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A Study of How Pursuit of Wealth Rank Distorts Risk Preferences

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Abstract

We theoretically explore the risk-taking behavior of two unequally-endowed risk-neutral agents who are presented with opportunities to play lotteries. We find that if the agents consider rank in the wealth distribution more important than wealth itself, then their risk preferences are distorted in a way that lowers their expected income, raises inequality and increases wealth-rank mobility. In equilibrium, the rich agent avoids some positive expected return lotteries and both agents gamble on some negative expected return lotteries. We simulate and graph equilibrium strategies to visualize how trying to get richer differs from trying to be richer than someone else.

Keywords: wealth rank, risk-taking, gambling, positional competition, relative income
JEL classification: C73, D31, D81, G11

1. Introduction

We study risk-taking behavior by two risk-neutral agents each of whom cares primarily about his rank in the wealth distribution and secondarily about how much wealth he has. In a previous study of the problem, we examined the intricate ways in which the pursuit of wealth rank distorts risk preferences, inducing agents to gamble on some negative expected return lotteries and to avoid some positive expected return investments. (Rtischev 2008) The model in that study, however, allowed the rich agent to take bigger risks than the poor agent, and the findings were thus contingent on such an inequality of opportunity. In this paper, we re-examine the problem in a different framework that gives both agents the same set of opportunities and also allows visualization of strategies.

Our investigation belongs to the strand of research on the connection between willingness to take risks and concern for relative position. Broadly speaking, we find that the poor takes on more risk while the rich plays it safe. This is generally consistent with prior theoretical results (Gregory 1980; Robson 1992, 1996; Stark 2019) as well as experimental evidence (Mishra, Barclay and Lalumière, 2014). Using a new modeling approach, we are able to depict the agents' risk strategies as regions in two-dimensional space and inspect in detail how the pursuit of rank shapes their risk strategies. This allows us to visually demonstrate that trying to get richer is very different from trying to be richer than someone else. It also reveals that, despite the overall tendency of the poor to gamble and the rich to play it safe, even the rich are prone to play some negative expected return lotteries as part of a strategy to defend high rank.

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Moreover, we find that, relative to pursuit of wealth, pursuit of wealth-rank lowers expected income, raises inequality and increases wealth-rank mobility.

The rest of the paper is organized as follows. The next section presents the model and Section 3 analyzes it in general. Section 4 derives strategies and outcomes in several baseline cases. Section 5 presents equilibrium analysis based on computer simulations. Section 6 concludes.

2. The model

There are two players: Abe endowed with $a>0$ and Bob endowed with $b>0$ dollars. Abe is richer: $d \equiv a-b>0$. A lottery ticket is offered to Bob: the ticket costs $y<b$ dollars, the winning prize is $x+y$ dollars, and the probability of winning is $1/2$. Thus, if Bob decides to buy the ticket, his wealth will either increase to $b+x$ or decrease to $b-y$. Simultaneously, another lottery ticket is offered to Abe: the price and prize are potentially different, but the probability of winning is also 1/2.

Abe and Bob don't see each other's lottery tickets but know that they are independent identically distributed draws from the uniform distribution of equiprobable binary lotteries with support on $\Omega = \{(x,y)|x,y \in (0,M]\}$, where $0<d<M<b$. Each agent must decide whether to buy the ticket he has been offered without any knowledge of the other's ticket or decision.

Alternatively, the game can be formulated in discrete space, as follows. Each player draws 2 balls (with replacement) from an urn containing M balls numbered from 1 to M. The first ball signifies the price of the lottery ticket y and the second ball signifies the possible net gain x. Below, we will use the continuous formulation for analysis and the discrete formulation for computer simulations.

Only one lottery is offered to each player and each must decide whether to play it or abstain. Each player's primary objective is to maximize the probability of becoming the richer player. Maximizing expected income is a secondary objective that each player considers only when the primary objective leaves him indifferent. Formally, if after the lottery Abe ends up with m_A dollars and Bob with m_B, then Abe's utility is

$$\mathrm{u}(m_A) = \begin{cases} m_A + V & if\, m_A \geq m_B \\ m_A & if\, m_A < m_B \end{cases} \tag{1}$$

and Bob's utility is

$$\mathrm{u}(m_B) = \begin{cases} m_B + V & if\, m_B > m_A \\ m_B & if\, m_B \leq m_A \end{cases} \tag{2}$$

where $V >> a+M$ is a big prize that brings more utility than money can buy.

3. Analysis

The game is essentially a winner-take-all tournament. Let m_i be the random variable representing the wealth of player $i \in \{A,B\}$ after the game. Denote the *ex ante* probability of Abe winning the tournament by $\pi \equiv Pr(m_A \geq m_B)$. Let $S_i \subseteq \Omega$ be the strategy of agent i, *i.e.*, the set of lotteries he would play if offered. Any strategy profile $\{S_A, S_B\}$ corresponds to an allocation of contest success probability (CSP) between the two players on the simplex $\{(\pi, 1-\pi) \mid \pi \in [0,1]\}$.

Let $F_i(m) \equiv Pr(m_i < m|S_i)$ be the cumulative distribution function of the post-game wealth of player i if he plays strategy S_i. Denote the corresponding probability density function by $f_i(m) = dF_i(m)/dm$.

Each player's optimal decision whether to play or pass on a given lottery depends on the distribution of the other player's *ex post* wealth given the other player's strategy. Suppose Abe is considering whether to play a particular lottery (x, y) given that Bob is playing strategy S_B which results in an *ex post* distribution of wealth $F_B(m)$. If Abe passes on the lottery, the probability of him remaining the richer man is $Pr(m_B \geq a) = F_B(a)$. If he plays the lottery, the probability of him remaining the richer man is

$$\frac{1}{2}F_B(a+x) + \frac{1}{2}F_B(a-y) \tag{3}$$

The critical threshold for Abe to play the lottery is thus

$$T_A(x,y) \equiv \frac{1}{2}F_B(a+x) + \frac{1}{2}F_B(a-y) - F_B(a) \tag{4}$$

If the threshold is positive, Abe plays the lottery. If it is negative, he abstains. If it is zero, he plays only if the lottery offers a positive expected return. We can thus express Abe's best response strategy as

$$S_A^{BR}(S_B) = \{(x,y) \in \Omega \,|\, T_A(x,y) > 0 \;\; or \; (T_A(x,y) = 0 \;\; and \; x > y)\} \tag{5}$$

Bob's best response can be expressed analogously.

Lemma 1. If a strategy S_i is a best response to some strategy of the opponent, *i.e.*, $S_i = S_i^{BR}(S_j)$, then for all $(x,y), (x',y') \in \Omega$ the following hold

(i) Every lottery offering a higher expected return than a lottery in the best response must also be part of the best response:

$$(x,y) \in S_i \Rightarrow (x',y') \in S_i \;\forall x' \geq x \;\forall y' \leq y$$

(ii) Every lottery offering a lower expected return than a lottery absent from the best response must also be absent from the best response

$$(x,y) \notin S_i \Rightarrow (x',y') \notin S_i \;\forall x' \leq x \;\forall y' \geq y$$

(iii) Let $s_i(x) \equiv \sup\{y \mid (x,y) \in S_i\}$ be the biggest loss that player i is willing to risk on a lottery in the hope of winning x. The function $s_i(x)$ is non-decreasing on (0, M) and traces the upper-left boundary (frontier) of S_i.

Proof.

(i) If $x' \geq x$ and $y' \leq y$, then $T_i(x',y') \geq T_i(x,y)$. Since (x,y) is in the best response set of strategies, $T_i(x,y) > 0$ and therefore $T_i(x',y') > 0$.

(ii) If $x' \leq x$ and $y' \geq y$, then $T_i(x',y') \leq T_i(x,y)$. Since (x,y) is not in the best response set of strategies, $T_i(x,y) < 0$ and therefore $T_i(x',y') < 0$.

(iii) Follows from (i) and (ii). QED

As shown in Figure 1, contest success probability can be computed by integrating probability density over the part of the 2M by 2M square event space that lies above the *45°* line. Specifically,

$$\pi = 1 - \int_{a-M}^{b+M} f_A(m_A) \int_{m_A}^{b+M} f_B(m_B) dm_B dm_A \tag{6}$$

which simplifies to

$$\pi = 1 - F_A(b+M) + \int_{a-M}^{b+M} f_A(m_A)\, F_B(m_A) dm_A \tag{7}$$

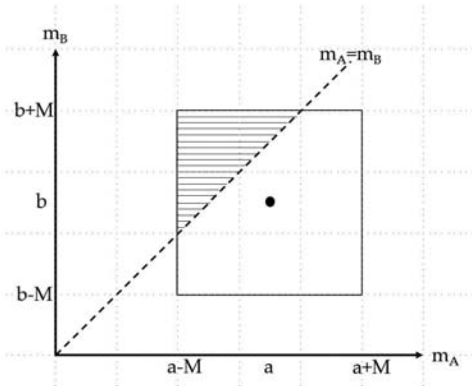


Figure 1. The event space is a 2M by 2M square centered on the endowment point (a, b). Abe remains the richest player at all outcomes that lie on or below the 45° line; all outcomes above the line correspond to Bob becoming the richer player.

By Lemma 1, best response strategies can be expressed in terms of boundary curves $s_A(x)$ and $s_B(x)$. Using these strategy curves and with reference to Figure 2, we can express Abe's cumulative distribution $Pr(m_A<m)$ by

$$F_A(m) = \begin{cases} F_A^{lose}(m), m < a \\ F_A^{win}(m), m \geq a \end{cases} \tag{8}$$

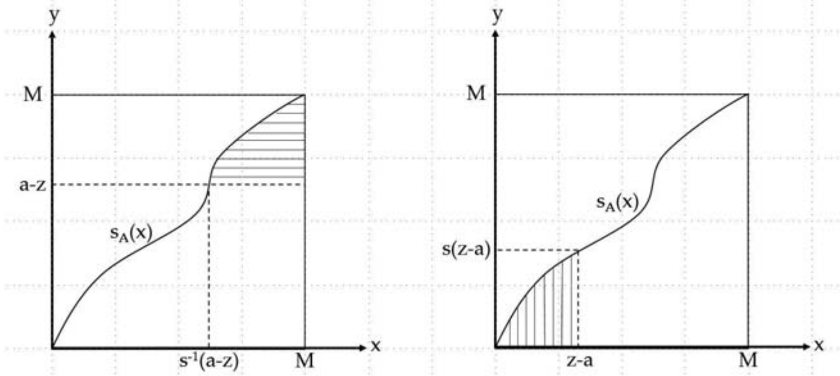
where

$$F_A^{lose}(m) = \frac{1}{2M^2} \int_{S_A^{-1}(a-m)}^{M} (s_A(x) - (a-m)) dx \tag{9}$$

$$F_A^{win}(m) = K_A + \frac{1}{2M^2} \int_0^{m-a} s_A(x) dx \tag{10}$$

and where the probability of Abe abstaining is given by

$$K_A = 1 - \frac{1}{M^2} \int_0^M s_A(x) dx \tag{11}$$



(i) $z<a$ (lottery loss) (ii) $z>a$ (lottery win)

Figure 2. The striped area in (i) represents all lotteries that can result in losses larger than a-z. The striped area in (ii) represents all lotteries that can result in winning less than z-a.

Differentiating (8) gives Abe's probability density function:

$$f_A(\mathrm{m}) = g_A(\mathrm{m}) + K_A\,\delta\,(\mathrm{m}-\mathrm{a}) \tag{12}$$

where δ is the Dirac delta function and

$$g_A(\mathrm{m}) = \begin{cases} g_A^{lose}\,(\mathrm{m})\ , m < a \\ g_A^{win}\,(\mathrm{m})\ , m > a \end{cases} \tag{13}$$

where

$$g_A^{lose}(\mathrm{m}) = \frac{M - s_A^{-1}(a-m)}{2\mathrm{M}^2} \tag{14}$$

$$g_A^{win}(\mathrm{m}) = \frac{s_A(m-a)}{2\mathrm{M}^2} \tag{15}$$

Bob's cumulative distribution and probability density functions can be expressed analogously.

Using these expressions for the distributions and densities, we can rewrite the contest success probability (7) as follows:

$$\pi = 1 - F_A^{win}(b+M) + K_A F_B^{win}(a) + \int_{a-M}^{b+M} g_A(m_A)\, F_B(m_A)\, dm_A \tag{16}$$

Since $a - M < b < a < b+M$, we can rewrite (16) to make explicit the various combinations of wins and losses by the two players:

$$\begin{aligned}\pi = 1 - F_A^{win}(b+M) + K_A F_B^{win}(a) \\ + \int_{a-M}^{b} g_A^{lose}(m_A)\, F_B^{lose}(m_A) dm_A \\ + \int_b^a g_A^{lose}(m_A)\, F_B^{win}(m_A) dm_A \\ + \int_a^{b+M} g_A^{win}(m_A)\, F_B^{win}(m_A) dm_A\end{aligned} \tag{17}$$

Expression (17) links players' strategies to contest success probabilities. The problem of finding best responses and equilibrium corresponds to maximizing (17) over the set of all possible strategy curves $s_A(x)$ and $s_B(x)$. Since it is difficult to solve this in general, we will examine various special cases and then use simulation to study the equilibrium.

4. Strategies and outcomes in baseline cases

We first examine three baseline cases in which both players are hard-wired to play the same strategy.

1. $S_i = \emptyset$: both players abstain from all lotteries. The allocation of CSP is $\pi = 1$.
2. $S_i = \Omega$: both players willing to play all lotteries. The probability density of m_A is uniformly distributed on $[a\text{-}M, a+M]$ and the cumulative distribution of m_A increases linearly from 0 to 1 on the same interval. The density and distribution of m_B are analogous. The allocation of CSP is
$$\pi = 1 - \frac{(2M-d)^2}{8M^2} \tag{18}$$
By symmetry of the lottery space, the expected gain from every positive-expected return lottery is offset by the expected loss from a corresponding negative-expected return lottery. Therefore, expected income is zero.
3. $S_i = \Lambda \equiv \{(x,y) \in \Omega \mid x>y\}$: both players maximize income by playing all lotteries with positive expected return and abstaining from all negative expected return gambles. The probability density and cumulative distribution functions for Abe are given by the following expressions (see Figure 3):
$$g_A^{lose}(m) = \frac{m+M-a}{2M^2}, \quad m \in [a-M, a) \tag{19}$$
$$g_A^{win}(m) = \frac{m-a}{2M^2}, \quad m \in (a, a+M] \tag{20}$$
$$F_A^{lose}(m) = \frac{(m+M-a)^2}{4M^2}, \quad m \in [a-M, a) \tag{21}$$
$$F_A^{win}(m) = \frac{3}{4} + \frac{(m-a)^2}{4M^2}, \quad m \in [a, a+M] \tag{22}$$

Bob's expressions are analogous, with b replacing a. The allocation of CSP is
$$\pi = \frac{5}{8} + \frac{5d}{12M} - \frac{d^2(6M+d)(2M-d)}{96M^4} \tag{23}$$

Players' expected incomes are $E[I_A] = E[I_B] = \frac{M}{12}$, where $E[I_A] \equiv E[m_A] - a$ and $E[I_B] \equiv E[m_B] - b$.

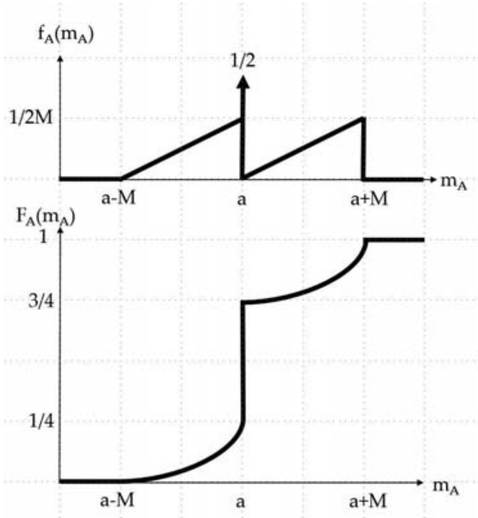


Figure 3. Probability density and cumulative distributions of Abe's post-game wealth if he plays the wealth-maximization strategy. Since he abstains from half the lotteries, the density includes a Dirac delta function with mass 1/2 at m_A = a.

We next examine best-response strategies in six baseline cases.

4. $S_B = \emptyset$: If Bob abstains from all lotteries, Abe's best response is to buy all positive expected return lotteries except those whose price is so large as to jeopardize his initial wealth advantage (Figure 4a left):

$$S_A^{BR}(\emptyset) = \{(x,y) \in \Omega \mid y < \min(d,x)\}$$

Abe's expected income is $E[I_A] = \frac{(d^2 - 3dM + 3M^2)d}{2M^2}$, which is less than M/12 that he could have earned if he didn't care about being leapfrogged and simply played all the positive expected return lotteries. By foregoing some expected income, Abe manages to keep CSP at $\pi = 1$, retaining his top rank for sure.

5. $S_A = \emptyset$: If Abe abstains from all lotteries, Bob's best response is to buy all lotteries with prize large enough to leapfrog Abe, and also positive-expected return lotteries with a smaller prize (Figure 4a right):

$$S_B^{BR}(\emptyset) = \{(x,y) \in \Omega \mid x > \min(d,y)\}$$

Bob's expected income is $E[I_B] = \frac{(d^2 - 3dM + 3M^2)d}{2M^2}$, which is less than M/12 that he could have earned if he didn't try to leapfrog Abe and simply played all the positive expected return lotteries. By foregoing some expected income, Bob manages to lower Abe's chances of retaining top wealth rank to $\pi = \frac{1}{2} + \frac{d}{2M}$.

6. $S_B = \Omega$: If Bob buys all lotteries, Abe's best response is (Figure 4b left):

$$S_A^{BR}(\Omega) = \{(x,y) \in \Omega \mid y < \min(M - d, x)\}$$

Abe's expected income is $E[I_A] = \frac{M^3 - d^3}{12M^2}$, which is less than M/12 that he could have earned if he didn't care about staying ahead of Bob and just tried to maximize income.

7. $S_A = \Omega$: If Abe buys all lotteries, Bob's best response is (Figure 4b right):

$$S_B^{BR}(\Omega) = \{(x,y) \in \Omega \mid x > \min(M - d, y)\}$$

Bob's expected income is $E[I_B] = \frac{M^3 - d^3}{12M^2}$, which is less than M/12 that he could have earned if he didn't care about getting ahead of Abe and just tried to maximize income.

8. $S_B = \Lambda$: If Bob maximizes income, Abe's best response is

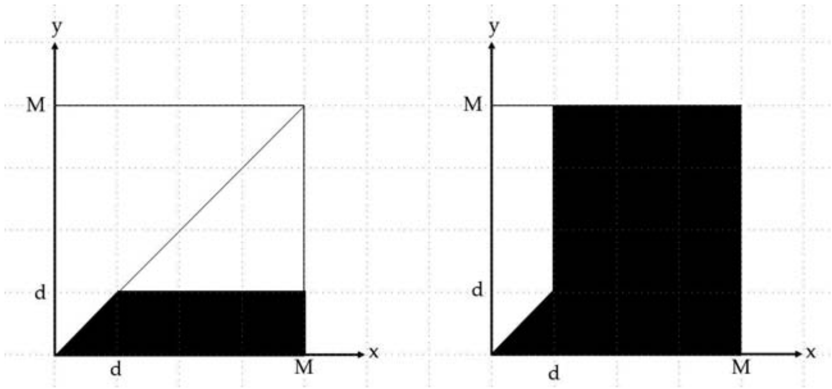
$$S_A^{BR}(\Lambda) = \{(x,y) \in \Omega \mid y < d \text{ and } (x + d)^2 + (y - d)^2 > 2d^2\}$$

This can be derived by substituting Bob's version of (21) and (22) into (4) and solving $T_A(x,y)=0$. As Figure 4c (left) shows, in response to an absolute wealth maximizer, a richer player who wants to keep his rank abstains from some positive expected return investments that may lead to a big loss but is willing to gamble on some small negative expected return lotteries.

9. $S_A = \Lambda$: If Abe maximizes income, Bob's best response is

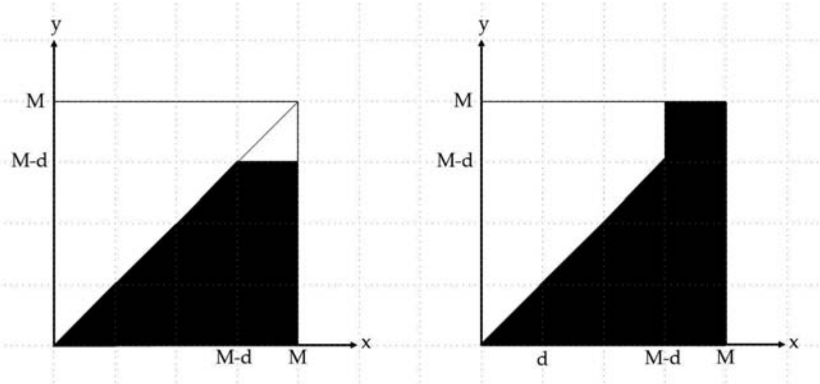
$$S_B^{BR}(\Lambda) = \{(x,y) \in \Omega \mid x > d \text{ or } (x + M - d)^2 + (y - (M - d))^2 > 2(M - d)^2\}$$

This can be derived by substituting (21) and (22) into Bob's version of (4) and solving $T_B(x,y)=0$. As Figure 4c (right) shows, in response to an absolute wealth maximizer, a poorer player who wants to leapfrog plays all positive expected return lotteries and many but not all of the negative expected return ones.



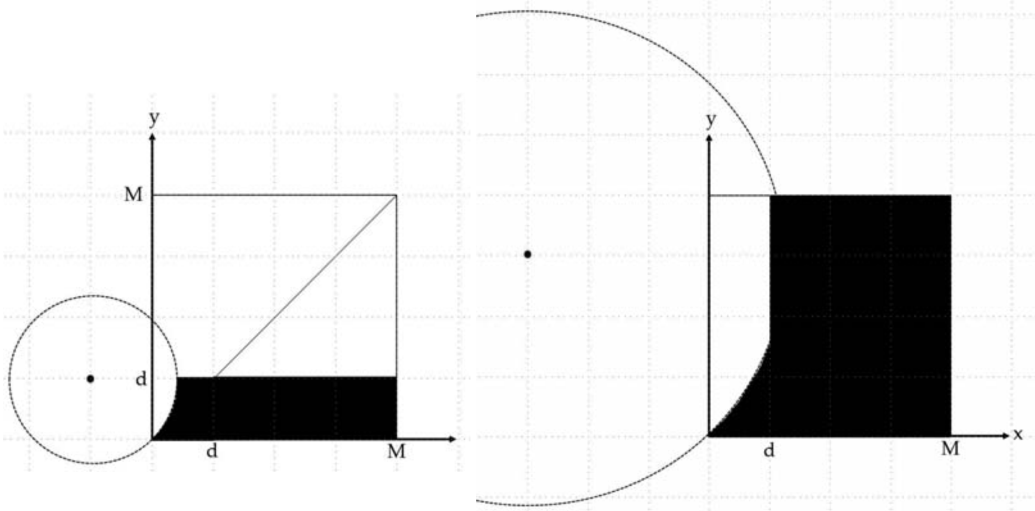
$S_A^{BR}(\emptyset)$ $S_B^{BR}(\emptyset)$

Figure 4a. Best response strategies when the other agent abstains from all lotteries (cases 4 and 5).



$S_A^{BR}(\Omega)$ $S_B^{BR}(\Omega)$

Figure 4b. Best response strategies when the other agent plays all lotteries (cases 6 and 7).



$S_A^{BR}(\Lambda)$ $S_B^{BR}(\Lambda)$

Figure 4c. Best response strategies when the other agent maximizes absolute wealth (cases 8 and 9). The circle on the left is centered at (-d, d) and has a radius of $d\sqrt{2}$. The circle on the right is centered at (d-M, M-d) and has a radius of $(M\text{-}d)\sqrt{2}$.

5. Equilibrium

Nash equilibrium is a strategy profile $\{S_A^{NE}, S_B^{NE}\}$ such that $S_A^{NE} = S_A^{BR}(S_B^{NE})$ and $S_B^{NE} = S_B^{BR}(S_A^{NE})$. We implemented the following simulation algorithm to find equilibrium strategies:

1. Initialize both players' strategies to absolute wealth maximization: $S_i = \Lambda$
2. Compute players' probability distributions of post-lottery wealth $f_i(m)$
3. Re-compute Abe's best response strategy, i.e., the strategy that maximizes the probability that $m_A \geq m_B$ given $f_B(m)$
4. Re-compute the probability distribution of post-lottery wealth for Abe: $f_A(m)$
5. Re-compute Bob's best response strategy, i.e., the strategy that maximizes the probability that $m_B > m_A$ given $f_A(m)$
6. Re-compute the probability distribution of post-lottery wealth for Bob: $f_B(m)$
7. Repeat 3-5 until both players' strategies stop changing or the maximum number of iterations has been reached

We programmed the algorithm in Ruby and ran the simulation with various sets of parameters. For all parameters and initial strategies that we tried, the algorithm quickly converged to an equilibrium.[2] We

2) For some parameter combinations, the algorithm converged to a cycle between two strategies. Since scaling the parameters proportionally eliminated such cycling, it appears that the cycling is an artifact of the discretization of strategy space.

will describe the results of a simulation with the following parameters: a=169, b=140, M=120. This simulation reached equilibrium in 13 iterations. The equilibrium strategies are shown in Figure 5 and closely resemble equilibria reached in other simulation runs using different parameter values. Note that in equilibrium both rich and poor agents play some negative expected return lotteries and that the rich agent but not the poor avoids some positive expected return lotteries. Figure 6 shows the equilibrium distributions of the players' post-game wealth. Overall, the poor agent takes on much more negative expected return risk than the rich agent. However, the poor agent is also more willing to make positive-expected return investments than the rich; in fact, unlike the rich, the poor agent does not abstain from any positive expected return lottery.

In this simulation, when both players maximize income, the probability that the rich stays rich is $\pi = 0.72$. Playing the game according to equilibrium strategies reallocates some CSP to the poor player. Specifically, in equilibrium $\pi = 0.65$, which means strategizing for rank raises the poor player's probability of leapfrogging from 0.28 to 0.35, a gain of *25%*.

Although playing the game according to equilibrium strategies increases wealth rank mobility, it reduces the expected income of both agents. Specifically, when using income maximization strategies, each agent's expected income is *10*. However, when playing rank-seeking equilibrium strategies, the rich and poor agent's expected incomes drop to *9.3* and *4.5*, respectively. Thus, by pursuing wealth rank instead of wealth, the rich player sacrifices expected income by 7% and the poor player by *55%*.

In the simulation, the pursuit of rank in the wealth distribution leads to greater wealth inequality. As Table 1 shows, the difference in initial endowments is 9.4% of total wealth. If the agents maximize income, the difference in their expected wealth comes down to 8.8% of the total expected ex-post wealth. However, if they pursue wealth rank, it rises to 10.5%. Thus, pursuing income reduces inequality but pursuing wealth rank increases it.

Table 1. Summary of simulation results with parameters a=169, b=140, M=120.

		Abstain	Max income	Wealth rank	percent change
Strategies		S= ∅	S= Λ	S^{NE}	
Abe's income	$E[m_A]$-a	0	10.0	9.324	-6.8%
Bob's income	$E[m_B]$-b	0	10.0	4.498	-55.0%
Abe's wealth	$E[m_A]$	169.0	179.0	178.324	-0.4%
Bob's wealth	$E[m_B]$	140.0	150.0	144.498	-3.7%
wealth gap	$\Delta = E[m_A - m_B]$	29.0	29.0	33.826	16.6%
total wealth	T	309.0	329.0	322.822	-1.9%
inequality ratio	Δ /T	0.0939	0.0881	0.1048	18.9%
Abe's CSP	π	1.0000	0.7210	0.6522	-9.5%
Bob's CSP	1- π	0.0000	0.2790	0.3478	24.7%

It is interesting to compare this equilibrium to best-responses against an income-maximizing opponent that we considered in general in Section 4, Cases 8-9 and Figure 4c. Figure 7 shows the simulated best-response strategies to an income-maximizing opponent for the same parameter values as the equilibrium simulation; Figure 8 shows the corresponding distributions of *ex-post* wealth. Figure 9 shows the difference between the equilibrium strategies and the best-responses to income maximization. As Figure 9a reveals, Abe plays many more positive expected return lotteries in equilibrium than when best-responding to an income-maximizing Bob. However, as Figure 9b reveals, Bob plays many more negative expected return lotteries in equilibrium than when best-responding to an income-maximizing Abe. Although the shapes of the equilibrium strategies resemble the best-response strategies to an income-maximizing opponent, in equilibrium both agents play more lotteries, take on more risk, and generate more variance in *ex-post* wealth.

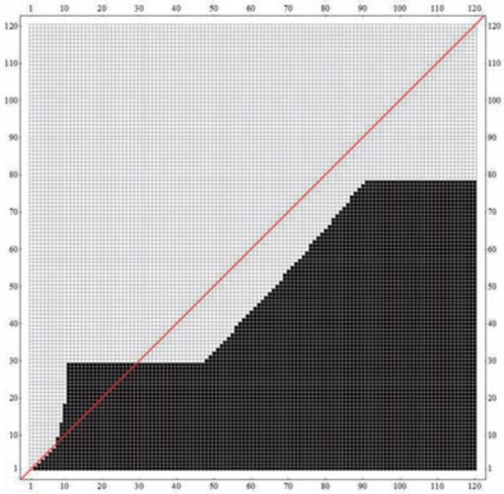


Figure 5a. Abe's equilibrium strategy. Abe plays some small negative-expected-return gambles and avoids many positive-expected-return investments. Parameter values: a=169, b=140, M=120. The horizontal axis is x (money gained upon winning lottery) and the vertical axis is y (money lost upon losing the lottery). Lotteries above (below) the diagonal line offer negative (positive) expected return.

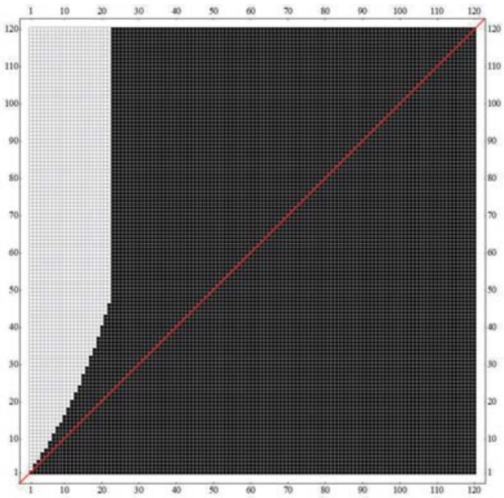


Figure 5b. Bob's equilibrium strategy. Bob does not avoid any positive-expected-return investments and plays many negative-expected-return gambles. Same parameter values as above.

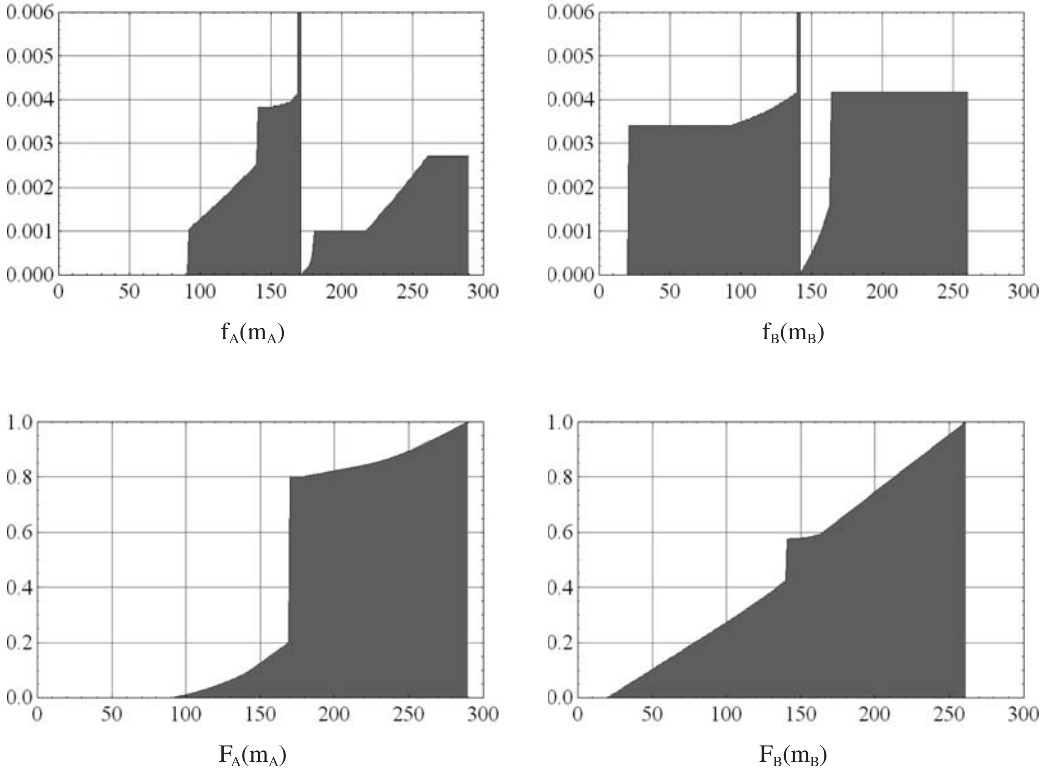


Figure 6. Probability density and cumulative distribution functions of post-game wealth corresponding to the equilibrium strategies in Figure 5. The point mass at endowment points a=169 and b=140 represents probability of abstaining from lotteries (peak not shown to scale). Parameter values: a=169, b=140, M=120.

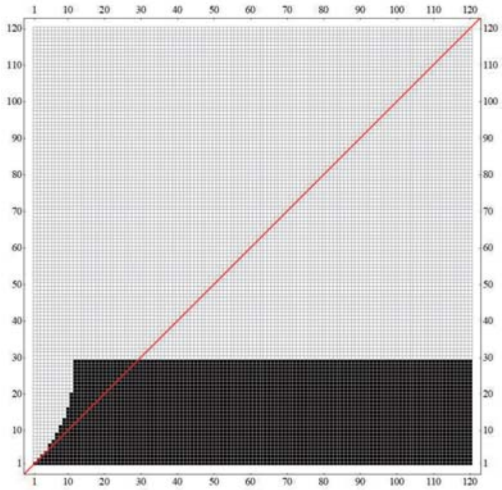


Figure 7a. Abe's best-response strategy when Bob maximizes his income. Parameter values: a=169, b=140, M=120.

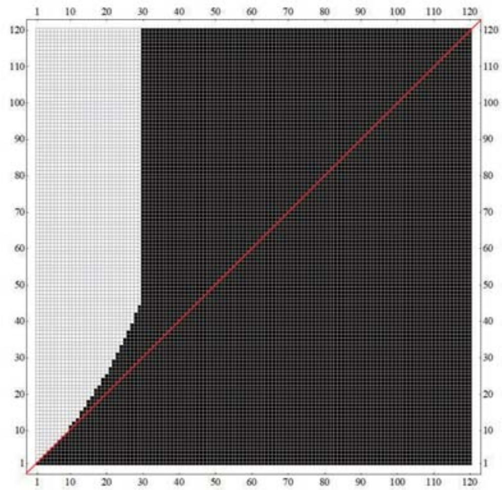


Figure 7b. Bob's best-response strategy when Abe maximizes his income. Parameter values: a=169, b=140, M=120.

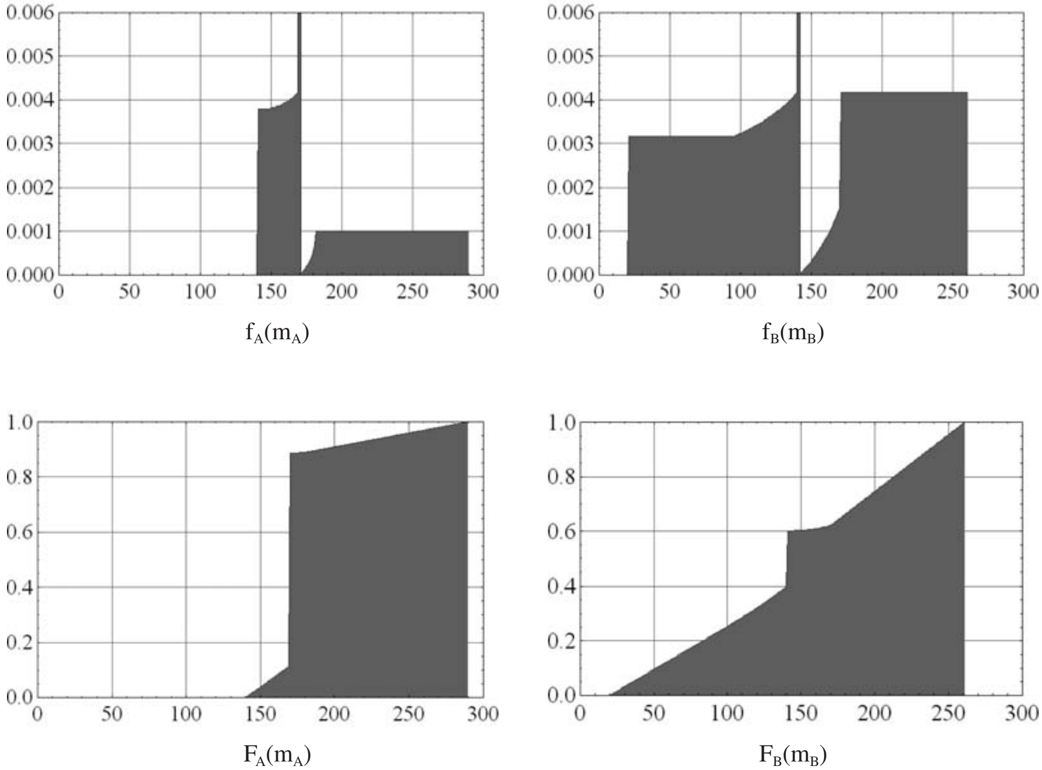


Figure 8. Probability density and cumulative distribution functions of post-game wealth corresponding to best-response strategies against an income-maximizing opponent shown in Figure 7. The point mass at endowment points a=169 and b=140 represents probability of abstaining from lotteries (peak not shown to scale). Parameter values: a=169, b=140, M=120.

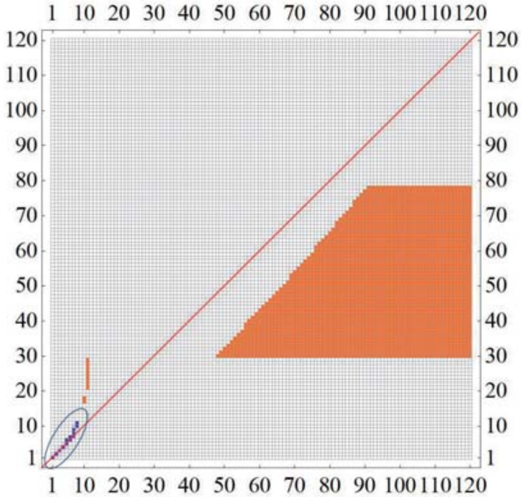


Figure 9a. Difference between strategies in figures 5a and 7a. Almost all of the shaded area represents lotteries played by Abe in equilibrium but not when best-responding to an income-maximizing Bob. Only the shaded lotteries circled in the lower left corner are not played in equilibrium but played when best-responding to an income-maximizing Bob.

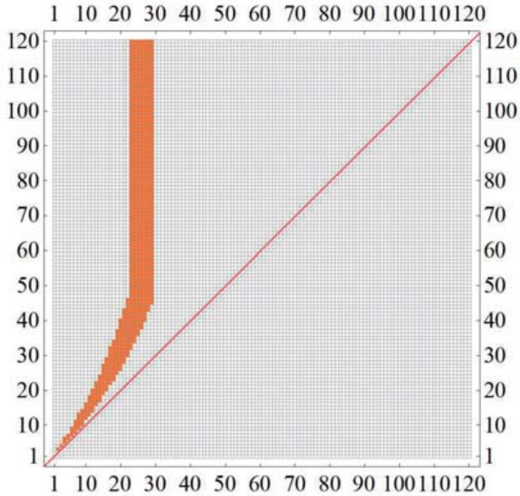


Figure 9b. Difference between strategies in figures 5b and 7b. Shaded area represents lotteries played by Bob in equilibrium but not when best-responding to an income-maximizing Abe.

6. Conclusion

We used a simple setting to explore how trying to get richer differs from trying to be richer than someone else. Specifically, we examined the risk-taking behavior of two unequally endowed risk-neutral agents who are presented with identical opportunities to play binary lotteries. We found that if the agents pursue rank in the wealth distribution as a primary objective and wealth as a secondary objective, their risk-taking behavior is distorted (relative to wealth-seeking) in a way that lowers their expected income and raises both inequality and wealth-rank mobility. In equilibrium, the poor agent is willing to bet on all positive expected return lotteries and on many of the negative expected return lotteries as well; the rich agent abstains from many positive expected return lotteries but is willing to gamble on a small number of negative expected return lotteries. It remains an open problem to generalize the analysis to many players and derive analytical expressions for the equilibrium strategies.

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