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Mathematical Reasoning versus Nature*

by

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It is an honour for me to give this lecture at the meeting of your Society and I heartily thank you for the invitation to give it.

A short formulation of the problem I want to discuss in this lecture can be given in the following 4 statements.

1) We all are now in the middle of the overwhelming ecological crisis. All have heard about it and about its possible consequences. Extinction of many species of animals; destruction of forests, by which our Planet is breathing; warming of the atmosphere that may in 30 or 50 years convert many of now fertile lands into deserts, that causes melting of arctic ice, so that ocean may flood many other countries; accumulation of atomic wastes which nobody knows how to dispose, and which will remain radioactive for thousands and even tens of thousands of years, etc., etc...—all this seriously threatens the existence of the living part of Nature—and man as its part.

2) This crisis is not some accidental mistake that can be easily corrected. On the contrary, it is a logical consequence of the development of a very specific civilization that has now spread all over the world. It is called *Technological Civilization* because it is based on application of technique in all possible fields of human activity. Technique is always considered as more reliable and more efficient than Nature and always replaces Nature if this is possible. One sociologist has even described it as an attempt by men to destroy the Nature and to replace it by an artificial Nature, based on technique.

3) The ideological basis of the Technological Civilizaion is the *Scientific Ideology* or *Scientism*. Its foundation is a belief in the existence of a small number of precisely formulated Laws of Nature; using them everything in the Nature can be predicted and manipulated. Cosmos is considered as a gigantic machine which can be operated if one knows the principles of its functioning. This Scientific Ideology plays the role of a religion of the Technological Civilization.

4) The principal dogma of the Scientific Ideology is *Mathematization*. It states that everything (or at least everything that is important) in the Nature can be measured, transferred into numbers (and other mathematical objects) and that by

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manipulation with them one can predict and master all events in Nature and Society.

Kant said that every branch of knowledge is as much a science as much mathematics it contains. Poincaré wrote that the final and perfect phase of every scientific theory is its mathematization.

In a sense one can say that we are living in a *Mathematical* Civilization—and may be dying with it. From this point of view it is natural for a mathematician to be interested in this interrelated questions.

The Scientific Ideology has now a long history. Already Galileo said that "The Book of Nature is written in the language of Geometry" (Geometry then meant Mathematics). About the same time Kepler wrote in a letter to his friend (1605): "My aim is to show that the celestial machine is to be likened not to a divine organism but to a clockwork". Descartes said that an animal is a machine and a century later Lammetry in his book "Man-machine" extended this principle to human beings.

However it was not until Newton's time that this mechanical conception of the world became a complete master over the minds. Newton and his contemporaries called his theory a "System of the World". It fascinated not only his contemporaries but many following generations. It seemed that it is possible to develop the whole picture of the Nature by means of a few laws from which everything can be deduced by means of solving differential equations, power-series expansions and other mathematical tools.

But most fascinated among all seemed Newton himself. It was not by chance that he called his principal work"Mathematical Principles of Natural Philosophy" ("Principia..."). At the end of it he proclaims that the same principles could be applied to include in his "System of the World" also the living beings. Newton writes: "And now we might add something concerning a certain most subtle spirit which pervades and lies hid in all gross bodies; by the force and action of which all sensation is excited, and the members of aminal bodies move at the command of the will, namely by the vibration of this spirit, mutually propagated along the solid filaments of the nerves, from outward organs of sense to the brain, and from the brain into the muscles. But these are things that cannot be explained in a few words." Evidently, Newton has in mind a mechanical theory of ether, and pretends that only lack of space prevents him to develop a mechanical theory of animal bodies and senses based on the theory of ether.

At the same time some anxious voices were heard. One asked whether there remains some place for God in this Mechanical System of Universe? One might ask even—for anything living? All Universe looked as a gigantic machine that operates exclusively on the ground of mechanical laws. And again one who was most anxious about this trend was Newton himself. Although Newton's religious convictions are up to now somewhat mysterious, it is quite certain that he was a deeply religious man. Surely the contradiction of his mechanical System of the World to his religious feelings was most painful to him. He has expressed this clearly in his correspondence. When he was about 50 years old, Newton went through a serious nervous crisis, some call it even a nervous illness. He was unable to sleep for several nights and days. His memory became confused. He was deeply depressed. The symptoms are very similar to those of a crisis, which the russian writer Lev Tolstoy went through at approximately the same age. It was conjectured that probably both crises had an ideological background.

Whatever it could be, in the later publications of Newton we encounter very interesting and new views. For instance in the second edition of "Principia" the famous "General Scholion" appears, where we find such statements as: "This six primary planets are revolted about the sun in circles concentric with the sun, and with motions directed towards the same parts (i.e. all in the same direction) and almost in the same plane. Ten moons are revolved about the earth, Jupiter, and Saturn, in circles concentric with them with the same direction of motion, and nearly in the plane of orbs of these planets; but it is not to be conceived that mere mechanical causes could give birth to so many regular motions, since the comets range over all parts of the heavens in very eccentric orbits; for by that kind of motions they pass easily through the orbs of the planets, and with great rapidity; and in their aphelions, where they move slowest, and are detained the longest, they recede to the greatest distances from each other, and hence suffer the least disturbance from their mutual attractions. This most beautiful system of the sun, planets, and comets could only proceed from the council and dominion of an intelligent and powerful Being." Here Newton clearly implies the most special constellation of the planets which ensures the stability of the solar system so that some planets don't fall at the sun and others aren't driven away like comets.

In his "Optics" Newton inserts some "Queries" such as: Query 23. (About the mechanical principle) "By this principle alone there never could have been any motion in the world. Some other principle was necessary for putting bodies into motion and now they are in motion some other principle is necessary for conserving the motion." In an unpublished "Queries" for one of the editions of "Optics": "If you think that the vis vitaliae is sufficient for conserving motion, prey tell me the experiments from whence you gather thy conclusion. Do you learn by any experiments that the beating of the heart gives no new motion to the blood... or that a man by his will can give no new motion to his body? etc. If so, tell me your experiments, if not your opinion is precarious. Reasoning without experience is very slippery." Obviously, the spirit and even the letter of these statements is contrary to what was written at the end of "Principia': it expresses a doubt about the possibility to explain the functioning of the animal organism by means of "mechanical principle". This principle is considered as applicable to unanimated part of Nature only—and even that with serious reservations. Of course, there always remains the difficult question about the borderline between Living and Unanimated Nature. In an unpublished manuscript "Oblivious Laws and Processes in Vegetation" Newton writes for instance: "The earth resembles a great animal or rather inanimate vegetable which draws in etheral breath for its daily refreshment in vital ferment and transpires again with gross exhalation. And according to condition of all other living beings ought to have its times of beginning, youth, old age, and parishing."

Newton was an extremely ambiguous thinker. His scientific ideas and religious believes were, as it seems, not in accordance: at least one can find in one and the same work contradicting views. But quite independent of his personal inner conflict his Mechanical System of the World had enormous influence. Quite correctly van der Waerden asserts that Newton and not any king of political figure was the central personality of his time.

It was Voltaire who created a cult of Newton in France, obviously using the authority of Newton in his struggle against the Church. However the culmination of scientific ideology and Newton-worship was attained in the school of Saint-Simon. In the first quarter of the XIX century Saint-Simon created a socialist system that lately influenced Marx and Engles. The creator of the philosophy of positivism Comte, was a pupil and some time a secretary of Saint-Simon. The German poet H. Heine was an ardent follower of the teaching of saint-simonism.

Scientific ideology was the basis of saint-simonism. Here are some typical statements of some leading figures among the saint-simonist. "The laws that rule the human society are just as exact as those that rule the fall of a stone". "The art and religion should be constructed with mathematical precision as one teaches "dans l'Ecole" to construct the bridges" ("L'Ecole" means the Politechnical School in Paris). Saint-Simon proposed to introduce a worship of Newton in special temples. The society should, according to his ideas, be ruled by the "Great Newtonian Council", consisting of the best mathematicians, physicists, chemists and physiologists of the world. As a president they had to choose a mathematician. All provinces should be ruled by "Small Newtonian Councils". It is interesting that among the closest followers of Saint-Simon were not only politicians and future revolutionaries but also some rich financiers who later founded the greatest French banks and railway companies. So that in all directions saint-simonism influenced the development of socialist teaching as well as of modern capitalism.

The mixture of scientism and technicism determines the spirit of modern ideology. The great biologist of the XX century K. Lorenz wrote: "One can apply the category of moral to a certain action only if it is directed toward something living. The modern man deals mostly with artificial objects. So he accustoms more and more to judge his actions from the point of view of criteria of efficiency and not of moral. That is why, if he encounters something living, he rapidly destroys it." A modern sociologist, S. Ramo, suggests: "We must now plan on sharing the earth with machines... We become partners. The machines require, for their optimal performance, certain patterns of society. We too have preferred arrangements. But we want what machine can furnish, and so we must compromise. We must alter the rules of society, so that we and they can be compatible." According to the principles of the ideology of the Technological Civilization, all phenomena of life that do not function according to the "optimal patterns" of the machines, i.e. can't be mechanized, are considered as unreliable and unimportant. More than that, they are gradually suppressed. L. von Bertalanffy writes: "This (may be with the exception of the atomic bomb) is the greatest invention of our century: the possibility to reduce a man to an automata, "buying" everything from tooth-paste and "Beatles" to presidents, atomic war and selfdestruction". The attitude, which considers every activity of a man as purely technical allows us to destroy forests and whales and to plan and to put into action socialist experiments on the scale of whole countries. A modern sociologist Mac Laish says: "After Hiroshima it was obvious that the loyalty of sciences was not to humanity but to truths—its own truth—and that the law of science was not the law of the good—but the law of the possible. What it is possible for science to know science must know."

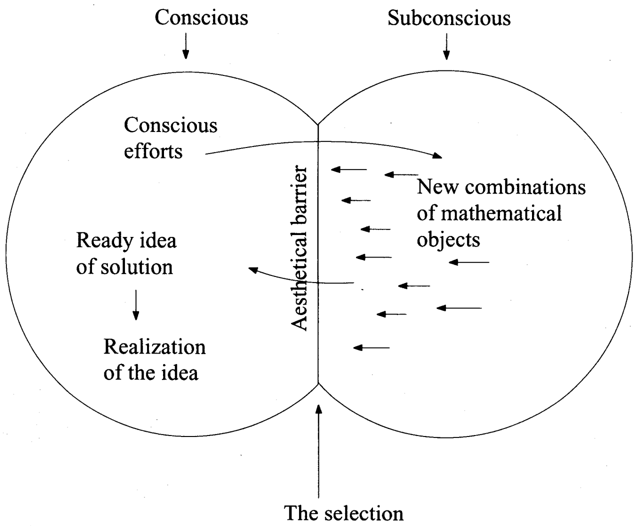
The central dogma of scientific ideology is the belief that everything is measurable, transformable into numbers, all life is translatable in the language of mathematics. This principle is contained already in the appeal of Galileo: "To measure everything that is measurable, and make measurable what is not measurable yet." Especially interesting is the second part of this programme; what should we do with love, compassion, courage or kindness? Obviously, all such features of human life have no place in this mathematized philosophy of the world. In the scientific ideology mathematization plays in principle the same role as standardization in technique. The simplest way to apply mathematics is counting. But one can count only homogeneous objects. Suppose we have, say, an apple, a flower, a dog, a house, a soldier, a girl, and the moon. We can count them, they are 7 in number—but 7 what? The only answer is; 7 subjects. So all the difference between the dog and the moon, the apple and the soldier have disappeared—they lost all their individuality and transformed in pure "subjects". Counting kills individuality. We have here a very primitive example but the idea is always the same. The other feature of mathematics which is very important for the scientific ideology is its ability to transform the solution of deep problems in standardized chains of reasoning. For instance, the squaring of the parabola was in Antiquity a problem which occupied a genius like Archimedes, which needed for its solution beautiful arithmetic identities. However now every high-school pupil can almost mechanically calculate the integral $\int_0^a x^n dx$ for arbitrary n. More than that, such calculation is easily done by a computer. There are plenty of such examples. One has a feeling that mathematics can be reduced to the work of some gigantic computer.

However, all of us who are working in mathematics, are convined that mathematical work is in principle different from the work of a computer. This question was a subject of an interesting debate between Poincaré and Hilbert that took place in the beginning of this century. Then the same problem was formulated differently: can mathematics be formalized? Hilbert's answer was "yes"—and he hoped to obtain in this way a proof of the consistency of Arithemetic. Poincaré didn't agree with him. Gödel's incompleteness theorem apparently solved the problem later in favour of Poincaré.

Poincaré stresses the role of intuition in mathematical reasoning. He says that mathematical reasoning contains in itself "a kind of creative power" and therefore differs from a chain of syllogisms. He especially implies on the mathematical (complete) induction, which, as he says, "contains an infinite sequence of syllogisms so to say compressed in one formula" His statements that methematician differs from a chess player or can't be replaced by some "mechanical device" leave impression that he simply lacked terminology in order to express his view in a short sentence: "mathematician can't be replaced by a computer."

Especially interesting is the role which Poincaré attributed to aesthetical feeling in mathematical work. Mathematical invention brings delight, he says, it is attractive just because of the aesthetical element contained in it. Should mathematics be only a collections of syllogisms, it would be understandable to almost everybody—a good memory would be sufficient. However it is known that most people grasp mathematics with great difficulty. According to Poincaré it is bacause the syllogism in Mathematics form a "structure" which has its beauty. One has to "see" this beauty in order to understand mathematics, which requires some aesthetical abilities, given only to few.

Poincaré suggests a very interesting scheme of the mathematical work. He connects it with the division of man's mind in conscious and unconscious parts (see picture). The process starts with the conscious efforts to solve some problem. This efforts rasise the activity of the subconscious.



There appears a great number of new combinations of methematical objects. They appear with tremendous velocity and in huge numbers. Now one should compare this part of the process with a work of a giant computer. But most part of this combinations has no use for the problem. They do not reach the conscious. Only very few of them are considered by the conscious. To reach the conscious, they undergo a selection, realized by the aesthetical principle. Some kind of aesthetical barrier allows for only small number of them to pass it. They emerge in the conscious as a ready idea of solution. Then it remains only a technical work of realization of this idea.

This scheme obviously resembles the picture of evolution based on mutations and natural selection, and probably arose under the influence of the last one. Much later, apparently being unaware of the views of Poincaré, Konrad Lorenz expressed analogous ideas. He considers life as "a process of learning", a "cognitive process". He stresses features, common to both processes, such as a "creative flash" or "act of creation" when after long roaming about, "almost instantaniously" a new idea or a new species appears. But one can reverse this parallel and consider evolution as functioning of some giant mind or soul of Nature. Conception of such "anima mundi" (soul of the cosmos) appeared in certain philiosophical and mystical teachings, by Plato and in Christiantiy. When I was young I had this idea of evolution as a process of thinking after reading Poincaré and it appeared to me being very attractive. Only much later have I learned that even before the time of Darwin the great naturalist L. Agassiz considered evolutions as "thinking of God". But if we pursue this analogy, how much beautiful picture gives the conception of Poincaré! As a decisive factor of evolution appears not a chance and "survival of the fittest" but an aesthetical criterion. A criterion which explains why Nature produces not only beautiful plants and animals but also solutions of the problem of adaptation of species which in their beauty can compete with most beautiful mathematical theories.

But for a working mathematician, it is hardly necessary to argue about the importance of aesthetical element in mathematics: in conversations of mathematicians one always hears: "this elegant proof", "this beautiful paper"... Every mathematician knows that in his work aesthetical feeling gives not only a delight which encourages and makes easier the necessary efforts but constitutes an effective working device which is no less important for him than purely logical reasoning. He will not pursue some line of thought because it leads to unsymmetric ugly formulas and he will believe in some hypothesis and spend a lot of efforts in trying to prove it because it is very beautiful. In this sense mathematics plays an opposite anti-technological role. We see that under the influence of the technological civilization beauty more and more disappears from our life: from painting and music, from the architecture of our cities, from the surrounding Nature—as beautiful butterflies, flowers and birds. Mathematics (with mathematical physics), however, remains almost the only island which retains this mysterious phenomenon in full amount. Jesus asked: "What is truth?" The notion of beauty is no less mysterious. However it is certain that it is some fundamental mode of reaction, just as important for most living beings as the notion of truth or moral is for human beings.

One friend of mine, a geologist, imparted me the following observation. Many species disappeared because of overdevelopment of some features that were first extremely useful for them: for instance, the huge armour of some giant tertiary reptiles. For Homo Sapiens this role can play his mind, the ability of cold, rational reasoning, unrestricted by moral or pity. Mathematics certainly has something to do with this ability of such algorithmic, machine-like reasoning. On the other hand it is intimately connected with aesthetical feeling which can serve as an antidote to this trend. And a mathematician has free choice to take part in one or other direction of the development of mankind.

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