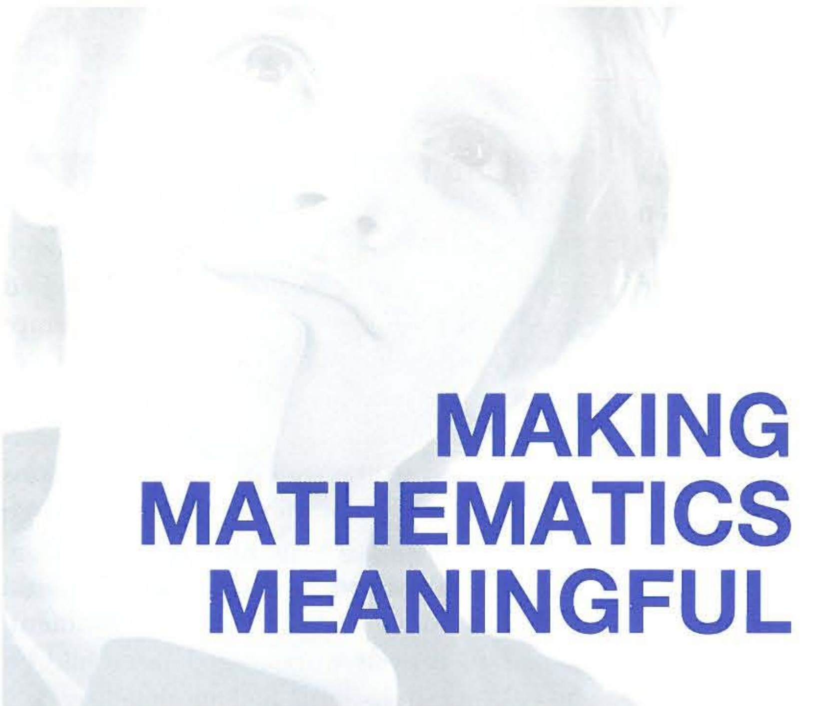

MAKING MATHEMATICS MEANINGFUL

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Using student-initiated problems to situate mathematics

Chris Brough and Nigel Calder share two scenarios in which students co-constructed their curriculum through generating their own problem solving and mathematical investigations. This article is a valuable exploration of the benefits of teachers working in partnership with their students.

Mathematics is everywhere—from the minute we open our eyes to check the alarm clock and calculate how many minutes extra we can afford to lie in bed, to measuring out our cereal for breakfast and estimating if we have enough petrol to make the 18 kilometre journey to work. As teachers of mathematics, we must ask ourselves if the way we teach reflects the real-life problem-solving situations our students will experience within their everyday world.

In many schools, mathematics is taught at a set time—it is something you 'do' between 9.30 and 10.30 am. Traditionally, lessons involve children being taught specific skills which are subsequently applied to teacher-created problems. Van den Heuvel-Panhuizen (2010) suggested that rather than beginning with an abstraction or definition, mathematics should be situated within rich problem-solving contexts that can be mathematised. Furthermore, studies have shown that when real-life problems are pursued and students have ownership during the learning process, motivation, achievement and persistence are enhanced (Bonotto, 2002; Lesh & Harel, 2003; Nolan & McKinnon, 2003). Using real-life problems also allows students to explore what might otherwise be abstract, disconnected concepts in an engaging, meaningful manner (Brown, Watson, Wright & Skalicky, 2011).

This paper examines how student-centred inquiry into real-life situations and problems can develop mathematical thinking and understandings. Data are drawn from a participatory action-based research study on student-centred curriculum integration (CI) where children pursued student-generated problem situations and were involved in the collaborative co-construction of curriculum (Brough, 2012). Mixed methods were used to collect data including: interviews, focus group meetings, electronic forums, observations, photographs and work samples. Although the learning that occurred spanned several curriculum areas, the focus of this article is the mathematics, in particular the measurement and geometric thinking.

The research project explored the principles and practices of student-centred CI. This teaching approach has a democratic pedagogy which places students at the centre of their learning and involves them in the collaborative co-construction of curriculum (for further reading refer to: Beane, 1997; Dowden, 2007; Fraser, 2000). Student-centred CI requires teachers to share power and work in partnership with their students. This is a challenging position for many teachers to embrace. In student-centred CI, learning is situated within learning contexts that are of relevance to students. In the most desirable circumstances, themes are raised by the students themselves and are often initiated as a result of genuine problem-solving contexts arising from within their immediate environment or, alternatively, a teachable moment or a question or issue that has been raised. The teacher's role in the co-construction process is to identify which issues have rich educational learning potential and are worthy of further inquiry, and to consider what scaffolding and explicit teaching will be required throughout the inquiry.

The scenarios to be discussed took place in two different Government primary schools which drew students from a range of cultural and socio-economic backgrounds. The first occurred in Mikayla's Year 0 (Foundation Year) class of five-year-olds and the second in Toni's Year 6 (Year 5 in Australia) class of ten-year-olds. The two scenarios are described in turn in the following sections

Scenario 1

The school playground had a drainage problem, so a large yellow digger had arrived at school to dig a sump hole in an attempt to resolve the issue. The appearance of a large noisy digger had understandably triggered the interest of Mikayla's students who rushed to the window asking questions and requesting that they be permitted to go outside to watch. In the initial phases of the project Mikayla researched how she would establish a democratic learning environment. She had actively encouraged her children to ask questions, and had modelled asking 'I wonder' questions, encouraged student curiosity, ideas, and thinking. Recognising that this student-initiated interest had the potential for rich learning, the class was led out onto the field where a flurry of further questions was asked and comments made: "How will they get the digger off the truck?", "What is he doing?", "Why is he digging a hole?", "How deep do you think the hole is?", "If he digs anymore he will end up in space", "Look, the dirt is changing colour as he digs".

Following a request from the students, the driver stopped work to explain what he was doing and why, and to answer the children's questions. During the conversation, the children had asked how deep the hole was and were informed that it was four metres deep. Despite the driver taking photographs, the children were unable to see directly into the hole, and not having an understanding of what a metre length looked like, they were unable to visualise this depth. This became the source of the mathematical inquiry. The children estimated the length of the hole by stretching out their hands and stepping varying predictions out on the courts. When Mikayla asked how they could find out for sure, the students decided that they should use the rulers. Discovering these were one metre long, they began laying out the rulers on the courts but then realised that they were one short. After much thought and deliberation, they determined they could reposition the first ruler to the end. The children compared their final measurement with initial estimates and then went on to measure in a variety of ways, including conventional and non-

Figure 1. The 4 metre sump hole in the school playground.

Figure 2. "How big is four metres?" Five year olds compare non-conventional measures with standardised measurement.

conventional measures. These included using their bodies, hands, metres and centimetres.

Having begun the inquiry unable to visualise or estimate with any degree of accuracy a length measuring 4 m, at the end of the problem-solving task the students had gained an understanding how long a metre is and how to measure length. They learned to order and compare lengths and were involved in estimating and making comparisons. Children were heard saying, "That is the same as three and a half of us" and "It is way bigger than we thought". They also gained an understanding of the need to measure in standardised units, with one child announcing, "Hey did you know these rulers are all the same length?" and the realisation that measuring with children who were different heights or foot strides would give slightly different answers. Counting and number strategies were also utilised during the measurement context, for example, using three and a half. The learning achieved went beyond New Zealand Curriculum level requirements with initial measurements taken in metric units rather than non-conventional measures (Ministry of Education, 2007) or the Australian Foundation year equivalent for Measurement and Geometry. During this task the children were involved in meaningful problem-solving, the inquiry was student-initiated and the children had a high level of ownership and engagement throughout the process. Photographs were used for story-writing, books on diggers were read, and a number of science experiments followed as a result of this real-life learning opportunity.

Scenario 2

The second scenario occurred in the Year 6 class. The learning context evolved from a discussion students were having about their classroom learning environment and how it could be enhanced. A group of students proposed that their prefabricated classroom was far too small and that it should be extended. The children immediately wanted to design plans for a new classroom and suggested that their ideas be taken to the school's Board of Trustees. Toni recognised the rich learning potential of the inquiry but wanted her students to realise that their ambitious idea may not come to fruition because of budget restrictions. The children were adamant that they wanted to pursue the project, suggesting that, "It would be fun and there would be lots of learning".

The students and teacher discussed what they would need to do in order to begin the extension. The class realised that they needed to determine the size of their current room to develop an understanding of area and to draw up plans. They discussed the most appropriate unit with which to measure large areas. Having determined that square metres were most appropriate Toni had her students make a piece of paper that was one metre square. Using this as a template the students estimated and measured various areas including the netball courts, and other outdoor spaces. When not an exact square metre, students measured in square centimetres as well. The teacher allowed her students to experiment and discussed why different answers were being generated for the same length. This required specific teaching to enable children to measure more accurately. Through the initial explorations, the children discovered that multiplying length by width was a more efficient strategy than counting and estimating. They applied this strategy when measuring the classroom. The children drew the existing plan into their square centimetre quad books, with each square representing a square metre. Discussions took place on the shape of the new classroom and the practicalities. In small groups, the children used multi-link cubes (each base representing one square metre) to make models of the floor areas of various-shaped rooms exploring annuluses, rectangles, triangles and circles. Interesting shapes and conversations around room height and measuring space led to a discussion about cubic metres and volume.

The children suggested that to measure volume, they could just measure up the walls, then the length, and up to the ceiling. One child then said: "But what does that mean? How does that show us the space?" The children discussed how you might measure in three-dimensions. Eventually, someone suggested, "What about us making a cube like the multilink cube but bigger?" The teacher challenged the students to make a one cubic metre from lengths of rolled up newspaper.

Figure 4. "How many cubic metres is our classroom?" Making a cubic metre and measuring.

They subsequently measured the length of the room's floor by rolling the cube along the walls to determine their length and then measured up the wall to enable them to determine the volume of the room using multiplication. However, the classroom had a pitched roof which offered a considerable challenge. Their suggestions for ways to resolve this included: "We could bend down the sides" and "Maybe we could make some other shapes". The teacher prompted them by asking them to consider halving the apex shape and see what they noticed. The groups took paper and made a triangle representing the apex the children explored reorganising the shape until one group discovered it made a rectangular shape. The group of students subsequently raced around sharing their discovery. The linear area was subsequently used to calculate the 3-D area by multiplying the length, width and height. Learning was consolidated and extended when the teacher challenged the children to apply their skills to measure the volume of the school swimming pool. This had a shallow and deep end and so was another trapesoid prism. To measure the depths they determined that lowering a stick into the pool and measuring the length of wet stick gave them the depth. They did this at each end.

The children continued to create various plans in their quad books and also made flow charts predicting the building process. A member of the council visited to provide expertise on costs, gaining consent, and the building process. This resulted in the mathematics extending beyond geometric thinking and measurement as the children calculated the costs of their designs. Consequently, the two-storey options were eliminated, ambitious designs were made smaller, and options for extending near the historic tree were also eliminated. Fundraising was also discussed with the children generating ideas for paying for the classroom.

Unfortunately, budgetary restrictions prevented the full extension becoming a reality for this group of children. However, a large covered veranda has since been added to this classroom. When reflecting on the experience the children reported that while they were disappointed that the classroom had not been not extended, they did not regret the project as it had been really enjoyable and they had learned a lot in the process. During the inquiry there was a high level of student engagement and motivation, while the conceptual mathematical understanding in geometry and measurement went well beyond curriculum requirements. For instance, the *Australian Curriculum* for Year 7 stipulates that students are expected to calculate volumes of rectangular prisms, and the mathematics content with which these students had engaged was beyond this. They were also immersed in calculating building costs and the application of a range of multiplicative and additive strategies to solve problems.

Implications

The mathematical inquiries in this project were student-led initiatives triggered by the children themselves. The teachers did not predetermine the content to be taught; rather, this emerged naturally from within the inquiry. Explicit teaching was still required with strategies and skills taught within the purposeful problem-solving setting. Initiating the inquiry themselves, students saw immediate relevance for the acquisition of particular skills and knowledge.

During this inquiry, the students had explored and effectively used two-dimensional representations of three-dimensional objects, enhanced their understanding of area and volume, and made connections between the various measurement representations. The mathematical thinking and knowledge gained through the class extension investigation involved students learning how to apply multiplicative strategies for area and volume, and how to measure using metric units for length, area and volume. Furthermore, the children needed to calculate various building costs that involved students having to apply additive and multiplicative strategies.

This approach to teaching mathematics requires teachers to look out for the teachable moment so learning is contextualised in a meaningful way. It requires teachers to adopt a more facilitative stance to their teaching whereby they ask questions that encourage and extend thinking, rather than providing all the answers and thinking for their students. It involves power-sharing as teachers listen and collaboratively co-construct learning. Adopting a student-centred approach is also reliant on teachers possessing excellent content knowledge as they scaffold students to complete the task at hand (Brough & Calder, 2012). A limitation for teachers can be highly prescriptive school-wide units and mandates which can inhibit opportunities to pursue spontaneous learning opportunities (Beane, 1997; Brough, 2012). The teachers in this project were relatively new to the profession, nevertheless they genuinely listened to their children, took risks and made learning meaningful, challenging and engaging for their students.

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