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A Splitting Property on a Set of Ideals and a Weak Saturation Hypothesis

by

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§ 0. Introduction

An ideal I over a regular uncountable cardinal κ is called "ν-saturated," if there is no collection $\{X_\alpha \mid \alpha<\nu\}$ such that $X_\alpha \notin I$ for every $\alpha<\nu$ and $X_\alpha \cap X_\beta \in I$ for $\alpha<\beta<\nu$. To consider "Ulam's problem (see [3]). Let the cardinality of S be $\aleph_1$. Can one define $\aleph_1$ σ-additive 0–1 measures on S so that each subset is measurable with respect to one of them? (This version was stated as problem 81 in [4].)"

A. Taylor extended the saturation property to a set of ideals and introduced a notation "$\langle\kappa: \lambda, \mu\rangle \longrightarrow \nu$." Then Ulam's problem is restated as

$$\text{"Does}\quad \langle\aleph_1: \aleph_1, \aleph_1\rangle \longrightarrow 2 \quad \text{hold?"}$$

This problem is still open now. In [7] Taylor got a strengthening result of K. Prikry's [6], that is,

$$\text{"SpH}_{\aleph_1} \quad \text{implies} \quad \langle\aleph_1: \aleph_1, \aleph_1\rangle \longrightarrow \aleph_2 \text{."}$$

In this paper we study a generalization of $\mathrm{SpH}_{\aleph_1}$, which will be written "$\{\kappa: \lambda, \mu\} \longrightarrow \nu$." This property is stronger than $\langle\kappa: \lambda, \mu\rangle \longrightarrow \nu$, and to speak the truth in [7] Taylor got that

$$\text{"SpH}_{\aleph_1} \quad \text{implies} \quad \{\aleph_1: \aleph_1, \aleph_1\} \longrightarrow \aleph_2 \text{."}$$

And we also discuss about Fodor's hypothesis for κ which is denoted by FH_κ. B. Baumgartner et al. had the next theorem [2].

$$\text{"SatH}_\kappa \quad \text{implies} \quad \text{FH}_\kappa\text{."}$$

We introduce "$\mathrm{SatH}_\kappa^\omega$," and show that $\mathrm{SatH}_{\aleph_1}^\omega$ implies $\mathrm{FH}_{\aleph_1}$. It will be easy to know that SatH_κ implies $\mathrm{SatH}_\kappa^\omega$. But we do not know whether $\mathrm{SatH}_\kappa^\omega$ implies SatH_κ or not.

Let κ be a regular uncountable cardinal number. An ideal over κ is a set I of subsets of κ satisfying the following conditions:

(1) $\phi \in I$,

(2) $\kappa \notin I$,

(3) if $X \in I$ and $Y \subseteq X$ then $Y \in I$,

(4) if $X\in I$ and $Y\in I$ then $X\cup Y\in I$.

Let λ be a cardinal number. An ideal I over κ is said to be λ-complete, if I satisfies the following condition:

$$\text{If}\quad \nu<\lambda \quad\text{and}\quad \{X_\alpha \mid \alpha<\nu\}\subseteq I,\quad \text{then}\quad \bigcup_{\alpha<\nu} X_\alpha\in I.$$

And an ideal I over κ is called uniform, if I satisfies the following condition:

If $X\subseteq\kappa$ and $|X|<\kappa$, then $X\in I$. ($|X|$ denotes the cardinality of X.)

Throughout this paper, an ideal means a uniform ideal over κ. Let I be an ideal over κ. We set

$$I^+=\{X \mid X\subseteq\kappa \text{ and } X\notin I\},$$

and say X has a positive measure or is a set of positive measure if $X\in I^+$. And let $A\in I^+$. we set

$$I|A=\{X \mid X\subseteq\kappa \text{ and } X\cap A\in I\}.$$

Then $I|A$ is an ideal over κ generated by $I\cup\{\kappa-A\}$. And it is easy to know that if I is λ-complete, then $I|A$ is also λ-complete. Let λ be a cardinal number, and $\mathscr{I}=\{I_\alpha \mid \alpha<\lambda\}$ be a collection of ideals over κ. We set

$$\mathscr{I}^+=\bigcap_{\alpha<\lambda} I_\alpha^+.$$

Let I be a κ-complete ideal over κ, then we call I is normal, if I satisfies the following:

If $X\in I^+$ and f is a function on X such that $f(\alpha)<\alpha$ for all $\alpha\in X$, $\alpha\neq 0$, then there is a $Y\subseteq X$ such that $Y\in I^+$ and f is constant on Y.

And another notations are standard. Let κ be a cardinal number the κ^+ is the least cardinal number $>\kappa$.

§1. Splitting property and saturation property

In this section we introduce a splitting property on a set of ideals. And we shall show that the large sets of normal κ-complete ideals has the saturation property or splitting property if every single normal ideal has the same one. Here "large" means that size of the completeness.

Definition ([7]). Let $\mathscr{R}$ be a set of ideals over κ, then the symbol

$$\langle\kappa:\lambda,\mu\rangle \xrightarrow{\mathscr{R}} \nu$$

means the following assertion:

If $\mathscr{I}=\{I_\gamma \mid \gamma<\lambda\}\subseteq\mathscr{R}$ is a set of at least μ-complete ideals over κ, then there is a collection $\{X_\alpha \mid \alpha<\nu\}\subseteq\mathscr{I}^+$ such that $X_\alpha\cap X_\beta\in\bigcap_{\gamma<\lambda}I_\gamma$ for $\alpha<\beta<\nu$.

If $\mathscr{R}$ is the set of all ideals over κ, we omit $\mathscr{R}$.

Definition ([7]). We say SatH_κ (saturation hypothesis for κ) holds if $\langle\kappa: 1, \kappa\rangle \longrightarrow \kappa^+$.

Hence it means every κ-complete ideal over κ is not κ^+-saturated.

Definition. $\mathscr{N}$ denotes the set of all normal ideals over κ.

Taylor showed the next theorem.

THEOREM 1 ([8]). *The following assertions are equivalent.*

(1) $\langle\kappa: \kappa, \kappa\rangle \xrightarrow{\mathscr{N}} \kappa^+$.

(2) $\langle\kappa: 1, \kappa\rangle \xrightarrow{\mathscr{N}} \kappa^+$.

(3) SatH_κ *holds.*

Definition. Let $\mathscr{R}$ be a set of ideals over κ, then the symbol

$$\{\kappa: \lambda, \mu\} \xrightarrow{\mathscr{R}} \nu$$

means the following assertion:

If $\mathscr{I}=\{I_\gamma \mid \gamma<\lambda\} \subseteq \mathscr{R}$ is a set of at least μ-complete ideals over κ, then there is a collection $\{X_\alpha \mid \alpha<\nu\} \subseteq \mathscr{I}^+$ such that $|X_\alpha \cap X_\beta|<\kappa$ for $\alpha<\beta<\nu$.

We also omit $\mathscr{R}$, if $\mathscr{R}$ is the set of all ideals over κ,

Definition ([7]). We say SpH_κ (splitting hypothesis for κ) holds if $\{\kappa: 1, \kappa\} \longrightarrow \kappa^+$.

LEMMA 2. $\{\kappa: \lambda, \mu\} \xrightarrow{\mathscr{R}} \nu$ *implies* $\langle\kappa: \lambda, \mu\rangle \xrightarrow{\mathscr{R}} \nu$.

Proof. Trivial.

The next lemma of Baumgartner et al. is helpful to study the relation between $\{\kappa: \kappa, \kappa\} \xrightarrow{\mathscr{N}} \kappa^+$ and $\{\kappa: 1, \kappa\} \xrightarrow{\mathscr{N}} \kappa^+$. Of course from left to right is trivial.

LEMMA 3 ([2]). *Let I be a normal ideal over κ and $\{X_\alpha \mid \alpha<\kappa^+\} \subseteq I^+$ such that $X_\alpha \cap X_\beta \in I$ for $\alpha<\beta<\kappa^+$. Then there is a collection $\{Y_\alpha \mid \alpha<\kappa^+\}$ such that $Y_\alpha \subseteq X_\alpha$ and $X_\alpha - Y_\alpha \in I$ for all $\alpha<\kappa^+$ and $|Y_\alpha \cap Y_\beta|<\kappa$ for $\alpha<\beta<\kappa^+$.*

COROLLARY 4. *The following* 2 *assertions are equivalent.*

(1) $\{\kappa: 1, \kappa\} \xrightarrow{\mathscr{N}} \kappa^+$.

(2) $\langle\kappa: 1, \kappa\rangle \xrightarrow{\mathscr{N}} \kappa^+$.

LEMMA 5. *Let $\mathscr{I}=\{I_\alpha \mid \alpha<\kappa\}$ be a set of normal κ-complete ideals over κ. Then*

$$I=\bigcap_{\alpha<\kappa} I_\alpha$$

is a normal κ-complete ideal over κ.

Proof. It is easy to check.

LEMMA 6. *$\langle\kappa\colon \kappa, \kappa\rangle \xrightarrow{\mathcal{N}} \kappa^+$ implies $\{\kappa\colon \kappa, \kappa\} \xrightarrow{\mathcal{N}} \kappa^+$.*

Proof. Let $\mathscr{I}=\{I_\alpha \mid \alpha<\kappa\}$ be a set of normal κ-complete ideals over κ. Then by hypothesis there is a collection $\{X_\alpha \mid \alpha<\kappa^+\}\subseteq\mathscr{I}^+$ such that $X_\alpha\cap X_\beta\in\bigcap_{\gamma<\kappa}I_\gamma$ for $\alpha<\beta<\kappa^+$. Set $I=\bigcap_{\gamma<\kappa}I_\gamma$. Then by Lemma 5, I is a normal κ-complete ideal over κ. So we get a collection $\{X_\alpha \mid \alpha<\kappa^+\}\subseteq I^+$ with $X_\alpha\cap X_\beta\in I$ for $\alpha<\beta<\kappa^+$. Thus by Lemma 3 there is a collection $\{Y_\alpha \mid \alpha<\kappa^+\}$ such that $Y_\alpha\subseteq X_\alpha$ and $X_\alpha-Y_\alpha\in I$ for all $\alpha<\kappa^+$ and $|Y_\alpha\cap Y_\beta|<\kappa$ for $\alpha<\beta<\kappa^+$. The proof will be complete if we have $\{Y_\alpha \mid \alpha<\kappa^+\}\subseteq\mathscr{I}^+$. Let assume that $Y_\alpha\in I_\beta$ for some $\alpha<\kappa^+$ and $\beta<\kappa$. Then $X_\alpha-Y_\alpha\in I\subseteq I_\beta$ implies

$$X_\alpha=(X_\alpha-Y_\alpha)\cup Y_\alpha\in I_\beta\,.$$

But this contradicts $X_\alpha\in {I_\beta}^+$.

Now we have the next theorem.

THEOREM 7. *The following assertions are equivalent.*

(1) $\{\kappa\colon 1, \kappa\} \xrightarrow{\mathcal{N}} \kappa^+$.

(2) $\{\kappa\colon \kappa, \kappa\} \xrightarrow{\mathcal{N}} \kappa^+$.

(3) $\langle\kappa\colon 1, \kappa\rangle \xrightarrow{\mathcal{N}} \kappa^+$.

(4) $\langle\kappa\colon \kappa, \kappa\rangle \xrightarrow{\mathcal{N}} \kappa^+$.

(5) SatH_κ *holds.*

Proof. (3) iff (4) iff (5) follows from Theorem 1. (1) iff (3) follows from Corollary 4. And (2) iff (4) follows from Lemma 2 and Lemma 6.

This theorem says that large sets of normal κ-complete ideals have the saturation property or splitting property if every single normal ideals have the same one. We do not know whether we can get the same matter or not, if "normal" is omitted. But Taylor showed small sets of ideals over κ have a certain saturation property, if every single ideals have the same one using the technique of Baumgartner et al. Here "small" means the size is less than the completeness.

We can easily modify his proof to assure the next theorem.

THEOREM 8. *If $\lambda<\kappa$, then the following assertions are equivalent.*

(1) $\{\kappa\colon 1, \lambda^+\} \longrightarrow \lambda^+$.

(2) $\{\kappa\colon \lambda, \lambda^+\} \longrightarrow \lambda^+$.

As for another equivalence of the same kind, we know the following.

Definition. Let I be an ideal over κ. A function f is an I-function if $\mathrm{dom}\,(f)\in I^+$.

Definition. Let $\mathscr{I}=\{I_\alpha \mid \alpha<\nu\}$ be a set of ideals over κ. A function f is an $\mathscr{I}$-

function if dom $(f) \in \mathscr{I}^+$.

THEOREM 9 ([5]). *Let λ be a cardinal number with $\lambda < \kappa = \mu^+$. Then the following assertions are equivalent.*

(1) *If I is a κ-complete ideal over κ, then there is a collection of I-functions $F = \{f_\alpha \mid \alpha < \kappa\}$ such that* range $(f_\alpha) \subseteq \mu$ *for all $\alpha < \kappa$ and*

$$\{\delta \mid f_\alpha(\delta) = f_\beta(\delta)\} \in I \qquad \textit{for} \quad \alpha < \beta < \kappa .$$

(2) *If $\mathscr{I} = \{I_\nu \mid \nu < \lambda\}$ is a set of κ-complete ideals over κ, then there is a collection of $\mathscr{I}$-functions $G = \{g_\alpha \mid \alpha < \kappa\}$ such that* range $(g_\alpha) \subseteq \mu$ *for all $\alpha < \kappa$ and*

$$\{\delta \mid g_\alpha(\delta) = g_\beta(\delta)\} \in \bigcap_{\nu < \lambda} I_\nu \qquad \textit{for} \quad \alpha < \beta < \kappa .$$

COROLLARY 10 ([5]). *Let λ be a cardinal number with $\lambda < \kappa = \mu^+$. Then* (1) *implies* (2).

(1) *If $\mathscr{I} = \{I_\nu \mid \nu < \lambda\}$ is a set of κ-complete ideals over κ, then there is a collection of $\mathscr{I}$-functions $G = \{g_\alpha \mid \alpha < \kappa\}$ such that* range $(g_\alpha) \subseteq \mu$ *for all $\alpha < \kappa$ and*

$$\{\delta \mid g_\alpha(\delta) = g_\beta(\delta)\} \in \bigcap_{\nu < \lambda} I_\nu \qquad \textit{for} \quad \alpha < \beta < \kappa .$$

(2) $\langle \kappa : \lambda, \kappa \rangle \longrightarrow \kappa$.

§2. $\mathrm{SatH}_\kappa^\omega$ and Fodor's property

Now we introduce a weak saturation hypothesis "$\mathrm{SatH}_\kappa^\omega$" and show that this implies Fodor's hypothesis when $\kappa = \aleph_1$.

Definition. Let I be a κ-complete ideal over κ. Then we say I satisfies Fodor's property if I satisfies the following:

If $\{X_\alpha \mid \alpha < \kappa\} \subseteq I^+$ then there is a collection $\{Y_\alpha \mid \alpha < \kappa\} \subseteq I^+$ such that $Y_\alpha \subseteq X_\alpha$ for all $\alpha < \kappa$ and $Y_\alpha \cap Y_\beta = \varnothing$ for $\alpha < \beta < \kappa$.

Definition. We say FH_κ (Fodor's hypothesis for κ) holds, if every κ-complete ideal I over κ satisfies Fodor's property.

THEOREM 11 ([2]). *Assume that $2^{\aleph_0} = \aleph_1$ and I is a normal $\aleph_1$-complete ideal over $\aleph_1$. And also assume that if for all $X \in I^+$ there is a collection $\{Z_\alpha \mid \alpha < \aleph_2\} \subseteq I^+$ such that $Z_\alpha \subseteq X$ for all $\alpha < \aleph_2$ and $\bigcap_{\alpha \in E} Z_\alpha \in I$ for each subset E of $\aleph_2$ of cardinality $\aleph_1$. Then I satisfies Fodor's property.*

LEMMA 12. *Let I be an $\aleph_1$-complete ideal over $\aleph_1$. Then the following assertions are equivalent.*

(1) *I has the following property*:

For all $X \in I^+$ there is a collection $\{Z_\alpha \mid \alpha < \aleph_2\} \subseteq I^+$ such that $Z_\alpha \subseteq X$ for all $\alpha < \aleph_2$ and $\bigcap_{\alpha \in E} Z_\alpha \in I$ for each subset E of $\aleph_2$ of cardinality $\aleph_1$.

(2) *For all* $X \in I^+$, $I|X$ *has the property formulated in* (1) *with respect to* I.

(3) *For all* $X \in I^+$, $I|X$ *has the following property*:

There is a collection $\{Z_\alpha | \alpha < \aleph_2\} \subseteq (I|X)^+$ *such that* $\bigcap_{\alpha \in E} Z_\alpha \in I|X$ *for each subset* E *of* $\aleph_2$ *of cardinality* $\aleph_1$.

Proof. (1)$\Rightarrow$(2): Let $X \in I^+$ and $Y \in (I|X)^+$. Since $Y \cap X \in I^+$, there is a collection $\{Z_\alpha | \alpha < \aleph_2\} \subseteq I^+$ such that $Z_\alpha \subseteq Y \cap X$ for all $\alpha < \aleph_2$ and $\bigcap_{\alpha \in E} Z_\alpha \in I$ for each subset E of cardinality $\aleph_1$ by (1). $Z_\alpha \cap X = Z_\alpha \in I^+$ implies

$$Z_\alpha \in (I|X)^+ .$$

And $(\bigcap_{\alpha \in E} Z_\alpha) \cap X = \bigcap_{\alpha \in E} Z_\alpha \in I$ implies

$$\bigcap_{\alpha \in E} Z_\alpha \in (I|X) .$$

Hence we get (2).

(2)$\Rightarrow$(3): Trivial.

(3)$\Rightarrow$(1): Let $X \in I^+$, then by (3) there is a collection $\{Z_\alpha | \alpha < \aleph_2\} \subseteq (I|X)^+$ such that $\bigcap_{\alpha \in E} Z_\alpha \in I|X$ for each subset E of $\aleph_2$ with $|E| = \aleph_1$. Set

$$X_\alpha = Z_\alpha \cap X \qquad \text{for} \quad \alpha \in \aleph_2 .$$

Then we have $X_\alpha = Z_\alpha \cap X \in I^+$ and

$$\bigcap_{\alpha \in E} X_\alpha = \bigcap_{\alpha \in E} (Z_\alpha \cap X) = \left(\bigcap_{\alpha \in E} Z_\alpha \right) \cap X \in I \qquad \text{for each} \quad E \quad \text{with} \quad |E| = \aleph_1.$$

Thus we get a collection $\{X_\alpha | \alpha < \aleph_2\} \subseteq I^+$ such that

$$X_\alpha \subseteq X \qquad \text{for all} \quad \alpha < \aleph_2 \quad \text{and} \quad \bigcap_{\alpha \in E} X_\alpha \in I$$

$$\text{for each} \quad E \subseteq \aleph_2 \quad \text{with} \quad |E| = \aleph_1.$$

Hence we have (1).

Remark. In (1) of this lemma, we can assume that a collection $\{Z_\alpha | \alpha < \aleph_2\} \subseteq I^+$ is a collection of pairwise distinct sets. Because, $\bigcap_{\alpha \in E} Z_\alpha \in I$ for each subset of $E \subseteq \aleph_2$ with $|E| = \aleph_1$ implies that if Z_α is same for $\alpha \in Y \subseteq \aleph_2$ then Y is at most countable. Then we can get a collection of pairwise distinct sets, because $\aleph_2$ is a regular cardinal number.

COROLLARY 13. *Assume that* $2^{\aleph_0} = \aleph_1$ *and* I *is a normal* $\aleph_1$*-complete ideal over* $\aleph_1$. *And also assume that for all* $X \in I^+$ *there is a collection* $\{Z_\alpha | \alpha < \aleph_2\} \subseteq (I|X)^+$ *such that*

$$\bigcap_{\alpha \in E} Z_\alpha \in I|X \quad \textit{for each} \quad E \subseteq \aleph_2 \quad \textit{with} \quad |E| = \aleph_1 .$$

Then I *satisfies Fodor's property.*

LEMMA 14. *Let I be a κ-complete ideal over κ. Assume that for any $X\in I^+$ there is a collection $\{Z_\alpha \mid \alpha<\kappa^+\}\subseteq(I\mid X)^+$ such that*

$$\bigcap_{\alpha\in E} Z_\alpha\in I\mid X \qquad \text{for each} \quad E\subseteq\kappa^+ \quad \text{with} \quad |E|=\aleph_0 .$$

Then for any $\{X_\alpha \mid \alpha<\kappa\}\subseteq(I\mid X)^+$ there is a $Y\in(I\mid X)^+$ such that

$$X_\alpha-Y\in(I\mid X)^+ \qquad \text{for all} \quad \alpha<\kappa .$$

Proof. Let $X\in I^+$ and $\{Z_\alpha \mid \alpha<\kappa^+\}\subseteq(I\mid X)^+$ be a collection of hypothesis. Assume that there is a collection $\{X_\beta \mid \beta<\kappa\}\subseteq(I\mid X)^+$ such that for all $\alpha<\kappa^+$ there is $\beta<\kappa$ such that $X_\beta-Z_\alpha\in I\mid X$. Then there is $\xi<\kappa$ and $W\subseteq\kappa^+$ with $|W|=\kappa^+$ such that $X_\xi-Z_\alpha\in I\mid X$ for all $\alpha\in W$. Because $I\mid X$ is κ-complete and $\aleph_0<\kappa$, we have that

$$\bigcup_{\alpha\in E} (X_\xi-Z_\alpha)\in I\mid X \qquad \text{for each} \quad E\subseteq W \quad \text{with} \quad |E|=\aleph_0 .$$

But by hypothesis $\bigcap_{\alpha\in E}Z_\alpha\in I\mid X$. Then we get

$$\bigcup_{\alpha\in E} (X_\xi-Z_\alpha)=X_\xi-\bigcap_{\alpha\in E} Z_\alpha\in(I\mid X)^+ .$$

Hence we get a contradiction. This completes the proof.

It is obvious that we can assume that the cardinality of E is $\alpha<\kappa$, in this lemma.

The next result was given by Taylor and independently by B. Balcar and P. Vojtáš [1].

THEOREM 15 ([8]). *Let I be an $\aleph_1$-complete ideal over $\aleph_1$. Then the following assertions are equivalent.*

(1) *I satisfies Fodor's property.*

(2) *Let $X\in I^+$. Then for any $\{X_\alpha \mid \alpha<\aleph_1\}\subseteq(I\mid X)^+$ there is a $Y\in(I\mid X)^+$ such that $X_\alpha-Y\in(I\mid X)^+$ for all $\alpha<\aleph_1$.*

Then we get the next lemma.

LEMMA 16. *Let I be an $\aleph_1$-complete ideal over $\aleph_1$. Then the next* (1) *implies* (2).

(1) *For any $X\in I^+$ there is a collection $\{X_\alpha \mid \alpha<\aleph_2\}\subseteq(I\mid X)^+$ such that*

$$\bigcap_{\alpha\in E} X_\alpha\in I\mid X \qquad \text{for each} \quad E\subseteq\aleph_2 \quad \text{with} \quad |E|=\aleph_0.$$

(2) *I satisfies Fodor's property.*

Proof. From Lemma 14 and Theorem 15.

Now we define a property weaker than SatH_κ, and show this property implies FH_κ when $\kappa=\aleph_1$.

Definition. We say $\mathrm{SatH}_\kappa^\omega$ holds if the following condition is satisfied:

If I is a κ-complete ideal over κ, then there is a collection $\{X_\alpha \mid \alpha<\kappa^+\}\subseteq I^+$ such that

$$\bigcap_{\alpha\in E} X_\alpha \in I \qquad \text{for each} \quad E\subseteq\kappa^+ \quad \text{with} \quad |E|=\aleph_0 .$$

Now we have

THEOREM 17. $\mathrm{SatH}^{\omega}_{\aleph_1}$ *implies* $\mathrm{FH}_{\aleph_1}$.

Proof. Let I be an $\aleph_1$-complete ideal over $\aleph_1$. And let $Y\in I^+$, then $I|Y$ is an $\aleph_1$-complete ideal over $\aleph_1$. Then by hypothesis there is a collection $\{X_\alpha \mid \alpha<\aleph_2\}\subseteq(I|Y)^+$ such that

$$\bigcap_{\alpha\in E} X_\alpha \in I|Y \qquad \text{for each} \quad E\subseteq\aleph_2 \quad \text{with} \quad |E|=\aleph_0 .$$

Hence by Lemma 16, I satisfies Fodor's property. This completes the proof.

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