(k_{ac}s + cs') \tag{19}$$

$$m\ddot{u} + k_t u = F_h \tag{20}$$

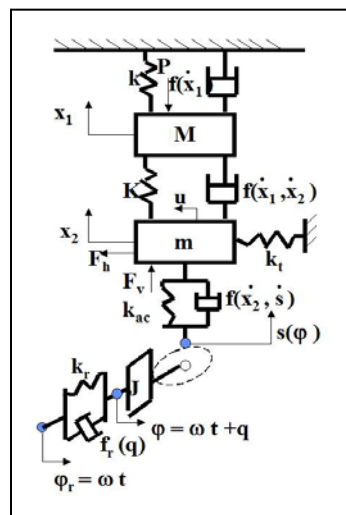


Figure 6: Dynamic model with four degrees of freedom, with torsional vibrations

### 3.7. Mono-dynamic damped dynamic model

Also in the paper Sava (1970), has presented a simplified dynamic model, amortized mono-mass (see figure 7).

The motion equation used has the form (21):

$$M\ddot{x} + c\dot{x} + (k + K)x = c\dot{s} + Ks - P \tag{21}$$

Which can be written more conveniently as (22):

$$x'' = A_1(y' - x') + \omega_1^2(y - x) - F \tag{22}$$

Where the coefficients  $A_1$ ,  $\omega_1^2$  and  $F$  are calculated with the expressions given in relation (23):

$$A_1 = \frac{ct_0}{M}; \omega_1^2 = \frac{(2K + k)t_0^2}{M}; F = \frac{Pt_0^2}{Ms_0} \quad (23)$$

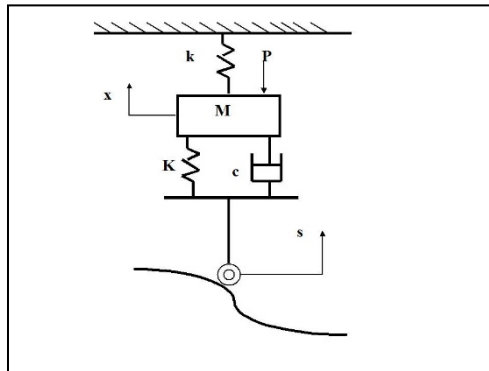


Figure 7: Mono-dynamic damped dynamic model

### 3.8. Dynamic damped two-mass model

In Figure 8 the bi-mass model proposed in the paper (SAVA, 1970) is presented.

The mathematical model is written (24, 25):

$$M\ddot{x}_1 + 2c\dot{x}_1 + (k + K)x_1 - c\dot{x}_2 - Kx_2 = -P(t) \quad (24)$$

$$m\ddot{x}_2 + 2c\dot{x}_2 + (K + k_{ac})x_2 - c\dot{x}_1 - Kx_1 = F_v + c\dot{s} + k_{ac}s \quad (25)$$

Equations (24-25) can be written as:

$$x_1'' = A_1(x_2' - 2x_1') + \omega_1^2(x_2 - x_1) - F \quad (26)$$

$$x_2'' = A_1(y' - 2x_2' + x_1') + \omega_2^2(y - x_2) + \mu\omega_1^2x_1 + [\mu F + (1 + \mu)y''](B_1 + B_2y' + B_3y) \quad (27)$$

where the notations (28) were used:

$$\mu = \frac{M}{m} \Rightarrow \text{the ratio of the two masses,}$$

$$\omega_2^2 = \frac{(k_{ac} + K)t_0^2}{m} \cong \frac{k_{ac}t_0^2}{m} \Rightarrow \text{the self dimensional pulse of the mass m,}$$

$$B_1 = \mu_1; B_2 = \frac{\mu_2 s_0}{\varphi_0}; B_3 = \mu_3 s_0 \quad (28)$$

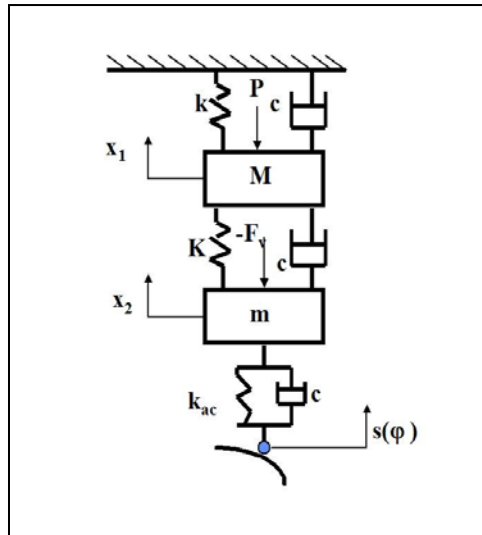


Figure 8: Dynamic damped two-mass model

### 3.9. A dynamic model with a single mass with torsional vibrations

In Figure 9 one can see a dynamic mono-mass model that also takes into account the torsional vibrations of the camshaft (SAVA, 1970).

The study points out that camshaft torsional vibrations has a negligible influence and can, therefore, be excluded from dynamic calculation models.

The same conclusion results from the work (SAVA, 1971) where the torsion model is studied in more detail.

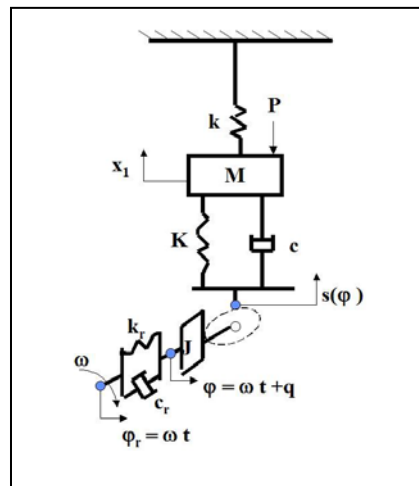


Figure 9: A dynamic model with a single mass with torsional vibrations

### 3.10. Influence of transverse vibrations

Tappet elasticity, variable length of the camshaft during cam operation, pressure angle variations, camshaft eccentricity, kinetic coupler friction, translation wear, technological and

manufacturing errors, system gaming, and other factors are factors that favor the presence of a transverse vibration of the rod weight (SAVA, 1970).

In the case of high amplitude vibrations, the response parameters to the last element of the tracking system will be influenced. Following Figure 10, it can be seen that if the curve a is the trajectory of the tip A, the point A will periodically reach point A', in which case the actual stroke of the  $y_r$  bar will change according to the law:  $y_r = y - y_v = y - u \cdot \tan v$ , where  $y$  is the longitudinal displacement of the tappet,  $u$  represents the transverse displacement of the mass  $m$ , of the tappet, and  $v$  is the pressure angle. The actual stroke,  $y_r$ , will change after the law (29):

$$y_r = y - y_v = y - u \tan v \quad (29)$$

The motion equation (dimensional) is written (30):

$$u'' + \frac{A_1 u}{(1 - A_2 y)^3} = [F + (1 + \mu) y''] (B_{11} + B_{21} y' + B_{31} y) \quad (30)$$

where were denoted by (31) the non-dimensional constants:

$$A_1 = \frac{3EI t_0^2}{ma^3}; \quad A_2 = \frac{s_0}{a}; \quad B_{11} = f_1 B_1; \quad B_{21} = f_1 B_2; \quad B_{31} = f_1 B_3 \quad (31)$$

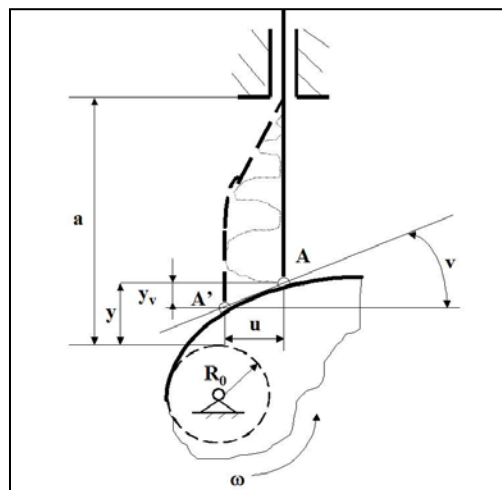


Figure 10: Influence of transverse vibrations

Also in the work (SAVA, 1970) the influence of the diameter of the rod, the lifting interval, the maximum length outside the tiller guides, the maximum lifting stroke and the various cam profiles on the A trajectory are analyzed.

Some conclusions:

It is noted that the reduction of the diameter of the rod of the tappet leads to the increase of the amplitude and the decrease of the average frequency of the transverse vibrations. Reducing the diameter of 1.35 times leads to an increase in amplitude of almost three times, and the average frequency decreases sensitively. Initial amplitudes are higher at the beginning of the interval, decreasing to the midpoint of the lifting interval, oscillation becoming insignificant, and towards the end of the rise due to the reduction of the length  $a$  by decreasing the  $y$  stroke the frequency increases and consequently the amplitude decreases from double to simple the beginning of the interval. Increasing the stick length beyond its 2.2 to 3 cm guides leads to an increase in vibration amplitude of about 25 times.

The law of motion without leaps in the input acceleration curve reduces the amplitude of the transverse vibration of the tappet. The author of the paper (SAVA, 1970) mentions that whatever the influence of the listed parameters is, for the cases considered, the amplitude values remain fairly small, and in case of reduced friction in the upper coupler, they can decrease even more. Consequently, the author of the paper (SAVA, 1970) concludes that the transverse vibrations of the follower exist and must draw the attention of the constructor only in the case of exaggerated values of the constants that characterize these vibrations. Regarding the distribution of internal combustion engines, the transverse vibration can be neglected without affecting the response parameters made at the valve.

### **3.11. Dynamic model with four degrees of freedom, with bending vibrations**

In the paper Koster (1974), has presented a four-degree dynamic model with a single oscillating motion mass, representing one of four degrees of freedom. The other three freedoms result from a torsional deformation of the camshaft, a vertical bending ( $z$ ), camshaft and a bending strain of the same shaft, horizontally ( $y$ ), all three deformations, in a plane perpendicular to the axis of rotation (see Figure 11). The sum of the momentary efficiency and the momentary losing coefficient is 1.

The work (KOSTER, 1974) is extremely interesting by the model it proposes (all types of deformations are being studied), but especially by the hypothesis it advances, namely: the cams speed is not constant but variable, the angular velocity of the cam  $\omega=f(\beta)$  being a function of the position of the cam (the cam angle of rotation  $\beta$ ).

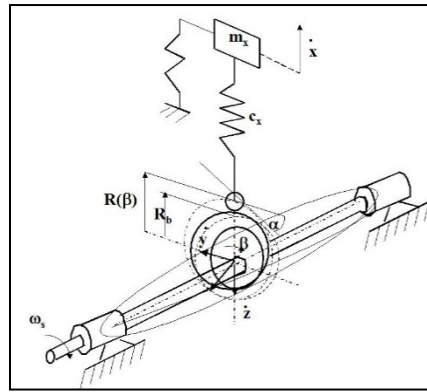


Figure 11: Dynamic model with four degrees of freedom, with bending vibrations

The angular velocity of the cam is a function of the position angle  $\beta$  (which we usually mark with  $\varphi$ ), and its variation is caused by the three deformations (torsion and two bends) of the shaft, as well as by the angular gaps existing between the source motors (drive) and camshaft.

The mathematical model taking into account the flexibility of the camshaft is the following; the rigidity of the cam between the cam and the cam is a function of the position  $\beta$  (cam angle of rotation), see the relationship (32):

$$\frac{1}{C(\beta)} = \frac{1}{C_x} + \frac{1}{C_z} + \left[ \frac{1}{C_\beta(\beta)} + \frac{1}{C_y} \right] \text{tg}^2 \alpha \quad (32)$$

$$\frac{1}{C_c} = \frac{1}{C_x} + \frac{1}{C_z} \quad (33)$$

Where  $1 / C_c$  see (33) is a constant rigidity given by the rigidity of the tappet ( $C_x$ ) and the cam ( $C_z$ ) in the direction of the tappet.

$$\frac{1}{C_{\tan}(\beta)} = \frac{1}{C_\beta(\beta)} + \frac{1}{C_y} \quad (34)$$

And  $1/C_{\tan}(\beta)$  see (34) represents the tangential stiffness,  $C_\beta$  being the torsional stiffness of the cam and  $C_y$  the flexural stiffness at the y axis of the cam, with  $C_\beta(\beta)$  given by the relation (35).

$$C_\beta(\beta) = \frac{K}{[R(\beta)]^2} \quad (35)$$

With (33) and (34) the relation (32) is rewritten in the form (36):

$$\frac{1}{C(\beta)} = \frac{1}{C_c} + \frac{tg^2 \alpha}{C_{\tan}(\beta)} \quad (36)$$

Where  $\alpha$  is the pressure angle, which is generally a function of  $\beta$ , and at flat tachets (used in distribution mechanisms), it has the constant value (zero):  $\alpha = 0$ .

The motion equation is written as (37):

$$m \cdot \ddot{x} + C(\beta) \cdot \dot{x} = C(\beta) \cdot h(\beta) \quad (37)$$

where  $h(\beta)$  is the motion law imposed by the cam.

The pressure angle,  $\alpha$ , thus influences (38):

$$tg \alpha = \frac{1}{R(\beta)} \frac{dh}{d\beta} \quad (38)$$

Where  $R(\beta)$  is the current radius, which gives the cam position (distance from the center of the cam to the cam contact point) and approximates by the mean radius  $R_{1/2}$ . The relation (38) can be put in the form (39); Where the average radius,  $R_{1/2}$ , is obtained with the formula (40):

$$tg \alpha = \frac{1}{R_{1/2}} \frac{\dot{h}}{\omega_s} \quad (39)$$

$$R_{1/2} = R_b + \frac{1}{2} h_m \quad (40)$$

$R_b$  is the radius of the base circle, and  $h_m$  is the maximum projected stroke of the tappet. This produces an average radius, which is used in the calculations for simplifications;  $\omega_s =$  machine angle, constant, given by machine speed. The equation (37) can now be written (41):

$$\ddot{x} = \frac{C_c \cdot [h(t) - x]}{m \cdot [1 + \frac{C_c}{C_{\tan}} (\frac{1}{R_{1/2}} \frac{\dot{h}}{\omega_s})^2]} \quad (41)$$

The solution of equation (41) is made for  $\alpha=0$ , with the following notations:

The period of natural vibration is determined with relation (42):

$$T_c = 2\pi \sqrt{\frac{m}{C_c}} \quad (42)$$



The period of the natural vibration period is obtained by the formula (43):

$$\tau = \frac{T_c}{t_m} \quad (43)$$

The slope during the lifting of the cam (44) is:

$$\operatorname{tg} \alpha_{mc} = \frac{h_m}{R_{1/2} \cdot \beta_m} \quad (44)$$

The shaft stiffness factor is obtained by the formula (45):

$$F_a = \frac{C_c}{C_{\tan}} \operatorname{tg}^2 \alpha_{mc} \quad (45)$$

With dimensional parameters given by (46);

$$H = \frac{h}{h_m}; X = \frac{x}{h_m}; T = \frac{t}{t_m}; \dot{H} = \frac{h}{h_m} t_m; \ddot{X} = \frac{x}{h_m} t_m^2 \quad (46)$$

The motion equation is written in the form (47):

$$\ddot{X} = \left(\frac{2\pi}{\tau}\right)^2 \cdot \frac{H - X}{1 + \dot{H}^2 \cdot F_a} \quad (47)$$

The nominal curve of the cam is known (48) and (49):

$$\dot{H} = \dot{H}(T) \quad (48)$$

$$H = H(T) \quad (49)$$

With (47), (48) and (49) the dynamic response is calculated by a numerical method.

The author of the paper (KOSTER, 1974) gives a numerical example for a motion law, corresponding to the cycloid cam (50):

$$H = T - \frac{1}{2\pi} \sin(2\pi T) \quad (50)$$

The work is especially interesting in how it manages to transform the four degrees of freedom into one, ultimately using a single equation of motion along the main axis. The dynamic model presented can be used wholly or only partially, so that on another classical or new dynamic model, the idea of using deformations on different axes with their cumulative effect on a single axis is inserted.

#### 4. RESULTS AND DISCUSSION

##### 4.1. A dynamic model with variable internal damping

Starting from the kinematic scheme of the classical distribution mechanism (see Figure 12), the dynamic, mono-dynamic (single degree), translatable, variable damping model (see Figure 13) is constructed, the motion equation of which is (PETRESCU, 2008):

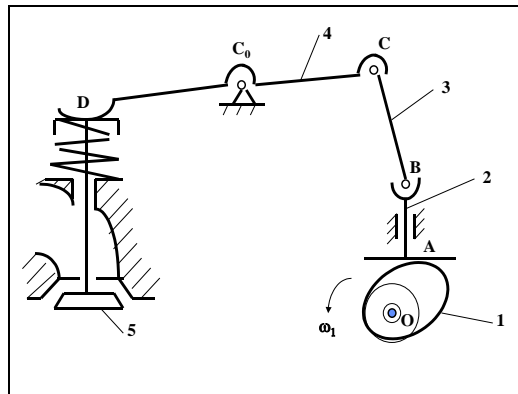


Figure 12: The kinematic scheme of the classic model, with distribution mechanism

$$M \cdot \ddot{x} = K \cdot (y - x) - k \cdot x - c \cdot \dot{x} - F_0 \quad (51)$$

Equation (51) is nothing else than the equation of Newton, in which the sum of forces on an element in a certain direction ( $x$ ) is equal to zero.

The notations in formula (51) are as follows:

M- mass of the reduced valve mechanism;

K- reduced elastic constants of the kinematic chain (rigidity of the kinematic chain);

k- elastic spring valve constant;

c - the damping coefficient of the entire kinematic chain (internal damping of the system);

$F \equiv F_0$  - the elastic spring force of the valve spring;

$x$  - actual valve displacement;

(the cam profile) reduced to the axis of the valve.

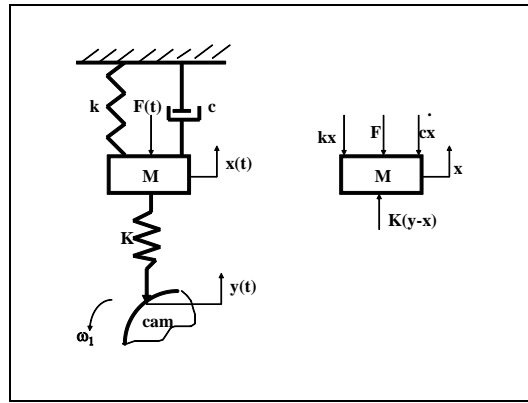


Figure 13: Mono - dynamic model, with internal depreciation of the variable system

The Newton equation (51) is ordered as follows:

$$M \cdot \ddot{x} + c \cdot \dot{x} = K \cdot (y - x) - (F_0 + k \cdot x) \quad (52)$$

At the same time the differential equation of the mechanism is also written as Lagrange, (53), (Lagrange equation):

$$M \cdot \ddot{x} + \frac{1}{2} \frac{dM}{dt} \cdot \dot{x} = F_m - F_r \quad (53)$$

Equation (53), which is nothing other than the Lagrange differential equation, allows for the low strength of the valve (54) to be obtained by the polynomial coefficients with those of the Newtonian polynomial (52), the reduced drive force at the valve (55), as well as the expression of c, ie the expression of the internal damping coefficient, of the system (56).

$$F_r = F_0 + k \cdot x = k \cdot x_0 + k \cdot x = k \cdot (x_0 + x) \quad (54)$$

$$F_m = K \cdot (y - x) = K \cdot (s - x) \quad (55)$$

$$c = \frac{1}{2} \cdot \frac{dM}{dt} \quad (56)$$

Thus a new formula (56) is obtained, in which the internal damping coefficient (of a dynamic system) is equal to half the derivative with the time of the reduced mass of the dynamic system.

The Newton motion equation (51, or 52), by replacing it with c takes the form (57):

$$M \cdot \ddot{x} + \frac{1}{2} \frac{dM}{dt} \cdot \dot{x} + (K + k) \cdot x = K \cdot y - F_0 \quad (57)$$

In the case of the classical distribution mechanism (in Figure 12), the reduced mass,  $M$ , is calculated by the formula (58):

$$M = m_5 + (m_2 + m_3) \cdot \left(\frac{\dot{y}_2}{\dot{x}}\right)^2 + J_1 \cdot \left(\frac{\omega_1}{\dot{x}}\right)^2 + J_4 \cdot \left(\frac{\omega_4}{\dot{x}}\right)^2 \quad (58)$$

formula in which or used the following notations:

$m_2$  = stick weight;

$m_3$  = the mass of the pushing rod;

$m_5$  = mass of the valve;

$J_1$  = moment of mechanical inertia of the cam;

$J_4$  = moment of mechanical inertia of the culbutor;

$\dot{y}_2$  = velocity of stroke imposed by cam law;

$\dot{x}$  = valve speed.

If  $i = i_{25}$ , the valve-to-valve ratio (made by the crank lever), the theoretical velocity of the valve (imposed by the motion law given by the cam profile) is calculated by the formula (59):

$$y \equiv \dot{y}_5 = \frac{\dot{y}_2}{i} \quad (59)$$

where:

$$i = \frac{CC_0}{C_0 D} \quad (60)$$

is the ratio of the crank arms.

The following relationships are written (61-66):

$$\dot{x} = \omega_1 \cdot x' \quad (61)$$

$$\ddot{x} = \omega_1^2 \cdot x'' \quad (62)$$

$$\dot{y}_2 = \omega_1 \cdot y_2' = \omega_1 \cdot i \cdot y' \quad (63)$$

$$\frac{\omega_1}{\dot{x}} = \frac{\omega_1}{\omega_1 \cdot x'} = \frac{1}{x'} \quad (64)$$

$$\omega_4 = \frac{\dot{y}_2}{CC_0} = \frac{\omega_1 \cdot y_2'}{CC_0} = \frac{\omega_1 \cdot y' \cdot i}{CC_0} = \frac{\omega_1 \cdot y' \cdot CC_0}{CC_0 \cdot C_0 D} = \frac{\omega_1 \cdot y'}{C_0 D} \quad (65)$$

$$\frac{\omega_4}{\dot{x}} = \frac{\omega_1 \cdot y'}{C_0 D \cdot \omega_1 \cdot x'} = \frac{1}{C_0 D} \frac{y'}{x'} \quad (66)$$

where  $y'$  is the reduced velocity imposed by the camshaft (by the law of camshaft movement), reduced to the valve axis.

With the previous relationships (60), (63), (64), (66), the relationship (58) becomes (67-69):

$$M = m_5 + (m_2 + m_3) \cdot \left(\frac{i \cdot y'}{x'}\right)^2 + J_1 \cdot \left(\frac{1}{x'}\right)^2 + J_4 \cdot \left(\frac{1}{C_0 D} \frac{y'}{x'}\right)^2 \quad (67)$$

or:

$$M = m_5 + \left[ i^2 \cdot (m_2 + m_3) + \frac{J_4}{(C_0 D)^2} \right] \cdot \left(\frac{y'}{x'}\right)^2 + J_1 \cdot \left(\frac{1}{x'}\right)^2 \quad (68)$$

or:

$$M = m_5 + m^* \cdot \left(\frac{y'}{x'}\right)^2 + J_1 \cdot \left(\frac{1}{x'}\right)^2 \quad (69)$$

We make the derivative  $dM/d\varphi$  and result the following relationships:

$$\frac{d\left[\left(\frac{y'}{x'}\right)^2\right]}{d\varphi} = \frac{2 \cdot y'}{x'} \cdot \frac{(y'' \cdot x' - x'' \cdot y')}{x'^2} = \frac{2 \cdot y'}{x'^2} \cdot (y'' \cdot x' - x'' \cdot y') = 2 \cdot \left(\frac{y'}{x'}\right)^2 \cdot \left(\frac{y''}{y'} - \frac{x''}{x'}\right) \quad (70)$$

$$\frac{d\left[\left(\frac{1}{x'}\right)^2\right]}{d\varphi} = \frac{2}{x'} \cdot \frac{-x''}{x'^2} = -2 \cdot \frac{x''}{x'^3} \quad (71)$$

$$\frac{dM}{d\varphi} = 2 \cdot m^* \cdot \left(\frac{y'}{x'}\right)^2 \cdot \left(\frac{y''}{y'} - \frac{x''}{x'}\right) - 2 \cdot J_1 \cdot \frac{x''}{x'^3} \quad (72)$$

Write the relationship (56) as:

$$c = \frac{\omega}{2} \cdot \frac{dM}{d\varphi} \tag{73}$$

which with (72) becomes:

$$c = \omega \cdot \left\{ [i^2 \cdot (m_2 + m_3) + \frac{J_4}{(C_0 D)^2}] \cdot \left(\frac{y'}{x'}\right)^2 \cdot \left(\frac{y''}{y'} - \frac{x''}{x'}\right) - J_1 \cdot \frac{x''}{x'^3} \right\} \tag{74}$$

or

$$c = \omega \cdot \left[ m^* \cdot \left(\frac{y'}{x'}\right)^2 \cdot \left(\frac{y''}{y'} - \frac{x''}{x'}\right) - J_1 \cdot \frac{x''}{x'^3} \right] \tag{75}$$

Where was noted:

$$m^* = i^2 \cdot (m_2 + m_3) + \frac{J_4}{(C_0 D)^2} \tag{76}$$

#### 4.2. Determination of motion equations

With relations (69), (62), (75) and (61), equation (52) is written first in the form (77), which develops in forms (78), (79) and (80):

$$M \cdot \omega^2 \cdot x'' + c \cdot \omega \cdot x' + (K + k) \cdot x = K \cdot y - F_0 \tag{77}$$

$$\begin{cases} \omega^2 \cdot x'' \cdot m_5 + \omega^2 \cdot m^* \cdot \left(\frac{y'}{x'}\right)^2 \cdot x'' + J_1 \cdot \left(\frac{1}{x'}\right)^2 \cdot x'' \cdot \omega^2 + \omega^2 \cdot x' \cdot m^* \cdot \left(\frac{y'}{x'}\right)^2 \cdot \left(\frac{y''}{y'} - \frac{x''}{x'}\right) - \\ x' \cdot \omega^2 \cdot J_1 \cdot \frac{x''}{x'^3} + (K + k) \cdot x = K \cdot y - F_0 \end{cases} \tag{78}$$

meaning:

$$\omega^2 \cdot m_5 \cdot x'' + \omega^2 \cdot m^* \cdot x' \cdot \left(\frac{y'}{x'}\right)^2 - \omega^2 \cdot m^* \cdot \left(\frac{y'}{x'}\right)^2 \cdot x'' + \omega^2 \cdot m^* \cdot y'' \cdot \frac{y'}{x'} + (K + k) \cdot x = K \cdot y - F_0 \tag{79}$$

And final form:

$$\omega^2 \cdot m_5 \cdot x'' + (K + k) \cdot x + \omega^2 \cdot m^* \cdot y'' \cdot \frac{y'}{x'} = K \cdot y - F_0 \tag{80}$$

which can also be written in another form:

$$\omega^2 \cdot \left( m_5 \cdot x'' + m^* \cdot y'' \cdot \frac{y'}{x'} \right) + (K + k) \cdot x = K \cdot y - F_0 \tag{81}$$

Equation (81) can be approximated to form (82) if we consider the theoretical input velocity  $y$  imposed by the camshaft profile (reduced to the valve axis) approximately equal to the velocity of the valve,  $x$ .

$$\omega^2 \cdot (m_s \cdot x'' + m^* \cdot y'') + (K + k) \cdot x = K \cdot y - F_0 \quad (82)$$

If the laws of entry with  $s$ ,  $s'$  (low speed),  $s''$  (low acceleration), equation (82) takes the form (83) and the more complete equation (81) takes the complex form (84):

$$\omega^2 \cdot (m_s \cdot x'' + m^* \cdot s'') + (K + k) \cdot x = K \cdot s - F_0 \quad (83)$$

$$\omega^2 \cdot (m_s \cdot x'' + m^* \cdot s'' \cdot \frac{s'}{x'}) + (K + k) \cdot x = K \cdot s - F_0 \quad (84)$$

#### 4.3. Solving the differential equation in two steps

The known differential equation, written in one of the above-mentioned forms, for example in form (85) (the equation was obtained through Taylor series developments), is solved twice. The first time is used for  $x'$  the value  $s'$  and for  $x''$  the value  $s''$ . In this way, the value  $x(0)$ , i.e. the dynamic displacement of the valve at step 0, is obtained. This displacement is derived numerically and  $x'(0)$  and  $x''(0)$  are obtained. The values thus obtained are introduced into the differential equation (which is used for the second consecutive time) and we obtain  $x(1)$ , i.e. the dynamic displacement of the sought valve,  $x$ , which is considered to be the final value. If one try to delete this process (for several steps), we will notice the lack of convergence towards a unique solution and the amplification of values at each pass (iteration). It is considered to solve the non-iterative equation, in two steps, but exactly and directly, the one-step solution, the second one, the first step being in fact a necessary mediation for the approximate determination of  $x'$  and  $x''$  values [54].

$$x = \frac{K \cdot s - k \cdot x_0 - m_s^* \cdot (D \cdot x'' + D' \cdot x') \cdot \omega^2 \cdot 0.001 - m_r^* \cdot (D \cdot s'' + D' \cdot s') \cdot \omega^2 \cdot 0.001 \cdot \frac{s'}{x'}}{K + k} \quad (85)$$

#### 4.4. The presentation of a differential equation, (dynamic model), which takes into account the mass of cam

Starting from the dynamic model presented, a new differential equation, describing the dynamic operation of the distribution mechanism from four-stroke internal combustion engines, will be obtained.

Practically, the formula expresses the reduced mass of the whole kinematic chain, and then changes the internal damping of the system,  $c$ , and automatically changes the entire dynamic (differential) equation, which entitles us to say that we are dealing with a new dynamic model, who also takes into consideration the chassis table.

The reduced mass  $M$  of the entire kinematic chain is now written in the form (86):

$$\begin{cases} M = m_5 + (m_2 + m_3).i^2 + J_1.\left(\frac{\omega_1}{\dot{X}}\right)^2 = m_{LS}^* + (m_2 + m_3).i^2 + J_1.\left(\frac{\omega_1}{\dot{X}}\right)^2 = \\ = m_{LS}^* + m_{LT}^* + J_1.\left(\frac{\omega_1}{\dot{X}}\right)^2 = m^* + J_1.\left(\frac{\omega_1}{\dot{X}}\right)^2 = m^* + \frac{m_1}{2}.r_A^2.\left(\frac{\omega_1}{\dot{X}}\right)^2 \end{cases} \quad (86)$$

The damping constant of the system now takes shape (87):

$$c = \frac{1}{2} \cdot \frac{dM}{dt} = \frac{1}{2} \cdot \left[ -2.J_1.\omega_1^2 \cdot \frac{\ddot{X}}{\dot{X}^3} + 2 \cdot \frac{m_1}{2} \cdot r_A \cdot r_A^I \cdot \frac{\omega_1^3}{\dot{X}^2} \right] \quad (87)$$

For the classical distribution mechanism the given value of (88) is found and is entered in the relation (87), which takes the form (89):

$$r_A \cdot r_A^I = (r_0^* + s + s'') \cdot s' \quad (88)$$

$$c = -J_1.\omega_1^2 \cdot \frac{\ddot{X}}{\dot{X}^3} + \frac{m_1}{2} \cdot (r_0^* + s + s'') \cdot s' \cdot \frac{\omega_1^3}{\dot{X}^2} \quad (89)$$

The differential equation (90) is still used:

$$M \cdot \ddot{X} + c \cdot \dot{X} + (K + k) \cdot X - K \cdot y + F_0 = 0 \quad (90)$$

The mass  $M$ , determined by (86) and the damping coefficient  $c$ , obtained with (89) in equation (90), is then introduced and one obtains a new differential equation (91), which is actually a new model dynamic base.

$$m^* \cdot \ddot{X} + J_1.\omega_1^2 \cdot \frac{\ddot{X}}{\dot{X}^2} - J_1.\omega_1^2 \cdot \frac{\ddot{X}}{\dot{X}^2} + \frac{m_1}{2} \cdot (r_0^* + s + s'') \cdot s' \cdot \omega_1^3 \cdot \frac{1}{\dot{X}} + (K + k) \cdot X - K \cdot y + F_0 = 0 \quad (91)$$

The differential equation (91) is written in the form (92) after the two identical terms that contain it on  $J_1$  are reduced:



$$m^* \cdot \ddot{X} + \frac{m_1^*}{2} \cdot (r_0^* + s + s'') \cdot s' \cdot \omega_1^3 \cdot \frac{1}{\dot{X}} + (K + k) \cdot X - K \cdot y + k \cdot x_0 = 0 \quad (92)$$

Using the transmission function, D and its first derivative, D', the differential equation (92), becomes equation (93):

$$m^* \cdot \omega_1^2 \cdot (x'' \cdot D + x' \cdot D') + \frac{m_1^*}{2} \cdot (r_0^* + s + s'') \cdot s' \cdot \omega_1^2 \cdot \omega_1 \cdot \frac{1}{x' \cdot D \cdot \omega_1} + (K + k) \cdot x - K \cdot y + k \cdot x_0 = 0 \quad (93)$$

Equation (93) is arranged in the form (94):

$$m^* \cdot \omega_1^2 \cdot D \cdot x'' + m^* \cdot \omega_1^2 \cdot D' \cdot x' + \frac{m_1^*}{2} \cdot \omega_1^2 \cdot \frac{(r_0^* + s + s'')}{D} \cdot \frac{s'}{x'} + (K + k) \cdot x - K \cdot s + k \cdot x_0 = 0 \quad (94)$$

Note x with  $s + \Delta x$ , (95):

$$x = s + \Delta x \quad (95)$$

With (95), equation (94) takes the form (96), where  $\Delta x$  represents the difference between the dynamic displacement x and the imposed s, both reduced to the valve axis:

$$\Delta x = \frac{-\omega_1^2 \cdot m^* \cdot [D \cdot x'' + D' \cdot x'] - \frac{m_1^*}{2} \cdot \omega_1^2 \cdot \frac{r_0^* + s + s''}{D} \cdot \frac{s'}{x'} - k \cdot (s + x_0)}{K + k} \quad (96)$$

To approximate the x' and x'' values, one use relations (97-100) and finally (99-100):

$$x' = \frac{dx'}{d\varphi} \Rightarrow dx' = x'' \cdot d\varphi \Rightarrow \Delta x' = x'' \cdot \Delta\varphi \cong s'' \cdot \Delta\varphi \quad (97)$$

$$x'' = \frac{dx''}{d\varphi} \Rightarrow dx'' = x''' \cdot d\varphi \Rightarrow \Delta x'' = x''' \cdot \Delta\varphi \cong s''' \cdot \Delta\varphi \quad (98)$$

$$x = s + \Delta x \Rightarrow x' = s' + \frac{d\Delta x}{d\varphi} = s' + \Delta \frac{dx}{d\varphi} = s' + \Delta x' \cong s' + s'' \cdot \Delta\varphi \quad (99)$$

$$x' = s' + \Delta x' \Rightarrow x'' = s'' + \frac{d\Delta x'}{d\varphi} = s'' + \Delta \frac{dx'}{d\varphi} = s'' + \Delta x'' \cong s'' + s''' \cdot \Delta\varphi \quad (100)$$

With relations (99) and (100), but also with approximation  $\frac{s'}{x'} \cong 1$ , equation (96) is written in the form (101):

$$\Delta x = \frac{-\omega^2 \cdot m^* \cdot [D \cdot (s'' + s''') \cdot \Delta\varphi + D' \cdot (s' + s'') \cdot \Delta\varphi] - \frac{m_1^*}{2} \cdot \omega^2 \cdot \frac{r_0^* + s + s''}{D} - k \cdot (s + x_0)}{K + k} \quad (101)$$

Equation (101) is ordered as (102):

$$\Delta x = \frac{-\omega^2 \cdot m^* \cdot [D' \cdot s' + (D + D' \cdot \Delta\varphi) \cdot s'' + D \cdot \Delta\varphi \cdot s'''] - \frac{m_1^*}{2 \cdot i^2} \cdot \omega^2 \cdot \frac{r_0 + s + s''}{D} - k \cdot (s + x_0)}{K + k} \quad (102)$$

Calculate  $\Delta x$  twice,  $\Delta x(0)$  and  $\Delta x \cdot \Delta x(0)$  gathered to generate  $x(0)$ , which is used to determine the variable angular velocity,  $\omega$ .

In  $\Delta x(0)$  equation  $\omega = \omega_n = \text{constant}$ .

In the second equation  $\Delta x$ ,  $\omega$  is determined using the first equation; for low speed  $x'$  and low acceleration  $x''$ , one now have two variants: either it can introduce directly all approximate values calculated with relations (99-100), or it can use  $x'(0)$  and  $x''(0)$  by direct (numerical) derivation of  $x(0)$ , which otherwise will only be used to find variable angular velocity,  $\omega$ .

With  $\Delta x$  gathered can to obtain the exact value of  $x$ , which one derive numerically and obtains the final (exact) values for reduced speed,  $x'$  and reduced acceleration,  $x''$ .

#### **4.5. Dynamic analysis for the sinus law, using the relationship (102), for the dynamic model considering the mass $m_1$ of the cam**

For this dynamic model (A3) there is a single dynamic diagram (Figure 14): Using the relation (102) obtained from the dynamic damping model of the variable system, considering the mass  $m_1$  of the cam, results the dynamic model A3 apply in the dynamic analysis presented in the diagram in Figure 14.

The SINus law is used, the engine speed,  $n = 5500$  [rpm], equal ascension and descent angles,  $\varphi_u = \varphi_c = 75^\circ$ , radius of the base circle,  $r_0 = 14$  [mm]. For the maximum stroke,  $h_T$ , equal to that of the valve,  $h_s$  ( $i = 1$ ), take the value of  $h = 5$  [mm]. A spring elastic constant,  $k = 60$  [N / mm] a valve spring compression,  $x_0 = 30$  [mm]. The mechanical yield is,  $\eta = 6.9\%$ .

The original model presented has the great advantage of accurately capturing vibrations within the analyzed system.

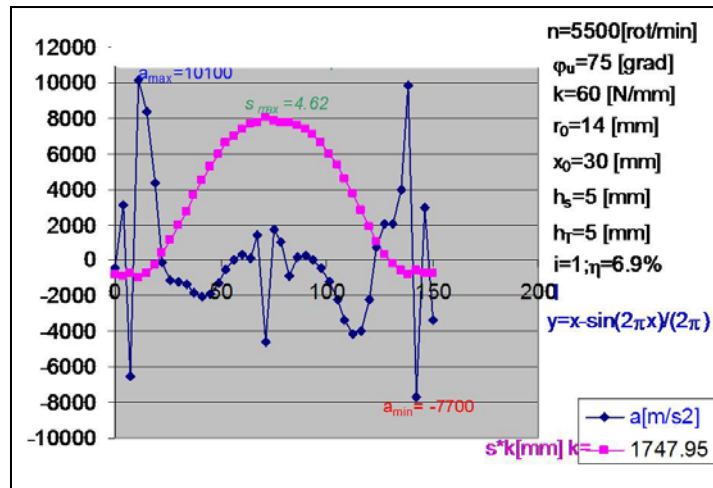


Figure 14: Dynamic analysis using the A3 dynamic model

## 5. CONCLUSIONS

The development and diversification of road vehicles and vehicles, especially of cars, together with thermal engines, especially internal combustion engines (being more compact, robust, more independent, more reliable, stronger, more dynamic etc.), has also forced the development of devices, mechanisms, and component assemblies at an alert pace. The most studied are power and transmission trains.

The four-stroke internal combustion engine (four-stroke, Otto or Diesel) comprises in most cases (with the exception of rotary motors) and one or more camshafts, valves, valves, and so on.

The classical distribution mechanisms are robust, reliable, dynamic, fast-response, and although they functioned with very low mechanical efficiency, taking much of the engine power and effectively causing additional pollution and increased fuel consumption, they could not be abandoned until the present. Another problem was the low speed from which these mechanisms begin to produce vibrations and very high noises.

Regarding the situation realistically, the mechanisms of cam casting and sticking are those that could have produced more industrial, economic, social revolutions in the development of mankind. They have contributed substantially to the development of internal combustion engines and their spreading to the detriment of external combustion (Steam or Stirling) combustion engines.

The problem of very low yields, high emissions and very high power and fuel consumption has been greatly improved and regulated over the past 20-30 years by developing and introducing modern distribution mechanisms that, besides higher yields immediately

deliver a high fuel economy) also performs optimal noise-free, vibration-free, no-smoky operation, as the maximum possible engine speed has increased from 6000 to 30000 [rpm].

The paper tries to provide additional support to the development of distribution mechanisms so that their performance and the engines they will be able to further enhance.

Particular performance is the further increase in the mechanical efficiency of distribution systems, up to unprecedented quotas so far, which will bring a major fuel economy.

Rigid memory mechanisms have played an important role in the history of mankind, contributing greatly to the industrial, economic, social changes in society, thus leading to a real evolution of mankind. Used in automated tissue wars, in cars as distribution mechanisms, automated machines, mechanical transmissions, robots and mechatronics, precision devices, and medical devices, these mechanisms have been real support for mankind along the time. For this reason, it considered useful this paper, which presents some dynamic models that played an essential role in designing rigid memory mechanisms.

The original model presented has the great advantage of accurately capturing vibrations within the analyzed system.

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## 8. NOMENCLATURE

$J^*$	is the moment of inertia (mass or mechanical) reduced to the camshaft
$J_{Max}^*$	is the maximum moment of inertia (mass or mechanical) reduced to the camshaft
$J_{min}^*$	is the minimum moment of inertia (mass or mechanical) reduced to the camshaft
$J_m^*$	is the average moment of inertia (mass or mechanical, reduced to the camshaft)
$J^{* \prime}$	is the first derivative of the moment of inertia (mass or mechanical, reduced to the camshaft) in relation with the $\varphi$ angle
$\eta_i$	is the momentary efficiency of the cam-pusher mechanism
$\eta$	is the mechanical yield of the cam-follower mechanism



$\tau$	is the transmission angle
$\delta$	is the pressure angle
$s$	is the movement of the pusher
$h$	is the follower stroke $h=s_{max}$
$s'$	is the first derivative in function of $\varphi$ of the tappet movement, $s$
$s''$	is the second derivative in raport of $\varphi$ angle of the tappet movement, $s$
$s'''$	is the third derivative of the tappet movement $s$ , in raport of the $\varphi$ angle
$x$	is the real, dynamic, movement of the pusher
$x'$	is the real, dynamic, reduced tappet speed
$x''$	is the real, dynamic, reduced tappet acceleration
$\ddot{x}$	is the real, dynamic, acceleration of the tappet (valve).
$v_{\tau} \equiv \dot{s}$	is the normal (cinematic) velocity of the tappet
$a_{\tau} \equiv \ddot{s}$	is the normal (cinematic) acceleration of the tappet
$\varphi$	is the rotation angle of the cam (the position angle)
$K$	is the elastic constant of the system
$k$	is the elastic constant of the valve spring
$x_0$	is the valve spring preload (pretension)
$m_c$	is the mass of the cam
$m_T$	is the mass of the tappet
$\omega_m$	the nominal angular rotation speed of the cam (camshaft)
$n_c$	is the camshaft speed
$n=n_m$	is the motor shaft speed $n_m=2n_c$
$\omega$	is the dynamic angular rotation speed of the cam
$\varepsilon$	is the dynamic angular rotation acceleration of the cam
$r_0$	is the radius of the base circle
$\rho=r$	is the radius of the cam (the position vector radius)
$\theta$	is the position vector angle
$x=x_c$ and $y=y_c$	are the Cartesian coordinates of the cam
$D$	is the dynamic coefficient
$\dot{D}$	is the derivative of $D$ in function of the time
$D'$	is the derivative of $D$ in function of the position angle of the camshaft, $\varphi$
$F_m$	is the motor force
$F_r$	is the resistant force.

## 9. AUTHORS' CONTRIBUTION

All the authors have contributed equally to carry out this work.

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