

# Web Publication of Three-Dimensional Animation Materials for Business Mathematics

## — 10 Graphics for Economics Mathematics Part 2 —

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### Abstract

The paper reports our web publication of visual teaching materials for business mathematics. The objective of the web site is to familiarize students with our visual approach for business mathematics at early stage. The significant advantage of visualization is that seeing the materials leads to better understanding of the mathematical process. Even if the students cannot understand the algebraic calculation process, they can intuitively understand the essence, only seeing the visual materials. The feature of our developed teaching materials is utilization of three-dimensional graphics and its animations. Just watching the animation, students can see the points, even if they have no idea about operations on the web. In the paper, we will illustrate three typical economics problems using three-dimensional animation which are a national income determination problem, an IS-LM analysis, and a Marshallian demand function.

### 1. Introduction

The paper reports our web publication of visual teaching materials for business mathematics. The objective of the web site is to familiarize students with our visual approach for business mathematics at early stage. The significant advantage of visualization is that seeing the materials leads to better understanding of the mathematical process. Even if the students cannot understand the algebraic calculation process, they can intuitively understand the essence, only seeing the visual materials. The feature of our developed teaching materials is utilization of three dimensional graphics and its animations. Just watching the animation, students can see the points, even if they have no idea about operations on the web.

In January 2011, we have published our first web site titled “Bond Mathematics by Graphics” [1]. The contents of the web site are mainly ones for bond mathematics in financial mathematics.

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Then, in autumn of 2011, we have published another web site titled "10 Graphics for Economics Mathematics Part 2"[2]. The contents are mainly ones for ordinary business mathematics problems and they are ones that we are using in our lectures. This paper reports the contents of this another 10 graphics site. In the paper, we will illustrate three typical economics problems using three-dimensional animation which are a national income determination problem, an IS-LM analysis, and a Marshallian demand function.

The following section illustrates teaching materials for national income determination problems. Section 3 explains teaching materials for IS-LM analysis. In section 4, we illustrate a constrained optimization problem called a Marshallian demand function using three-dimensional graphics and animation. Finally, section 5 gives concluding remarks and describes the direction of our future work.

## 2. National Income Determination

Macroeconomics is concerned with the equilibrium level of national income. In the national income determination problem, setting up the national economy model, we find the equilibrium levels of the national income[3].

Let us solve the problem:

Problem: Given that consumption  $C=100+0.6 Y$ , the investment  $I=40$ , and the government expenditure  $G=40$ , where  $Y$  represents the national income. From the theory, we know the relation  $Y=C+I+G$ . Find the equilibrium level of national income. Then how is the effect on the equilibrium level if the government expenditure  $G$  rises to be 60 from 40?

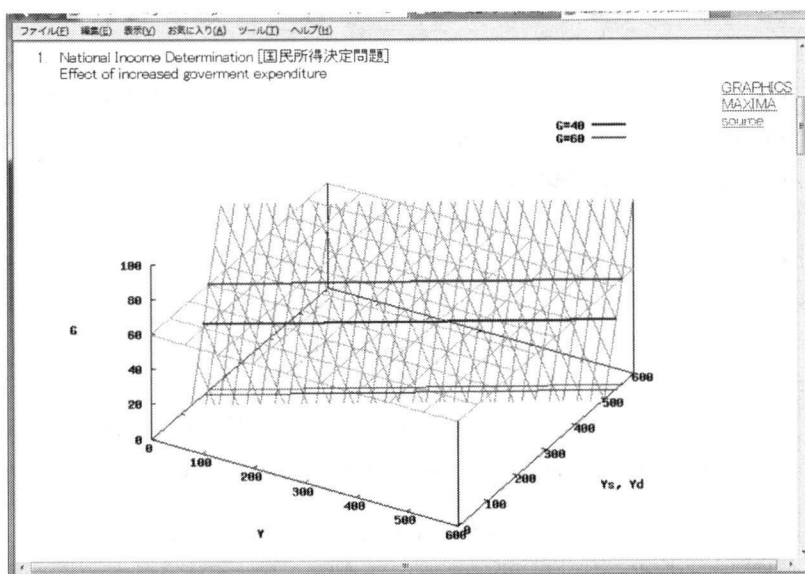


Figure 1. A three-dimensional gif animation for explanation of a shift effect by the government expenditure in the national income determination problem.

The point is to grasp the effect on the equilibrium levels by the increase of  $G$ . To put emphasis on the change of  $G$ , we shall illustrate a three-dimensional graphics with  $G$  as the perpendicular axis as shown in Figure 1. The other axes are the national income  $Y$  and  $Y_s$  and  $Y_d$  where  $Y_s$  represents the total supply of the nation and  $Y_d$  represents the total demand of the nation. From the theory,  $Y_s$  and  $Y_d$  are as follows:

$$Y_s=Y, \quad Y_d=C+I+G$$

Then, we calculate the function of  $Y_d=Y_d(Y, G)$ .

$$Y_d=140+0.6Y+G$$

This  $Y_d$  plane is illustrated in Figure 1 as a shaded plane. In Figure 5, another plane corresponding to  $G=60$  are illustrated. The  $G=60$  plane is parallel to the 2-D plane of  $Y$  and  $Y_d$ . Then we take the intersection between these two planes. In other words, we cut the 3-D  $Y_d$  plane by the  $G=60$  plane in a horizontal direction.

The result is the  $Y_d(Y)$  line at the  $G$  value. By projecting the line to the 2-D plane of  $Y$  and  $Y_d$ , we can obtain the function of  $Y_d=Y_d(Y)$ . Similarly, we can obtain the  $Y_d$  line at  $G=40$ .

Finally, we draw the supply line  $Y_s=Y$  on the 2-D plane of  $Y$  and  $Y_d$  as shown in Figure 2. As the effect of the increase of  $G$ , the equilibrium level increases from 450 to 500.

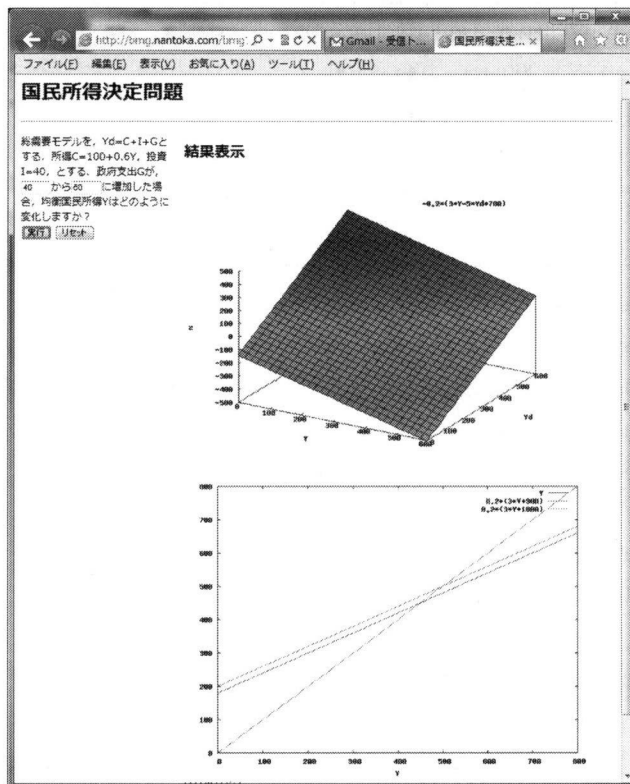


Figure 2. The teaching material for the national income determination in which a student can modify the given values of the government expenditure. The graphics are illustrated instantly.

This is a typical shift problem which often appears in economics mathematics. In existing textbooks of economics mathematics, by only using the 2-D plane of  $Y$  and  $Y_d$  illustration, the shift are explained<sup>1</sup>. However, when we investigate an effect of the third parameter's increase, 3-D graphics are effective. On our web site titled "10 Graphics for Economics Mathematics Part 2", the shift process is illustrated by a gif animation. The reason why we made the gif animation is we would like to implement how to operate on the graphics for computer-sensitive students. Because the operations would include a rotation, a cutting by a plane, and a projection, we thought someone could not handle them well.

For developing a gif animation like this, we used MAXIMA which is a free software [4]. As illustrated here, a three-dimensional animation is effective when we explain the third parameter's effect on a noticeable parameter (e. g., national income).

### 3. IS-LM Analysis

Another shift effect example is illustrated on our web titled "10 Graphics for Economics Mathematics Part 2" which is an IS-LM analysis. To analyse IS and LM schedules, the interest rate  $r$  is introduced and the equilibrium of the money market has to be considered[3]. Let us solve the following sample problem.

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Problem: Given the commodity market relations as follows:

$$Y_d = C + I, C = 0.7Y + 200, I = -100r + 100$$

In addition, given the money market relations as follows:

$$M_d = 0.2Y + 400 - 200r$$

What is the effect on the equilibrium level of income if the money supply  $M_s$  increases from 580 to 590?

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We call the equation relating  $Y$  and  $r$  in the commodity market "IS schedule". On the other hands, the equation relating  $Y$  and  $r$  in the money market is called "LM schedule". From the theory, at the equilibrium level,  $M_s = M_d$  in the money market. In the problem, the IS and LM schedules are calculated as follows:

$$r = -\frac{(3Y - 3000)}{1000}, \quad r = \frac{(Y - 5M_s + 2000)}{1000}$$

Solving them simultaneously,

$$M_s = \frac{(Y - 1000r + 2000)}{5}$$

In our teaching materials, the function of  $M_s$  is illustrated as shown in Figure 3. The shaded plane is the  $M_s$  function. The perpendicular axis is  $M_s$  and other two axes are  $Y$  and  $r$ . In the Figure 3, we cut the  $M_s$  plane by the  $M_s = 590$  plane in a horizontal direction. On the 2-D plane of  $Y$  and  $r$ , this intersection line is projected as the right -sided line. This is a LS schedule from the equilibrium money market.

On the other hands, we draw the IS schedule on the same projected 2-D plane as shown in Figure 4. From Figure4, we can find that the increase of  $M_s$  increases the equilibrium level of the national income.

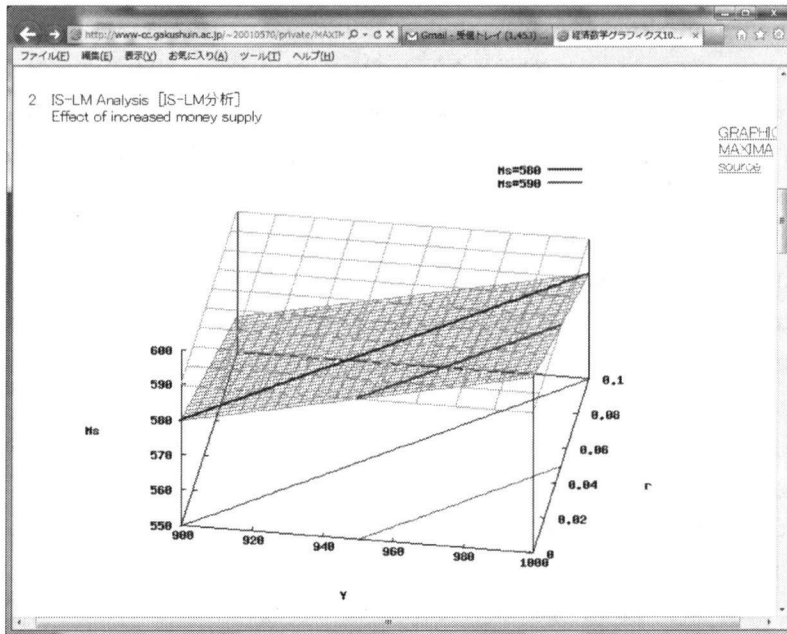


Figure 3. A three-dimensional animation for explanation of a LM schedule shift by the  $M_s$  change effect.

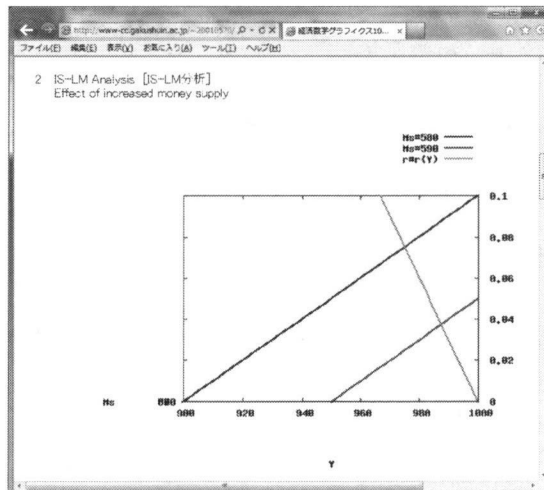


Figure 4. A simple explanation only by a 2-D graph for an IS-LM analysis.

Concerning the IS-LM analysis, the web offers the page in which a student can modify the  $M_s$  value and draw the 3-D and 2-D graphics as shown in Figure 2.

#### 4. Marshallian Demand Function

A Marshallian demand function expresses the amount of a commodity that a consumer will buy as a function of commodity prices (e.g.,  $p_1$  and  $p_2$ ) and an available income/budget  $M$ [5]. For example, the function is expressed as  $f(M, p_1, p_2)$  for two kinds of commodities. The function is named after Alfred Marshall.

This problem is a constrained maximization problem of the consumer's utility level. The utility level expresses a consumer's satisfaction level. In the case of two kinds of commodities, the unknowns are the two amounts/quantities of the commodities at the utility maximum point. We call the function of the quantity of the commodity at the maximum point the Marshallian demand function. Namely, the Marshallian demand function is derived by means of utility maximization subject to a budgetary constraint.

Suppose that a utility function is given as  $u(x, y) = \sqrt{x} + \sqrt{y}$  subject to  $p_1 \times x + p_2 \times y = M$ , where  $M$  is the budget available,  $p_1$  and  $p_2$  are prices of commodity 1 and 2, and  $x$  and  $y$  are quantities of commodity 1 and 2.

In general, you may change the utility function, so that the function can fit your case. For example,  $u(x, y) = xy$ ,  $u(x, y) = \sqrt{xy}$ , or  $u(x, y) = \log(xy)$  would be available.

We know that algebraically we can solve a constrained optimization problem using the Lagrange function as follows:

$$L = u + \lambda \cdot \{M - (p_1 \cdot x + p_2 \cdot y)\}$$

Thus, in this case,

$$\frac{\partial}{\partial x} L = \frac{1}{2\sqrt{x}} + \lambda \{-p_1\} = 0$$

$$\frac{\partial}{\partial y} L = \frac{1}{2\sqrt{y}} + \lambda \{-p_2\} = 0$$

Solving these expressions simultaneously, we can the expression of the Lagrange multiplier  $\lambda$  as follows:

$$\lambda = \frac{\frac{1}{2\sqrt{x}}}{p_1} = \frac{\frac{1}{2\sqrt{y}}}{p_2}$$

In general, at the maximum point, the  $\lambda$  value can be expressed with the marginal utility  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$  as follows:

$$\lambda = \frac{1}{p_1} \cdot \frac{\partial u}{\partial x} = \frac{1}{p_2} \cdot \frac{\partial u}{\partial y}$$

At the maximum point, the Lagrange multiplier is expressed as the marginal utility with respect to the quantity per JPY (Japanese Yen) of the price concerning each commodity.

From this relation between  $x$  and  $y$ , find  $y$  as follows:  $y = \frac{xp_1^2}{p_2^2}$

Insert this function  $y$  to the budget condition expression, and we can obtain the expression of  $M$

as follows:

$$M = \frac{p_1 x (p_2 + p_1)}{p_2}$$

Thus,

$$x = \frac{M}{\frac{p_1 (p_2 + p_1)}{p_2}}$$

Similarly, we can obtain the following function of y.

$$y = \frac{M}{\frac{p_2 (p_1 + p_2)}{p_1}}$$

These two expressions of x and y are called the Marshallian demand functions.

We illustrate our three-dimensional graphics for this constrained optimization problem in Figure 5. In the graphics, the utility function  $u(x, y)$  is illustrated as a curved surface. In addition, a plane is illustrated. The plane is corresponding to the budget condition  $p_1 \times x + p_2 \times y = M$

The intersection curve between the utility curved surface and the condition plane is also illustrated in Figure 5. Under the budget condition, we can select innumerable x and y value pairs on the intersection curve. Then, we can visually spot the maximum point on the intersection curve. In Figure 5, the point is marked by a small sphere.

The graphics operation is facilitated with three sliders of commodity prices  $p_1$  and  $p_2$  and the budget amount M. Our students can move these three values by the sliders. Then the plane will be moved following the slider values. When M becomes bigger, the maximum point of the utility also becomes bigger. On the graphics, the values of x, y, and  $\lambda$  at the maximum point change following the slider movement. The values are ones calculated by the Lagrange multiplier method. In addition, on the graphics, we can confirm that the value of  $\lambda$  is equal to the marginal utility per JPY of the price. In Figure 5, the value is 0.001497 approximately.

Such kind of interactive graphics illustration would help students understand the relationships. On the web publication, we offered an explanation video material in which we operated sliders. Both a Japanese version video and an English version video are available on our web. This graphics is developed in Maple which is a commercial mathematics software. In Gakushuin University, students can use Maple freely.

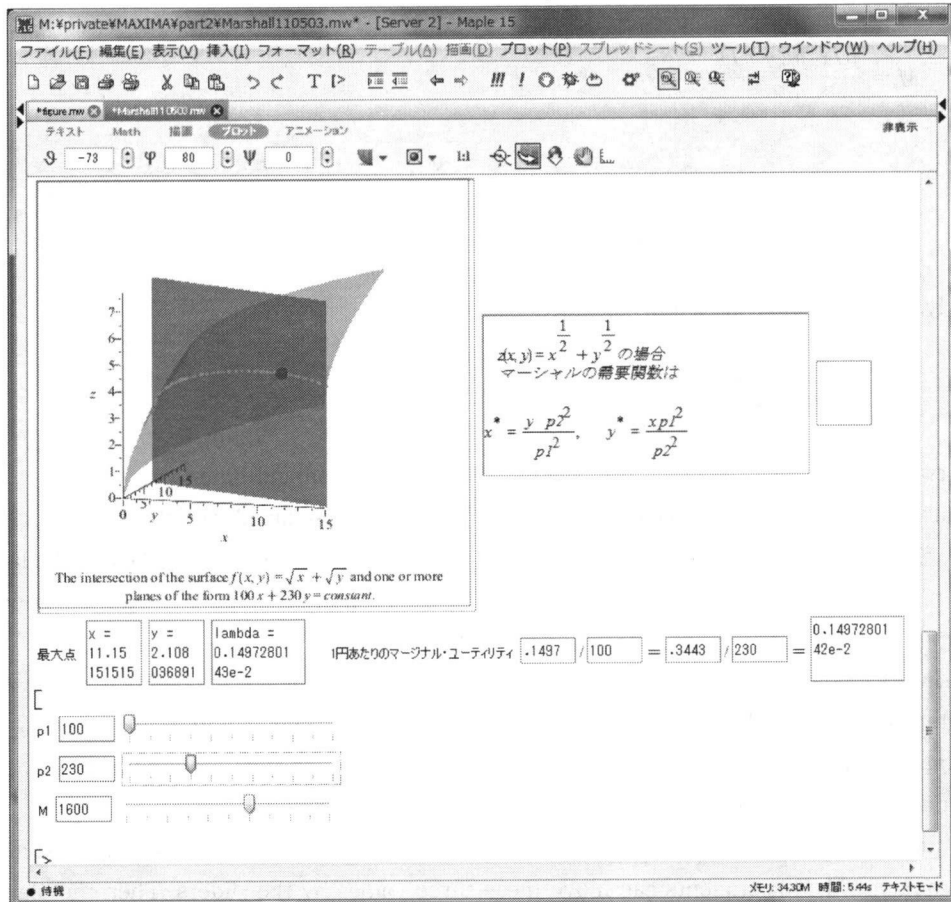


Figure 5. A three-dimensional graphics for Marshallian demand function

In general, the Lagrange multiplier  $\lambda$  in the Marshallian demand function problem at the maximum point can be expressed as follows:

$$\frac{\partial}{\partial M} u = \lambda$$

In general, the Lagrange multiplier is given by the ratio of the changes: approximately change in optimum value of  $u$  over change in  $M$ . Namely we can say that the Lagrange multiplier  $\lambda$  is expressed as the effect on a utility level by the fluctuation of budget amount at the maximum point. In other words, that is the cost-utility performance.

On the web, we illustrate visually this  $\lambda$ . The three-dimensional graphics in Figure 6 shows the gradient of  $u$  at the maximum point by an arrow mark. The arrow mark can also be changed interactively by the slider moving. As the students can change their viewpoints, rotating the graphics, at the proper viewpoint, they can perceive  $\Delta u$  over  $\Delta M$  from the change in slope. Then they can understand the bigger  $M$  becomes, the smaller the cost-utility performance becomes.



The graphics in Figure 6 is illustrated by Mathematica which is a commercial mathematical software widely used.

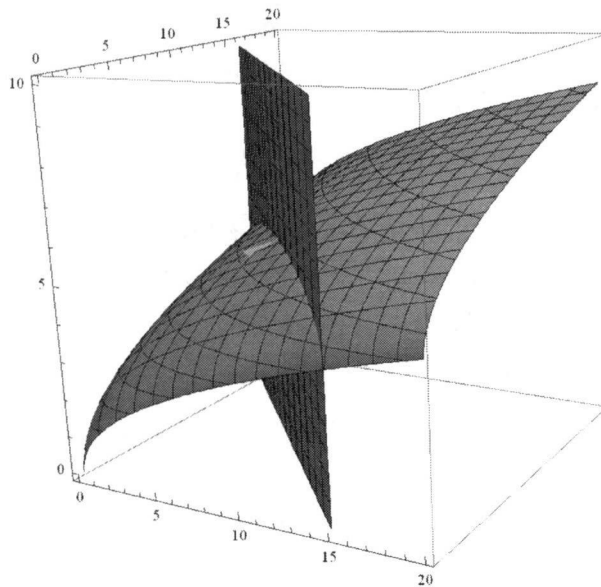


Figure 6. A three-dimensional graphics for explanation of the Lagrange multiplier.

The Lagrange multiplier is explained in many mathematical textbooks[6, 7]. However, as far as we know, we have never seen this kind interactive teaching material. Therefore we would like to put an emphasis of the effectiveness with the slider movement to explain the Lagrange multiplier.

## 6. Conclusion

The paper reports our web publication of visual teaching materials for business mathematics. The objective of the web site is to familiarize students with our visual approach for business mathematics at early stage. The significant advantage of visualization is that seeing the materials leads to better understanding of the mathematical process. In pour lectures of business mathematics, students are given both algebraic and visual approach teaching. Even if the students cannot understand the algebraic calculation process, they can intuitively understand the essence, only seeing the visual materials. The feature of our developed teaching materials is utilization of three-dimensional graphics and its animations.

In the paper, we illustrated three typical economics problems using three-dimensional animation which are a national income determination problem, an IS-LM analysis, and a Marshallian demand function. In the first and second problems, a shift effect is explained. In economics Mathematics, such shift effect problems are often discussed. To explain the effect, a three-dimensional animation is effective to make students understand the process: taking the

fluctuated parameters as the perpendicular axis, drawing the 3-D graphics with other two parameters, cutting the 3-D plane with the fixed value plane of the parameter, taking the intersection between these two planes, and finally projecting the intersection on the 2-D plane.

In the third example of the constrained optimization problem, we illustrated an interactive graphics facilitated sliders. By moving the budget amount, the student can understand the fluctuation of the maximum point which leads to understand the meaning of the Lagrange multiplier. We would like to put an emphasis of the effectiveness with the slider movement in business mathematics lectures.

We have published the web titled "10 Graphics for Economics Mathematics Part 2" which included these three teaching materials. The objective of the web is to make such a visual approach in mathematics teaching much more popular in Japan. We will continue to make various visual materials for business mathematics lectures.

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