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# Solution of the Round Table Problem for the Case of $(p^k+1)$ Persons

by

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## Introduction

In 1905, an English genius puzzlist Dudeney [2] proposed the "Round Table Problem," as follows:

"Seat the same $n$ persons at a round table on $(n-1)(n-2)/2$ occasions so that no person shall ever have the same two neighbours twice. This is, of course, equivalent to saying that every person must sit once, and only once, between every possible pair."

This problem may look easy at a first glance, but one will soon find it quite difficult to solve. Dudeney [3] declared that he had found a subtle method for solving all cases of $n$, but he died without publishing it. To the authors' knowledge, solutions are known only for the cases of $n=p+1$ and $n=2p$, where $p$ is any prime number [5].

In this paper, we give a solution of the round table problem for the case of $n=p^k+1$, where $k$ is any positive integer, by using a cyclic permutation of the projective linear group PGL$(2, p^k)$.

## Preliminaries

To begin with, we will give some basic concepts on the projective linear group PGL$(2, p^k)$ [1]. Let $p$ be a prime and $k$ be a positive integer. We consider a Galois field GF$[p^k]$. Following the usual notation, we denote

$$q=p^k, \qquad K=\mathrm{GF}[q], \qquad K^*=K-\{0\}.$$

We denote by PG$(1, q)$ the one-dimensional projective space over $K$. The $(q+1)$ points in PG$(1, q)$ may be represented by the $q$ symbols $[1, x]$, where $x$ runs through GF$[q]$, and the additional symbol $[0, 1]$. We think of PG$(1, q)$ as the set $K\cup\{\infty\}$ where $\infty$ is the image of $[0, 1]$ under the bijection $[x_0, x_1]\leftrightarrow x_1/x_0$. We put $K^+=K\cup\{\infty\}$ hereafter.

The projective linear group PGL$(2, q)$, that is PGL$(2, p^k)$, is defined as a permutation group over $K^+$, whose elements are given by

$$x\to(\alpha x+\beta)/(\gamma x+\delta), \qquad x\in K^+, \quad \alpha, \beta, \gamma, \delta\in K, \qquad \alpha\delta\neq\beta\gamma.$$

Thus, the number of elements in PGL(2, $q$) is $(q+1)q(q-1)$. It is known that PGL(2, $q$) constitutes a triply transitive group over $K^+$. Let

$$f(r)=r^2-ar-b\,, \qquad a\in K\,, \quad b\in K^*$$

be a minimum polynomial for the extended Galois field GF[$q^2$] over $K$. Then, the sequence of points $x_0, x_1, x_2, \cdots$ generated by

$$x_j=(ax_{j-1}+1)/bx_{j-1}\equiv P(x_{j-1})\,, \qquad j=1, 2, 3, \cdots \tag{1}$$

runs through all points of $K^+$ once and only once, and returns to the starting point $x_0$. This can be proved as a special case of Singer's theorem [4]. We call this sequence fundamental cyclic sequence by the permutation $P$, and denote it by

$$X=(x_0, x_1, x_2, \cdots, x_q)\,. \tag{2}$$

And $P$ is called generating permutation of the fundamental cyclic sequence $X$.

## Some properties of cyclic sequences

For the fundamental cyclic sequence $X$ in Equation (2), we define the induced cyclic sequence $sX+t$ by

$$sX+t=(sx_0+t\,, sx_1+t, sx_2+t, \cdots, sx_q+t)\,, \qquad s\in K^*\,, \qquad t\in K\,. \tag{3}$$

Of course, we regard $sx_0+t$ follows $sx_q+t$. Every point in the fundamental cyclic sequence by $P$ is represented by

$$x_j=P(x_{j-1})=P(P(x_{j-2}))=P^2(x_{j-2})=\cdots=P^j(x_0)\,, \qquad j=0, 1, 2, \cdots, q\,.$$

Then, the points in the induced cyclic sequence $sX+t$ are written as

$$sP^j(x_0)+t\,, \qquad j=0, 1, 2, \cdots, q\,.$$

Define a permutation $R$ by

$$R(x)=sx+t\,, \qquad x\in K^+\,.$$

Then, by using the relations

$$sP^j(x_0)+t=[RP^jR^{-1}]R(x_0)=[RPR^{-1}]^jR(x_0)\,, \qquad j=0, 1, 2, \cdots, q\,,$$

the generating permutation for $sX+t$ is given by $RPR^{-1}$. So, in this connection, we call both fundamental cyclic sequence and induced cyclic sequence simply cyclic sequence, if no confusion occurs.

LEMMA 1. *Let $s$ and $t$ run through all the elements in $K^*$ and $K$ respectively, and generate $q(q-1)$ cyclic sequences $sX+t$ from $X$. Let $y_0, y_1, y_2$ be any three points in $K^+$. Then the ordered triple $\{y_0, y_1, y_2\}$ appears once and only once in some cyclic sequence as consecutive points.*

*Proof.* The fact that $y_0$, $y_1$, $y_2$ appear in some cyclic sequence as consecutive points means that one can choose $s$ $(\in K^*)$, $t$ $(\in K)$, and $j$ $(0 \leqq j \leqq t)$ so that the following equations hold:

$$y_0 = sP^j(x_0) + t\ ,$$
$$y_1 = sP^{j+1}(x_0) + t\ ,$$
$$y_2 = sP^{j+2}(x_0) + t\ .$$

Here, $sP^j(x_0)+t$ is a linear transformation of $x_0$, and the total number of such linear transformations obtained by varying $s, t, j$ is $(q+1)q(q-1)$. Hence, if we prove $s=s'$, $t=t'$, and $j=j'$ from the relation

$$sP^j(x)+t = s'P^{j'}(x)+t'\ , \qquad x \in K^+\ ,$$

then $(q+1)q(q-1)$ points of $sP^j(x)+t$ constitute a triply transitive group and the lemma holds.

First, let

$$sP^j(x_a)+t = s'P^{j'}(x_a)+t' = \infty\ .$$

Then, $P^j(x_a) = P^{j'}(x_a) = \infty$ and $j=j'$ holds. Next, let $P^j(x_b)=0$. By the equality

$$sP^j(x_b)+t = s'P^j(x_b)+t'\ ,$$

we get $t=t'$. From this, $s=s'$ is easily obtained. Q.E.D.

For a cyclic sequence $X$, we define inverted cyclic sequence $\bar{X}$, as a sequence in which the order of $X$ is inverted. The inverted cyclic sequence of $X$ in Equation (2) is

$$\bar{X} = (x_q,\ x_{q-1},\ x_{q-2},\ \cdots,\ x_1,\ x_0)\ .$$

LEMMA 2. *Let $X$ be a fundamental cyclic sequence generated by*

$$x_j = P(x_{j-1}) = (ax_{j-1}+1)/bx_{j-1}\ .$$

*If we get an induced cyclic sequence $sX+t$ from $X$, then its inverted cyclic sequence is given by*

$$\overline{sX+t} = -sX + sa/b + t\ . \tag{4}$$

*Proof.* Since $X$ is generated by $P(x)=(ax+1)/bx$, $\bar{X}$ is generated by $P^{-1}(x) = 1/(bx-a)$. Let

$$R(x) = -x + a/b\ ,$$

then

$$P^{-1}(x) = RPR^{-1}(x)\ .$$

This means that the inverted cyclic sequence $\bar{X}$ is induced by $R$, and $\bar{X}$ is given by

$$\bar{X} = -X + a/b\ .$$

Thus, we obtain

$$\overline{sX+t}=s\bar{X}+t=-sX+sa/b+t\,.$$

Q.E.D.

## Solution of the round table problem for $n=p^k+1$

By Lemma 1 and Lemma 2, we can easily get a solution of the round table problem for $n=p^k+1$. Running $s$ and $t$ through $K^*$ and $K$ respectively, we construct, from a fundamental cyclic sequence $X$, $q(q-1)$ induced cyclic sequences

$$sX+t\,,\qquad s\in K^*\,,\qquad t\in K\,. \tag{5}$$

For each cyclic sequence $sX+t$, its inverted cyclic sequence $-sX+sa/b+t$ is also contained in Equation (5) as a different cyclic sequence. Therefore, if we remove one from each pair of mutually inverted cyclic sequences, the remaining $q(q-1)/2$ cyclic sequences constitute a solution of the round table problem.

The proof is almost self-evident. For any three points $y_0, y_1, y_2$ in $K^+$, one of the ordered triples $\{y_0, y_1, y_2\}$ and $\{y_2, y_1, y_0\}$ appears once and only once in the $q(q-1)/2$ cyclic sequences.

In order to obtain a solution explicitly, we will find all pairs of mutually inverted cyclic sequences in Equation (5). Let $\omega$ be a primitive root of GF$[p^k]$, and express the elements of $K$ by

$$0,\ \omega^0,\ \omega^1,\ \omega^2,\ \cdots,\ \omega^{q-2}\,.$$

Then, from a cyclic sequence $X$, we obtain the following $q$ cyclic sequences:

$$sX,\ sX+\omega^0,\ sX+\omega^1,\ sX+\omega^2,\ \cdots,\ sX+\omega^{q-2}\,. \tag{6}$$

Since

$$-s=(p-1)s\,,$$

any two cyclic sequences in Equation (6) are not mutually inverted when $p>2$. Their inverted cyclic sequences are represented by

$$-sX,\ -sX+\omega^0,\ -sX+\omega^1,\ -sX+\omega^2,\ \cdots,\ -sX+\omega^{q-2}\,.$$

Here, the order of inverted cyclic sequences is not necessarily same as the order of the original cyclic sequences in Equation (6). Thus, if we suitably select $(q-1)/2$ elements in $K^*$, and let $s$ run through these elements, $q(q-1)/2$ cyclic sequences

$$sX,\ sX+\omega^0,\ sX+\omega^1,\ sX+\omega^2,\ \cdots,\ sX+\omega^{q-2}$$

constitute a solution of the round table problem for $n=p^k+1$.

Since

$$\omega^{(q-1)/2}=-1\,,$$

the elements of $K^*$ are expressed as

$$\pm\omega^0,\ \pm\omega^1,\ \pm\omega^2,\ \cdots,\ \pm\omega^{(q-3)/2}\ .$$

THEOREM 1. *Let $p$ be an odd prime, $k$ a positive integer, and put $q=p^k$ and $n=q+1$. Take a cyclic permutation of order $n$ from the projective linear group* PGL(2, $q$), *and let $X$ be the fundamental cyclic sequence generated by it. Then, $q(q-1)/2$ cyclic sequences of*

$$\omega^i(X+t)\ ,\qquad 0\leqq i\leqq(q-3)/2\ ,\qquad t\in K$$

*constitute a solution of the round table problem for $n$ persons, where $\omega$ is a primitive root of* GF$[p^k]$.

For $p=2$, the relation $s=-s$ holds, and two mutually inverted cyclic sequences always appear in a pair in Equation (6). And if $s\neq s'$, $sX+t$ and $s'X+t'$ are never mutually inverted for any $t$ and $t'$. Therefore, if $s$ runs through all the elements in $K^*$ and $t$ runs through just a half of the element of $K$, we can remove every one of the mutually inverted cyclic sequences. But, this is still quite tedious and we will continue our considerations a little further.

Let $X=(\infty, x_1, x_2, \cdots, x_q)$ be a cyclic sequence generated by the permutation $P$, which is defined in Equation (1). Then we have

$$x_1=a/b\ ,\qquad x_q=0\ ,\qquad x_1+x_q=a/b\ . \tag{7}$$

If we assume

$$x_j+x_{q+1-j}=a/b \tag{8}$$

and use the relations

$$x_{j+1}=P(x_j)=(ax_j+1)/bx_j\ ,$$
$$x_{q-j}=P^{-1}(x_{q+1-j})=1/(bx_{q+1-j}-a)\ ,$$

we obtain

$$x_{j+1}+x_{q-j}=a/b\ .$$

Thus, by mathematical induction, Equation (8) holds for $j=1, 2, \cdots, [(q+1)/2]$, where $[z]$ denotes the largest integer not greater than $z$.

From the above argument, for the case of $p=2$, two cyclic sequences $X+t$ and $X+t+a/b$ (or, more generally, $s(X+t)$ and $s(X+t+a/b)$) are mutually inverted. Therefore, let $a/b$ be a base of $K$, then the elements of $K$ are classified into two classes, one of which contains $a/b$ in its polynomial representation. Thus, if $t$ runs through one of the above classes and $s$ through $K^*$, $q(q-1)/2$ cyclic sequences of the form $sX+t$ constitute a solution of the round table problem for $n=2^k+1$.

Especially, if we substitute $aX/b$ by $X$, Equation (8) becomes

$$x_j+x_{q+1-j}=1\ . \tag{9}$$

We call a cyclic sequence having this property normalized cyclic sequence. It is easily shown that the permutation by which the normalized cyclic sequence is generated is given by

$$\{x+(b/a^2)\}/x .$$

Now, if we express the elements of Galois field by using the minimum polynomial, they are classified into two classes according as the unit is contained in its polynomial representation or not. So, by the above results, we obtain the following theorem:

THEOREM 2. *Let $k$ be a positive integer, $q=2^k$ and $n=q+1$. Take a cyclic permutation of order $n$ from the projective linear group* PGL(2, $q$), *and construct a normalized cyclic sequence $X$ from it. Let $\omega$ be a primitive root of* GF[$2^k$], *and let $t$ run through $2^{k-1}$ values of*

$$t=\omega(\delta_{k-2}\omega^{k-2}+\delta_{k-3}\omega^{k-3}+\ \cdots\ +\delta_1\omega+\delta_0) ,$$

*where $\delta_j$, $j=0, 1, 2, \cdots, k-2$, are either* 0 *or* 1. *Then, the $q(q-1)/2$ cyclic sequences $\omega^i(X+t)$, $i=0, 1, 2, \cdots, q-2$, constitute a solution of the round table problem for $n=2^k+1$.*

## Examples

*Example* 1. $n=5$ (by GF[$2^2$]).

Let $q=2^2$ and construct the extended Galois field GF[$q^2$] from GF[$q$]. Let $\omega$ be a primitive root of GF[$q$] and let $\omega^2+\omega+1$ be a minimum polynomial for GF[$q$], then we can take

$$f(r)=r^2+r+\omega$$

as a minimum plynomial for GF[$q^2$]. Therefore, $a=-1$, $b=-\omega^2$, and $P(x)=(x-\omega^2)/x$. Let $x_0=\infty$ and construct a normalized cyclic sequence $X$. Then we have

$$X=(\infty, \omega^0, \omega^1, \omega^2, 0) .$$

Take 0 and $\omega^1$ as two values of $t$ in Theorem 2, we obtain the following six cyclic sequences as a solution of the round table problem for $n=5$:

$$\begin{aligned}
\omega^0(X+0)&=(\infty, \omega^0, \omega^1, \omega^2, 0) ,\\
\omega^1(X+0)&=(\infty, \omega^1, \omega^2, \omega^0, 0) ,\\
\omega^2(X+0)&=(\infty, \omega^2, \omega^0, \omega^1, 0) ,\\
\omega^0(X+\omega^1)&=(\infty, \omega^2, 0, \omega^0, \omega^1) ,\\
\omega^1(X+\omega^1)&=(\infty, \omega^0, 0, \omega^1, \omega^2) ,\\
\omega^2(X+\omega^1)&=(\infty, \omega^1, 0, \omega^2, \omega^0) .
\end{aligned}$$

If we denote 0, 1, 2, $-\infty$ instead of $\omega^0$, $\omega^1$, $\omega^2$, 0, respectively, the above sequences are represented by

$$(\infty, 0, 1, 2, -\infty),$$
$$(\infty, 1, 2, 0, -\infty),$$
$$(\infty, 2, 0, 1, -\infty),$$
$$(\infty, 2, -\infty, 0, 1),$$
$$(\infty, 0, -\infty, 1, 2),$$
$$(\infty, 1, -\infty, 2, 0),$$

which correspond to

$$(1, 2, 3, 4, 5),$$
$$(1, 2, 4, 5, 3),$$
$$(1, 2, 5, 3, 4),$$
$$(1, 3, 2, 5, 4),$$
$$(1, 4, 2, 3, 5),$$
$$(1, 5, 2, 4, 3),$$

given by Dudeney [2].

*Example 2.* $n=9$, 17, 33.

Since we can easily obtain solutions of the round table problem for $n=9$, 17, 33 by the same procedure as in Example 1, we only list the minimum polynomials for $GF[2^k]$, $(k=3, 4, 5)$, the normalized cyclic sequences $X$'s, and the subsets of $K$ which give $2^{k-1}$ values of $t$ in Theorem 2.

For $n=9$:
Minimum polynomial $=\omega^3+\omega+1$,
$X=(\infty, \omega^0, \omega^2, \omega^5, \omega^3, \omega^1, \omega^4, \omega^6, 0)$,
4 values of $t=\{0, \omega^1, \omega^2, \omega^4\}$ .

For $n=17$:
Minimum polynomial $=\omega^4+\omega+1$,
$X=(\infty, \omega^0, \omega^3, \omega^{12}, \omega^8, \omega^{13}, \omega^4, \omega^5, \omega^7, \omega^9, \omega^{10}, \omega^1, \omega^6, \omega^2, \omega^{11}, \omega^{14}, 0)$
8 values of $t=\{0, \omega^1, \omega^2, \omega^3, \omega^5, \omega^6, \omega^9, \omega^{11}\}$

For $n=33$:
Minimum polynomial $=\omega^5+\omega+1$,
$X=(\infty, \omega^0, \omega^{15}, \omega^{16}, \omega^{20}, \omega^{10}, \omega^{13}, \omega^{19}, \omega^2, \omega^7, \omega^{30}, \omega^{21},$
$\omega^{29}, \omega^6, \omega^1, \omega^{12}, \omega^{23}, \omega^{18}, \omega^{27}, \omega^{26}, \omega^3, \omega^{25}, \omega^{17},$
$\omega^{22}, \omega^5, \omega^{11}, \omega^{14}, \omega^4, \omega^8, \omega^9, \omega^{24}, 0)$
16 values of $t=\{0, \omega^1, \omega^2, \omega^3, \omega^4, \omega^6, \omega^7, \omega^9, \omega^{12}, \omega^{13}, \omega^{19}, \omega^{20},$
$\omega^{21}, \omega^{24}, \omega^{28}, \omega^{30}\}$

*Example* 3. $n=10$ (by $GF[3^2]$).

We shall take again normalized cyclic sequences, though it is not obligatory for $p>2$. Take $\omega^2+\omega+2$ as a minimum polynomial for $GF[3^2]$ and $P(x)=(x-\omega^7)/x$ as a generating permutation of the fundamental cyclic sequence, then we have

$$X=(\infty, \omega^0, \omega^5, \omega^1, \omega^3, \omega^4, \omega^6, \omega^2, \omega^7, 0)\,.$$

From this sequence, we obtain the following 36 cyclic sequences as a solution of the round table problem for $n=10$:

$$\begin{aligned}
&\omega^0(X+0)=(\infty, \omega^0, \omega^5, \omega^1, \omega^3, \omega^4, \omega^6, \omega^2, \omega^7, 0)\\
&\omega^1(X+0)=(\infty, \omega^1, \omega^6, \omega^2, \omega^4, \omega^5, \omega^7, \omega^3, \omega^0, 0)\\
&\omega^2(X+0)=(\infty, \omega^2, \omega^7, \omega^3, \omega^5, \omega^6, \omega^0, \omega^4, \omega^1, 0)\\
&\omega^3(X+0)=(\infty, \omega^3, \omega^0, \omega^4, \omega^6, \omega^7, \omega^1, \omega^5, \omega^2, 0)\\
&\omega^0(X+\omega^0)=(\infty, \omega^4, \omega^2, \omega^7, \omega^5, 0, \omega^1, \omega^3, \omega^6, \omega^0)\\
&\omega^1(X+\omega^0)=(\infty, \omega^5, \omega^3, \omega^0, \omega^6, 0, \omega^2, \omega^4, \omega^7, \omega^1)\\
&\omega^2(X+\omega^0)=(\infty, \omega^6, \omega^4, \omega^1, \omega^7, 0, \omega^3, \omega^5, \omega^0, \omega^2)\\
&\omega^3(X+\omega^0)=(\infty, \omega^7, \omega^5, \omega^2, \omega^0, 0, \omega^4, \omega^6, \omega^1, \omega^3)\\
&\omega^0(X+\omega^1)=(\infty, \omega^7, 0, \omega^5, \omega^4, \omega^6, \omega^3, \omega^0, \omega^2, \omega^1)\\
&\omega^1(X+\omega^1)=(\infty, \omega^0, 0, \omega^6, \omega^5, \omega^7, \omega^4, \omega^1, \omega^3, \omega^2)\\
&\omega^2(X+\omega^1)=(\infty, \omega^1, 0, \omega^7, \omega^6, \omega^0, \omega^5, \omega^2, \omega^4, \omega^3)\\
&\omega^3(X+\omega^1)=(\infty, \omega^2, 0, \omega^0, \omega^7, \omega^1, \omega^6, \omega^3, \omega^5, \omega^4)\\
&\omega^0(X+\omega^2)=(\infty, \omega^3, \omega^7, \omega^0, \omega^1, \omega^5, 0, \omega^6, \omega^4, \omega^2)\\
&\omega^1(X+\omega^2)=(\infty, \omega^4, \omega^0, \omega^1, \omega^2, \omega^6, 0, \omega^7, \omega^5, \omega^3)\\
&\omega^2(X+\omega^2)=(\infty, \omega^5, \omega^1, \omega^2, \omega^3, \omega^7, 0, \omega^0, \omega^6, \omega^4)\\
&\omega^3(X+\omega^2)=(\infty, \omega^6, \omega^2, \omega^3, \omega^4, \omega^0, 0, \omega^1, \omega^7, \omega^5)\\
&\omega^0(X+\omega^3)=(\infty, \omega^5, \omega^6, \omega^4, \omega^7, \omega^2, \omega^0, \omega^1, 0, \omega^3)\\
&\omega^1(X+\omega^3)=(\infty, \omega^6, \omega^7, \omega^5, \omega^0, \omega^3, \omega^1, \omega^2, 0, \omega^4)\\
&\omega^2(X+\omega^3)=(\infty, \omega^7, \omega^0, \omega^6, \omega^1, \omega^4, \omega^2, \omega^3, 0, \omega^5)\\
&\omega^3(X+\omega^3)=(\infty, \omega^0, \omega^1, \omega^7, \omega^2, \omega^5, \omega^3, \omega^4, 0, \omega^6)\\
&\omega^0(X+\omega^4)=(\infty, 0, \omega^3, \omega^6, \omega^2, \omega^0, \omega^7, \omega^5, \omega^1, \omega^4)\\
&\omega^1(X+\omega^4)=(\infty, 0, \omega^4, \omega^7, \omega^3, \omega^1, \omega^0, \omega^6, \omega^2, \omega^5)\\
&\omega^2(X+\omega^4)=(\infty, 0, \omega^5, \omega^0, \omega^4, \omega^2, \omega^1, \omega^7, \omega^3, \omega^6)\\
&\omega^3(X+\omega^4)=(\infty, 0, \omega^6, \omega^1, \omega^5, \omega^3, \omega^2, \omega^0, \omega^4, \omega^7)\\
&\omega^0(X+\omega^5)=(\infty, \omega^2, \omega^1, 0, \omega^6, \omega^3, \omega^4, \omega^7, \omega^0, \omega^5)\\
&\omega^1(X+\omega^5)=(\infty, \omega^3, \omega^2, 0, \omega^7, \omega^4, \omega^5, \omega^0, \omega^1, \omega^6)\\
&\omega^2(X+\omega^5)=(\infty, \omega^4, \omega^3, 0, \omega^0, \omega^5, \omega^6, \omega^1, \omega^2, \omega^7)\\
&\omega^3(X+\omega^5)=(\infty, \omega^5, \omega^4, 0, \omega^1, \omega^6, \omega^7, \omega^2, \omega^3, \omega^0)\\
&\omega^0(X+\omega^6)=(\infty, \omega^1, \omega^4, \omega^3, \omega^0, \omega^7, \omega^2, 0, \omega^5, \omega^6)\\
&\omega^1(X+\omega^6)=(\infty, \omega^2, \omega^5, \omega^4, \omega^1, \omega^0, \omega^3, 0, \omega^6, \omega^7)\\
&\omega^2(X+\omega^6)=(\infty, \omega^3, \omega^6, \omega^5, \omega^2, \omega^1, \omega^4, 0, \omega^7, \omega^0)\\
&\omega^3(X+\omega^6)=(\infty, \omega^4, \omega^7, \omega^6, \omega^3, \omega^2, \omega^5, 0, \omega^0, \omega^1)\\
&\omega^0(X+\omega^7)=(\infty, \omega^6, \omega^0, \omega^2, 0, \omega^1, \omega^5, \omega^4, \omega^3, \omega^7)\\
&\omega^1(X+\omega^7)=(\infty, \omega^7, \omega^1, \omega^3, 0, \omega^2, \omega^6, \omega^5, \omega^4, \omega^0)\\
&\omega^2(X+\omega^7)=(\infty, \omega^0, \omega^2, \omega^4, 0, \omega^3, \omega^7, \omega^6, \omega^5, \omega^1)\\
&\omega^3(X+\omega^7)=(\infty, \omega^1, \omega^3, \omega^5, 0, \omega^4, \omega^0, \omega^7, \omega^6, \omega^2)
\end{aligned}$$

*Example* 4. $n=26, 28, 50$.

Since we can easily obtain solutions of the round table problem for $n=26, 28, 50$

by the same procedure as in Example 3, we only list the minimum polynomials, the generating permutations $P(x)$'s, and the normalized cyclic sequences $X$'s.

For $n=26$:
Minimum polynomial $=\omega^3+3\omega+3$
$P(x)=(x-\omega^{23})/x$
$X=(\infty, \omega^0, \omega^2, \omega^{14}, \omega^{22}, \omega^{15}, \omega^{13}, \omega^{19}, \omega^{20}, \omega^5, \omega^{12}, \omega^7, \omega^{17}, \omega^6, \omega^{16}, \omega^{11}, \omega^{18}, \omega^3, \omega^4, \omega^{10}, \omega^8, \omega^1, \omega^9, \omega^{21}, \omega^{23}, 0)$

For $n=28$:
Minimum polynomial $=\omega^3+2\omega+1$
$P(x)=(x-\omega^{21})/x$
$X=(\infty, \omega^0, \omega^{15}, \omega^{23}, \omega^{10}, \omega^{19}, \omega^{25}, \omega^3, \omega^{17}, \omega^{20}, \omega^{16}, \omega^7, \omega^9, \omega^8, \omega^{13}, \omega^{12}, \omega^{14}, \omega^5, \omega^1, \omega^4, \omega^{18}, \omega^{22}, \omega^2, \omega^{11}, \omega^{24}, \omega^6, \omega^{21}, 0)$

For $n=50$:
Minimum polynomial $=\omega^2+6\omega+3$
$P(x)=(x-\omega^{47})/x$
$X=(\infty, \omega^0, \omega^{30}, \omega^{28}, \omega^{33}, \omega^{36}, \omega^{26}, \omega^{42}, \omega^{25}, \omega^9, \omega^{29}, \omega^{41}, \omega^3, \omega^{43}, \omega^{23}, \omega^{16}, \omega^{35}, \omega^2, \omega^{27}, \omega^{13}, \omega^{46}, \omega^7, \omega^8, \omega^{37}, \omega^{15}, \omega^{32}, \omega^{10}, \omega^{39}, \omega^{40}, \omega^1, \omega^{34}, \omega^{20}, \omega^{45}, \omega^{12}, \omega^{31}, \omega^{24}, \omega^4, \omega^{44}, \omega^6, \omega^{18}, \omega^{38}, \omega^{22}, \omega^5, \omega^{21}, \omega^{11}, \omega^{14}, \omega^{19}, \omega^{17}, \omega^{47}, 0)$

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## Appendix. Schedules for $n=13\sim25$

Mr. Henry H. Dudeney [2] declared in his book that a solution of the round table problem was possible for any number of persons and he had recorded schedules for every number up to 25 persons inclusive and for 33. However, he did not give any method for solving the problem and only showed schedules for every number up to 12 persons inclusive. It seems that a solution of the problem is possible for any number of persons, because a schedule for any small number of persons can be obtained either by some theoretical method or by the method of trial and error. One of the authors, G. Nakamura, collects some schedules for small numbers of persons. In this appendix, we give schedules for every number from 13 to 25 persons. The schedules are obtained by Kiyasu-zen'iti for 13 persons, by G. Nakamura for 15, 16, 19, and 22 persons, by K. Koba for 17 persons, by Y. Kushida for 21, 23, and 25 persons. The schedules for 14, 18, 20, and 24 persons are easily obtained by an elementary method.

In order to introduce a simple expression, we first give known schedules for 5 and 6 persons. A schedule for 5 persons is given by

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |
| 0 | 1 | 3 | 4 | 2 |
| 1 | 2 | 4 | 0 | 3 |
| 2 | 3 | 0 | 1 | 4 |
| 3 | 4 | 1 | 2 | 0 |
| 4 | 0 | 2 | 3 | 1 |

Except the first column, every number descends in cyclical order. So, we briefly express this schedule by

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | |
| 0 | 1 | 3 | 4 | 2 | --- C.S. |

(C.S.: 0→1→2→3→4→0)

where C.S. means cyclical shift. Another schedule for 5 persons is

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | | 0 | 2 | 1 | 4 | 3 |
| 0 | 1 | 3 | 4 | 2 | | 0 | 3 | 1 | 2 | 4 |
| 0 | 1 | 4 | 2 | 3 | | 0 | 4 | 1 | 3 | 2 |

which is equivalent to the previous schedule. It is seen that 0 and 1 are repeated down the column and the other numbers descend in cyclical order. Thus, we briefly express this by

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | --- C.S. |
| 0 | 2 | 1 | 4 | 3 | --- C.S. |

(C.S.: 2→3→4→2)
(0 and 1: repeaters)

A schedule for 6 persons is given by

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 5 | 3 | 4 | | 0 | 1 | 3 | 4 | 5 | 2 |
| 0 | 2 | 3 | 1 | 4 | 5 | | 0 | 2 | 4 | 5 | 1 | 3 |
| 0 | 3 | 4 | 2 | 5 | 1 | | 0 | 3 | 5 | 1 | 2 | 4 |
| 0 | 4 | 5 | 3 | 1 | 2 | | 0 | 4 | 1 | 2 | 3 | 5 |
| 0 | 5 | 1 | 4 | 2 | 3 | | 0 | 5 | 2 | 3 | 4 | 1 |

In this case, the number 0 is repeated down the column and the other numbers descend in cyclical order. So, we express this by

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 5 | 3 | 4 | --- C.S. |
| 0 | 1 | 3 | 4 | 5 | 2 | --- C.S. |

(C.S.: 1→2→3→4→5→1)
(0: repeater)

$n=13$

0 2 1 4 11 6 9 8 7 10 5 12 3 --- C.S.

0 2 5 10 8 7 11 4 3 12 1 6 9 --- C.S.

0 2 6 9 10 5 3 12 7 8 1 11 4 --- C.S.

0 2 8 7 1 3 12 9 6 4 11 10 5 --- C.S.

0 2 4 11 5 10 1 9 6 12 3 7 8 --- C.S.

0 2 3 12 4 11 8 7 9 6 10 5 1 --- C.S.

(C.S.: 2→3→4→5→6→7→8→9→10→11→12→2)

(0 and 1: repeaters)

$n=14$

0 1 11 3 9 5 7 13 6 8 4 10 2 12 --- C.S.

0 2 9 6 5 10 1 13 12 3 8 7 4 11 --- C.S.

0 3 7 9 1 2 8 13 5 11 12 4 6 10 --- C.S.

0 4 5 12 10 7 2 13 11 6 3 1 8 9 --- C.S.

0 5 3 2 6 12 9 13 4 1 7 11 10 8 --- C.S.

0 6 1 5 2 4 3 13 10 9 11 8 12 7 --- C.S.

(C.S.: 0→1→2→3→4→5→6→7→8→9→10→11→12→0)

(13: repeater)

$n=15$

1 2 3 14 4 13 5 12 6 11 7 10 8 9 0 --- C.S.

1 2 0 3 14 6 11 13 4 7 10 12 5 8 9 --- C.S.

1 2 11 6 9 8 7 10 5 12 3 14 0 4 13 --- C.S.

1 2 9 8 3 14 10 7 6 11 4 13 0 5 12 --- C.S.

1 2 5 12 0 6 11 9 8 14 3 4 13 7 10 --- C.S.

1 2 13 4 9 8 12 5 6 11 0 7 10 3 14 --- C.S.

1 2 7 10 0 8 9 13 4 5 12 14 3 6 11 --- C.S.

(C.S.: 2→3→4→5→6→7→8→9→10→11→12→13→14→2)

(0 and 1: repeaters)

$n=16$

0 1 14 3 12 5 10 7 8 9 6 11 4 13 2 15 --- C.S.

0 2 13 6 9 5 10 4 11 8 7 12 3 1 14 15 --- C.S.

0 3 12 2 13 1 14 5 10 9 6 8 7 11 4 15 --- C.S.

0 4 11 12 3 5 10 13 2 6 9 14 1 7 8 15 --- C.S.

0 5 10 11 4 6 9 1 14 7 8 2 13 12 3 15 --- C.S.

0 6 1 11 7 5 2 10 8 4 14 13 9 3 12 15 --- C.S.

0 7 8 10 5 13 2 1 14 4 11 3 12 9 6 15 --- C.S.

(C.S.: 1→2→3→4→5→6→7→8→9→10→11→12→13→14→15→1)

(0: repeater)

$n=17$

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | |
| 0 | 2 | 1 | 7 | 12 | 13 | 9 | 3 | 11 | 6 | 14 | 8 | 4 | 5 | 10 | 16 | 15 | --- C.S. |
| 0 | 3 | 7 | 1 | 8 | 6 | 5 | 2 | 4 | 13 | 15 | 12 | 11 | 9 | 16 | 10 | 14 | --- C.S. |
| 0 | 4 | 12 | 8 | 1 | 15 | 10 | 11 | 3 | 14 | 6 | 7 | 2 | 16 | 9 | 5 | 13 | --- C.S. |
| 0 | 5 | 13 | 6 | 15 | 1 | 3 | 9 | 10 | 7 | 8 | 14 | 16 | 2 | 11 | 4 | 12 | --- C.S. |
| 0 | 6 | 9 | 5 | 10 | 3 | 1 | 13 | 15 | 2 | 4 | 16 | 14 | 7 | 12 | 8 | 11 | --- C.S. |
| 0 | 7 | 3 | 2 | 11 | 9 | 13 | 1 | 12 | 5 | 16 | 4 | 8 | 6 | 15 | 14 | 10 | --- C.S. |
| 0 | 8 | 11 | 4 | 3 | 10 | 15 | 12 | 1 | 16 | 5 | 2 | 7 | 14 | 13 | 6 | 9 | --- C.S. |

(C.S.: 0→1→2→3→4→5→6→7→8→9→10→11→12→13→14→15→16→0)

$n=18$

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 15 | 3 | 13 | 5 | 11 | 7 | 9 | 17 | 8 | 10 | 6 | 12 | 4 | 14 | 2 | 16 | --- C.S. |
| 0 | 2 | 13 | 6 | 9 | 10 | 5 | 14 | 1 | 17 | 16 | 3 | 12 | 7 | 8 | 11 | 4 | 15 | --- C.S. |
| 0 | 3 | 11 | 9 | 5 | 15 | 16 | 4 | 10 | 17 | 7 | 13 | 1 | 2 | 12 | 8 | 6 | 14 | --- C.S. |
| 0 | 4 | 9 | 12 | 1 | 3 | 10 | 11 | 2 | 17 | 15 | 6 | 7 | 14 | 16 | 5 | 8 | 13 | --- C.S. |
| 0 | 5 | 7 | 15 | 14 | 8 | 4 | 1 | 11 | 17 | 6 | 16 | 13 | 9 | 3 | 2 | 10 | 12 | --- C.S. |
| 0 | 6 | 5 | 1 | 10 | 13 | 15 | 8 | 3 | 17 | 14 | 9 | 2 | 4 | 7 | 16 | 12 | 11 | --- C.S. |
| 0 | 7 | 3 | 4 | 6 | 1 | 9 | 15 | 12 | 17 | 5 | 2 | 8 | 16 | 11 | 13 | 14 | 10 | --- C.S. |
| 0 | 8 | 1 | 7 | 2 | 6 | 3 | 5 | 4 | 17 | 13 | 12 | 14 | 11 | 15 | 10 | 16 | 9 | --- C.S. |

(C.S.: 0→1→2→3→4→5→6→7→8→9→10→11→12→13→14→15→16→0)
(17: repeater)

$n=19$

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 0 | 3 | 18 | 14 | 7 | 12 | 9 | 8 | 13 | 6 | 15 | 17 | 4 | 5 | 16 | 10 | 11 | --- C.S. |
| 1 | 2 | 5 | 16 | 8 | 13 | 11 | 10 | 14 | 7 | 0 | 15 | 6 | 18 | 3 | 4 | 17 | 9 | 12 | --- C.S. |
| 1 | 2 | 7 | 14 | 16 | 5 | 0 | 17 | 4 | 15 | 6 | 9 | 12 | 18 | 3 | 10 | 11 | 8 | 13 | --- C.S. |
| 1 | 2 | 9 | 12 | 16 | 5 | 6 | 15 | 13 | 8 | 3 | 18 | 0 | 4 | 17 | 11 | 10 | 7 | 14 | --- C.S. |
| 1 | 2 | 11 | 10 | 0 | 12 | 9 | 4 | 17 | 13 | 8 | 5 | 16 | 18 | 3 | 14 | 7 | 6 | 15 | --- C.S. |
| 1 | 2 | 13 | 8 | 0 | 14 | 7 | 17 | 4 | 11 | 10 | 18 | 3 | 6 | 15 | 9 | 12 | 5 | 16 | --- C.S. |
| 1 | 2 | 15 | 6 | 11 | 10 | 5 | 16 | 3 | 18 | 8 | 13 | 0 | 9 | 12 | 14 | 7 | 4 | 17 | --- C.S. |
| 1 | 2 | 17 | 4 | 13 | 8 | 7 | 14 | 5 | 16 | 0 | 6 | 15 | 11 | 10 | 9 | 12 | 3 | 18 | --- C.S. |
| 1 | 2 | 3 | 18 | 4 | 17 | 5 | 16 | 6 | 15 | 7 | 14 | 8 | 13 | 9 | 12 | 10 | 11 | 0 | --- C.S. |

(C.S.: 2→3→4→5→6→7→8→9→10→11→12→13→14→15→16→17→18→2)
(0 and 1: repeaters)

$n=20$

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 17 | 3 | 15 | 5 | 13 | 7 | 11 | 9 | 19 | 10 | 8 | 12 | 6 | 14 | 4 | 16 | 2 | 18 | --- C.S. |
| 0 | 2 | 15 | 6 | 11 | 10 | 7 | 14 | 3 | 18 | 19 | 1 | 16 | 5 | 12 | 9 | 8 | 13 | 4 | 17 | --- C.S. |
| 0 | 3 | 13 | 9 | 7 | 15 | 1 | 2 | 14 | 8 | 19 | 11 | 5 | 17 | 18 | 4 | 12 | 10 | 6 | 16 | --- C.S. |
| 0 | 4 | 11 | 12 | 3 | 1 | 14 | 9 | 6 | 17 | 19 | 2 | 13 | 10 | 5 | 18 | 16 | 7 | 8 | 15 | --- C.S. |
| 0 | 5 | 9 | 15 | 18 | 6 | 8 | 16 | 17 | 7 | 19 | 12 | 2 | 3 | 11 | 13 | 1 | 4 | 10 | 14 | --- C.S. |
| 0 | 6 | 7 | 18 | 14 | 11 | 2 | 4 | 9 | 16 | 19 | 3 | 10 | 15 | 17 | 8 | 5 | 1 | 12 | 13 | --- C.S. |
| 0 | 7 | 5 | 2 | 10 | 16 | 15 | 11 | 1 | 6 | 19 | 13 | 18 | 8 | 4 | 3 | 9 | 17 | 14 | 12 | --- C.S. |
| 0 | 8 | 3 | 5 | 6 | 2 | 9 | 18 | 12 | 15 | 19 | 4 | 7 | 1 | 10 | 17 | 13 | 14 | 16 | 11 | --- C.S. |
| 0 | 9 | 1 | 8 | 2 | 7 | 3 | 6 | 4 | 5 | 19 | 14 | 15 | 13 | 16 | 12 | 17 | 11 | 18 | 10 | --- C.S. |

(C.S.: 0→1→2→3→4→5→6→7→8→9→10→11→12→13→14→15→16→17→18→0)
(19: repeater)

$n=21$

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 19 | 2 | 17 | 4 | 15 | 3 | 16 | 5 | 14 | 7 | 12 | 9 | 10 | 11 | 8 | 20 | 6 | 13 | 18 | 1 | --- C.S. |
| 0 | 19 | 10 | 9 | 20 | 5 | 14 | 11 | 8 | 13 | 6 | 15 | 4 | 12 | 7 | 1 | 18 | 3 | 16 | 17 | 2 | --- C.S. |
| 0 | 19 | 4 | 15 | 8 | 11 | 12 | 7 | 16 | 3 | 1 | 18 | 5 | 14 | 9 | 10 | 13 | 6 | 17 | 2 | 20 | --- C.S. |
| 0 | 19 | 6 | 13 | 12 | 7 | 18 | 1 | 5 | 14 | 20 | 11 | 8 | 17 | 2 | 4 | 15 | 10 | 9 | 16 | 3 | --- C.S. |
| 0 | 19 | 8 | 11 | 16 | 3 | 5 | 14 | 13 | 6 | 2 | 17 | 10 | 9 | 18 | 1 | 20 | 7 | 12 | 15 | 4 | --- C.S. |
| 0 | 19 | 20 | 10 | 9 | 1 | 18 | 11 | 8 | 2 | 17 | 12 | 7 | 3 | 16 | 13 | 6 | 4 | 15 | 14 | 5 | --- C.S. |
| 0 | 19 | 12 | 7 | 5 | 14 | 17 | 2 | 10 | 9 | 3 | 16 | 15 | 4 | 8 | 11 | 1 | 18 | 20 | 13 | 6 | --- C.S. |
| 0 | 19 | 14 | 5 | 9 | 10 | 4 | 15 | 18 | 1 | 13 | 6 | 8 | 11 | 3 | 16 | 20 | 17 | 2 | 12 | 7 | --- C.S. |
| 0 | 19 | 16 | 3 | 13 | 6 | 10 | 9 | 7 | 12 | 20 | 4 | 15 | 1 | 18 | 17 | 2 | 14 | 5 | 11 | 8 | --- C.S. |
| 0 | 19 | 18 | 1 | 17 | 2 | 16 | 3 | 20 | 15 | 4 | 14 | 5 | 13 | 6 | 12 | 7 | 11 | 8 | 10 | 9 | --- C.S. |

(C.S.: 1→2→3→4→5→6→7→8→9→10→11→12→13→14→15→16→17→18→19→1)

(0 and 20: repeaters)

$n=22$

| | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 21 | 2 | 19 | 4 | 17 | 6 | 15 | 8 | 13 | 10 | 11 | 12 | 9 | 14 | 7 | 16 | 5 | 1 | 20 | 3 | 18 | --- C.S. |
| 0 | 21 | 4 | 17 | 8 | 13 | 12 | 9 | 16 | 5 | 20 | 1 | 3 | 18 | 7 | 14 | 11 | 10 | 2 | 19 | 6 | 15 | --- C.S. |
| 0 | 21 | 15 | 6 | 9 | 12 | 3 | 18 | 5 | 16 | 11 | 10 | 17 | 4 | 2 | 19 | 8 | 13 | 14 | 7 | 20 | 1 | --- C.S. |
| 0 | 21 | 8 | 13 | 16 | 5 | 3 | 18 | 11 | 10 | 19 | 2 | 6 | 15 | 14 | 7 | 1 | 20 | 9 | 12 | 17 | 4 | --- C.S. |
| 0 | 21 | 10 | 11 | 20 | 1 | 9 | 12 | 19 | 2 | 8 | 13 | 18 | 3 | 7 | 14 | 17 | 4 | 6 | 15 | 16 | 5 | --- C.S. |
| 0 | 21 | 9 | 12 | 18 | 3 | 6 | 15 | 10 | 11 | 1 | 20 | 13 | 8 | 4 | 17 | 16 | 5 | 7 | 14 | 19 | 2 | --- C.S. |
| 0 | 21 | 3 | 18 | 6 | 15 | 9 | 12 | 8 | 13 | 5 | 16 | 2 | 19 | 20 | 1 | 17 | 4 | 14 | 7 | 11 | 10 | --- C.S. |
| 0 | 21 | 20 | 1 | 19 | 2 | 18 | 3 | 17 | 4 | 16 | 5 | 15 | 6 | 14 | 7 | 13 | 8 | 10 | 11 | 9 | 12 | --- C.S. |
| 0 | 21 | 7 | 14 | 2 | 19 | 16 | 5 | 9 | 12 | 4 | 17 | 18 | 3 | 11 | 10 | 6 | 15 | 20 | 1 | 13 | 8 | --- C.S. |
| 0 | 21 | 16 | 5 | 11 | 10 | 4 | 17 | 20 | 1 | 15 | 6 | 8 | 13 | 3 | 18 | 19 | 2 | 12 | 9 | 7 | 14 | --- C.S. |

(C.S.: 1→2→3→4→5→6→7→8→9→10→11→12→13→14→15→16→17→18→19→20→21→1)

(0: repeater)

$n=23$

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 21 | 22 | 2 | 19 | 4 | 17 | 6 | 15 | 8 | 13 | 10 | 11 | 12 | 9 | 14 | 7 | 16 | 5 | 18 | 3 | 20 | 1 | --- C.S. |
| 0 | 21 | 4 | 17 | 8 | 13 | 12 | 9 | 16 | 5 | 20 | 1 | 3 | 18 | 7 | 14 | 11 | 10 | 15 | 6 | 22 | 19 | 2 | --- C.S. |
| 0 | 21 | 15 | 6 | 10 | 11 | 16 | 5 | 1 | 20 | 17 | 4 | 2 | 19 | 8 | 13 | 22 | 14 | 7 | 9 | 12 | 3 | 18 | --- C.S. |
| 0 | 21 | 8 | 13 | 16 | 5 | 19 | 2 | 6 | 15 | 3 | 18 | 11 | 10 | 14 | 7 | 1 | 20 | 9 | 22 | 12 | 17 | 4 | --- C.S. |
| 0 | 21 | 10 | 11 | 20 | 1 | 9 | 12 | 19 | 2 | 8 | 13 | 18 | 3 | 22 | 7 | 14 | 17 | 4 | 6 | 15 | 16 | 5 | --- C.S. |
| 0 | 21 | 14 | 7 | 5 | 16 | 12 | 9 | 19 | 2 | 10 | 11 | 3 | 18 | 17 | 4 | 8 | 13 | 1 | 20 | 22 | 15 | 6 | --- C.S. |
| 0 | 21 | 12 | 9 | 3 | 18 | 15 | 6 | 8 | 13 | 20 | 1 | 22 | 11 | 10 | 4 | 17 | 16 | 5 | 2 | 19 | 14 | 7 | --- C.S. |
| 0 | 21 | 16 | 5 | 11 | 10 | 22 | 17 | 4 | 12 | 9 | 6 | 15 | 1 | 20 | 7 | 14 | 2 | 19 | 18 | 3 | 13 | 8 | --- C.S. |
| 0 | 21 | 18 | 3 | 5 | 16 | 8 | 13 | 11 | 10 | 19 | 2 | 1 | 20 | 4 | 17 | 7 | 14 | 15 | 6 | 12 | 9 | 22 | --- C.S. |
| 0 | 21 | 20 | 1 | 19 | 2 | 18 | 3 | 17 | 4 | 22 | 16 | 5 | 15 | 6 | 14 | 7 | 13 | 8 | 12 | 9 | 11 | 10 | --- C.S. |
| 0 | 21 | 2 | 19 | 6 | 15 | 20 | 1 | 11 | 10 | 17 | 4 | 16 | 5 | 8 | 13 | 14 | 7 | 3 | 18 | 22 | 12 | 9 | --- C.S. |

(C.S.: 1→2→3→4→5→6→7→8→9→10→11→12→13→14→15→16→17→18→19→20→21→1)

(0 and 22: repeaters)

$n=24$

| | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 21 | 3 | 19 | 5 | 17 | 7 | 15 | 9 | 13 | 11 | 23 | 12 | 10 | 14 | 8 | 16 | 6 | 18 | 4 | 20 | 2 | 22 | --- C.S. |
| 0 | 2 | 19 | 6 | 15 | 10 | 11 | 14 | 7 | 18 | 3 | 22 | 23 | 1 | 20 | 5 | 16 | 9 | 12 | 13 | 8 | 17 | 4 | 21 | --- C.S. |
| 0 | 3 | 17 | 9 | 11 | 15 | 5 | 21 | 22 | 4 | 16 | 10 | 23 | 13 | 7 | 19 | 1 | 2 | 18 | 8 | 12 | 14 | 6 | 20 | --- C.S. |
| 0 | 4 | 15 | 12 | 7 | 20 | 22 | 5 | 14 | 13 | 6 | 21 | 23 | 2 | 17 | 10 | 9 | 18 | 1 | 3 | 16 | 11 | 8 | 19 | --- C.S. |
| 0 | 5 | 13 | 15 | 3 | 2 | 16 | 12 | 6 | 22 | 19 | 9 | 23 | 14 | 4 | 1 | 17 | 11 | 7 | 21 | 20 | 8 | 10 | 18 | --- C.S. |
| 0 | 6 | 11 | 18 | 22 | 7 | 10 | 19 | 21 | 8 | 9 | 20 | 23 | 3 | 14 | 15 | 2 | 4 | 13 | 16 | 1 | 5 | 12 | 17 | --- C.S. |
| 0 | 7 | 9 | 21 | 18 | 12 | 4 | 3 | 13 | 17 | 22 | 8 | 23 | 15 | 1 | 6 | 10 | 20 | 19 | 11 | 5 | 2 | 14 | 16 | --- C.S. |
| 0 | 8 | 7 | 1 | 14 | 17 | 21 | 10 | 5 | 3 | 12 | 19 | 23 | 4 | 11 | 20 | 18 | 13 | 2 | 6 | 9 | 22 | 16 | 15 | --- C.S. |
| 0 | 9 | 5 | 4 | 10 | 22 | 15 | 17 | 20 | 12 | 2 | 7 | 23 | 16 | 21 | 11 | 3 | 6 | 8 | 1 | 13 | 19 | 18 | 14 | --- C.S. |
| 0 | 10 | 3 | 7 | 6 | 4 | 9 | 1 | 12 | 21 | 15 | 18 | 23 | 5 | 8 | 2 | 11 | 22 | 14 | 19 | 17 | 16 | 20 | 13 | --- C.S. |
| 0 | 11 | 1 | 10 | 2 | 9 | 3 | 8 | 4 | 7 | 5 | 6 | 23 | 17 | 18 | 16 | 19 | 15 | 20 | 14 | 21 | 13 | 22 | 12 | --- C.S. |

(C.S.: 0→1→2→3→4→5→6→7→8→9→10→11→12→13→14→15→16→17→18→19→20→21→22→0)

(23: repeater)

$n=25$

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 23 | 2 | 21 | 4 | 19 | 6 | 17 | 8 | 15 | 10 | 13 | 12 | 11 | 14 | 9 | 16 | 7 | 18 | 5 | 20 | 3 | 22 | 1 | 24 | --- C.S. |
| 0 | 23 | 4 | 19 | 8 | 15 | 12 | 11 | 16 | 7 | 20 | 3 | 1 | 22 | 5 | 18 | 9 | 14 | 24 | 13 | 10 | 17 | 6 | 21 | 2 | --- C.S. |
| 0 | 23 | 6 | 17 | 24 | 12 | 11 | 18 | 5 | 1 | 22 | 7 | 16 | 13 | 10 | 19 | 4 | 2 | 21 | 8 | 15 | 14 | 9 | 20 | 3 | --- C.S. |
| 0 | 23 | 8 | 15 | 16 | 7 | 1 | 22 | 9 | 14 | 17 | 6 | 2 | 21 | 10 | 13 | 18 | 5 | 3 | 24 | 20 | 11 | 12 | 19 | 4 | --- C.S. |
| 0 | 23 | 10 | 13 | 20 | 3 | 7 | 16 | 17 | 6 | 24 | 4 | 19 | 14 | 9 | 1 | 22 | 11 | 12 | 21 | 2 | 8 | 15 | 18 | 5 | --- C.S. |
| 0 | 23 | 12 | 11 | 1 | 22 | 13 | 10 | 24 | 2 | 21 | 14 | 9 | 3 | 20 | 15 | 8 | 4 | 19 | 16 | 7 | 5 | 18 | 17 | 6 | --- C.S. |
| 0 | 23 | 14 | 9 | 5 | 18 | 19 | 4 | 10 | 13 | 1 | 22 | 15 | 8 | 6 | 17 | 20 | 3 | 11 | 12 | 2 | 21 | 16 | 24 | 7 | --- C.S. |
| 0 | 23 | 16 | 7 | 9 | 14 | 2 | 21 | 24 | 18 | 5 | 11 | 12 | 4 | 19 | 20 | 3 | 13 | 10 | 6 | 17 | 22 | 1 | 15 | 8 | --- C.S. |
| 0 | 23 | 18 | 5 | 13 | 10 | 8 | 15 | 3 | 20 | 21 | 2 | 16 | 7 | 11 | 12 | 6 | 24 | 17 | 1 | 22 | 19 | 4 | 14 | 9 | --- C.S. |
| 0 | 23 | 20 | 3 | 17 | 6 | 14 | 9 | 11 | 12 | 8 | 15 | 24 | 5 | 18 | 2 | 21 | 22 | 1 | 19 | 4 | 16 | 7 | 13 | 10 | --- C.S. |
| 0 | 23 | 22 | 1 | 21 | 2 | 20 | 3 | 24 | 19 | 4 | 18 | 5 | 17 | 6 | 16 | 7 | 15 | 8 | 14 | 9 | 13 | 10 | 12 | 11 | --- C.S. |
| 0 | 23 | 9 | 14 | 13 | 10 | 2 | 21 | 18 | 5 | 15 | 8 | 20 | 3 | 19 | 4 | 6 | 17 | 12 | 11 | 24 | 7 | 16 | 22 | 1 | --- C.S. |

(C.S.: 1→2→3→4→5→6→7→8→9→10→11→12→13→14→15→16→17→18→19→20→21→22→23→1)

(0 and 24: repeaters)

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