Body fractions:
A physical approach to fraction learning

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outlines a different

way of exploring

fractions: using a

physical approach.

Many students experience great difficulty understanding the meaning of fractions (Anthony & Walshaw, 2007; Behr, Lesh, Post & Silver, 1983; Davis, Hunting & Pearn, 1993; Lamon, 2007; Verschaffel, Greer & Torbeyns, 2006; Young-Loveridge, Taylor, Hawera & Sharma, 2007). For many students who have spent their early mathematics lessons focusing on counting (whole) numbers, recognising that there are many numbers between those whole numbers called fractional numbers, is quite revolutionary. The foundation of understanding fractions is the idea that they are parts of a whole. The fact that one whole object can be divided into many equal parts, with each part having a name relative to the original whole, opens up a whole new realm of number understanding for the students.

Students often comment that they find fractions to be meaningless and confusing (Zevenbergen, Dole & Wright, 2004). Their learning has frequently been based on rules and procedural computation, while conceptual understanding has often been minimal. Understanding what a fraction means and how to operate with fractions (i.e., addition, subtraction, multiplication, division) is often daunting for many students. Even more intimidating is the appreciation of how fractional knowledge can be successfully applied in everyday life.

The issue of how students come to understand fractions is quite contentious. Recently Steffe and Olive (2010) presented an argument that children's understanding of fractions involves the reorganisation of their knowledge of discrete quantity (i.e., "how many" with whole numbers) based on their construction of number sequences, rather than developing fraction understanding independently of whole-number understanding. Fractions require students to think about not just "how many?" (*discrete* quantity) but also "how much?" (*continuous* quantity). Steffe and Olive (2010) found that children who had not developed generalised (whole) number sequences were unable to progress in their fraction knowledge, no matter how effective the teaching strategies. According to Thompson (2010), "children impose segmentations on continuous quantities and reassemble them as measured quantities" using their understanding of generalised number sequence, and this comes from the development of "spatial operations with continuous quantities" (p. xiii).

On the *New Zealand Number Framework*, fractions are one component of knowledge, as well as being part of the strategy domain of proportion and ratios (Ministry of Education, 2008). On the fraction knowledge domain, students first need to be able to order unit fractions (Stage 5 on the framework), before coordinating numerators and denominators (Stage 6), before identifying equivalent fractions (Stage 7). On the strategy domains for addition/subtraction and multiplication/division, operations with fractions and decimals follow expertise at operations with whole numbers (Stage 7; see New Zealand Ministry of Education, 2008). On the proportion and ratio domain, there is a progression from finding a fraction of a set using repeated addition (Stage 5), to finding a fraction of a set using division and multiplication (Stage 6), to finding a whole after being given the part (Stage 7). This progression in understanding is also noted by Zevenbergen et al (2004), who emphasise the importance of finding fractions of a whole as a basis of other fraction and decimal knowledge.

Supporting students' fraction understanding using concrete materials

There is a substantial body of literature showing that concrete materials help students to understand mathematical concepts (e.g., Clements & McMillen, 1996; Fennell & Rowan, 2001; Pape & Tchoshanov, 2001; Zevenbergen et al, 2004). According to Swan and Marshall (2010), mathematics manipulative material includes any "object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (p. 14). This definition presupposes that the materials consist of objects outside of the students themselves.

Finding a range of suitable real world models as contexts for teaching mathematical ideas is recognised as part of good teaching. However, as Zevenbergen et al (2004) note, finding accurate real life models that refer to fractions is often difficult. This paper presents an activity in which students use their own bodies to represent fractional quantities. While it is acknowledged that the use of the body does not show the pieces as being equal in size (an important criterion when defining fractions of a whole), it is a useful, fun model that illustrates an important concept. Because the body (or more precisely the arm span) is used to symbolise the fraction pieces, the size of those pieces is limited to the whole, halves and quarters. While discussion at a later date may develop to include other equivalent fractions (for example eighths and sixteenths), it is not possible to show these within this activity. Students enjoy the kinaesthetic approach to the learning associated with this activity and as a result may remember it better than other more familiar activities.

Body fractions

In order to carry out this activity, the students need to be standing with enough space around them to enable them to move about freely. The activity begins by establishing with students the values of the fractions that can be represented by different body poses using the arms to signify particular fractional quantities. For example: one-whole is represented by stretching out both arms to the sides horizontally; one-half is represented by stretching out just one arm; one-quarter is represented by stretching out one arm up to the elbow then bending the forearm over the upper arm so the fingers touch the corresponding shoulder.

Using these three representations, students can individually, or in groups, construct different fractional quantities. For example, an individual student when asked to show one-half might extend one arm, or might show it as two-quarters (see Figure 1). The students should be asked to ensure that when constructing a given fractional number no student is left out of a group. This means that students in a particular group may need to reconstruct their original

to indicate one-quarter). A group of three students could use the three poses to show one-whole plus one-half plus one-quarter, whereas a group of four students could show three-halves plus one-quarter, and a group of five students could show two-halves plus three-quarters.

I have found this activity to work equally well with Year 5 to 8 students, and with pre-service and in-service teacher education students. The activity seems to generate a lot of discussion among members of each group, and sometimes there is disagreement among group members until everybody in the group is convinced that their representation accurately depicts the target quantity. It can provide a very nice opportunity for argumentation, including explanation and justification (see Hunter, 2010).

Once students have formed their groups, they may be given a fraction and asked to work out how many different ways they can make that quantity within their group. For example, a group of five students might make one-and-one-half as four-quarters and one-half (see Figure 2), or 15 quarters as three-wholes plus one-half plus one-quarter (see Figure 3).

Figure 1. A child showing two-quarters to make one-half in total using body fractions.

Figure 2. Children showing one-and-one-half with four-quarters plus one-half using body fractions.

representation to include an individual who was not initially part of a group. An example of this might be that when asked to show one whole-and-three-quarters, a pair of students could show one whole (one student with both arms outstretched) and three-quarters (one student with one arm outstretched to show one-half and the other arm bent

As students become familiar with the activity demonstrating multiple groupings of the whole, half and quarter, this can be extended to the understanding of discrete models for fractions and later further developed to show the relationship between standard fractions, decimal fractions and percentages.

Figure 3. Children showing 15 quarters using body fractions.

Extending body fractions to set values

A variation on the basic body fractions activity is to allocate a whole number value to each of the fractional poses. For example, the one-whole pose could be allocated a set value of 20, the one-half pose would then be worth 10, and the one-quarter pose worth 5. Students can then be asked in their groups to construct a quantity such as 15, or 45, or 70, etc. At this point, students often work out how many members are required in the group by skip counting in either fives or tens, having identified the appropriate pose for a particular fractional piece. This can provide fruitful discussion among students as they determine the representation of a given total. Frequently they will arrive at the appropriate group formation only to find that one or two class members are not in a group. As

the class 'rule' is that no-one is allowed to be left out, they then need to regroup in some way to allow for any extra students to be included. An example of this might be when instructed to show 30 (when the whole is worth 20), two children could simply show one person as one-whole and one person as one-half (see Figure 4). However, another group may notice one student not in a group so reconstruct their body fractions to show one-whole and two-quarters (see Figure 5).

Figure 4. Children showing one-and-one-half as one-whole plus one-half using body fractions.

Figure 5. Children showing one-and-one-half using one-whole plus two-quarters, with body fractions.

Different groups may then be asked to share with the rest of the class how they formed their groups. At this point it is a good idea to ask a student to record on a white board both the fraction representation of the group and the numeric value of each of the fractions (see Figure 6) as the students share how they formed their given number. This part of the activity can consolidate the students' understanding and often allows other class members to see the many different ways a fraction with a set value can be formed. It is a good idea when students are recording their group construction they align the numeric value with the fraction representation written directly below (see Figure 6).

When asked to form a representation of 75 (given 1 whole equals 20)

(a) A group of 5 students may say:

$$75 = 20 + 20 + 20 + 10 + 5$$
$$= 1 + 1 + 1 + \frac{1}{2} + \frac{1}{4}$$
$$= 3\frac{3}{4}$$

(b) A group of 8 students may say:

$$75 = 20 + 15 + 15 + 5 + 5 + 5 + 5 + 5$$
$$= 1 + \frac{3}{4} + \frac{3}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$
$$= 1 + 1\frac{1}{2} + 1 + \frac{1}{4}$$
$$= 3\frac{3}{4}$$

Figure 6. Recording the numeric value of a set using fractional representation.

Extending body fractions to percentages and decimals

Once the students have mastered body fractions using standard fractions and set values, the activity can be extended to percentages and decimal fractions. Given that one-whole is 100% and one-half is 50% and one-quarter 25%, the students can now show various percentages using their arms. For example 275% might be shown as: one-whole plus one-whole plus three-quarters; or one-whole, plus one-half, plus-one half,

plus one-half, plus one-quarter, etc. This allows for consolidation of the conversion between fractions and percentages and is good for showing students that percentages, like fractions, can be greater than one-whole.

The body fraction activity can then be further developed to include a mixture of fractions, decimals and percentages and the conversions between them. When students record their group formation using one fractional representation (e.g., decimals), they could also record the representation in an alternative fractional model (e.g., percentages). For example after having formed a group showing 2.75 they could record this as a decimal, fraction and percentage (see Figure 7).

A group is asked to make 2.75 and record their formation

$$2.75 \quad = \quad 1 \quad + \quad 1 \quad + \quad 0.5 + 0.25$$
$$2\frac{3}{4} \quad = \quad 1 \quad + \quad 1 \quad + \quad \frac{1}{2} \quad + \quad \frac{1}{4}$$
$$275\% = 100\% + 100\% + 50\% + 25\%$$

Figure 7. Recording the group formation as a decimal, fraction and percentage.

Conclusion

While the body fraction activity is limited to understanding of wholes, halves and quarters, it has many applications beyond the simple representation of those particular quantities. It enables students to experience multiple representations of the whole, half and quarter as fractions greater than the one whole, something which many students struggle with. For example, some students find it difficult to accept that you can have (15 quarters) and that this is a legitimate representation of a fraction. It also allows for the extension into the set value and decimal values of the equivalent fraction.

Often students have had their fraction learning restricted to rules and procedural computation with little or no conceptual

representation. A major advantage of the body fractions activity is that students often remember vividly their participation in this activity and its connection with different fractional quantities.

Acknowledgement

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