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THE INFLUENCE OF LIFE EXPECTANCY IN THE DETERMINATION OF PENSION FUND INVESTMENTS

Farah Kristiani[1]

[1]Department of Mathematics,Faculty of Information Technology and Sciences
Parahyangan Catholic University
Jln. Ciumbuleuit 94, Bandung – Indonesia
Email: farah@unpar.ac.id

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Abstract

Preparating one's retirement is important for many people, because an employee will often have no salary anymore after retiring, but his/her daily needs are still remained. It is not easy to calculate or estimate the amount of investment funds that must be saved during the active working years to make up for the lack of funds after one's retirement. This is still the case, even when the case when he/she will receive a pension from the company where he/she worked. In this research we further examine how the life expectancy plays an important role in calculating the amount of necessary fund investments. The results of these calculations are compared with the results without life expectancy. From this analysis, it can be concluded that the required pension fund investments calculated with the life expectancy is much less than the one without life expectancy. This happens because the required pension fund investments are only estimated by assuming that he/she is still alive.

Keywords: present value, annuity, pension, investment

Abstrak

Persiapan memasuki usia pensiun penting untuk dilakukan mengingat seorang karyawan tidak akan mempunyai pemasukan dari gaji setelah dia pensiun, tapi pengeluaran untuk kebutuhan hidup sehari-hari masih tetap ada. Kesulitan yang sering muncul adalah menghitung berapa besaran dana investasi yang harus dilakukan pada saat masih aktif bekerja supaya mencukupi kekurangan dana setelah seorang karyawan pensiun meskipun ia telah menerima juga tunjangan pensiun dari perusahaan tempatnya bekerja. Pada penelitian ini ditelaah lebih lanjut bagaimana ekspektasi hidup berperan dalam menghitung besaran dana investasi ini. Hasil perhitungannya dibandingkan dengan perhitungan biasa yang tidak menerapkan ekspektasi hidup di dalamnya. Dari hasil analisa, dapat disimpulkan bahwa besaran dana pensiun yang dihitung dengan menerapkan ekspektasi hidup, jauh lebih kecil dibandingkan dengan besaran dana pensiun yang dihitung tanpa ekspektasi hidup, karena antisipasi dana investasi pensiun hanya dilakukan selama orang tersebut diasumsikan masih hidup.

Kata Kunci: nilai tunai, anuitas, pensiun, investasi.

1. Introduction

According to [1], people who worked many years, of course want at some moment to enjoy a decent retirement, both mentally and materially, provided that there are sufficient funds for their living and lifestyle. In [1] it is further explained that in such a pension plan, the savings or investments are dominant. In retirement planning, one needs to know for sure what lifestyle can be permitted after retirement and how much money is needed to finance it.

The same reference also explaines that often people tend to depend too much on the company pension fund or on a pension fund provided by the government (eg Social Security). It is important to evaluate the value that can be realistically expected from the pension fund. After that, it can be decided whether the funds are sufficient to meet one's needs after retiring. If these funds are not sufficient, one has to collect additional funds through an investment program.

In [2], techniques are explained on how to estimate the necessities of life after retirement. The current research describes the information needed to estimate possible shortages of funds. Preparation of these funds will be called the pension fund investments. Calculation of the shortages of funds can be done in two ways, with or without taking into account a person's life expectancy. The life probability is taken from the 2007 Mortality Table published by Social Security USA. After that, we shall analyze the results of the calculations.

2. Method

2.1. Future Lifetime Distribution and Life Table

Following [3], we denote someone who is now aged x years old by *(x).* Let X be a random variable for *the age-at-death of (x).* Then [3] shows that the random variable *T(x)*, which is *the time until death* or *future lifetime* of *(x),* can be obtained by means of the following relationship:

$$T(x) = X - x.$$

Let $F_{T(x)}(t)$ be the distribution function of *T(x)*

$$F_{T(x)}(t) = \Pr\left(T(x) \le t\right) = {}_tq_x,$$

and $S_{T(x)}(t)$ the survival function of *T(x),* so

$$S_{T(x)}(t) = 1 - F_{T(x)}(t) = {}_tp_x.$$

Again from [3] we know that a life table usually contains tabulations, such as individual ages, basic functions q_x, l_x, d_x, and, possibly, additional derived functions. Here, l_x represents *the expected number of survivors from age 0 to age x,* and d_x represents *the number of deaths between ages x and x+1.*

The functions${}_tq_x$ and ${}_tp_x$ are related to the function l_x by the formulas

$$ {}_tp_x = \frac{l_{x+t}}{l_x}, \tag{1} $$

and

$$ {}_tq_x = 1 - \frac{l_{x+t}}{l_x}. \tag{2} $$

2.2. Annuity

In [4], the annuity is defined as a series of payments made at equal time intervals over a certain time period. Based on the certainty of payment, one distinguishes two types of annuity, namely *certain annuity* and *life annuity.* Certain annuity is an annuity that should be paid over a certain time period, e.g. mortgage payments on a home. Life annuity is an annuity that should be paid as long as someone remains alive, e.g. insurance premiums that are paid for as long as someone is still alive in certain time period.

2.3. Certain Annuity

Annuity due is an annuity period in which payments are made at the beginning of the period for n periods. The *present value* at point $t = 0$ can be determined by

$$ \ddot{a}_{\overline{n}|} = \frac{1 - v^n}{d} \tag{3} $$

where $d = 1 - v$, $v = 1/(1 + i)$, and i is the rate of interest per period.

In addition to certain annuity that always is the same amount, there is also a certain annuity in which the amount of payment can change geometrically. Consider an annuity due with a term of n periods in which the first payment is 1 and succesive payments increase in geometric progression with common ratio *1+i.* The present value of this annuity is

$$ 1 + (1 + i)\, v + (1 + i)^2 v^2 + \cdots + (1 + i)^{n-1} v^{n-1} = n \tag{4} $$

with $v = 1/(1 + i)$.

2.4. Life Annuity

Following [3], the definition of life annuity is a series of payments made continuously or at equal intervals (such as months, quarters, and years) for as long as a given life survives. It may be temporary, that is, limited to a given number of years, or it may be payable for the whole duration of life. The payment may be due at the beginning of each payment interval or at the end.

The present value of random variable of an n-year temporary life annuity due of 1 per year is

$$ Y = \begin{cases} \ddot{a}_{\overline{k+1}|}, & 0 \le k < n; \\ \ddot{a}_{\overline{n}|}, & k \ge n. \end{cases} $$

and its *actuarial present value* is

$$\ddot{a}_{x:\overline{n}|} = E\left[Y\right] = \sum_{k=0}^{n-1} v^k . {}_kp_x. \tag{5}$$

2.5. Pure Endowment Insurance

In [5], a description of an n-year pure endowment insurance can be found. In this type of insurance, benefits are payable at the end of the n^{th} year, provided that the insured survives this term. Accordingly,

$$Z = \begin{cases} 0, & 0 \leq k < n; \\ v^n, & k \geq n. \end{cases}$$

where Z is the present value of the future payment of a unit of money if there is a risk.

The actuarial present value of Z is

$$A_{x:\overline{n}|}^{\;\;1} = E\left[Z\right] = v^n. \ {}_kp_x. \tag{6}$$

2.6. The Determination of Pension Fund Investments

2.6.1. The Determination of Investments with Certain Annuity

As mentioned in Section 1, in retirement planning, one needs to know for sure what lifestyle can be permitted after retirement and how much money is needed to finance it. Most people do not know how to calculate the necessity monthly expenses and how to estimate it in the future. Here, the way to estimate the necessity monthly expenses in the future will be explained, so the retirement planning would be better [2].

First, assume that one wants to retire at r years old. Next, count the necessity monthly expenses in the present. Then, estimate the inflation that will occure until the moment of retirement. The estimated necessity monthly expenses after retiring can now be computed by *accumulated function.*

Now, the accumulated function will be explained in more detail. We denote as before, someone who is now aged x years old by *(x)* and his/her monthly needs by Rp *B(x)* / month. If the assumption of inflation is i/year, then a year later he/she has monthly needs :

$$B\left(x+1\right) = B\left(x\right)\left(1+i\right). \tag{7}$$

In the second year, monthly life needs will become

$$\begin{aligned} B\left(x+2\right) &= B\left(x+1\right)\left(1+i\right) \\ &= B\left(x\right)\left(1+i\right)^2. \end{aligned}$$

At the time of retirement, at age r years old, the monthly life needs can be estimated as

$$\begin{aligned} B\left(r\right) &= B\left(r-1\right)\left(1+i\right), \\ &= B\left(x\right)\left(1+i\right)^{r-x}. \end{aligned} \tag{8}$$

These needs will increase each year because of the inflation. The necessary funds from the time of the retirement at time r until death can be estimated at time r by

$$B_{tot}\left(r\right) = B\left(r\right)\left(\omega - r\right). \tag{9}$$

Equation (9) follows from the annuity equation from equation (4) that has equal geometrical ratio and discount factor on *(1+i)* [4]. The time period that is used in this calculation is from the age of retirement *(r)* until the assumed age of death (ω).

For example, assume that someone who is 30 years old, has monthly needs Rp 2.000.000,00 / month. If the assumption of inflation is 6%/year and the preferred age of retirement is 56, then equation (8) implies the following monthly needs :

$$\begin{aligned} B\left(56\right) &= 2.000.000.(1 + 0,06)^{56-30} \\ &= 9.098.765,93. \end{aligned}$$

From the result of this calculation, it can be seen that the need for adequate funding of daily living expenses at the time of retirement become larger, while an income from salary is no longer obtained. The difference is often expected to be covered by retirement benefits from the company where he/she worked. Unfortunately, often the pension benefits received by an employee are usually not sufficient. Let the pension benefits received by an employee be R per month, then the total value of R from the beginning up to his/her death when ω, judging its value at time of r follows from

$$R.\ddot{a}_{\overline{\omega-r}|} = R.\frac{1-v^{\omega-r}}{1-v}, \tag{10}$$

with $v = \frac{1}{1+i}$.

From equations (9) and (10) the shortage of funds needed to cover the total needs in retirement can be calculated. Let the shortage of funds at time r be denoted by *K(r)*. Then

$$K\left(r\right) = B\left(r\right)\left(\omega - r\right) - R.\frac{1-v^{\omega-r}}{1-v}. \tag{11}$$

If an employee wants to save for the retirement funding shortfall, it is advisable to save it from early on, to make it less burdensome. One can set aside some of the income for savings or make a profitable investments in strict self discipline.

Suppose P estimates the amount of monthly savings to be made by the employee and i_{inv} is the investment rate chosen by the employee. Using equations (3) and (11), given the present value at time of x, the value of P taking into account all shortage of funds at r, follows from the following equation :

$$\begin{aligned} K\left(r\right).{v_{inv}}^{r-x} &= P.\ddot{a}_{\overline{r-x}|}, \\ P &= \frac{K\left(r\right).{v_{inv}}^{r-x}}{\ddot{a}_{\overline{r-x}|}}, \end{aligned} \tag{12}$$

with $v_{inv} = \frac{1}{1+i_{inv}}$ and $\ddot{a}_{\overline{r-x}|}$ is determined by equation (3) but i replaced by i_{inv}.

2.6.2. The Determination of Investments with Life Annuity

Life annuity is an annuity that should be paid as long as someone remains alive [3]. In this section, not only the effect of life annuity on the determination of an investment fund will be discussed but also the comparison of the amount of annual savings will be made.

Some calculations are still the same, but for certain calculations, life probability from the 2007 Mortality Table published by Social Security USA [6] will be used.

First, accumulated function will be explained in detail. As before, denoted someone who is now x years old by *(x)* and his/her monthly needs by Rp *B(x)* / month. Assume an inflation of i/year and he/she is still alive a year later. Then the monthly needs can be calculated by

$$B_p\left(x+1\right)=B\left(x\right)\left(1+i\right).p_x.$$

Following the same reasoning as before, at the time of retirement the estimated needs per month will be

$$\begin{aligned} B_p\left(r\right) &= B_p\left(r-1\right)\left(1+i\right)._{r-x}p_x, \\ &= B\left(x\right)\left(1+i\right)^{r-x}._{r-x}p_x, \end{aligned}$$

provided that this person is still alive at the age of r years.
Using equation (1), the equation above can be related to data from Life Table as follows

$$B_p\left(r\right)=B\left(x\right)\left(1+i\right)^{r-x}.\frac{l_r}{l_x}\ . \tag{13}$$

These needs will increase each year because of inflation. The total life necessities from the time of retirement until death can be estimated at time r by

$$\begin{aligned} B_{p_{tot}}\left(r\right) &= B_p\left(r\right).\left(1+i\right)^k.v^k.\sum_{k=0}^{\omega-r-1}{}_kp_r, \\ &= B_p\left(r\right).\sum_{k=0}^{\omega-r-1}{}_kp_r, \\ &= B_p\left(r\right).\sum_{k=0}^{\omega-r-1}\frac{l_{r+k}}{l_r}. \end{aligned} \tag{14}$$

Let the pension benefits received by an employee be R. Then by using equation (6) and assuming that the employee stays alive until age , the total value of R from the beginning up to the moment before he/she dies at age ω, judging its value at time of r follows from

$$\begin{aligned} R.\ddot{a}_{r:\overline{\omega-r}|} &= R.\sum_{k=0}^{\omega-r-1}v^k._kp_r, \\ &= R.\sum_{k=0}^{\omega-r-1}v^k.\frac{l_{r+k}}{l_r}. \end{aligned} \tag{15}$$

From equations (15) and (16), the shortage of funds needed to cover the total funds needs after retiring can be calculated. If we denote the shortage of funds at time r be $K_p(r)$, then

$$K_p(r) = B_p(r) \,.\, \sum_{k=0}^{\omega-r-1} \frac{l_{r+k}}{l_r} - R. \sum_{k=0}^{\omega-r-1} v^k . \frac{l_{r+k}}{l_r}. \tag{16}$$

Next, let P_p estimate the monthly savings made by the employee and i_{inv} the investment rate chosen by the employee. Using equations (5), (6) and (16), given the present value of $x,$ and taking into account all shortage of funds at r, the value of P_p can be calculated by :

$$\begin{aligned} K_p(r) \,.{v_{inv}}^{r-x} \,.{}_{r-x}p_x &= P_p.\ddot{a}_{x:\overline{r-x}|}, \\ P_p &= \frac{K_p(r) \,.{v_{inv}}^{r-x} \,.{}_{r-x}p_x}{\ddot{a}_{x:\overline{r-x}|}}, \end{aligned} \tag{17}$$

where $v_{inv} = \frac{1}{1+i_{inv}}$ and $\ddot{a}_{x:\overline{r-x}|}$ is calculated with equation (5) but with i replaced by i_{inv}.

3. Results and Discussions

3.1. The Determination of Investments with Certain Annuity

In this section, for some cases the results will be shown of the calculations to determine the pension fund investment by using certain annuity. Some of the variable's values are assumed to refer to a condition close to real.

The assumptions used in the following example/calculation are : the inflation rate (i) is 6% / year, the necessities of life per month $B(x)$ for someone who is now 25 years old is Rp 2.000.000,00, the planned retirement age (r) is 56, and the pension benefit that will be received at retirement (R) is Rp 4,000,000.00 / month. The data for the expected maximum age for a person (ω), retrieved from the Life Table, is up to the age of 120 years.

The calculation to see how much the investment funds each month need to be prepared by a person will be done for ages (x), with $x = 25$, 30 and 40. The investment rates (i_{inv}) which will be used are also different, namely 6%, 7% and 15%, since the investment characteristics for each person may also vary.

Estimation for the amount of monthly savings to be made by the employee are calculated by equation (12). The results are given in Table 1.

3.2. The Determination of Investments with Life Annuity

In this section, for some cases the results will be shown of the calculations for determining the pension fund investment, but now by applying the life expectancy and life annuity. Some of the value of variables also assumed to refer to a condition close to real.

Table 1. Monthly Savings without Life Annuity (Rp)

x \ i_{inv}	6%	7%	15%
25	7.901.978,01	6.503.562,10	1.232.956,93
30	8.186.666,42	6.985.907,42	1.816.730,12
40	9.414.482,58	8.585.566,92	3.998.358,32

The assumptions used are : the applicable inflation rate (i) is 6% / year, the necessities of life per month $B(x))$ for someone who is now 25 years old is Rp 2.000.000,00, the planned retirement age (r) is 56, and the pension benefit that will be received at retirement age (R) is Rp 4,000,000.00 / month. The data for the expected maximum age for a person (ω), retrieved from the Life Table, is up to the age of 120 years.

The calculation to show much the investment funds each month needed by a person of ages (x), will be done for $x = 25$, 30 and 40. We use several values of investment rates (i_{inv}), namely 6%, 7% and 15%, as the investment characteristics for each person are also typically different.

The estimate of the amount of monthly savings to be made by the employee can be calculated by equation (17). The results are provided in table below:

Table 2. Monthly Savings with Life Annuity (Rp)

x \ i_{inv}	6%	7%	15%
25	2.263.770,06	1.860.386,47	346.872,76
30	2.275.844,25	1.939.524,71	500.574,05
40	2.277.083,43	2.074.936,83	961.056,43

3.3. Analysis

From Tables 1 and 2, it can be seen that the monthly savings increase for fixed investment rate and increasing age. If the age is fixed and the investment rate used is higher, the monthly savings which should be set aside will be smaller.

The next question is about how much savings should be set aside by an employee for pension investments. This research shows how to calculate the savings an employee should prepare.

Analysis can be done by looking at the comparison between equations (11) and (16), wherein each of the equation is used to calculate the amount of monthly savings. Equation (16) is influenced by ${}_{r-x}p_x < 1$ as multiplier of the present value of $K_p(r)$. Present value of $K_p(r)$ should be multiplied by ${}_{r-x}p_x$ because it will only be used if x can reach the age of r with probability ${}_{r-x}p_x$. Assumption used in equation (11) is different because it does not take into account a person's life probability. This difference in assumption which then affects the amount of savings, that is $P_p < P$.

An example for the case $x = 25$ and $i_{inv} = 15\%$, it is obtained that the difference is quite significant. Monthly savings with life annuity for this case is Rp 346.872,76 that is smaller than monthly savings without life annuity which is Rp 1.232.956,93. All the calculation results can be seen in Tables 1 and 2.

It turns out that if the life probability and life annuity are applied in the calculations of the pension investments, the amount of savings will be much smaller. The reason is that for those savings, the magnitude of the necessities of life will only be used if a person is still alive, so anticipation is only made to cover the shortfall for the person who is still alive.

4. Conclusion

After observing the cases presented in the Section 3, it can be concluded that the amount of shortage of funds needed at the retirement has already been taken into account from now by using several assumptions.

The calculations can be done in two ways, with or without taking into account a person's life expectancy. Actuarial theory can be applied directly when calculating life annuities and life benefits.

From the examples, it can be seen that the preparation of pension fund investments will be huge if it ignores a person's life expectancy. In other words, it is more profitable if the person's life expectancy is applied to any funds in future calculations.

Based on the calculations of the monthly savings in Tables 1 and 2, it can be concluded that if the pension investments start faster, then the amount of monthly savings to be set aside are also getting smaller. Regular valuation methods should be routinely performed in anticipation of possible shortages of investment.

The following suggestions may be useful for further research. First of all, it is advisable to use Life Tables used for residents of Indonesia so that the assumptions in the calculations above are closer to the real conditions in Indonesia. Secondly, research can be further equipped with examples of investments that are often used along with the illustrations of the rate of return and risk that may be encountered.

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