Understanding teaching practice in support of Non-English-Speaking-Background (NESB) students' mathematics learning

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ABSTRACT

Research is needed that gives special attention to the experiences of mathematics teachers of Non-English-Speaking-Background (NESB) students. This need prompted me to investigate four of my tertiary mathematics classes that included a number of NESB students, through practitioner research.

The main data collection methods for the study were audio-taping classroom discussions, journaling of my experiences, and student group interviews. The data were analysed around three main domains: classroom social norms, sociomathematical norms and classroom mathematical practices.

The first key finding was that NESB students can shift from being less-active to more active participants in mathematical activity when the teacher works with them to jointly constitute classroom social norms that attach importance to, make it safe to and encourage student participation. In this study, these were volunteering to share ideas, explaining and justifying contributions, and asking questions. Establishing these social norms occurs gradually, and NESB students can adopt them over time.

The second key finding indicates that changes can occur in strategies NESB students use to solve mathematics problems that are embedded in everyday contexts, if they are involved in the joint construction of sociomathematical norms that value the negotiation of mathematical meaning. Over time, and with support, students can develop the ability to support their thinking, and come to understand what counts as mathematical problem-analysis, explaining and justifying mathematically, and communicating mathematically. The authority to evaluate the authenticity of mathematical contributions can be distributed among NESB students and between students and the teacher, if students are positioned as creators and evaluators of mathematical knowledge.

The third key finding is that the specific ways of acting and reasoning that are appropriate for a particular mathematics topic evolve when the teacher and NESB students discuss problems and solutions related to that specific mathematical idea. Instead of relying on memorized procedures for solving problems, students

progressively rely on their conceptual understanding of specific mathematical ideas and practices.

The findings challenge the general view that NESB students will necessarily be passive and prefer to learn by memorising information. They indicate that NESB students can develop as active learners, embracing expectations and obligations that they will contribute ideas and negotiate meaning, rather than follow what the teacher says. Over time, a number of my NESB students developed autonomy in mathematical problem solving and their ability to solve mathematics problems and contribute to class discussion improved.

In light of the findings, I propose that NESB students' mathematics learning may be enhanced by a focus on initiating and guiding joint constitution of classroom social norms that value and encourage student participation in the social construction of knowledge and sociomathematical norms that promote conceptual understanding through negotiation of mathematical meaning. I further propose that NESB students' mathematics learning may be supported by guiding collective construction of classroom mathematical practices concerned with the specific ways of reasoning and acting needed in particular mathematics topics.

The findings of this study have relevance and offer fresh insights for mathematics teachers, researchers and tertiary institutions into how NESB students can be supported to learn mathematics. Further research in this area could examine practices of other mathematics teachers involved with NESB students.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

My initial interest in this research arose from my teaching experiences, over a long period of time, with students who learn mathematics in English, and yet English is not their first language. Often, these students say to me: "Mathematics is a difficult subject;" "The problem is with understanding the question;" "Word problems are confusing and a waste of time;" "Language is our problem because English is not our first language;" "You are the teacher, so tell us the method for the answer." I tell them not to expect me to do the thinking for them, but to learn by exploring, suggesting solutions, reflecting and sharing their ideas with others in the class. It is this mismatch between my view and students' views about how mathematics should be taught and learned that led to this study.

In order to put the impetus for this thesis in perspective, I begin by presenting a summary of my background and my motivation for the study. Next, the need for study is spelt out, in terms of the problems faced by Non-English-Speaking-Background (NESB) students studying mathematics and the challenges faced by their teachers. Under research design I state the research question, briefly outline the practitioner research design, and signal the theoretical perspective that has enabled me to gain insight and understanding into my teaching practice. Finally, I present an overview of thesis chapters.

1.2 My background and motivation for this study

I am a mathematics teacher, with long experience teaching mathematics. I taught mathematics to high school students in Zimbabwe for 11 years. These students' first language is not English, which is the language of instruction. Since 2001, however, I have been teaching mathematics to NESB students at a university in New Zealand. These students are studying Year 13 secondary school mathematics before they start their degree studies. Throughout this period, I have made a number of informal observations about my practice and from these informal observations I have conjectured that the problems in my mathematics classrooms may be to do with

student reluctance to participate in discussions, their uneasiness with collaboration, and difficulties with contextualized problems. Rather than benefiting from contextualized problems, which I anticipated would help students link mathematics with their lives, I have observed the confusion, frustration even despair some of my students experience when faced with these problems. On the other hand, some students blossomed in my classes. It appears that the social context in my classroom can enable and it can constrain how NESB students learn mathematics. Consequently, the impetus for this research is not a one-time event; rather it draws from experiences that stretch over my entire mathematics teaching career.

In my mathematics classes I have always wanted to develop my students' ability to solve contextualised mathematics problems, to talk and communicate about mathematics ideas, to express their own mathematical thinking and judgements and to take ownership of their learning. Furthermore, I have wanted to develop student dispositions towards currently accepted ways of doing mathematics. Rather than learn mathematics by committing information and rules to memory without understanding, I have always wanted my students to learn mathematics, with conceptual understanding, through collective construction of knowledge.

Seeing that my students struggle, I began to think about ways to explore my classroom to gain a deeper insight and understanding of my own teaching practice. I therefore decided to gain a better understanding of my own practice by embarking on conscious reflection and evaluation of my professional behaviour. This meant researching my own teaching. I hoped that a better understanding of my teaching practice could help me tackle some of the problems faced by NESB students studying mathematics, and be of interest to others with similar concerns.

1.3 The need for the study

Mathematics teaching is an important area of research because teachers have direct influence on what mathematics is taught and, how it is taught and learned (McCrone, 2005; Anthony & Walshaw, 2007). If we are really concerned about NESB students' mathematics learning, we need to investigate the teaching practice of their

mathematics teachers. Put another way, there is need to research how the issues associated with teaching mathematics to NESB students can be better addressed.

1.3.1 Problems faced by NESB mathematics students

The problems faced by students who learn mathematics in a language different from their first language have been acknowledged the world over (e.g. Adler, 1999; Bernardo, 2002; Biggs, 1999; Gutierrez, 2002; Moschkovich, 2002). The problems students face are associated with their language proficiency, their beliefs about the teaching and learning of mathematics, and the mismatch between their expectations and those of their teachers. Sometimes NESB students' learning problems occur because mathematical problems or questions and other learning materials are written using assumptions about what the typical local student might be familiar with in terms of ways of working and the contexts and materials involved. These assumptions may be incorrect for NESB students. For example, a study carried out in Spain found that NESB students and\or new arrivals in Spain tended to have difficulties understanding and using "classroom norms that the mathematics teacher and most local students consider[ed] shared" (Planas & Gorgorió, 2004, p. 20). Biggs (1996) and Leung (2001) have identified that Asian students studying in foreign countries find the nature of co-operative learning and collaboration in these countries differs from those in the Chinese education system.

In the New Zealand education system, particularly at the secondary school level, the delivery model tends to be dominated by collaborative small group activities and\or whole-class discussions, rather than teacher transmission. These teaching and learning approaches may not be familiar to and\or fit with NESB students' expectations, and so they can struggle to fit in or adjust. For example, in a New Zealand study of NESB students studying calculus at Year 13 level, Edwards (2003) found that Chinese students expected teachers to be available for extra support after class time, as was the case in their home country where the school day was extended significantly. Edwards noted that teacher use of humour and a learner-centred teaching style could cause NESB students discomfort and confusion rather than helping them relax and feel at home. Edwards also found that some NESB students' limited proficiency in English caused them embarrassment when interacting with

teachers or other English-speaking people. Similar conclusions were arrived at by Neville-Barton and Barton (2005) in their study of NESB mathematics students at five high schools and one university in New Zealand. They also concluded that when contextualised problems are used in teaching mathematics, NESB students are disadvantaged due to, among other factors, language difficulties. How these issues might be addressed is a concern of this study.

Since classroom norms and expectations along with the tasks used have important consequences for students' performance and the distribution of learning opportunities this study focuses on the ways classroom social norms, sociomathematical norms, and classroom mathematical practices (Cobb, Stephan, McClain & Gravemeijer, 2001; Yackel & Cobb, 1996; Yackel, Cobb & Wood, 1991) can facilitate or constrain NESB students' mathematics learning. Based on literature review and my teaching experience, I assume that NESB students may have not been exposed to the teaching style that values and encourages negotiation of mathematical meaning.

1.3.2 The practitioner's problem

The poor performance of NESB students in mathematics is often blamed on their mathematics teachers who are said to be ill-prepared to offer appropriate instruction for these students. NESB students come from a wide variety of social and cultural backgrounds, thereby posing huge challenges to mathematics teachers if they are to meet individual needs. Linguistically NESB students can be very diverse, meaning that NESB students in mathematics classrooms are likely to have many different first languages (Walshaw & Anthony, 2007), giving rise to communication and other related problems. Adler (2001), Khisty (1995), Neville-Barton and Barton (2005), and Moschkovich (1999), for example, have, in separate studies, identified the challenges to teachers associated with teaching mathematics in multilingual classroom contexts. These findings suggest that a mathematics teacher faces extra demands due to the multiple first languages among NESB students (Enyedy, Rubel, Castellon, Mukhopadhyay, Esmonde & Secada, 2008).

Given the current emphasis on international education, mathematics teachers, the world over, cannot wait for NESB students to master English (the language of instruction) before they teach them mathematical content (Campbell, Adams &

Davis, 2007). A teacher of NESB students is now expected to prepare activities that enable students to simultaneously learn the language of instruction and formal mathematical language (Anthony & Walshaw, 2009; Moschkovich, 2002). In addition, teachers are expected to identify linguistic demands that affect NESB students' learning, and adjust their instruction to "accommodate" cultural and linguistic issues associated with NESB students.

Research focusing specifically on mathematics teachers of NESB students is not readily available but teachers understanding their own teaching practice is gaining interest in mathematics education worldwide (Planas & Gorgorió, 2004). Practitioners are being urged to subject their practice to rational interrogation (Parker, 1997) and the emerging importance of teacher practitioner research is evident from publications about methods of conducting such research (van Zee, 2006). The perception is that researching one's own practice has the benefit of deliberately focusing on and generating knowledge in the context of practice (Pritchard, 2002). In schools, colleges, and universities, teachers see the development of a research-base grounded understanding of their educational efforts as key to their improving the services they offer (Zeichner & Noffke, 2001). This study fits with this trend. I considered it was essential that I systematically study my own teaching practice to develop a research-informed understanding of the ways in which I might better support my students' mathematics learning.

1.4 Research design

This section states the research question guiding this study, gives a brief outline of a practitioner research design and signals the theoretical perspective used.

1.4.1 The research question

The research question that guides this study is:

> What can I do to my teaching practice to enhance my students' mathematics learning?

In order to address the question What can I do to improve my practice? Mason (2002) suggests that participants or practitioners, among other things, "explore what their

practice consists of as opposed to their ideals and wishes; and what action they take and what the effects are of taking those actions" (Mason, 2002, p.173). This is what this study intends to do. Following Mason I am proposing to explore my actions, aspirations, fears, and motivations with respect to my teaching practice. The main objective of this study therefore is to interrogate my experience; to record, analyze and reflect on episodes or events that occur as I teach and to report a story of my professional learning about my own practice as it enables and constrains the learning of my NESB students.

More specifically, the objectives of the study are:

1.   To document, reflect on and interpret teaching and learning events that occur during the research period;

2.   To reflect on my own actions during selected events in my mathematics classes;

3.   To reflect on my students' actions during the same events in my mathematics classes;

4.   To build new understandings of myself and my practice, with special focus on the implications for NESB student mathematics learning;

5.   To identify and explore the impact of new teaching practices on students' mathematics learning; and

6.   To produce a sequence of writings (or story) about my professional learning as my practices evolve during the research period, in the hope that this will be of benefit to other mathematics teachers of NESB students.

My wish in this study is to develop an understanding of my teaching practice in a manner that enables me to work with, and enhance the mathematics learning of my NESB students who have come to New Zealand to study Mathematics with Calculus and\or Mathematics with Statistics, for at least one semester, before gaining admission to study for a university degree. The word *new*, in some of my objectives, is used to mean new to the teacher (i.e. me) and I expect that the understandings will be new to other mathematics teachers of NESB students at tertiary level. It is closely

tied to the overall research question. Explicit focus on initiating and guiding the development of classroom norms and mathematical practices that value, encourage and provide opportunities for negotiation of mathematical meaning was new to me. So, the understandings of the impact on student learning of implementing certain teaching strategies will be a new experience for me.

Although my students are involved in this study, the main concern is my teaching practice. I therefore pursue an inward focus directed at my own actions as a mathematics teacher of NESB students, rather than focus exclusively on the students' classroom behaviour and their responses to the curriculum. This thesis documents the journey I travelled while trying to satisfy my wish to enhance NESB students' mathematics learning. In it I engage in theory and link this with my practice. To do this in a logical way, I need a fitting research design and theoretical perspective for the study. Next, I provide a brief overview of a practitioner research design used in this study.

1.4.2 A practitioner research design

Practitioner research is inquiry into one's own practice in teaching or teacher education (Cochran-Smith & Donnell, 2000; Steele & Widman, 1997). It can be used to fine-tune one's strategies and practice (Cochran-Smith & Lytle, 1999) and hopefully, become a more effective practitioner than before. In broad terms it refers to a range of activities people carry out as they seek "knowledge or understanding while pursuing or improving a social practice in which they regularly engage" (Pritchard, 2002, p. 3). Carr and Kemmis (1986) contend that such self-reflective enquiries are undertaken by participants (e.g. teachers, students, principals) in certain situations (including educational) in order to improve the rationality of their own situation or educational practices, their understanding of these practices, and the situations in which the practices are carried out. This approach is appropriate for this study, and hence, is used to enable me answer my research question as I focus on my development while trying out different teaching strategies with my students.

This study investigates my teaching practice at a tertiary institution in New Zealand as I work with NESB students to develop their mathematical knowledge, before they gain admission to do undergraduate studies. Data collection includes audio-taping my

words during teaching. Data are also collected through journaling of reflections on lessons and through student group-interviews. All data are analysed using an interpretive methodology. Data and data analysis provide significant evidence required to answer the research question. Although research data are generated at one particular place, I am able to bring a broader practitioner perspective on NESB students' mathematics learning to this research. A theoretical perspective that guides this study is discussed next.

1.4.3 Theoretical perspective

The theoretical perspective that allows me to talk about my teaching practice in a coherent manner is sociocultural theory (Cobb, Gravemeijer, Yackel, McClain & Whitenack, 1997). A sociocultural perspective emphasizes the social and cultural nature of learning mathematics. Specifically, the study will use and build on Cobb et al.'s (2001) understanding of mathematics classrooms as involving the interplay of social norms (e.g. forms of participation), sociomathematical norms (normatives specific to mathematics), and mathematical practices (normatives specific to a mathematical idea, e.g. geometric sequence). This team's understanding is that together, these three aspects constitute the social perspective of learning. In addition their understanding is that the mathematics classroom involves a psychological or individual perspective. The psychological perspective is concerned with the nature of the individual student's reasoning in communal classroom activities. Cobb and his colleagues coordinated these two perspectives and argued that they are interactively related. They accounted for the individual, the group and the interplay between them. Since my research is about my teaching practice and social aspects of student learning as it occurs in the context of the classroom, my main focus is on the social aspects of Cobb and his team's theoretical perspective.

This study is concerned with classroom social norms that offer opportunities for NESB student participation in mathematical activity. Social norms refer to those aspects of social interactions that become normative expectations in the classroom (Yackel, Rasmussen & King, 2000). They include that students are expected to explain and justify their thinking. By immersing myself in my teaching practice and examining the social norms that are established by my NESB students and myself I

hope to gain a better understanding of my teaching practice and at the same time enhance their learning.

In addition to classroom social norms, this study is interested in sociomathematical norms. Sociomathematical norms are normative aspects specific to mathematical activity. They are those social norms that are specifically related to mathematics. Sociomathematical norms include collective understandings of what constitutes "as mathematically different, sophisticated, efficient and elegant" (Yackel, 2001, p. 13).

Classroom mathematical practices are concerned with normative ways of reasoning and symbolising about particular mathematical ideas (Bowers, Cobb & McClain, 1999; Cobb et al., 2001). They are part and parcel of mathematical activity, but unlike social and sociomathematical norms, they are specific to particular content. In view of this, I further propose to examine some classroom mathematical practices that are established and developed during shared particular mathematical topics. Students' individual mathematical interpretations and reasoning within a topic can influence collective or shared classroom practices that evolve as part of a mathematical topic.

Taken together, these three constructs (classroom social norms, sociomathematical norms, and classroom mathematical practices) will form the framework I use to describe and inform changes in my teaching practice to enhance NESB students' mathematics learning in my mathematics classes.

I began the study hoping it would serve a dual purpose. That is, the ongoing findings would support my ability to use reflective strategies in my teaching practice and support my NESB students' learning of mathematics. At the same time, my wish was that findings would be relevant to the teaching and learning of mathematics, in general. It is my desire that the study address the gap in literature concerning ways in which mathematics teachers of NESB students and their teaching practice can be supported.

1.5 Overview of thesis chapters

This thesis is divided into nine chapters. This chapter, Chapter1, has provided an introduction to the study. I began by presenting a synopsis of my background and motivation for the study before signalling some of the problems faced by NESB

mathematics students.   Some of the practitioner's challenges were identified and described. The research question was stated and the practitioner research discussed in brief. Finally, the chapter outlined the theoretical perspective that guides this study.

Chapter 2 reviews the literature on the issues currently facing mathematics education. Key aspects of each issue are outlined in relation to how they impact mathematical activity and might support or not NESB students' mathematics learning. Mathematics is proposed as a human activity and learning mathematics as a social and cultural activity. Chapter 3 discusses, in more detail, the theoretical perspective that underpins this study, namely, the sociocultural perspective. A specific framework for analyzing the classroom microculture is detailed. This identifies three significant domains for this study and each domain is discussed, separately.

In Chapter 4, the methodology and methods used in this study are outlined. The main theoretical influences on the methodology of the study as well as the processes of data collection and analysis are discussed. The chapter highlights issues to do with ethics, trustworthiness and generalisability in practitioner research.

Chapter 5 presents and analyses data associated with classroom social norms. Examples are used that clarify and illustrate how classroom social norms were initiated and guided. Episodes illustrate the shifts that occurred in NESB students' participation over time. Data presentation and analysis continues in Chapter 6, where the focus is on sociomathematical norms. Examples show how sociomathematical norms for contextualized problem solving were collectively constituted. In addition, they show the changes in the ways NESB students participated in this mathematical activity. Data related to classroom mathematical practices is presented and analysed in Chapter 7. In this chapter, examples demonstrate the ways I initiated and guided the establishment of particular classroom mathematical practices exclusive to different topics. They also illustrate the changes in students' specific ways of acting and reasoning in particular mathematical ways associated with a topic.

Chapter 8 discusses the findings in relation to the research question, the literature reviewed and the sociocultural perspective. Lessons emerging from the study are discussed in relation to the three domains of interest in this study: classroom social norms, sociomathematical norms, and classroom mathematical practices.

Finally, Chapter 9 summarises main conclusions about teaching NESB mathematics students, arrived at in this study. This last chapter also sets out limitations of the study, implications of findings, directions for further research, and concluding remarks.

CHAPTER 2

ISSUES FACING MATHEMATICS EDUCATION

2.1 Introduction

The research focus for this study arose from my personal interest in non-English-speaking-background (NESB) students' mathematics learning but it was shaped by the issues currently facing mathematics education. The main issues are: a diversity of views about the nature of mathematics, pedagogical issues to do with mathematics, and how to better support NESB students' mathematics learning. This chapter reviews literature relevant to these matters to identify gaps with respect to teaching and learning approaches to support NESB students' mathematics learning.

Differing views about the nature of mathematics have an influence on the ways in which mathematics is approached in the classroom. In particular, the product view, process view, and cultural view of mathematics have a huge effect on how mathematics is taught and learned. Each of these views is outlined and discussed in relation to classroom practice. Next, I deliberate on issues associated with mathematics pedagogy under the headings the role of contextualized mathematical problems, communicating mathematically, and developing student autonomy in mathematics. Issues specific to NESB students' mathematics learning are of particular interest in this study. In order to gain a deeper and better understanding of the impact of mathematics classroom culture on NESB students, some of the major concerns are identified and discussed from both a theoretical and practical point of view. In particular, the discussion focuses on academic culture and language. I propose that an understanding of all these matters provides a sound basis for investigating the ways NESB students can be assisted with their mathematics learning. In the final section, I provide a summary of the chapter.

2.2 Views of the nature of mathematics

There is a wide range of views about the nature of mathematics (Dossey, 1992; Lerman, 1983; Nickson, 1994; Presmeg, 2007). From this range, I have identified three broad views as predominant in the literature and of relevance to my study.

They are the product view, the process view and the cultural view. Although these views overlap, I will discuss them separately, in order to better highlight particular points that impact on teaching and learning mathematics.

2.2.1 Product view of mathematics

The product view characterizes mathematics as a fixed body of knowledge that consists of concepts, principles and skills (Dossey, 1992; Ernest, 1996). It has specific content, methods, language and answers. From this perspective, mathematics is essentially a product, with a distinctive knowledge structure (Burton, 1995). The product view does not treat mathematics as something that changes and grows (Nickson, 1994). It regards mathematics as being the same over time, everywhere and for everyone, hence, universal (Ernest, 1990). The product view treats mathematics as abstract and value-free (Leung, 2001; Lerman, 1990). It considers mathematics to be separated from human activity and context-free. Mathematics is regarded as objective truth with formal rules. Mathematical truths are discovered through deductive arguments or methods. The product view does not perceive mathematics as a way of knowing and interpreting our experiences. Instead, its goal is to provide rigorous systems to warrant the unquestionable certainty of mathematical knowledge. This has implications for classroom practice.

Teacher classroom practice that is aligned with the product view focuses on helping students acquire the distinctive structure of mathematics. It emphasizes mathematical content, rules, language and correct deductive methods and answers. Mathematical concepts or methods are meticulously transmitted by the teacher (Wong, 2002). In mathematics classrooms, all authority resides with teacher. Teaching involves highly structured lessons and rigorous mathematical language. Teaching practice committed to the product view pays no attention to context. Instead it focuses on formal rules. These rules can be proved deductively using correct arguments (Burton, 1995).

Classroom practice disregards human values. At the same time it emphasizes conformity (Leung, 1995). Mathematics ideas are not negotiated (Burton, 1995; Ernest, 1996). Instead, instruction stresses using the same method and getting the same answers from everyone when solving the same mathematical problem. Methods are learned first; uses and applications come later.

In a classroom aligned to a product view of mathematics, students learn the distinctive structure of mathematics. They acquire mathematical content and learn procedures for solving tasks. This can lead students to think that mathematics is a structured collection of mathematical facts and methods to be acquired by memorizing or rote learning. Furthermore students can think that mathematics is transmitted by the teacher to the students and that all the authority rests with the teacher. Students could think that each mathematical task has a unique, fixed and objective answer. This can lead students to believe that it is more important to know *how* not *why* (Boaler, 2002). In addition, they can perceive mathematics as a formal game with no connection with human activity. This could lead them to think that mathematics is value-free.

Literature shows that although the notion that mathematics is a body of infallible and objective truth is now questioned by many mathematicians and philosophers, there is still wide acceptance of this view by society, mathematics teachers, and students (Benn & Burton, 1996). A number of mathematics teachers continue to align their teaching with the product view. However, an increasing number of people see mathematics as a social construct, a process. This view is discussed next.

2.2.2 Process view of mathematics

In contrast to the product view that characterizes mathematics as absolute, the process view regards mathematics as a social construct that is open to change and development (Ernest, 2010a; Hersh, 1998; Nickson, 1994). The process view considers mathematics to be a human activity. It is an outcome of social processes (Cobb, Yackel & Wood, 1992). From the process view, mathematics is a social process that involves investigation. It is open to revision in terms of concepts (Hersh, 1998). This impacts classroom instruction.

Teaching practice committed to the process view focuses on the social practices of jointly negotiating and constructing mathematical meaning, in small-groups and the whole-class (Yackel & Cobb, 1996). In addition, instruction focuses on the process of doing mathematics in context, rather than on mathematics content itself (Leung, 2001). Apart from this, teaching uses problem solving and investigational approaches (Cobb, Yackel & Wood, 2011; Lerman, 1990). It utilizes real-life or practical

problems. Teacher classroom practice stresses the process of solving the task rather than getting the final 'correct' answer. It pushes for exploring, questioning and the merits of mathematical thinking (Moschkovich, 2007). Mathematics is sometimes done individually, but at other times shared, challenged, questioned and discussed (Burton, 1995; Moschkovich, 2002). In mathematics classrooms that adhere to the process view, students experience mathematics as a human activity. They share their work and explain their thinking. Students engage in mathematical investigations and experience collectively solving practical or real-life problems. Simultaneously, they can recognize that mathematical knowledge can be subjected to scrutiny, reconsideration, revision and refinement (Cobb et al., 2001). These experiences can lead them to think that mathematical meanings can be explored and revised. Students can also think that mathematical meaning is subject to change or improvement (Lerman, 1996; Nickson, 1994).

By itself, the process view is limited because it fails to adequately account for what mathematics is, that is, mathematics is value-laden. The 'value' aspect of how mathematical knowledge comes into being implies that mathematics is not only a social phenomenon but also a cultural one (Nickson, 1994). The next section discusses the cultural view of mathematics.

2.2.3 Cultural view of mathematics

A cultural view of mathematics locates knowing and doing mathematics within sociocultural activity, and recognizes the differences in how this activity is organized within different communities and interpreted by different individuals (Barton, 1996; Cobb & Hodge, 2002). The cultural view regards mathematics as an inherently cultural activity that is tied to particular cultural practices and so, mathematical knowledge is considered to be cultural (Bishop, 1988; Lerman, 1996; Moschkovich, 2002). From this perspective, mathematics is value-laden and situated in social contexts, but not universal (Dengate & Lerman, 1995; Lerman, 1983; Presmeg, 2007). This view acknowledges that people develop mathematical knowledge through social and cultural institutions (Nasir, Hand & Taylor, 2008). Consequently, mathematics is not the same everywhere and for everyone. Mathematics comes to life when human beings *do mathematics*. What people come to know is *embodied* in

action as individuals manage themselves and their goals within the environments they operate in (Cobb & Bowers, 1999).

From a cultural view of mathematics, teaching practice recognizes that culture matters in mathematics teaching and learning (Nasir et al., 2008; Sfard & Prusak, 2005). Teacher classroom practice committed to the cultural view acknowledges that the requirements of what it means to know mathematics, to be a mathematics learner, and to be an effective mathematics student in a particular classroom context vary (Presmeg, 2007). Teaching practice can recognize that mathematics learning and knowing differ across cultural groups. In addition, it can recognize the diversity in the students' ways of knowing and different ways to learning (Diversity in Mathematics Education, 2007). Consequently, pedagogy can create a range of opportunities so that all learners have access to mathematical knowledge, irrespective of their cultural and experiential background (Gutstein, 2003; Gutiérrez, 2002). In this way, classroom practice can create classroom environments that support various forms of knowledge and empower all students to think and reason mathematically (Moschkovich, 2002; Nasir et al., 2008).

Teaching can highlight the link between mathematics and human values. The wide variety of mathematical practices, values, and identities learners bring to the mathematics classroom can be accommodated and recognized (Sullivan, Mousley & Zevenbergen, 2004, 2006). Classrooms committed to the cultural view of mathematics can be viewed as sites of local cultural processes. They can be regarded as social spaces that position different kinds of discourse and reasoning as forms of learning and knowing that have authority (Cobb & Hodge, 2002).

Teaching practice can create conditions aimed at bridging in-school and out-of-school mathematical knowledge (Cobb & Nasir, 2002; Martin, 2000; Moschkovich, 2007; Nasir et al., 2008). Instruction focuses on uncovering students' competences and drawing on their backgrounds, experiences and informal strategies or everyday practices (Carraher, Carraher & Schliemann, 1985; Lave, 1988; Saxe, 1988, 1991). In addition, teacher classroom practice can integrate students' out-of-school practices with academic mathematics practices (Lubienski, 2002). In doing this, instruction can highlight the connection between cultural and academic or school mathematics. This

needs to be done without overusing, overemphasising or overvaluing everyday practices and sacrificing school mathematics.

In a classroom based around a cultural view of mathematics, students can get the teacher's assistance in addressing issues to do with the boundaries between academic and cultural knowledge that could create barriers for their mathematics learning (Sullivan, Zevenbergen & Mousley, 2003). Students can experience classroom practices that offer them encouragement and support as they move across out-of-school cultural contexts and classroom contexts that help to shape and constitute their mathematics learning (Moschkovich, 2007). They can learn to merge the practices of out-of-school cultural contexts and mathematics classroom contexts. To do this, NESB students need to learn to reconcile their past experiences in mathematics classrooms in their home countries and mathematical practices in the host countries - New Zealand in this study. While doing this, students can come to think that mathematics is a cultural activity and that mathematical knowledge is cultural.

In mathematics classrooms aligned to the cultural view, teaching practice uses a discourse approach to mathematics learning that allows consideration of the different points of view students bring to mathematical discussions (Moschkovich, 2002). Instruction pushes for learning mathematics to occur through social interaction, sharing ideas, joint negotiation of meaning and participation by all members of the class (Nasir & Hand, 2006). It can promote engagement in discussions that transcends vocabulary acquisition to enable students to engage in communicating concepts and developing mathematical content (Diversity in Mathematics Education, 2007). Teaching distributes the authority to contribute and to judge mathematical contributions to all members of the classroom (Yackel & Cobb, 1996).

Instruction should not merely focus on what individual students lack, such as mathematical language or English language, but on how students draw on multiple resources to communicate their point of view (Moschkovich, 1999). In order to accommodate various ways of communicating ideas, teacher classroom practice can shift between different communication practices. It can broaden the conception of competent student participation to accommodate students' use of resources such as

gestures, objects and their first languages (Moschkovich, 2007; Nasir, 2002). No one form of participation should be privileged over others.

Students in classrooms aligned to the cultural view of mathematics, can experience learning mathematics as a cultural activity. They can be encouraged to use ideas from their cultural and mathematical background (Civil, 2002; Diversity in Mathematics Education, 2007; Presmeg, 2007). The resources and knowledge they bring and utilize in mathematical activities can be recognised, valued and legitimized by other students and the teacher (Moschkovich, 2002). So, students can experience having their contributions validated and authorized by members of the classroom. In this case, students can come to think that mathematics is not universal; rather it is negotiated in a cultural context. In addition, students can recognize that mathematics is associated with human values. Students can have the opportunity and support to broaden their conceptions of mathematics and mathematical knowledge (Leung, 1995). They have the chance to develop an appreciation of mathematics and mathematics learning as social and cultural activities.

To sum up, collectively and individually, the product, process and cultural views, the conception of the nature of mathematics, the view of the nature of mathematics teaching, and the model of the process of learning mathematics, have different implications for the ways classroom mathematics teaching and learning is conceptualised and managed. Put another way, the view of mathematics that is emphasized in a particular classroom influences the ways mathematics is learned in that classroom. If a mathematics teacher emphasises content this tends to lead students to focus on memorising this. If they emphasise process, students might focus on negotiating and creating their own mathematical knowledge. If they emphasise culture, students may be disposed to negotiating mathematical knowledge in cultural contexts. The view of mathematics adopted for this study is outlined in the next section.

2.2.4 The view of mathematics adopted for this thesis

The view of mathematics adopted for this study is a combination of the process and cultural views of mathematics, and to a lesser degree the product view. That is, mathematics is regarded as a dynamic, continually growing field of human creation

and invention, a cultural product (Ernest, 2010b). The emphasis is also on students engaging in problem solving, active construction of understanding and negotiation of meaning, and the teacher as facilitator. In this adopted view, mathematics is a process of enquiry and coming to know, but not a finished product, since its results are open to revision. It is viewed as a dynamically organised structure situated in a social and cultural context. The product aspects involving mathematical content, facts, symbols or notation, are not treated as separate entities or product, but an integral part of the knowledge construction process. On the basis of my understanding from literature, I consider the combination of the process and cultural view of mathematics having a dominant role, and the product view taking a minor one, to be the most appropriate for my study.

In addition to a diversity of views about the nature of mathematics, there are a number of well documented issues to do with the nature of effective mathematics pedagogy. These are of particular interest in this study given its focus. A selection of the issues that have particular relevance to this study, based on understanding from literature, is discussed in the next section.

2.3 Issues associated with mathematics pedagogy

Mathematics education is facing a number of pedagogical issues that are linked with different views about the nature of mathematics and the diversity amongst students of mathematics. Some of these issues, as evidenced in the latest handbook of research in mathematics teaching and learning (Lester, 2007) include technology, problem solving, access and equity, assessment, communication, and autonomy. Like Lester, I consider the issues around mathematics problem solving, communication, and student autonomy important and needing addressing.

Contextualised mathematics problems are discussed first because they provide a base for engaging in communicating about mathematics ideas. This discussion is followed by that of communicating mathematically. Developing student autonomy relies heavily on students' ability to communicate mathematically, so it is discussed last. Following this, in the next section, issues to do with NESB students' mathematics learning are deliberated on.

2.3.1 The role of contextualized mathematical problems

In this study, the term 'contextualized problem' refers to a word problem in which the mathematical task is embedded in a context where 'context' is used to mean a real or imagined situation (Chapman, 2006; Meyer, Dekker & Quetelle, 2001). Using contextualised mathematics problems appears justified for this study since the view of mathematics being adopted stresses the significance of context. Also, mathematics teachers are being encouraged to use "realistic contexts in order to make mathematics more meaningful and accessible for all students" (Sullivan, Zevenbergen & Mousley, 2003, p. 107). Although some researchers (e.g. Fraivillig, Murphy & Fuson, 1999; Harvey & Averill, 2012) have found evidence that support the use of contextualised mathematics problems to support learning, others (e.g. Cooper & Dunne, 1998; Planas and Gorgorió, 2004) have some reservations. This section deliberates on some of the pedagogical opportunities and challenges associated with using contextualised mathematics problems in mathematics learning.

Pedagogical opportunities

Contextualized mathematical problems can play a significant role in mathematics teaching and learning by motivating students to learn mathematics, being a source of opportunities for mathematical reasoning and thinking, anchoring student understanding, and illustrating potential applications (Chapman, 2006; Lubienski, 2007). A context, embedded in a contextualized problem can lure students into a problem situation, arouse their curiosity and interest, and may compel them to explore the mathematics that is necessary to answer the challenges posed (Harvey & Averill, 2012; Meyer et al., 2001). Verschaffel (2002) described the goal for using contextualised problems as "to bring reality into the mathematics classroom, to create occasions for learning and practising the different aspects of applied problem solving, without the practical… inconveniences of direct contact with the real world situation" (p. 64). Contextualized problems can help reduce or eliminate the abstraction or complexity often connected with mathematics (Boaler, 1993, 1998; Sierpinska, 1995). The embedding of mathematics in a context may make the task seem more realistic to students and provide vital information to make the task more understandable to the students (van den Heuvel-Panhuizen, 2005). Contextualised

mathematics problems can enhance the "transfer of mathematical learning through demonstration of the links between school mathematics examples and real world problems" (Boaler, 1994, p. 552). They allow linking academic mathematics with real life. Those problems where all learners feel represented (Cooper & Dunne, 1998, 2000; Planas & Gorgorió, 2004; Sullivan et al., 2003) can make the learning of mathematics become more meaningful to them.

Along with motivating students, a contextualized problem can act as a source of opportunities for discussions, mathematical reasoning and thinking, (Lubienski, 2000; Sullivan et al., 2003). Contexts create learning environments where students can come to grips with the basic mathematical ideas by interrogating their own reasoning (Gravemeijer & Doorman, 1999). The 'real' situations, in contextualized problems, offer occasions for students to become creative, critical-minded, and socially and culturally aware (Chapman, 2006; Dapueto & Parenti, 1999). In addition to making it possible for students to construct an understanding of important mathematics ideas, contextualised mathematics problems create opportunities for students to develop positive attitudes towards the subject and themselves as mathematics thinkers (Boaler & Greeno, 2000; Cobb, Gresalfi & Hodge, 2009). In a study of a group of middle school students, in USA, Turner, Gutiérrez and Sutton (2011), concluded that contextualised mathematics problems allow for collective problem solving, thereby creating opportunities for various "students to take on active problem-solving roles, including communicating mathematical thinking, connecting multiple representations, and justifying solution strategies" (p. 227).

Contextualized problems can anchor the student's mathematical understanding by offering openings for students to collaborate and engage in reflective thinking (Sullivan et al., 2003). The context in a contextualized problem can provide individuals with a model to help them understand and remember the new mathematics they will have acquired (Meyer, et al., 2001). Sometimes contextualized problems illustrate potential applications of mathematics to people's lives (Boaler, 2002; Lave, 1996; Saxe, 1991; Sullivan, Zevenbergen & Mousley, 2002). Hence, these problems can facilitate the learning of mathematics in ways that transcend the boundaries (Lave, 1996; Nasir et al., 2008) that are generally found between the mathematics classroom and the real world. Students are able to see how mathematics

is generated from contexts, and applied to problems in their own lives. They have the chance to perceive mathematics as socially negotiable in context, rather than a remote body of knowledge (Koirala, 1999).

In New Zealand a study of eight pairs of students aged 11 and 12 (Irwin, 2001) found that students who worked on contextualized problems made significantly more progress in their knowledge of decimals than those who worked on non-contextual problems. In addition, Irwin found that contextualized problems created an environment where students could use (to their benefit) out-of-school, cultural, or historical experience (or knowledge) rather than formal mathematics, to solve the problem. In spite of the learning opportunities that can be created from using contextualized mathematical problems, certain pedagogical challenges can also arise.

Pedagogical challenges

The challenges related to using contextualized mathematical problems include learners' lack of familiarity with the contexts involved, demands (on students) of the language used in the problem and, the boundaries between the everyday or cultural knowledge embedded in the problem and academic or school mathematics (Civil, 2002). Other challenges involve creating and selecting relevant contextualized problems and the ambiguity of some contextualized problems (Boaler, 1993). These instructional challenges can occur separately or in different combinations.

The social and cultural aspects of contextualized problems can create challenges related to learners' lack of familiarity with the context involved (Sullivan et al., 2003; Planas & Gorgorió, 2004). Under some circumstances learners fail to interpret or understand the problem, and it becomes a challenge for them. Instructional challenges can also arise when learners have difficulties understanding the language used in a contextualized mathematical problem (Campbell, Adams & Davis, 2007). This extra burden to students' mathematics learning poses a challenge for all involved. Reconciling everyday or cultural knowledge in a contextualized problems and academic mathematics can be hard for some learners (Nasir et al., 2008). When this happens, using a contextualized problem gives rise to an instructional challenge. Finding appropriate contextualized problems can be a challenge for teachers. A study of students in USA (Lubienski, 2007) showed that the same contextualized problem

satisfied the learning needs of some students but not others. In a United Kingdom study of 11-14 year olds, Cooper and Dunne (2000) found that students working on contextualised problems in assessment used their knowledge of familiar contexts in ways that were not intended by the question. So, the use of contextualised mathematics problems disadvantaged some students. Similarly, unclear or vague contextualized problems can bring about a challenge when learners misunderstand the problem or encounter other issues related to an ambiguous task. Boaler (1993) contends that wrong choice of contextualised problems can lead learners to confusion and may significantly disadvantage some students.

Although the issues around the use of contextualized mathematical problems are important and worth discussing, a related and equally significant pedagogical issue is that of communicating mathematically. This matter is deliberated on in the next section.

2.3.2 Communicating mathematically

The issues around communicating mathematically include what it means to be able to communicate mathematically, why it is important and implications for classroom practice. The term communicating mathematically is being used in this thesis to mean using mathematical language and representations to formulate and express mathematical ideas in written, oral and diagrammatic form in a way that is acceptable to the wider mathematical community (Gould, 2008). Communicating mathematically involves more than having the ability to apply mathematical conventions and linguistic formulations appropriately. It includes "coming to know mathematics at a deeper level (i.e. internalization) and coming to think mathematically" (Khisty & Morales, 2004, p. 4). Communicating mathematically comprises a particular type of discourse or talk and register (Pimm, 1987). It has specific language functions that are relevant to mathematics. Depending on the context, the meanings that emerge in discourse are multiple, changing, situated, and determined socially and culturally (Cobb, Wood & Yackel, 1993). Communicating mathematically and doing mathematics are inseparable. Both involve acting, as well as using tools, symbols and objects.

Research that stresses student participation in classroom mathematical activity tends to focus on and highlight the importance of communicating mathematically (Lampert & Cobb, 2003). In their study, Cobb et al. (1993), for example, found that with the guidance of the teacher, students developed abilities that enabled them to engage in genuine mathematical communication, both in small-group and whole-class discussions. The study suggested that ability to communicate mathematically enabled these students to contribute effectively in the negotiation of mathematical meaning and better understanding of the mathematics involved. In a separate study, Forman (1996) compared students working individually with those who worked cooperatively on the same level mathematics problems in middle school classrooms, in U.S.A. She concluded that students participated in mathematical discourse in increasingly substantial ways as they began to "understand the skills, norms, values and ideas that are shared by mathematically literate adults" (Forman, 1996, cited in Lampert & Cobb, 2003, p. 239). This finding also illustrated the significance of being able to communicate mathematically. Both Cobb and his team's and Forman's findings demonstrated the importance of initiating students into ways of communication used by mathematically literate experts. These findings are consistent with Tate's (2008) and other researchers' findings who, through research, concluded that mathematical communication creates additional opportunities for students to learn and explore mathematics.

In a classroom that values and is committed to communicating mathematically, instruction can encourage and support development of students' ability to communicate mathematically (NCTM, 2000). It can promote communication as a vehicle for creating mathematical meaning as a means and an end in learning mathematics (Huang, Normandia & Greer, 2005). Classroom teacher practice can develop learners' ability to communicate their mathematical thinking coherently and clearly to their classmates and the teacher through taking a more active role in classroom mathematical activities (Dixon, Egendoerfer & Clements, 2009; Forman, McCormick & Donato, 1998). To a large extent, instruction can advocate mathematics learning as a communicative process that takes place in social contexts (Cobb, Yackel & Wood, 1992; Forman 2003). In this classroom teaching can encourage participation in mathematical activity and assist students as they make

attempts to communicate in ways that "mathematically competent people talk and act" (Moschkovich, 2002, p. 199).

Teacher classroom practice can create opportunities for students to develop academic language proficiency needed for cognitively demanding mathematical tasks (Campbell et al., 2007). It can facilitate knowledge development of both the discourse and the register of the subject. The language of mathematics can be specifically taught and developed over a period of time (Cummins, 1991; Khisty & Morales, 2004). In this classroom, the teacher can encourage students to use the language of mathematics when sharing mathematical thinking (White, 2003; Kazemi & Stipek, 2001). Teacher classroom practice can structure communication patterns in ways that encourage students to engage with mathematics in ways that facilitate development of students' ability to use mathematical language appropriately – as a thinking tool and for communicating ideas. Pedagogy can promote utilization of mathematical language in spoken and written discourse for academic functions such as analyzing, explaining, justifying, presenting arguments and evaluating mathematical content ideas and processes (NCTM, 2000, 2007; Solomon & Rhodes, 1995). The teacher's own talk, or what Khisty and Chval (2002) referred to as the teacher's pedagogic discourse in the context of mathematics, can play a significant role of modelling mathematical communication. In addition to this, the teacher can revoice student responses using more technical or mathematical terms. The nature of the teacher's pedagogic discourse can affect the process by which students could develop their control of the discourse of mathematics.

Pedagogy committed to communicating mathematically recognizes how some students use aspects of competent mathematical communication that go beyond the vocabulary list by encouraging them to use multiple resources such as gestures, objects and everyday experiences to communicate their thinking (Moschkovich, 2007). In addition, teaching recognizes that failure to communicate mathematically does not necessarily indicate students' lack of mathematical understanding. Rather, a student's current mathematical understanding or lack of confidence can, for example, prevent that student from communicating mathematical thinking or can constrain the types of mathematical communication they can engage in. So, instruction can help

students manage language demands that may constrain their ability to communicate their mathematical ideas.

Pedagogy can create conditions that facilitate small-group and whole-class discussions (Cobb et al., 1993; Kazemi & Stipek, 2001; Tatsis & Koleza, 2008). Discussions can offer students opportunities both to learn mathematical language from others and to practise communicating mathematically as they contribute ideas and provide reasons, explanations and justifications, etc. (Anthony & Walshaw, 2009; Moschkovich, 1999). Classroom practice can afford learners opportunities to engage in extended classroom interactions that can allow them to experience mathematical communication through using it and hearing from their peers and the teacher how mathematical discourse, especially academic language in mathematics, is used (Khisty & Morales, 2004). While doing this, students can have the opportunity to organize and consolidate their mathematical thinking through communication with other members of the classroom community. They can debate each other's mathematical thinking. In the process, they could refine both their understanding and their ability to communicate mathematically. Students can learn mathematics as they make attempts to communicate mathematically (Cobb et al., 2001; Lubienski, 2000). At the same time, the teacher can assess students' mathematical understanding, identify gaps in their understanding and offer appropriate support, in part, through students' communication of their thoughts (McKenzie, 2001). In addition, students can get socialized to accept new norms of interaction, and learn new meanings of mathematical words and symbols, as they work collaboratively on mathematical problems. They can manifest their mathematical knowledge by exhibiting their ability to communicate in ways shared by the wider mathematical community (Yackel & Cobb, 1996).

In this type of classroom, students not only can they develop the ability to communicate mathematically they can take charge of the discourse of mathematics. In addition, students may recognize that learning mathematics and communicating mathematically are reflexively related.

Many mathematics educators acknowledge the close relationship between encouraging students to communicate mathematically and using classroom questioning as one of the main teaching strategies.

Classroom questioning

Questioning is an important teaching strategy for creating a classroom environment that is conducive to the development of students' mathematical thinking (Burns, 1985; van Zee, Iwasyk, Kurose, Simpson, & Wild, 2001). The term *question* is used to describe "any request for a response from students on a mathematical issue where the response is expected in a relatively short period of time, typically only a few seconds" (Speer, Smith, & Horvath, 2010, p. 109). Both students and the teacher can stimulate students' development of mathematical knowledge in mathematics through posing classroom questions (Kazemi, 1998; Knuth & Peressini, 2001; Martino & Maher, 1999; Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008).

Teacher questioning

Teacher questioning is an important aspect of teaching and classroom conversations (Chin, 2006). It can assist students construct conceptual knowledge (Franke, Webb, Chan, Ing, Freund & Battey, 2009; Harrop & Swinson, 2003) and promote students' understanding of mathematical ideas (Harrop & Swinson, 2003; Sahin & Kulm, 2008). The teacher contributes to the process of student conceptual change and extension of student ideas through asking questions in support of "student reinvention and extension" of a mathematical idea (Martino & Maher, 1999, p.54).

Teacher questioning can be used to initiate discussion, elicit student thinking and encourage students to elaborate on their answers and ideas. It supports students in making their thinking public, as they try to detail their strategies for solving particular mathematics problems or connect with their classmates' strategies (Franke, Web, Chan, Ing, Freund & Battey, 2009). Teacher questioning can engage and challenge students' thinking through asking students to justify their ideas (NCTM, 1991). The teacher's questioning also has a number of other functions; for example, to review material and diagnose student ideas (Chin, 2007). It serves to uncover reasons for errors students make and the underlying misconceptions which can lead to these.

Through skilful and timely questioning of students' developing ideas, the teacher can estimate a student's level of understanding, as well as promote justification and generalization on the part of the student (Martino & Maher, 1999). Teacher questioning is intended to guide student thinking, to test knowledge of facts and to provide feedback to the teacher about student understanding of the material being discussed (van Zee & Minstrell, 1997; Chin, 2007).

In a study of teacher questioning during a class discussion about measurement, in USA, van Zee and Minstrell (1997) concluded that the teacher used 'reflective tosses' effectively to help students clarify their contributions, consider various view points, and monitor their own thinking. van Zee and Minstrell (1997) used the term *reflective toss* to refer to a situation where the teacher throws the responsibility for thinking back to a student by posing a question in response to a student contribution. A reflective toss consists of the sequence, student statement, teacher question, additional student statement (Chin, 2006). So, teacher questions are formulated in a way that highlights important mathematical ideas (Franke et al., 2008), and shifts responsibility and authority for judging answers from the teacher to all students (van Zee & Minstrell, 1997; Chin, 2007). Questioning should be flexible, giving room for the teacher to adjust his/her questions to accommodate student contributions (Chin, 2006). The teacher needs to ask follow-up questions following on a student's initial response or explanation. Such follow up questions help to build on the student's ideas (Franke et al., 2009).

Prompting and probing questions

Prompting questions nudge students to publicly speak their thoughts (Anghileri, 2006). They are intended to "gain insight into students' thinking, promoting their autonomy, and underpinning the mathematical understanding that is generated" (p. 43). Prompting questions are valuable only if the teacher is responsive to students' intentions rather than his or her own.

Probing questions are intended to explore what students think about the topic being discussed (Dillon, 1988; Graesser & Person, 1994), guide towards specific understandings (van Zee & Minstrell, 1997), elicit discussion, or check on progress

(Gall, 1984; Mehan, 1985). They have instructional purposes which include extending students' knowledge, encouraging explanations and elaborations, and promoting deeper mathematical thinking (Sahin & Kulm, 2008). They ask for clarification, justification, or explanation. Probing questions ask students to apply prior knowledge to a current problem or idea, and ask students to justify their ideas.

In a USA study involving elementary mathematics classrooms, Franke et al. (2008) found that teacher questioning involving asking a sequence of probing questions "frequently helped students provide a complete explanation after they provided an explanation that was not correct or complete" (p. 390). Sequences of probing questions were found to benefit all participants in the class. They enabled the teacher to fully understand students' level of thinking and make more informed instructional decisions. In addition, they helped students being questioned to clarify, solidify, and correct their thinking. Furthermore, probing sequences of specific questions gave opportunity to other students, in class, to connect their own thinking with what was being said, potentially enabling them to correct their misconceptions.

Student questioning

"Students have questions" (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008, p. 381). Many students, however, lack the ability to effectively pose questions without explicit instruction in posing questions (Martinello, 1998). Some research (e.g. King, 1994; Lampert, 1990; van Zee, Iwasyk, Kurose, Simpson, & Wild, 2001) has shown that when students are taught and encouraged to generate questions, their comprehension of material improves, they make links between thoughts in the content of the lesson and connect those links to their prior knowledge. Piccolo and his team report research that found that students who posed questions, in classroom interactions, were more flexible with the content and demonstrated greater understanding of the ideas being discussed.

The use of questioning as one of the main teaching strategies in this study, required that both the students and the teacher pose questions that support student learning. The teacher would need to pose prompting and probing questions that diagnose and extend student thinking - questions that elicit student ideas and encourage them to

explain and justify their contributions. Students would need to learn to formulate and pose their own questions in ways that could assist them to learn mathematics. When questioning is one of the key teaching strategies, the teacher needs to make important decisions about how long to wait for student responses.

Wait time

One of the important teaching strategies closely related to questioning is wait time (Marzano, 1993). It is regarded by many educators as an effective teaching strategy (Speer et al., 2010). The term *wait time* was first defined by Rowe (1974) as pausing for a few seconds after asking students a question and before expecting them to give an answer. Some studies (e.g. Rowe, 1974, 1986; Tobin, 1987) have shown that with longer wait time (3 to 5 seconds) students become more engaged, make richer, more productive responses that lead to better quality classroom discussions, the teacher formulates more higher level questions, talks less but asks more probing questions. Other significant effects of extended wait time noted by Rowe and Tobin, and based on their separate studies, are that teacher interruptions to student discussions become less, there is an increase in student active participation, higher cognitive learning is more likely to occur, and both the students and the teacher have the opportunity to think and process their ideas before articulating them.

In a study involving science students in Australia, Tobin (1987) found that longer wait time can facilitate higher cognitive learning across elementary, middle and high school science. In another study of ten mathematics classes, also in Australia, (Tobin, 1986) found that longer wait time (3 to 5 seconds), in whole class settings was associated with higher achievement in mathematics and better quality classroom discussions. Wait time provided students the opportunity to cognitively process orally presented information before making it public. Students used wait time to think and process their ideas. They were more able and willing to respond to teacher solicitations and provided more detailed responses. The teacher asked more appropriate, probing questions that gave rise to additional opportunities for students to verbally participate.

So, with longer wait time in this study, the quality of classroom discussions is likely to improve and NESB students' mathematics learning enhanced. Also, students get the opportunity to cognitively process their ideas before articulating them to the whole class. Furthermore, the teacher may be inclined to probe for additional student contributions (e.g. explanations and clarifications), NESB student contributions may increase, and the quality of the learning environment may improve.

A number of mathematics educators are, however, of the view that communicating mathematically is associated with another major issue in mathematics education, that of autonomy in mathematics. This issue is deliberated on next.

2.3.3 Developing student autonomy in mathematics

Developing student autonomy in mathematics is an issue in education in general because a goal of education in the last 30 years or so has been to enable students to become independent learners (Darling-Hammond, 2000; Warfield, Wood & Lehman, 2005). This issue is particularly pertinent for mathematics education at the moment when the goal is to enable students to think creatively and flexibly about mathematical concepts and solve mathematics problems with understanding (NCTM, 1989, 2000). Autonomy can be described as "the *ability* to make decisions for oneself, about right and wrong in the… [social] realm and about truth and untruth in the intellectual realm, by taking relevant factors into account, independently of reward and punishment" (Piaget 1973 cited in Kamii, 2004, p. 45). This definition highlights the capability to engage in independent social and intellectual decision making. This notion of autonomy is analogous to Dewey's (1938) use of the term 'self-control' in his progressive view which states that "The ideal aim of education is creation of power of self-control" (p. 64). DeVries and Kohlberg (1987) define autonomy more concisely as "the capacity to create rules"

(p. 32). What is significant in this definition is its emphasis on the ability to generate original rules, rather than rely on prescribed ones. Deci (1995) describes autonomy as "acting volitionally with a sense of choice and a willingness to behave responsibly in accordance with one's interests and values" (p. 9). The main aspect of Deci's definition is the significance of choice. Choice, in a broad sense of the term, is a key

aspect of autonomy. To describe the notion of autonomy Greeno (2007) and other researchers used the term conceptual agency. According to these researchers conceptual agency involves selecting methods and developing meanings and relations between various concepts and principles (Gresalfi & Cobb, 2006). This is in contrast to the term disciplinary agency, also used by Greeno, which is concerned with applying established methods. With conceptual agency, the emphasis is on making decisions, exploring, strategizing, choosing methods, and considering and developing meanings and relations of concepts and principles (Gresalfi & William, 2009).

What is significant is that all these descriptions are consistent amongst themselves, and with Yackel and Cobb (1996) who define autonomy with respect to an individual's participation in classroom activities. In addition, the portrayals are compatible with Cobb (1999) who describes autonomy in mathematics as being "synonymous with the gradual movement from relatively peripheral participation in classroom activities to more substantial participation, in which students rely on their own judgments rather than those of the teacher" (p. 8). From this perspective, autonomy in mathematics should, therefore, be treated as an attribute of an individual's way of participating in a group, rather than a feature of a student's actions.

To develop student autonomy in mathematics, teaching practice needs to create situations in which students engage in exploring mathematical ideas, making conjectures around those ideas, and justifying their mathematical thinking (Warfield et al., 2005). This involves students reflecting on their mathematical situations and communicating their thinking with others (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier & Human, 1997).

In a classroom committed to autonomy in mathematics, pedagogy can promote making independent social and intellectual decisions. Instruction can create conditions that facilitate both social and intellectual autonomy (Dixon et al., 2009; Yackel & Cobb, 1996). It can support learners become socially autonomous through contributing to the negotiation of classroom expectations and obligations (Kamii, 2004; Cobb et al., 1993; Gresalfi & Williams, 2009). Classroom activities can occur in a social context that provides an environment encouraging students to engage in

cooperative interactions with other students and the teacher (Kamii, Clark & Dominick, 1994). Teaching can create classroom "interactions of mutual respect and cooperation" (Carter, 2004, p.3) and refrain from coercion or control. In addition to encouraging students to be active participants in classroom activities, teacher classroom practice can position them to be accountable for their behaviour and to take ownership of their learning. Teaching can emphasize self-control. In addition, it can support students to develop self-confidence required for participating autonomously in mathematical activity (Gresalfi & Williams, 2009). Teacher classroom practice can provide opportunities for students to practise governing their own behaviours by taking relevant factors into account when deciding what is, or is not, acceptable (Dewey, 1938; DeVries, 1997; Kamii, 2004). In this classroom, students can decide *when* to make a contribution and *what* constitutes an acceptable contribution or behaviour. The prevailing social factors can affect "how a novice's performance becomes less dependent upon *other-regulation* (regulation by others) and becomes more self-regulated over time" (Forman, 2003, p. 334).

Pedagogy needs to encourage making personal mathematical decisions or judgments (Moschkovich, 2007; Nasir & Hand, 2006). In addition to encouraging intellectual decision making, instruction needs to encourage the devolution of responsibility to judge mathematical contributions (Yackel & Cobb, 1996). Students can contribute to this devolution by increasingly getting involved in making mathematical judgments. Gradually, both the teacher and students can have the authority to validate mathematical knowledge. Through exchanging viewpoints with others, students can construct certain beliefs and values (Cobb et al., 2001; Kamii et al., 1994) that can enable them to form personal judgments. In the course of teacher-guided classroom interactions, students can practise drawing on their own intellectual capabilities and consider all relevant factors before making mathematical decisions and judgments in mathematical activities. Ownership and empowerment could be fostered in students as they make decisions about what can be meaningful to them (Dixon et al., 2009; Lo, Wheatley & Smith 1994). In turn, students can develop the ability to participate as increasingly autonomous in mathematical activity.

When classroom practice emphasizes autonomy, mathematics teaching needs to aim to produce students whose understanding and abilities in the subject cannot be limited

to skillful performance of prescribed steps, but transcend following acquired or established skills, rules and procedures (Gresalfi & Williams, 2009). Instruction should promote the capacity to create rules and methods for solving tasks or problems and not to rely on established formulae, rules or procedures (Boaler, 1998; Greeno, 2006; Kazemi & Stipek, 2001). In addition, teacher classroom practice should allow students freedom to choose or use their own approaches to problems and develop meanings and relationships between concepts (McClain & Cobb, 2001). In this classroom, students can experience creating or negotiating or choosing rules, strategies or solution methods for themselves to use in specific classroom situations. They should have access, mandate, and ability to use, adapt, and combine available resources in unusual ways (Greeno, 2007). Students' actions can depend on the choices they can make in which conceptual resources can be evaluated, appropriated and modified for the purpose of their current activity.

In a mathematics classroom committed to autonomy in mathematics, pedagogy needs to promote reliance on own thinking and intellectual reasoning (Dixon et al., 2009; Greeno, 2006; Lo et al., 1994). Teacher classroom practice can support students to attain higher levels of reasoning. At the same time, it can discourage them from relying on the authority of the teacher or text book for source of information and decisions on legitimacy of mathematical actions (Kamii, 2004). Instruction needs to create opportunities that press students to exchange viewpoints or mathematical thinking with other members of the class. In this classroom, students can learn to debate, think critically, and evaluate mathematical contributions (Kamii et al., 1994; Yackel & Cobb, 1996).

Commitment to the development of intellectual autonomy in mathematics requires that teacher classroom practice promote conceptual agency (Greeno, 2007). In this classroom students should have the opportunity to develop and exercise conceptual rather than disciplinary agency. With this ability, students can become active participants who can act autonomously. They can be able to operate with conceptual agency in mathematical activity (Gresalfi et al. 2006). The teacher needs to create opportunities for students to develop the ability to act authoritatively and accountably in mathematical activity (Greeno, 2006). Students can be accountable for their thinking, explanations, reasons, etc. They can have the opportunity to practise

authoritative and accountable actions that utilize and conform to the general concepts and principles of mathematics. Students' knowing in mathematics could be measured by the extent to which they can be capable of authorship, evaluation, and modification of concepts, methods, and resources of the realm of mathematics (Greeno, 2007).

When pedagogy emphasizes development of student autonomy, students can, in the course of classroom interactions, construct specifically mathematical beliefs and values that can enable them to operate effectively in mathematical activity (Cobb et al., 2001). In this classroom, the ways students are positioned relative to other members of the classroom, and to the mathematical content they engage in, can in part determine whether or not they can become autonomous in mathematics (Gresalfi et al., 2006).

The issues discussed so far affect every mathematics student, but my major concern in this study is NESB students' mathematics learning. Paying special attention to matters surrounding NESB students' learning seems reasonable and relevant. So, the next section looks, specifically and in more detail, at issues concerning NESB mathematics students.

2.4 Issues to do with NESB students' mathematics learning

Issues affecting NESB students' mathematics learning have been examined for some time now (e.g. Adler, 1999; Moschkovich, 2002; Sullivan et al., 2003). Many of these issues are ongoing and continue to influence NESB students' mathematics learning. Among these issues are academic culture and language (Andrade, 2006; Lewthwaite, 1996). These two issues are of particular concern to this thesis because they have been found by other researchers (e.g. Gorgorió & Planas, 2001) to be key factors in mathematics learning, but also create obstacles for NESB students' mathematics learning (Campbell et al., 2007; McNeal & Simon, 2000), the focus of this study.

2.4.1 Academic culture

The term academic culture is used, in this study, to encompass the "knowledge and patterns of behaviour" (McNeal & Simon, 2000, p. 477) associated with the activities of learning and classroom life. Academic culture includes ways of perceiving, acting

on, interpreting, evaluating and defining problems, objects, actions and events, in a learning environment. In addition, it incorporates ways of defining and assessing justifications and explanations involved in the learning process. Underlying an academic culture are the perceived ways in which mathematical knowledge can be gained (Meaney, 2002). Participants in an academic culture require knowledge of what constitutes acceptable participation for that group. They need to have an understanding of their roles, obligations and expectations. Two broadly defined academic cultures can be identified: Western and non-western. The term western countries is used, in this research, to refer to Anglo-Saxon countries, namely, Australia, Canada, New Zealand, United Kingdom and USA. These countries have a lot to share in terms of cultural values. All other countries are referred to as non-western.

Academic cultures in some non-western (e.g. Asian) and western mathematics classrooms can be categorized in terms of the following dichotomies: product oriented or process oriented; an emphasize on rote learning or an emphasize on meaningful learning; authoritarian or employs student-centred learning; utilizes extrinsic motivation or applies intrinsic motivation; uses whole-class exposition teaching or encourages sharing ideas and negotiating meaning (Leung, 2001). These dichotomies portray the extreme situations; in reality individual classroom academic cultures lie on a continuum. The position on the continuum divides academic cultures into either non-western or western.

An academic culture that is product oriented emphasizes content (Wong, 2002). In contrast, a process oriented academic culture is aligned with the process view of mathematics and stresses the process nature of gaining mathematical knowledge. An academic culture can highlight committing information to memory without understanding. Alternatively, it can stress learning with understanding, before committing to memory (Leung, 2001). An academic culture can place authority, related to knowledge and patterns of behaviour concerned with the activities of learning in the classroom, on the teacher. In contrast, authority can be shared by all students and the teacher, and student-centred learning is valued and used. An academic culture can endorse and utilize extrinsic motivation (need for high academic achievement, need to satisfy parents' or teacher's expectations, good grades or

examination performance) or it can promote student interest in learning the subject (Gao & Watkins, 2002). Some academic cultures attach importance to exposition of knowledge to the whole-class while others value sharing and negotiation of meaning. These and other characteristics of an academic culture have significant implications for classroom practice involving NESB students.

Implications for classroom practice

In mathematics classrooms with NESB students, classroom practice needs to help learners manage the differences between non-western and western academic cultures. Pedagogy needs to attend to any academic culture clash (Bishop, 2002; Seah, 2002) that might arise due to the differences that exist between the academic cultures. The term academic culture clash is being used to refer to a situation in which the teacher and NESB students can differ significantly in their views of mathematics or mathematical knowledge and the ways the subject should be learned (McNeal & Simon, 2000). It occurs when the teacher's expectations conflict with those that students from non-western academic cultures may have come to anticipate in a mathematics class, to the extent that there could be a lack of shared basis for meaningful interactions, understanding and communication between the teacher and the students (Campbell et al., 2007).

When NESB students are involved, teaching can turn academic culture clash into an educational symbiosis by utilizing the strengths they bring to the class (McNeal & Simon, 2000). Classroom practice cannot treat NESB students as deficient. Instead, pedagogy can focus on uncovering students' competencies and resources and build on them (Moschkovich, 2007). As Jilk's (2009) study of Latino students in USA suggested, students' communication skills and desire to verbalize their opinions could become strengths in classroom mathematical communities.

Instruction can help students to transition from a non-western academic culture to a western one (FitzSimons, 2002). It can support students who can be acculturated to a non-western classroom practice (McNeal & Simon, 2000), as they make attempts to fit in a western academic culture. It can treat the situation NESB students can be in as a significant cultural issue involving "the cultural nature of both the mathematical knowledge and the means of meditating that knowledge" (Bishop, 2002, p. 121).

Simultaneously, instruction can manage a culturally evolved system (with symbols and artefacts) as well as culturally held ideas that structure classroom actions and interactions (Nasir, 2006) in a western academic culture. Teaching practice can recognize that these students cannot belong to a homogenous group, even those students coming from the same country (Leung, 2001). So, instruction can accommodate learners' different experiences which can be intertwined with unique values of bilingualism and academic cultures that can shape who they can be (Jilk, 2009).

Classroom practice can promote knowledge and patterns of behaviour associated with learning mathematics in a classroom environment that is consistent with the process and cultural nature of mathematics rather than the product. Teaching can promote the process of gaining mathematical knowledge (Fraivillig, Murphy & Fuson, 1999), and downplay acquisition of the subject matter (Kazemi, 1998; Leung, 2001). In addition, it can encourage learning mathematics by doing and construction of knowledge. Pedagogy can encourage learners to "engage in doing mathematics while participating actively in a discourse community" (Gallos, 2006, p. 207). It can create conditions that promote dialogic classroom communication with emphasis on communicating mathematics ideas (Holmes, 2004). In addition to this, teaching can promote meaningful and enjoyable learning but discourage memorizing without meaningful understanding (Wong, 2002). It can advocate mathematical conceptual understanding (Kazemi & Stipek, 2001), before committing to memory, if ever there can be need to commit anything.

Instruction can promote student-centred learning rather than place importance on the authority of the teacher (Wong, 2002). Teaching cannot largely be authoritarian. It can regard teachers not as experts or scholars, but facilitators of learning (Moschkovich, 2007). Instruction can encourage substantial communication among students, in small-groups or whole-class discussion (Gallos, 2006). In addition, instruction can encourage learners to offer and explain their mathematical ideas and to justify their answers or solutions, (Cobb et al., 2001). Pedagogy can stress and support elaboration of one's thinking and can encourage productive mathematical arguments to take place (White, 2003). Teaching can encourage students to be constructive critics of each other's ideas, who listen to other members of the

classroom community, and identify and challenge what they can either agree or disagree with (Gresalfi & Williams, 2009). It can encourage diverse and creative contributions and endeavour to make this the norm for classroom activity. Authority to contribute knowledge and judge it can be shared among all the members of the classroom community. Teaching can encourage taking ownership of own learning. At the same time, teaching can use intrinsic motivation instead of extrinsic motivation. It can make attempts to arouse learners' interest in mathematics. Instruction can, however, not employ external push such as fear of poor examination performance or society's cultural expectations (Leung, 1995; Wong, 2002).

Teaching involving NESB students cannot be dominated by teacher expositions and extensive teacher-directed explaining and questioning in large-group settings (Gallos, 2006). It cannot provide explicit solution methods and rules. Neither can mathematics concepts be meticulously transmitted by the teacher. Instead, content can be treated as a set of ideas to be interrogated by all members of the classroom community. So, teaching practice can promote discourse practices - interpreting each other's actions and contributions, and joint negotiation of mathematical meaning (Fraivillig et al., 1999; Kazemi & Stipek, 2001; Moschkovich, 2002; Nasir, 2002).

In mathematics classrooms committed to the western academic culture, NESB students can experience learning mathematics in a broadly constructed western classroom context. Students can experience learning mathematics in classrooms where ideas and the knowledge they can bring to class can be recognized, valued, used and sometimes questioned (Jilk, 2009). They can learn mathematics as a process involving joint construction of mathematical knowledge. So, they could think that mathematical knowledge is a social construction that involves a process.

NESB students can learn mathematics meaningfully, without the pressure to commit information to memory. Students can be positioned conceptually relative to mathematics and its learning through the ways in which content can be organized to enable "particular mathematical insights and understandings" (Gresalfi & Williams, 2009, p. 313). These students can experience a relatively unrestricted conception of mathematics that can affect their general strategy for mathematical problem solving (Wong, 2006). They cannot practice specific mathematical procedures before gaining

the deep meaning attached to the concepts involved (Leung, 2001). Consequently, they can recognize that mathematics can be learned with understanding and without memorizing.

When mathematics instruction is aligned to a western academic culture, NESB students can be subjected to learner-centred learning and can exercise authority. Students can learn mathematics in classrooms where direct, explicit and authoritative moves (generally from the teacher) (Leung, 1995) can be avoided. Instead, they can engage in solving tasks, either individually or in small groups. In addition to this, they can be exposed to qualitatively different mathematical experiences that can stimulate them to think and offer ideas. These students can participate in various ways and use their own different solution methods to solve problems (Boaler, 1998; Cobb et al., 2001; White, 2003). As they work in groups, students cannot rely on and insist upon the authority of the teacher (Gallos, 2006). Rather, students can experience and exercise shared authority (Cobb, Gresalfi & Hodge, 2009; Greeno, 2006). They can be positioned to consume, create and judge new mathematical knowledge (Gresalfi & Williams, 2009; Lubienski, 2002; White, 2003). Through this, NESB students can appreciate that they can take charge of their own learning as well as share authority with other members of the classroom.

Students can have opportunities to learn mathematics out of interest for the subject. This could lead them to think that mathematics can be learned, not just for academic achievement, but for personal satisfaction or interest. Apart from this, NESB students can learn in the context of sharing and negotiating mathematical meaning rather than direct exposition or transmission of mathematical knowledge. So, they can experience being active rather than passive participants - exploring each other's thinking, valuing others' ideas, and engaging in student-student interaction (Moschkovich, 1999; White, 2003). Consequently, students could recognize that mathematical knowledge belongs to everyone and, it can be shared or negotiated.

It is, however, useful to mention that Trends in International Mathematics and Science Study (TIMSS) research reports suggest that, typically, NESB students from some Asian countries outperform Western students in mathematics and can score high marks in international mathematics tests (Leung, 2002, 2005). These reports

indicate mathematics success of non-Western compared to Western students. So, some people may argue that continuing to teach such students in the style to which they have been accustomed may suit their mathematics learning habits and maintain the quality of their learning. Given this, it is also important to note that during my long career teaching NESB students, mathematics, I found that students struggle with application problems. Being able to apply concepts in real-life contexts has been problematic for many of my NESB students. Yet, in their degree studies, for example, in Engineering there are lots of application problems. So, this study developed from my hypothesis that continuing to teach NESB students in the style they are used to might not be of long term benefit to them.

In addition to concerns associated with academic culture, certain issues around language affect NESB students' mathematics learning (FitzSimons, 2002; Meaney, 2002), and for NESB students, in particular. Some of these issues are discussed next.

2.4.2 Language

Language is a crucial learning resource in the mathematics classroom as it influences *how* students learn mathematics, and ultimately *what* they learn (Moschkovich, 2002). It is used to both explore and display mathematical knowledge. It enables the development of successful participation in mathematics activity through shared meanings and understandings among members of the classroom community. Interaction and communication is possible if individuals share common understandings that they use as an implicit basis for reference when speaking to one another (Bruner, 1996). The formal language of mathematics can, however, exclude and alienate certain groups of socio-cultural groups from engaging actively in mathematical activity (Meaney, 2002).

Irrespective of their main language, NESB students have difficulties understanding and engaging with each other or the teacher because of differential communicative competence in the language of instruction, i.e. English (Adler, 1995). NESB students' levels of proficiency in the English language affect their ability to actively and effectively participate in mathematics classrooms (Moschkovich, 2002). Some studies have suggested that, for these students, language exerts extra demands on an already complex situation of learning mathematics (Barwell, 2009; Campbell et al.,

2007; Halai, 2009). To help students cope with learning obstacles associated with language, some mathematics educators have proposed a focus on code switching in mathematics classes with NESB students.

Code switching

The term *code switching* is used to describe alternation in the use of two or more languages in a single speech act (Adler, 1998; Baker, 1993). Code switching may consist of only one word, a phrase, a sentence or several sentences (Zazkis, 2000). Code switching has been noted by a number of researchers as a valuable communication resource for teaching and learning and to foster mathematical understanding (Adler, 1998; Clarkson, 2005, 2006; Khisty, 1995; Moodley, 2007; Moore, 2002; Moschkovich, 2002). In some studies (Setati & Adler, 1998) mathematics teachers were observed to code switch in order to focus or regain the student's attention, to translate or clarify information, to reinforce lesson material, and to reformulate and model mathematical language (Setati, 2002). On the other hand, NESB students code switch for various reasons including to seek clarification and to provide an explanation, or for elaboration, and to express ideas, or even to reiterate an idea (Moschkovich, 2005; Zazkis, 2000). Some students use code switching to repeat what has been said in English, in exact or modified form to, for example, add emphasis, to ensure understanding of what has been said, and to verify and/or build vocabulary (Moodley, 2007). Code switching can promote both student-student and student-teacher interactions in classrooms involving NESB students (Setati, 2002).

A number of researchers in mathematics education have advocated for the use of NESB students' first languages as resources for teaching and learning mathematics (Adler, 2001; Moschkovich, 2002; Setati, 2005). These researchers see NESB students' first language(s), as a vital tool for thinking and communication. They argue that NESB students' first languages provide support needed by these students as they continue to simultaneously learn and develop proficiency in the language of instruction (i.e. English) and learn mathematics (Setati, Adler, Reed, & Bapoo, 2002). These researchers add that the mathematics teacher has an important role of guiding NESB students to move from a stage where they can talk about mathematics in their

first language to a stage where they can use the formal language of mathematics, in English, and effectively engage in conceptual mathematics discussions, in English. I was aware of this potential in this study.

Implications for classroom practice

In the mathematics classroom teaching can focus on uncovering NESB students' language competencies (Moschkovich, 2007). Instruction can acknowledge and legitimize student verbalization as a resource for teaching and learning purposes, or a tool for thinking and display of mathematical knowledge (Adler, 1999). At the same time, NESB students' language can be used as a resource in the teaching\learning process. Teaching can allow students who share the same main language to use it, in small-group activities, to express their mathematical ideas using code switching (Moschkovich, 2002).

Teaching practice can create conditions that allow and encourage all students to have access to the language of instruction and the language of mathematics (Adler, 1997). Rather than leave the learning of language implicit, classroom practice can balance the attention given to explicit language of the mathematics classroom and mathematical understanding. Language development can become an integral part of learning mathematics, rather than an explicit focus of study (Adler, 1999). The teacher can manage tensions in the use of formal mathematical language and informal language on one hand, and in the use of the language of instruction on the other hand. He or she can support students' movement between talk used for thinking as students work on the mathematics task and talk that can be used as display of mathematical knowledge. Additionally, the teacher can make explicit and intentional moves aimed at helping to bridge what students say and conventional mathematical language.

Classroom instruction can stress sharing, exploring and displaying ideas. In addition, it can encourage negotiation and joint construction of mathematical meaning. The teacher can provide students the support they can need to develop the ability to communicate, in mathematical activity, with understanding (Kazemi & Stipek, 2001). Classroom instruction can encourage student-student and teacher-student conversation (Fraivillig et al., 1999). In addition, it can encourage NESB students to

participate in mathematical discussions where they can grapple with mathematical content, even if they lack the correct language, and even if they can switch from English to their main language (Adler, 1997). In the conversations, classroom practice can allow NESB students to use aspects of competent mathematical communication that go beyond the vocabulary list (Moschkovich, 2002). Language practices of the classroom can be used to scaffold students' passage into the mathematical discourse. Teaching can encourage students learn to participate in valued language practices, such as, being explicit and precise, and using specific language in particular contexts (Moschkovich, 2002). Productive mathematical conversations can be centred on purposeful mathematical talk with genuine student contribution (White, 2003).

Students can experience using the language of instruction and the language of mathematics as a psychological tool and a social and cultural tool for sharing, exploring, negotiating and constructing mathematical knowledge. They can "use the language of mathematics to express ideas precisely" (NCTM, 2000). In addition, students can learn mathematics as discussions move freely between their main languages and English (Campbell et al., 2007). They can have access to the appropriate language coupled with the teacher's support in developing the ability to 'say it' (Adler, 1999). Students can attend to both the mathematical ideas to be learned and to a language they can be trying to master. In the process they can develop the ability to 'talk mathematics'. They can become more fluent and able to express their ideas when mathematical talk becomes a classroom norm, negotiated and changed by members of the classroom community (Moschkovich, 2005). Students may think that language is a social and cultural tool for learning mathematics.

2.5 Chapter Summary

This chapter has reviewed literature on issues currently facing mathematics education. By giving special attention to views about the nature of mathematics, pedagogical challenges, and issues to do with NESB students' mathematics learning, I have highlighted the importance of these matters in mathematics teaching and

learning. Each concern was outlined and then discussed on the basis of how it might affect the ways in which NESB students can be supported in mathematical activity.

Literature to do with the product, process and cultural views of the nature of mathematics was discussed. I stressed the impact each conception has on what and how mathematics is taught and learned in the classroom. For each view, I identified the main elements that do support or hinder NESB students' mathematics learning. The social and cultural nature of mathematical activity was discussed in some detail. The argument was made that it is essential for NESB students to regard mathematics as a human activity and as both a process and a cultural activity that is personally and socially constructed through interaction and negotiation. On issues around pedagogy, a case was made that contextualized problems have an important role in mathematics classrooms. In addition, communicating mathematically, and developing student autonomy in mathematics were discussed in view of their importance in mathematics learning. Academic culture and language were discussed as issues that specifically impact on NESB students. Taken together, the three issues and their aspects mould the classroom participation structure and the negotiation of mathematical meaning.

There is, however, a lack of research that focuses specifically on the practice of teachers of NESB mathematics students. Research is, therefore, needed that addresses this problem. In spite of this, the insight I gained from reviewing literature helped guide the direction of my study. I came to understand that I need theories that help me to appreciate and cope with the social and cultural nature of student learning. The theoretical perspective that allowed me to successfully do this was the sociocultural approach.

CHAPTER 3

THEORETICAL PERSPECTIVE

3.1 Introduction

In this chapter, I discuss the learning theories that have served as sources of ideas, enabling insight into mathematics classroom learning processes. Starting with cognitive psychology theories, each learning theory is discussed in relation to how it conceptualizes the learner and what each perspective contributes to an understanding of the learning process. A sociocultural perspective to learning is outlined and discussed in view of its clarifying power on current issues with regard to mathematics learning. The rationale for adopting a sociocultural theory in this study is outlined.

My chosen theoretical perspective is Cobb's (1997, 2001) emergent sociocultural theory. The emergent perspective, in particular, helped me to understand the classroom practices that developed in this study. Cobb et al.'s formulation acknowledges that individual and social and cultural processes are reflexively related. It offers potential insight into the problems facing teachers of NESB students. In this chapter, Cobb's classroom social norms, sociomathematical norms and classroom mathematical practices are identified as significant constructs for understanding participation and learning in and through mathematical activity in a classroom of NESB students. Each is outlined and discussed, and research that focuses on each notion is deliberated on. The chapter concludes with a summary.

3.2 Learning theories

This section deliberates on cognitive psychology theories, constructivist approaches and sociocultural perspectives. Each theory is discussed separately in terms of implications for student learning in mathematics classrooms. In addition, the limitations of each perspective and their link with NESB students' mathematics learning are discussed.

3.2.1 Cognitive psychology theories

Cognitive or psychological learning theories have their roots in the work of Piaget, who observed and monitored individual children. What emerged from his analysis of

data was a descriptive account of children's "cognitive development in terms of sequential stages tied to age and maturity" (Lyle, 2000, p. 46). For Piaget, the stages of development were the same for all individuals and proceeded in a linear and hierarchical fashion.

According to Piaget, the individual learner constructs his\her knowledge schemes subject to, and filtered through, past and current experiences. He used the term schemata to refer to knowledge structures or constructs and ways of perceiving, understanding, and thinking about the world. Piaget described mental development or learning as a process of equilibrium in response to external stimuli (Marlowe & Page, 2005). He theorized that while interacting with the environment, the student assimilates components of the external world into his\her existing cognitive structures (schemata). If the new experiences do not fit the existing structures, the student will change the structures to accommodate the new information. The process of assimilation and accommodation create equilibrium (Piaget, 1967\1971). Hence, in Piaget's view, mathematical knowledge is meaningful when students construct it themselves. Cognitive psychology is thus based on the assumption that the individual student's development "involves qualitative changes in their mathematical reasoning" (Cobb, 2007, p. 21).

In the mathematics classroom, if teachers were to apply Piaget's views strictly, they would wait for students to attain a certain developmental age before introducing particular mathematical concepts. However, in practice, teachers introduce the same concepts to students of varying ages in their classrooms, even though students may be at different levels of cognitive development. A teacher who is committed to Piaget's views focuses on each individual student as he\she constructs their own mathematical knowledge. Such a situation creates huge demands for the teacher. Implementing Piaget's theory in full is particularly problematic when teaching NESB students because mathematics learning is inherently social and cultural, and these students bring to class experiences of various academic cultures and languages. Teachers of NESB students, therefore, need to adjust their application of the theory in order to accommodate students' transition from different academic cultures to that embraced by the host country.

Cognitive development theories, such as that proposed by Piaget, have been criticized by some learning theorists. For instance, Donaldson (1978, as cited in Lyle, 2000) concluded that children's ability to solve problems depends on more than intellectual development - it depends on ways in which tasks are presented, and on the interpersonal relationships among the participants. Donaldson and her team demonstrated that children have the ability to think in mathematical ways, in embedded contexts. She added that one of the barriers to children's learning is the requirement to engage in dis-embedded or context-free thinking.

Piaget's model is further criticized for claiming that learning is an individual activity, and is linear and sequential (Ernest, 1991). It is also criticized for ignoring the importance of cultural context in learning (Cobb et al., 1992). Focusing on the restricted domain of logico-mathematical intelligence, which is only one aspect of child development, is another weakness associated with Piaget's development theory (Almeida & Ernest, 1996).

In short, the psychological perspective's focus on the individual has been criticized for not adequately and fully explaining the nature of student learning. What is needed is to pay special attention to the social and cultural forces (Anderson, 2007) that influence student learning. Cognitive development theories have failed to adequately explain learning processes. Consequently, cognitive perspective limitations have led to a shift, by some educators, from these theories towards constructivist perspectives.

3.2.2 Constructivist approaches

In this section, I discuss two types of constructivists: personal or individual, and social. I deliberate on the ways each approach characterises learning before discussing some of its limitations.

Personal\individual constructivist

The constructivist perspective emerged in the light of Piaget's work (Brown, 2001). A central tenet of the constructivism is that the learner actively constructs mathematical knowledge from existing beliefs and experiences, rather than simply receiving it readymade (Lerman, 1996). It is because an individual constructs their own meanings and understandings of mathematical concepts and problems that the

emphasis in constructivist perspective is not on the transmission of mathematical knowledge but on "questioning, investigating, problem generating, and problem solving" (Marlowe & Page, 2005, p. 8). Learning involves using information and thinking processes to develop and build, or even change an individual's meaning for and understanding of the concepts and ideas. In this perspective, knowledge development is seen as a cognitive process (Simon, 1995). The individual integrates current experiences with past experiences, and what she\he already knows about the subject at hand. Every individual learner constructs their own meaning and learning about mathematical issues, problems and topics. Coming to know is an adaptive process that organizes experiential reality (Brown, 2001). Since individuals have had different experiences, their understandings, their interpretations and their knowledge constructs of mathematical concepts cannot be exactly the same.

A personal or individual constructivist perspective is criticised for focusing exclusively on the processes by which the individual constructs their own knowledge while ignoring the influence of context (Lerman, 1996). The perspective is further criticised for limiting the consideration of social and cultural influences on the construction of knowledge (Airasian & Walsh, 1997). It disregards interpersonal aspects of mathematics teaching and learning (Cobb, Wood & Yackel, 1991). Another criticism is that, by focusing only on the individual learner, it does not offer a complete explanation of students' learning of mathematics (Lerman, 1996). In addition, the perspective neither offers suggestions for teaching mathematics (Simon, 1995); nor does it indicate how the suitability or unsuitability of teaching strategies can be determined. Hence, a personal constructivist perspective fails to adequately cater for the view of mathematics learning embraced for this study.

Social constructivist

 When it is recognized that learning is a social process, constructivism can be extended to social constructivism (Ernest, 1996; Cobb & Yackel, 1996). In the social constructivist view, knowledge development, and hence learning, is a social process mediated by interaction (Ernest, 1999, 2006; Kim, 2001; Simon, 1995). Individuals construct meaning during interactions with each other and with the context. This perspective recognizes the interplay between the social and the personal in learning.

In a social constructivist approach, people jointly construct knowledge under particular social and interaction conditions.

The social constructivist approach recognizes that mathematical knowledge and concepts grow and change (Cobb & Yackel, 1996). This perspective stresses the genesis of mathematical knowledge, not its justification. It links subjective and objective mathematical knowledge, with each contributing to the creation and re-creation or renewal of the other (Brown, 2001). It can therefore, be argued that social construction moves knowledge development forward. In the mathematics classroom, students would be expected to participate in social interactions in the joint construction of personally meaningful mathematical knowledge.

Critics of the social constructivism point out the confusion that might arise from a multiplicity of individual meanings (Airasian & Walsh, 1997). In a mathematics classroom, it would be difficult to come to some sort of consensus on mathematics matters because each student could be developing a unique understanding of concepts involved. In addition, like a personal constructivist perspective, a social approach fails to incorporate both individual and collective understanding (Yackel & Cobb, 1996). Furthermore, it does not include suggestions for teaching or suggest what kind of interventions the teacher could make. In view of these shortcomings, a social perspective is not suitable for helping to understand my teaching practice and NESB students' mathematics learning.

3.2.3 Sociocultural perspectives

Sociocultural research has raised questions about the validity of theories of cognitive development that emphasize the individual rather than interaction (Mercer, 1999). In contrast to cognitive theorists, the focus of sociocultural theorists is on "the process by which people develop particular forms of reasoning as they participate in established cultural practices" (Cobb, 2007, p. 22). Sociocultural theory has emerged as an alternative approach to the "dichotomies of the social and the psychological/individual" (Planas & Gorgorió, 2004, p. 21). The main goal of "a sociocultural approach is to explicate the relationship between human action, on one hand, and the cultural, institutional, and historical situations in which this action occurs, on the other" (Wertsch, del Rio & Alvarez, 1995, p. 11).

Sociocultural perspectives are increasingly being used to understand mathematics learning in a way that places culture as a centre (Nasir & Hand, 2006). These perspectives assume that social and cultural processes are core processes to learning (Cobb, 2007). In addition, they assert the importance of "local activity settings" in students' learning. Approaches that rely on the sociocultural constructs for inspiration are premised on the idea that knowledge is socially constructed "as individuals participate in culturally organized activities, activities that involve values, norms, goals,…within which people co-create activity through talk and action" (Nasir & Cobb, 2002, p. 95). They are guided by the view that mathematics learning occurs through a process of mathematical communication in social and cultural contexts.

A sociocultural perspective characterizes mathematics as inherently social and cultural (Cobb & Yackel, 1996). It regards learning as a social and cultural activity, as well as a dynamic process (Brown, 2001; Lubienski, 2002). This perspective allows foregrounding of the role of social activity and cultural tools in mathematics learning. The theory permits mathematical activity, thought and learning to be considered as mediated cultural tools in the activity (Forman, 2003). A sociocultural perspective recognises peer collaboration, and learning and shared understandings in social and cultural contexts (Moschkovich, 2002). It treats "discourse as being at the centre of human intellectual development" (Lerman, 1996, p.147). Although mathematics is generally performed in the social sphere, a sociocultural perspective considers negotiation to be "a process of mutual appropriation in which the teacher and students continually co-opt or use each other's contribution" (Cobb & Yackel, 1996, p. 186). Furthermore, a sociocultural theory permits recognition of cultural and historical contexts, beliefs, and a shift from decontextualized, universal representation of social interaction, language, and cognition (Forman, 2003) towards a theory highlighting interconnections between cultural institutions, social practices and interpersonal relationships. A sociocultural perspective recognizes that learning is a normative venture that involves constraining and enabling "access to value-laden resources that affect the level and kinds of participation" (Kumpulainen & Renshaw, 2007, p. 111). It acknowledges the close ties between theory and practical application (Boaler, 2002) and recognises the link between instructional practices and learning outcomes.

Some sociocultural theorists describe learning as participation in social contexts (e.g. Greeno, 2003). This characterization is consistent with Sfard (1998) and other theorists who not only highlight the social aspect of learning, but also recognize the importance of both the individual and the social factors in mathematics learning. Sfard (2003) used two terms, *acquisition* and *participation*, to describe learning. The term, acquisition, provides insight into the development of individual knowledge and helps to account for the diversity in performance of individual students. Participation, on the other hand, is required in order to adequately describe and theorize the role of social settings, the nature of inter-subjectivity, and the notion that knowledge is situated in social settings (Lerman, 1996). This version advocates participation in social or communal activities and recognizes the link between the individual and the social. Participation is perceived as both a process and an outcome of learning, rather than as acquisition of knowledge or conceptual change (Sfard, 1998). It includes elements of both the social and the individual (Sfard, 2003) since the individual student operates in a social context. These two elements should be considered together. They are interdependent and maintain their distinctiveness in a dynamic, complementary relationship. In this perspective, the central organizing premise in learning is social and cultural rather than reasoning. In the mathematics classroom, the teacher supports individual students as they participate in social or communal activities. This is a major issue in this study.

There are socioculturalists who use participation in mathematical discourse practices to describe learning mathematics (Forman, 2003). In this characterization, the central issues involve negotiation of normative aspects of mathematical activity and mathematical practices, the focus of this study. In this perspective, attention should be directed at developing the learners' ability to communicate and participate in the negotiation of different aspects of the classroom microculture (or classroom social context). In the mathematics classroom, "students learn to mathematize situations, communicate about situations, and use resources for mathematizing and communicating" (Moschkovich, 2002, p.197). Forman (2003) emphasizes communication in social and cultural contexts and recognizes historical contexts and beliefs as important factors in mathematics learning. Communication enables sharing of mathematical ideas and mathematical argumentation to take place. The term

argumentation is used in this study to mean "using debate to resolve conflict and arrive at a common understanding" (Anthony & Walshaw, 2009, p.19).

Another version of the sociocultural perspective describes learning as participation in developing normative aspects of the classroom microculture (Cobb et al., 1997). This version highlights collective construction of normative social practices related to the student participation in classroom activities, normative aspects specific to mathematics, and normative aspects specific to a mathematical idea. It recognizes that mathematics is inherently social, mathematics learning is a social activity that occurs in social contexts, and mathematical knowledge is emergent (Cobb et al., 2001). In this perspective, learning is a process involving negotiation of meaning. Implementation of this perspective requires students to be active participants in joint constitution of normative aspects of the mathematics (Yackel & Rasmussen, 2003). In this perspective, students are viewed as part of the learning environment.

These three variations of the sociocultural perspective are not distinct, but overlap and complement each other. All three perspectives emphasise the importance of participation in learning. They, however, vary in their emphasis of the significance of social and cultural aspects of learning, and their acknowledgement of learning as a process. From these three perspectives, to understand NESB students' learning it is necessary to follow an individual's participation in particular activities and how they draw on artefacts, tools and social others to solve mathematics problems (Lerman, 1996; Simon, 1995). The three perspectives allow that effective participation can be constrained by the students' past diverse social and academic cultures and so teachers need to devise ways in which students' diverse backgrounds are used as strengths in advancing their learning.

3.3 Rationale for adopting a sociocultural theory in this study

This study is about my teaching practice and NESB students' mathematics learning. In order to better understand this practice, mathematics classrooms, and NESB learners, I needed a theoretical perspective that provided some understanding of the relationship between group and individual mathematical activity. A sociocultural perspective meets this condition, making it suitable for adopting in this study.

Sociocultural theory's attention to social and cultural aspects of learning, each of which is highly significant in this research, makes the perspective attractive for this study. Furthermore, since a sociocultural theory accommodates the view of mathematics embraced in this study, it became appropriate that it be adopted for this study.

Two aspects of sociocultural theory stand out to make it particularly relevant to my study. The first is highlighting the importance of social and cultural factors in students' learning and development (Nasir et al., 2008). The second is stressing the significance of studying the change process rather than merely documenting the final outcome of that change process. Two themes emerge from this. The first theme concerns how the nature of teaching strategies may influence students' mathematics learning. The second is concerned with how doing mathematics involves communicating mathematical ideas (Anthony & Walsh, 2009). A sociocultural approach is relevant to the extent that it identifies the fundamental link between learning, and the structures and functions of instructional activity, and how learning requires communicating in a social context (Moschkovich, 2007).

A sociocultural theory allows recognition of NESB students' varied academic cultures and different forms of communicating ideas (e.g. using artefacts, tools and symbols, oral and written language) (Campbell et al., 2007; Moschkovich, 2007). This is crucial since every student comes with experiences and various ways of expressing their thinking. In addition, it provides a lens through which the local production of the academic culture can be recognized, taken apart and analysed. By recognising that the academic culture of the mathematics classroom is created moment-to-moment during interactions as people participate in (and recreate) cultural practices (Nasir & Hand, 2006), a sociocultural perspective allows consideration of the academic cultural norms NESB students bring to the classroom.

A sociocultural perspective permits regarding mathematics learning as not just about gaining new mathematical knowledge, but about the student's personal transformation – about becoming (Lerman, 1996). It allows learning to be considered as shifts in how students view their relationships with peers and with mathematics and how they come to view themselves as learners.

The perspective provides insight that offers the potential to enable understanding mathematics classrooms with NESB learners. This perspective focuses attention on student participation in mathematical activity, mathematical communication, and development of students' mathematical knowledge (Moschkovich, 2002), in order to gain an understanding of how joint construction of mathematical meaning relates to an individual student's mathematics learning (Cobb et al., 2001).

Out of my own experience and the research studies, this study is premised on the notion that to improve my teaching practice I would need to help my NESB students into new ways of doing mathematics. I hypothesised that NESB students, in this study, would need to be initiated into mathematical classroom processes, focused on contextualised and non-contextualised mathematics problems, communication, and autonomy, as emergent phenomena rather than ready-made practices. The theory that allows me to better understand how this might be done is Cobb's emergent perspective. This is discussed next.

3.4 Cobb's emergent perspective

Cobb (1997) and his colleagues explored ways to account for students' mathematical development as it occurred in social contexts of primary classrooms, over the course of their work. Cobb's emergent perspective coordinates two separate perspectives on classroom activity: the social, and psychological or individual (Bowers et al., 1999; Cobb et al., 2001; Cobb, 2007). He developed a three-level framework that includes key aspects of a social perspective to learning and three corresponding components of a psychological (individual) perspective. The framework is reproduced below in Table 3.1.

Table 3.1   An interpretive framework for analyzing communal and individual mathematical activity and learning (Cobb, Stephan, McClain & Gravemeijer, 2001, p. 119)

Social Perspective	Psychological Perspective
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical interpretations and reasoning

The social perspective consists of three aspects: classroom social norms, sociomathematical norms and classroom mathematical practices (discussed in section 3.4). These focus on students' ways of acting, reasoning, and arguing "that have been established as normative in a classroom community" (Cobb, 2007, p. 29). The individual perspective comprises three components. The first is concerned with students' individual beliefs about their roles and roles of others in mathematical activity. The second focuses on students' mathematical beliefs and values. Mathematical interpretations and reasoning make up the third component. The individual perspective is concerned with "the nature of the individual students' reasoning or… the specific ways of participating in communal classroom activities" (p. 29). From this perspective, as we develop social aspects, the individual student's reasoning is perceived as an act of participation in normative activities. Three key ideas associated with the emergent perspective are a reflexive relation between the social and individual perspectives, emergent aspects, and non-hierarchical levels of focus.

A reflexive relation between the social and individual perspectives

The emergent approach views social perspectives and individual perspectives as linked rather than separate processes (Cobb et al., 2001). It acknowledges the individual and the social as complementary where "individual thought and social and cultural processes are considered to be reflexively related, with neither attributed absolute priority over the other" (Cobb et al., 1997, p. 152). The individual and social

are not just interdependent; they are co-dependent, neither can exist without the other (Yackel & Rasmussen, 2003). The learning process, human action and the social\cultural context are inseparable. The emergent framework "acknowledges the individual and the social as recursively related" (Southerland, Kittleson, Settlage & Lanier, 2005, p. 1032). Students' individual construction of knowledge relies on the process of socialization into the academic culture.

Emergent aspects

Within the emergent perspective, classroom processes are treated as emergent phenomena (Cobb, 2007). The focus is on developing, rather than fully established, individual, and social and cultural processes. At the local, classroom level, the emergent approach views reproduction of culture as a process of emergence in which students' constructive activities and mathematical practices in which they are participating co-evolve. All the components of the emergent framework are perceived as continually emerging.

Levels not hierarchical

Cobb and his colleagues (2001) emphasise that the three levels of focus in the emergent perspective are not hierarchical. They do not suggest any order of importance. Different levels are used to highlight particular points related to mathematics learning. For the purposes of this study, the framework will be referred to as the three-level framework. The view embraced in this study is that mathematics learning is a social activity that occurs in social contexts. Consequently, my major focus is on collective mathematical learning of the classroom community, rather than just individual students. Hence, social aspects are foregrounded and discussed in more detail. This is done to highlight particular points associated with each component. Individual aspects are backgrounded, and their role in collective learning acknowledged.

In order to further clarify the distinction between the three components of the mathematics classroom microculture (or under the social perspective), each is now briefly delineated.

3.4.1 Classroom social norms

Classroom social norms are expectations and obligations regarding classroom participation that are jointly constructed or negotiated by the teacher and the students (Bowers et al., 1999; Cobb et al., 1997; Cobb et al., 2009; Yackel et al., 1991). They are "taken-as-shared beliefs that constitute a basis for communication and make possible the smooth flow of classroom interactions" (Cobb et al., 1993 as cited in Yackel & Rasmussen, 2003, p. 160). 'Taken-as-shared' is used to show that "members of the classroom community, having no direct access to each other's understanding, achieve a sense that some aspects of knowledge are shared but have no way of knowing whether the ideas are in fact shared" (Simon, 1995, p. 116). Classroom social norms are norms or standards of participation or general discourse patterns of the classroom, and an aspect of the classroom microculture (Cobb et al., 2001).

Social norms serve as the means by which higher knowledge is socially and individually developed (Dixon et al., 2009). The notion of norm is used to include what is acceptable for that particular classroom. Hence, in this study, the term social norm is used to describe expressions of normative expectations and responsibilities in a specific classroom.

General classroom social norms can be referred to as social constructs that involve a taken-for-granted (i.e. needing no explanation or justification) sense of when it is appropriate to make a contribution in class (Cobb et al., 1992). For a teacher, they are about fostering acceptable behaviour in the local situation. These norms are concerned with the interpretation of the classroom participation structure or classroom action and interaction that are not specific to mathematics, but apply to any subject (Cobb et al., 2001; Cobb & Hodge, 2002). Classroom social norms become normative over time. Several studies (e.g. Cobb et al., 2001; Dixon et al., 2009; Yackel & Cobb, 1996) found that constructing classroom social norms supports development of student social autonomy.

Social norms are not necessarily fixed (Cobb et al., 1991). They are subject to change. The emergent perspective correlates classroom social norms against students' general beliefs about their roles and the nature of mathematical activity. A student's

beliefs are the individual understandings of normative expectancies which they utilize to appraise a situation (Yackel & Rasmussen, 2003). Classroom social norms and a student's beliefs are reflexive, co-evolve, and are "mutually enabling and constraining" (Yackel et al., 2000, p. 277).

3.4.2 Sociomathematical norms

The term sociomathematical norm is used to describe norms or standards of the classroom actions and interactions that are specific to the mathematical activity and guide mathematical work (Franke, Kazemi & Battey, 2007; Yackel & Rasmussen, 2003). Sociomathematical norms label the classroom social constructs particular to mathematics. They are jointly developed by the teacher and the students. These norms are about the actual process by which students and the teacher contribute mathematical ideas. They guide the quality of mathematical discourse in the classroom (Yackel & Cobb, 1996). Sociomathematical norms, therefore, go beyond classroom social norms that are concerned with managing classroom discourse. Since they are interactively constituted by each classroom community, sociomathematical norms may differ from one classroom to the next.

Although they are specific to mathematics, sociomathematical norms cut across areas of mathematical content by dealing with mathematical qualities of statements and solutions. They guide mathematical argumentation and acceptable ways of reasoning with tools and symbols in mathematical activity (Cobb et al., 2009). In the emergent approach, a feature is termed argumentation in which mathematical reasoning is based on explanation and justification of mathematical thinking (Cobb et al., 1997). Students can, therefore, negotiate mathematical meaning through the process of argumentation. Examples of sociomathematical norms include an acceptable mathematical explanation, an acceptable mathematical solution, a different mathematical solution, similar mathematical solutions and an efficient solution.

Sociomathematical norms are not predetermined criteria introduced into mathematics classroom from outside; they form when explanations and justifications are made acceptable (Yackel & Cobb, 1996). Sometimes, acceptability is made possible when explanations and justifications can be interpreted in terms of actions on mathematical objects that were practiced.

The prevailing taken-as-shared basis for mathematical communication can serve as conditions against which students explain and justify their thinking (Yackel & Cobb, 1996). Joint understanding of what an acceptable mathematical explanation or statement is evolves over time. However, developing a taken-as-shared understanding of a particular sociomathematical norm can be highly problematic (Cobb et al., 2001). For instance, some students may struggle with what criteria to use when judging mathematical difference or same solution. Sometimes, the criteria emerge spontaneously in the course of interactions with the students. At other times, the teacher may need to make it explicit for both the teacher and the students (Yackel et al., 2000).

As a representative of the wider mathematics community, the teacher legitimizes the ongoing negotiation of what is acceptable mathematical activity. However, in the classroom that emphasizes sociomathematical norms, the teacher shares this role with the students (Yackel & Cobb, 1996). What the classroom community legitimizes as acceptable mathematical activity influences students' implicit understandings.

The constitution of sociomathematical norms is captured or shaped by the existing classroom culture. Like social norms, sociomathematical norms are continually regenerated and modified by the students and the teacher through their ongoing coordinated actions (Cobb et al., 2001; Cobb & Yackel, 2011). Through conversation and reflection, sociomathematical norms can be clarified and refined. So, sociomathematical norms are emergent. Since these reflective activities have the potential to contribute to students' mathematical learning, it can be argued that sociomathematical norms and learning mathematics are reflexively related.

The emergent perspective matches sociomathematical norms with students' mathematical beliefs and values (Cobb & Yackel, 1996). Specifically mathematical beliefs and values that students construct (individual perspective) enable them to participate in the negotiation of sociomathematical norms (social perspective) (Cobb et al., 2001). Conversely, while contributing to the development of sociomathematical norms, students reorganize their own mathematical beliefs and values. These two aspects co-develop.

Some researchers in mathematics education have argued that sociomathematical norms determine what mathematics students learn, in terms of content, and what they ultimately believe about the nature of mathematical knowledge and the ways in which it is learned (Wood, 1999). In addition, some studies (e.g. Lopez & Allal, 2007; McClain & Cobb, 2001; Sekiguchi, 2005; Yackel et al., 2000) have demonstrated that establishing sociomathematical norms can foster negotiation of mathematical meaning and the development of student intellectual autonomy in mathematics. These findings show the importance of developing sociomathematical norms in this study.

3.4.3 Classroom mathematical practices

Classroom mathematical practices are concerned with specific taken-as-shared ways of acting, reasoning, arguing, and symbolizing regarding particular mathematical ideas (Cobb & Hodge, 2002; Cobb et al., 2001; Bowers et al., 1999). They are social constructs that involve a communal sense of what constitutes acceptable actions, reasons, arguments and symbols relating to specific mathematical topics. This is in contrast to sociomathematical norms that cut across all mathematics content areas. A classroom mathematical practice imposes "a set of constraints on acceptable solutions as well as conceptual resources for constructing solutions or arguments" (Greeno, 2003, p. 326). For instance, mathematical practices are concerned with what constitutes a solution to a quadratic equation.

Like social norms and sociomathematical norms, classroom mathematical practices are collectively constructed and developed by the classroom community (Bowers et al., 1999). Generally, mathematical practices evolve as the teacher and students discuss topic problems and solutions. During the establishment of a mathematical practice, students are obliged to explain and justify their actions and interpretations. At some point, collective mathematical understanding is achieved and the activity of interpreting becomes institutionalized as a classroom mathematical practice that is beyond justification (Cobb et al., 2001). Students would then be acting in a taken-as-shared reality in which they do not need to justify their actions around a particular set of content practices. From the emergent perspective, a mathematical practice is viewed as an emergent phenomenon rather than a fully-established way of reasoning and communicating into which students are to be initiated.

When viewed from an emergent perspective, there is a reflexive relation between classroom mathematical practices (social perspective) and individual students' ways of interpreting and reasoning or students' mathematical conceptions (psychological\individual perspective) (Bowers et al., 1999; Cobb et al., 2001). Consequently, the emergence of collective mathematical practices and individual students' learning co-exist (McClain & Cobb, 2001). The emergent perspective associates classroom mathematical practices with individual students' mathematical interpretations and reasoning related to a particular content topic. Students contribute to the development of classroom mathematical practices as they restructure their individual mathematical activities. Concurrently, the "reorganizations are enabled and constrained by the students' participation in mathematical practices" (Cobb & Yackel, 1996, p. 180).

Commitment to development of classroom mathematical practices impacts on classroom practice. In separate studies, Bowers, Cobb and McClain (1999), Rasmussen, Stephan and Allen (2004), and Stephan and Rasmussen (2002) found that the development of classroom mathematical practices supports students' mathematics learning, as well as development of their social and intellectual autonomy. These matters are of particular relevance to this study.

In order to cope with the analysis and interpretation of learning in this study, a framework or an analytic tool is needed that is compatible with the sociocultural theory (Cobb et al., 1997), and that addresses both social and individual perspectives on mathematics learning. The emergent framework allows for this (Cobb et al., 2001).

3.5 A framework for this study

In this study, I adopted Cobb's (1997) sociocultural emergent perspective (discussed in section 3.4) and relied heavily on it for key concepts and methodological guidance (Cobb & Yackel, 1996; Cobb et al., 1997; Cobb et al., 2001). It is, however, important to note that there are major differences between Cobb and his colleagues' investigation, and my study. While Cobb and his team of researchers investigated mathematics learning at primary school level, my study is concerned with mathematics learning at tertiary level.

Distinguishing three key aspects of the classroom microculture

Consistent with Cobb's emergent perspective, a framework for this study recognizes the relationship between social and individual processes of learning. It distinguishes three key aspects of the classroom microculture (classroom social norms, sociomathematical practices and mathematical practices) and three corresponding components of the individual or psychological perspectives. The adopted framework allows me to treat the individual and social aspects as complementary components of the learning process. It allows me to focus explicitly on each of the three key aspects of the classroom microculture. In addition, it permits me to pay special attention to the social aspects of mathematics learning and teaching without disregarding significant contributions emanating from students' individual constructions (Southerland et al., 2005). The social perspective is brought to the fore, thereby reflecting the relative emphasis given to mathematics as a social process and the importance that is attached to collective learning in this study. This move enables me to highlight particular points about each social aspect, the aim of this study. In addition, this move is needed to enable better understanding of the interactions between me, the teacher, and students in mathematical activity, and about how learning happens. By drawing on the emergent formulation, I am able to acknowledge that mathematical activity is culturally situated (Lerman, 1999; Nasir & Hand, 2006). The perspective allows me to treat students' mathematical reasoning as participation in social and cultural classroom activities.

Like Cobb's emergent approach, the framework for this study, acknowledges that the difference between classroom social norms, sociomathematical norms and classroom mathematical practices is subtle. This difference can be explained by way of examples. For instance, in this study, explaining and justifying one's contribution or solution is an example of a classroom social norm. However, when the focus is on providing mathematical reasons or interpretations to clarify and justify a contribution or solution, the normative understanding is a sociomathematical norm. Explaining and justifying mathematically is an example of a sociomathematical norm. The statement 'counting the number of arrows' describes a classroom mathematical practice when the focus is on the mathematics topic 'arrangements and selections'.

Social aspects and individual processes are reflexive

A framework for this study recognises that in classroom mathematical activity social aspects and individual processes are reflexive. The framework enables me to accept this reflexive relationship but still give precedence to social and cultural processes over individual psychological processes. It makes it possible for me to examine how the individual components shape the social aspects, as well as to better understand the extent to which social processes shape, and are shaped by, individual learning. This framework permits me to recognize that students reorganize their mathematical ways of reasoning and knowing as they construct personal mathematical knowledge in social and cultural contexts, and as they participate in practices of the mathematics classroom. It approves of my viewing my NESB students as "reasoning with tools while participating in and contributing to the development of communal practices" (Cobb, 2007, p. 30). This move assists me to better understand the intersection between the individual and group mathematical knowledge construction of my NESB mathematics students.

Social and sociomathematical norms and mathematical practices are emergent

The framework for this study recognises that social and sociomathematical norms, and mathematical practices are emergent. Consistent with Cobb's perspective, this framework allows me to treat social norms, sociomathematical norms and mathematical practices constituted in mathematics classrooms as evolving phenomena that are continually regenerated by the students and the teacher during ongoing classroom interactions. It allows me to recognize that students do not inherit pre-packaged and intact mathematical ideas. At the same time, the framework makes it possible for me to acknowledge that students need to engage in tasks which give rise to new understandings of what might be seen as old ideas. This framework allows me to recognize that my students' mathematical learning has to do with becoming initiated into the conventional cultural usage (Brown, 2001), over time. It enables me to recognize that learning involves a process of sense-making and negotiation of mathematical meaning. In addition, it allows the use of different forms of participation to sustain evolving classroom norms and mathematical practices.

By recognizing the emergent nature of mathematics learning, the perspective offers guidance on ways to better understand how NESB students might learn mathematics in an academic culture that values and encourages negotiation of meaning. The adopted framework for this study makes it possible to attend to, and account for the diverse ways in which NESB students participate in mathematics classrooms and how this participation changes over time. It allows me to bring the diversity in NESB students' mathematical reasoning to the fore, and situate these students' diversity in the social context of their participation in communal mathematical activity.

Levels in the three-level framework are not hierarchical

Like Cobb's (1997; 2001) emergent perspective, the framework for this study regards the three levels of focus (classroom social norms, sociomathematical norms and classroom mathematical practices) as being non-hierarchical. It considers social and sociomathematical norms and mathematical practices to be complementary to each other. The framework for this study, therefore, allows me to recognize that social and sociomathematical norms and mathematical practices are not ranked, develop at the same time and affect student learning.

Previous studies that used the emergent perspective successfully

Other researchers have successfully applied Cobb's emergent perspective in their studies. Simon, Tzur, Heinz, Kinzel and Smith (2000), for example, were guided by the emergent perspective in their study involving the practice of mathematics teachers in transition, in the USA. They found coordinating the social and cognitive aspects of the emergent approach fundamental to their research as a whole. These researchers acknowledged that the emergent perspective enabled them to focus particularly on the cognitive aspects and gain a better understanding of the perspectives of teachers who participated in their research. In this study, I use the emergent perspective to allow me to focus especially on the social aspects and gain better understanding of my teaching practice and NESB students' collective learning.

Southerland et al. (2005) applied the emergent perspective in their research, arguing that one important characteristic of this framework is that it offers insight into how group construction of meaning relates to individual learning. The emergent

perspective enabled these researchers to account for the individual, the group and the interplay between the two camps. Hershkowitz and Schwarz (1999) extended application of the emergent perspective to middle school classrooms, describing the approach as "a powerful theory for describing cognitive development within classrooms" (p. 149). These researchers contend that there is a link, albeit indirect, between social and individual processes. By coordinating the social perspective and the individual psychological perspective, they were able to achieve a broad perspective on individual and collective mathematical knowledge development. In my study, it is also argued that the approach can provide some insight into understanding how collective negotiation of mathematical meaning relates to individual NESB student's mathematical reasoning. I use the adopted framework to allow me to link the social and individual aspects and better understand how NESB students might learn mathematics collectively, in an academic culture characterized by explanations and justifications, and negotiation of meaning. The adopted framework allows me to elucidate the relationship between human action and the social\cultural context in which the action takes place.

In a study involving tertiary students, Yackel, Rasmussen and King (2000) drew on the emergent approach for theoretical orientation. The perspective allowed them to analyse collective learning that emerged in a classroom community of undergraduate mathematics students. Focusing on social and sociomathematical norms - both in the social perspective, they were able to pay attention to the classroom environment in general and the mathematics environment specifically. The emergent perspective allowed these researchers to regard the relationship between the sociomathematical norms that were constituted by the class and, the students' and the teacher's mathematical beliefs and values, as reflexive, and mutually enabling and constraining. These researchers contend that the reflexivity between social and individual aspects that is recognized by the emergent perspective allowed analysis of the classroom community in a way that enabled better understanding of the social and sociomathematical norms regarding explanation. The perspective enabled them to document the emerging social norm of 'students explain their thinking and attempt to make sense of others' thinking'. In this study, the adopted emergent perspective allows explicit attention to social aspects of learning and teaching, and better insights

into understanding how communal learning might evolve in classrooms with NESB students.

In a case study involving third-grade mathematics students, Bowers et al. (1999) adapted the emergent perspective. This move enabled them to "document students' mathematical development in the social context of a classroom over a prolonged period of time" (p. 25). The perspective allowed these researchers to record communal classroom mathematical practices in which the students participated and the development of their individual understandings while participating in those evolving practices. In addition, the perspective enabled them to clarify the relationship between students' contributions to the developing mathematical practices and their ongoing negotiation of social and sociomathematical norms. As with Bowers and her colleagues, the emergent perspective makes it possible for me, in this study, to document communal development of social norms, sociomathematical norms and mathematical practices. The perspective provides direction on how these norms and mathematical practices might evolve in a classroom with NESB students.

More recently, when Rasmussen and Marrongelle (2006) carried out research involving undergraduate mathematics students in the USA, their framework was grounded in the emergent perspective. The emergent approach enabled them to work from the premise that meaning is constructed through interaction and that learning is a process involving interaction between the learner and her or his surroundings. In addition, the approach allowed the researchers to better understand and specify significant aspects of the proactive role of the teacher to create and sustain learning conditions that enable students to learn mathematics with understanding and\or make movements in their views about mathematics learning and teaching. These observations reinforce the viewpoint that the emergent perspective fits well within this study. They confirm that the adopted emergent perspective allows analysis of social aspects, in this study, allowing better understanding of my role, as teacher, of supporting the development of NESB students' conceptual orientation to an academic culture that values and encourages negotiation of mathematical meaning.

By using the emergent perspective, this study is able to add a cultural dimension to Cobb and his team's focus on classroom social and sociomathematical norms and

mathematical practices. The perspective makes it possible to link the three social aspects of the classroom microculture to cultural ideas about how learning happens and how mathematics happens.

3.6 Chapter summary

This chapter has discussed some learning theories and has set out the theoretical framework that guided this study. Sociocultural theory was identified as a significant perspective for understanding mathematical activity. This theory helped me to think in different ways about issues to do with NESB students' mathematics learning and my own teaching practice. It helped me question myself, in more ways than one and ask: What can I do to enhance my students' mathematics learning? Literature on sociocultural theories assisted me to think about a plausible way to investigate my practice.

Cobb's (1997) emergent perspective views the individual and the social as reflexively related. It offers insights that enable understanding of mathematics classrooms and learners. I have introduced Cobb's three main domains of interest to this study: classroom social norms, sociomathematical norms and classroom mathematical practices. Social norms are regularities in classroom social interactions that are established on implicitly shared group agreements. They are used to interpret the classroom participation structure and are not confined to mathematics classrooms. Sociomathematical norms deal with the normative aspects of classroom action and interaction that are specific to mathematics. Constructing sociomathematical norms helps create coherent classroom mathematical discussions and model academic discourse of mathematics. Mathematical practices are concerned with particular mathematical ideas. Development of classroom mathematical practices is accompanied by the actual learning process. I have proposed that the constitution of classroom social, sociomathematical norms and classroom mathematical practices needs to be considered in classrooms with NESB students. I further proposed that these norms and practices need to support NESB students' development of social and mathematical autonomy.

In order to examine the construction of classroom social norms, sociomathematical norms and mathematical practices in a systematic manner, I need credible research

methodology and methods. The next chapter outlines and discusses the research methodology and methods in this study.

CHAPTER 4

RESEARCH METHODOLOGY AND METHODS

4.1 Introduction

This chapter lays down the research design for the study. In the first section, quantitative and qualitative research approaches are identified as the two main research orientations in mathematics education. I outline these before focusing on qualitative research. Within this orientation, naturalistic and interpretive methodologies are discussed as appropriate approaches for investigating one's own practice. Practitioner research is then described and discussed in some detail. It is identified as appropriate for investigations involving professional development and professional learning. Next, the research methods used to collect and analyse data in practitioner research are discussed and their relevance to fostering deeper understanding of one's practice is stressed. Issues related to the ethics, trustworthiness and generalizability of practitioner research are identified.

In the second main section, the specific research design and the methods used to collect and analyse data for this study are presented and discussed. Issues to do with the trustworthiness and generalizability of this study are addressed. Finally, a chapter summary is presented.

4.2 Research Methodology

This section presents an overview of quantitative and qualitative research orientations, a description and discussion of practitioner research, and an outline of some of the methods that can be used to collect and analyse data in practitioner research. Ethical issues around practitioner research are highlighted. The section ends with a discussion of trustworthiness and generalizability in practitioner research.

4.2.1 Quantitative and qualitative research

Quantitative and qualitative research are two of the main research orientations in mathematics education. Quantitative research can be described as a type of "research in which the researcher decides what to study; asks specific questions; collects quantifiable data from participants; analyzes these numbers using statistics; and

conducts the inquiry in an unbiased, objective manner" (Creswell, 2008, p. 46). Quantitative approaches emphasize measurement and analysis of cause-and-effect between variables (Denzin & Lincoln, 2005). They tend to address research problems that require a depiction of trends or an explanation of any relationship among variables. In these approaches, objectivity is stressed and, time-, context- and value-free generalisations are desired and possible. Quantitative research rejects notions of choice, freedom, individuality and moral responsibility (Cohen, Manion & Morrison, 2007).

Unlike quantitative research, in qualitative research the questions tend to be broad and general, and researchers seek to learn and understand from participants (Creswell, 2008). Qualitative research tends to address problems requiring an exploration in which little is known about the problem. Its major concern is an in-depth understanding of the phenomenon in question rather than generalizations and universal truths (Denzin & Lincoln, 2005; Higgs & Cherry, 2009). Qualitative research approaches place importance on the information that is gathered in the real-life context as the action unfolds, tentative interpretation and meaning-making of reality, and acknowledges the role of the researcher, rather than on controlling events and seeking to establish cause and effect (Creswell, 2009; Polkinghorne, 2005). Hypotheses and definitions are allowed to emerge in context as the study develops. Qualitative research approaches acknowledge that any research will be partially complete (Lankshear & Knobel, 2005). It is value-laden.

In qualitative research, the assumption is that knowledge is context and time dependent and, that time- and context-free generalisations are undesirable and not possible because meaning is continuous and evolving over time (Cohen, Manion & Morrison, 2011; Drew, Hardman, & Hosp, 2008). Researchers aligned with qualitative research contend that multiple-constructed realities exist. In qualitative orientation, it is not possible to completely separate causes and effects since logic flows from specific to general. Neither is it possible to entirely distinguish knower and known because the "subjective knower is the source of reality" (Johnson & Onwuegbuzie, 2004, p. 14). Qualitative approaches stress the close relationship between the researcher and the social reality being studied (Denzin & Lincoln, 2003).

The researcher and participant are interdependent. Also, the researcher and all other participants are changed by the process of inquiry (Drew et al., 2008).

Qualitative research approaches have, however, some limitations which the researcher needs to be aware of and have measures put in place to try and minimize their effects. One such shortcoming is that, in qualitative research, it may be difficult to understand what is happening due to the complexity of the natural settings (Drew et al., 2008). Another is that results of qualitative research may not be generalizable to other groups of people. Also, the integrity of definitions and hypotheses may be less clear as this only become apparent in the course of the study, and analysis of results may be very time-consuming. However, the importance of gathering data in natural settings cannot be outweighed by these limitations.

Two orientations to qualitative research are relevant to this study. These are naturalist inquiry and interpretivist inquiry.

Naturalistic inquiry

Naturalistic inquiry is a qualitative research approach that seeks to explore and understand phenomenon in natural settings (Denzin & Lincoln, 2005). This approach relies on gathering information about events, processes, programmes, issues, and activities as they occur *in situ* within real-life contexts (Lankshear & Knobel, 2005). Naturalistic inquiry is principally concerned with how participants experience, understand, interpret, and participate in their usual social and cultural contexts. The central perspective of the naturalistic inquiry stresses the importance of participants' views and the setting (e.g. the mathematics classroom) in which the participants express these (Cohen et al., 2011; Creswell, 2008). Naturalistic inquiry aims to provide rich and detailed descriptions of people in action (e.g. teacher, students) in their regular local settings (Lankshear & Knobel, 2005).

Participants' behaviour is recorded in natural settings, as it evolves. Information is gathered by talking directly with the participants and observing them behave and act within their natural setting (Creswell, 2009). In a mathematics classroom, for example, data is collected as students interact in small groups with the teacher, during whole-class discussions, and as they work individually.

A naturalistic approach is suited to studies that seek answers to questions that seek to examine unfolding events; focus on a broad analysis of entire phenomenon or context; and require exploration of reasons for behaviour and ways in which behaviour unfolds (Drew et al., 2008). The main limitation of the naturalistic inquiry is the cumbersome and time consuming nature of data collection and analysis (Lankshear & Knobel, 2005).

Interpretivist inquiry

Interpretivist inquiry is a qualitative research orientation that emphasises interpreting and understanding participants' actions within natural settings (Cohen et al., 2011). Interpretivist approaches focus on the action of the participants, in social contexts. Interpretive inquiry aims to understand participants' interpretations of the situation. It is concerned with searching local meanings and understanding human experience through the eyes of the participants themselves. Interpretive researchers want to capture variation in local settings, using fine-grained descriptions of settings and actions, and through interpretations of how participants make sense of their sociocultural contexts and activities. In interpretivist orientation, theory emerges from the interpretation of data. The developing theory "becomes sets of meanings which yield insight and understanding of people's behavior" (Cohen, Manion & Morrison, 2000, p. 23).

Consistent and distinguishing features of interpretive research include privileging the insider's perspective and focusing on understanding the sociocultural processes in natural settings (Cohen et al., 2000), such as how participants learn and/or teach mathematics in classrooms. So, interpretive researchers record interactions in natural settings, conduct interviews, and review written artefacts such as personal, reflective journals. Interpretive inquiry aims for particularizability rather than generalizability (van Zee, 2006). It seeks to describe, analyse, and interpret features of a specific situation "preserving its complexity and communicating the perspectives to participants" (Borko, Liston & Whitcomb, 2007, p. 3). According to the interpretive approach, individuals have the unique ability to describe, clarify and interpret their experiences, as they are lived and constituted in awareness, and represent them to themselves (Cohen et al., 2007; Polkinghorne, 2005). In the mathematics classroom,

for example, students can describe their understanding of the teaching they experience. The main limitation of interpretive research is the lack of shared conceptual frameworks and designs (Borko et al., 2007). This makes it difficult to combine findings and make comparisons across studies, even when studies of similar experience are involved.

In this study, the term qualitative research encompasses aspects of both naturalistic and interpretive inquiries. The focus is on collecting data in natural settings (i.e. mathematics classrooms), and interpreting and understanding participants' (i.e. my students' and my) experiences and actions.

Research that focuses on natural settings, interpretation of participants' actions and interactions in social contexts, and in which the practitioner is the researcher, is often termed practitioner research.

4.2.2 Practitioner research

In this study, the term *practitioner research* is used in its most general sense to refer to "the array of educational genres where the practitioner is the researcher, the professional context is the research site, and practice itself is the focus of study" (Cochran-Smith & Donnell, 2006, p. 503). It takes practitioner research as systematic enquiry in an educational setting, done by someone working in that setting, the findings of which are made available to other stakeholders (Menter, Elliot, Hulme, Lewin, & Lowden, 2011). Practitioner research often accommodates an understanding of how researchers are practically "related to the situations they investigate" (Brown, 2003), where their actions, as teacher-researchers, are seen as an essential part of the situation being described (e.g. Adler, 1993; Brown, 1994; Schön, 1983). Practice is examined from inside, by the teacher, with the knowledge generated primarily to understand and improve the practitioner's practice within the local context (Borko et al., 2007; Pritchard, 2002). The knowledge generated, however, can also be used beyond the local context when it is shared with the wider community of educators and teachers.

Practitioner research has different versions and variations that emerge from different research traditions and social movements (Dadds & Hart, 2001; Zeichner & Noffke,

2001). The more common forms of practitioner research in education include action research (Carr & Kemmis, 1986; Corey, 1953; Stenhouse, 1985), self-study (Adler, 1996; Loughran, 2004, 2007; Olson, 2000), teacher research (Anderson & Herr, 1999; Cochran-Smith & Lytle, 1999), the scholarship of teaching and learning (Boyer, 1990), and the use of practice as a site for research (Cochran-Smith & Donnell, 2006; Shulman, 2000).

Despite the variation among various versions of practitioner research, important features are shared by all versions. These include that practitioner is the researcher, the professional context is site of the study, blurred boundaries between research and practice, and intentionality and systematicity. These are described next.

Practitioner as the researcher- the professional site as the research site

In all versions of practitioner research, the main defining feature is that the practitioner is simultaneously the researcher (Menter et al., 2011). Also, across all variations of practitioner research, the researcher's professional setting is the site of inquiry (Borko et al., 2007). The focus of investigation is problems and issues that arise from the professional practice (Bartlett & Burton, 2006), not from a study of literature, as often is the case in traditional research on teaching and learning. In this study, I am teacher and the researcher, and my mathematics classrooms are both the professional context and the natural setting for the study.

Boundaries between research and practice

Since the practitioner is simultaneously a researcher, and the professional context is the site for inquiry, the boundaries between research and practice are often unclear (Cochran-Smith & Lytle, 2004; Hargreaves, 1996), creating vital opportunities for reflection on and improvement of the practice of mathematics teaching. When a mathematics teacher engages in teacher research or self-study, for example, the distinction between mathematics teaching and research becomes vague and opportunities for reflecting on and improving teaching practice arise as part of both research and teaching (Ball, 1995; Cochran-Smith & Lytle, 2009). Blurred boundaries between teaching and research make it possible for practitioners to "construct their own questions, interrogate their assumptions and biographies, and

continuously re-evaluate whether a particular solution or interpretation is working and find another if it is not" (Cochran-Smith & Donnell, 2006, p.510). Blurring of boundaries and changed roles can have the potential to produce innovative research, as well as new kinds of mathematical knowledge (Borko et al., 2007).

Intentionality and systematicity

All variations of practitioner research share the features of *intentionality* and *systematicity* (Borko et al., 2007; Cochran-Smith & Lytle, 2004). Intentionality refers to the planned and deliberate nature of practitioner research, while systematicity is used to refer to organized ways of collecting data, keeping records of experiences and events inside and outside the context of practice, and analysing the data that has been gathered and recorded. Practitioner researchers intentionally focus on a research question of concern associated with their practice (Lankshear & Knobel, 2005). In addition to documenting students' learning, practitioner researchers systematically document their own teaching and learning. They record their planning, evaluation processes, their questions, interpretative frameworks, change in their views over time, issues they see as predicaments, and recurring themes.

Although for many years teachers have engaged in practitioner research "to improve their teaching and learning, to develop and refine their practice, innovate and evaluate their teaching" (Campbell & Jacques, 2003, cited in Bartlett & Burton, 2006, p. 396), there has been criticism of this research approach's general quality.

Criticisms of practitioner research

Although it has gained a measure of acceptance and standing over the last quarter century, some critics argue that there are epistemological and methodological problems associated with practitioner research (Bartlett & Burton, 2006). Common criticisms focus on the methods, the science, the political, and the personal or professional aspects (Cochran-Smith & Lytle, 2004). Practitioner research is criticized on the grounds of knowledge generation and use, validity and generalizability, and appropriateness of researcher roles and sites of research (Cochran-Smith & Donnell, 2006).

Criticism concerning knowledge generation and use oppose the idea that practitioner research is developing a form of research that contributes to a new epistemology of practice that is governed by different criteria and epistemological traditions (Cochran-Smith & Lytle, 2004). They challenge, on methodological grounds, the view that practitioner researchers have the skill, or the analytic ability to do research about their own practice or in their own professional context. Critics belonging to this camp challenge the possibility that the practitioner can function effectively as the researcher in their professional setting. They argue that practitioner researchers are not equipped with essential basic knowledge, skills or understanding of research methodology to be able to conduct research of any significant value (Bartlett & Burton, 2006). Critics maintain that practitioner research ignores the importance of the experience and expertise of the researcher on the outcome of the research process. In addition, the critics question the practitioner-researcher's ability to satisfy the usual criteria of qualitative research and "transcend the self" (Huberman, 1996, cited in Cochran-Smith & Donnell, 2006, p. 513). Like the critics who question practitioner research on the grounds of knowledge generation, those opposed to its methods of knowledge generation assume that practitioner-researchers are bound by the same methodological criteria as those of more traditional research, and so are not engaged in the emergence of a new genre of research (Cochran-Smith & Donnell, 2006).

Regarding validity and generalizability, some critics claim that practitioner research is idiosyncratic and that educational research needs to synthesize findings across settings rather than analyse findings from a single classroom, course or program (Anderson & Herr, 1999). These critics argue that since large samples, uniform procedures, cross-site comparisons, controlling and testing for the impact of specific variables, and a focus on educational outcomes are absent in practitioner research, this approach fails to meet the requirements of research, particularly the property of rigour.

Critics concerned with practitioner research's purpose and ends, and its political and ideological bases, condemn it for being too political, instead of being apolitical, neutral, non-ideological, and value-free (Cochran-Smith & Donnell, 2006). They charge practitioner research with being advocacy, activism, or political manipulation by disenfranchised individuals rather than research. The critics focusing on personal

or professional development aspects view practitioner research as a vehicle for individuals' personal or professional development, rather than a mode of knowledge generation or critical praxis (Anderson & Herr, 1999). Critics often challenge the apparent equating of the teacher's experience and beliefs with knowledge. Practitioner-researchers are criticized for being too personal, in that they focus on the person in an ego-centred sense (Bartlett & Burton, 2006). Critics argue that research should be more than self-discovery. It should include tangible evidence that problematic issues of teaching and learning have been tackled, head-on. In short, the criticism of practitioner research is essentially tied to questions about what counts as knowledge, evidence, effectiveness, and research. Despite these criticisms and limitations, practitioner researcher was considered viable for this study because in addition to its strengths discussed earlier in this section, its data collection methods enable the answering of my research question. The next section discusses data collection methods in practitioner research.

4.2.3 Data collection methods in practitioner research

This section discusses the main data collection methods in practitioner research that have relevance to this study. In practitioner research, several data collection methods are typically used in conjunction with each other in order to produce a thorough and rigorous piece of research (Burton & Bartlett, 2005; Denzin & Lincoln, 2005; Drew et al., 2008). These methods are audio-taping classroom dialogue, journaling experiences, interviews, and the collection of students' work.

Audio-taping classroom dialogue

Audio-taping classroom dialogue is one of the common methods of collecting data in practitioner research (Burton & Bartlett, 2005). Practitioner-researchers use audio-tapes to record group discussions and other class interactions, student and teacher presentations, and their talk with students. Audio-taping generates data by capturing contextualized spoken data (Creswell, 2009). Mathematical activity and classroom talk, for example, can be audio-taped or video-taped as they unfold in the context being studied. Audio-tape can preserve "the complexity of and relationship between the interactions, activities and language uses that take place during the course of the event" (Lankshear & Knobel, 2005, p. 195). The audio tape rolls and records while

the practitioner researcher goes about the business of teaching the students as usual. The practitioner researcher must, however, be aware of the limitations of audio-taping. For example, sometimes it may be impossible to record all talk that takes place in the classroom because of the size of the room. Another disadvantage is that transcribing audio-taped data is time-consuming and the researcher needs to allow time for this (Drew et al., 2008). For these reasons, the use of more than one data collection method is highly recommended. Journaling is an often used method in practitioner research.

Journaling

Journaling belongs to the category of documents that are commonly used in qualitative practitioner research as a source of data or data itself (Anderson, Herr & Nihlen, 2007). It is an intentional, quiet, reflective, human action that is contextually situated (York-Barr, Sommers, Ghere, & Montie, 2001). The three key features of journaling are that it is socially and contextually situated, records intentional reflective human actions, and engages participants in interrogating aspects of teaching and learning by documenting experience (Lyons & LaBoskey, 2002). Journals can include such items as date and time, a short description of an episode (with details of important aspects of the event), and an analysis (York-Barr et al., 2001). The significance and implications of what happened may be included.

Journaling is focused on trying to represent and understand educational experience (Clandinin & Connelly, 2000). It is intended to produce a sequence of writing or journal entries that tell a story of what is going on in a particular context (Brown, 2003; Lyons & LaBoskey, 2002). A sequence of journal entries captures the experience of the research process in a tangible way (Brown, 2001), acting as a narrative technique and recording events or experiences, observations, thoughts, questions, interpretations and feelings that have significance to the practitioner researcher (Anderson et al., 2007). It is a research tool that can be used to capture reflections and encounters (Creswell, 2008).

Journaling does not only involve an account, history, or narration of a story; rather it is related to knowing and knowledge construction. Journaling can facilitate a kind of reflection that is ordinarily difficult to carry out (Mattingly, 1991). Journaling while

reflecting may provoke a powerful consideration of the ordinarily tacit body of the constructs that inform practice. Since journaling and interpreting fragments of experience are interlinked, descriptions of events and analyses or interpretations of incidents can be interwoven.

Advantages and limitation of journaling

Journaling has both strengths and limitations. One of the advantages of keeping journal entries is that they can be re-read many times over (Lyons & LaBoskey, 2002). Journal entries can help a researcher to recall thoughts and experiences at different times of the research period. They can assist the researcher "to hold an idea or experience still for reflection" (Goldsmith & Schifter, 1997, p.43). Problem-solving strategies that worked in the past, and were documented as journal entries, can be used as references and analogies when dealing with current problems.

Journaling allows the practitioner-researcher to enter into dialogue with the self. The practitioner-researcher can focus on (1) interactions in the classroom that require reflection, (2) evaluative comments, and (3) the way forward (Borg, 2001; 2006). Another advantage is that, through journaling, practitioner-researchers (1) articulate and rationalise concerns; explore situations, (2) acknowledge, express, and examine feelings, (3) describe classroom events and procedures, (4) formulate plans and decide actions, (5) describe and evaluate progress (or lack of it), (6) capture, explore and pursue ideas, and (7) structure thoughts. Journals allow the practitioner-researcher to undertake critical reflection upon practice, particularly the close relationship between the professional and personal (Littlewood, 1995). Journaling allows practitioner-researchers to document what actually happened, their opinion about the event, and the way forward. They can include both professional and personal aspects of classroom life.

Production of journals can lead to expanding awareness, insights about one's practice and understanding of changed practices, making connections between theory and practice, and formation of new hypotheses for action (York-Barr et al., 2001). A journal can inform the teacher-researcher about changing thoughts and new ideas. It provides a rich source of data on the daily occurrences of a classroom (Anderson et

al., 2007). The teacher can use a journal as a reflective tool in examining his\her teaching or documenting his\her thinking on certain student(s) or issues.

Journaling highlights the importance of meaning, and the knowledge of situations, contexts and particulars of a specific group of students, of the school, and so on, rather than context-free and universal rules about teaching (Lyons & LaBoskey, 2002). It seeks the meaning of experience, rather than the narrow epistemological question of how to know the truth. In journaling, the teacher-researcher's knowledge is made conscious, public, and open to scrutiny by the researcher and others. Life in the classroom is tested and refined over time.

In the mathematics classroom, the teacher and\or the student may construct knowledge through journaling classroom experience. A journal encourages the teacher and\or students to describe, interpret and reflect. It offers private space for an individual to honestly recount and review experience. This means that the data is reasonably credible. At the same time, it makes invisible thoughts visible and helps clarify the journaler's thinking. A teacher-researcher can use a journal to document particular events or to monitor strategies or activities that worked well for students, and those that were less successful during a lesson and reflect on them with a view to improving practice.

Brown (2003) contends that practitioners who research their classrooms effect changes through their actions in the classroom itself, by generating writing commenting on their classroom practice. Journaling provides the teacher-researcher with a means of describing experience or practice as well as a way of identifying and clarifying beliefs, perspectives, challenges, and wishes for practice (York-Barr et al., 2001). Through journaling, the practitioner researcher's story is told and analysed in a highly self-conscious way. The practitioner's descriptions of classroom practice have the potential to effect changes in the reality attended to by the practitioner. The writing that is produced can be seen as responding to past actions as well as guiding future action. In describing his\her classroom, the practitioner affects the way he\she sees it, thus the way he\she acts in it and the way he\she subsequently describes it. In engaging in this cyclic process, the teacher-researcher passes through a sequence of perspectives, each susceptible to a variety of later interpretations.

Journal data has the advantage of being ready for analysis without the need for transcription. The teacher-researcher's writing can be processed as data and the journal entries can be scrutinized as an integral aspect of practice and instrumental in the process of practitioner-led change (Borg, 2001; Brown, 2003).

In practitioner research, it is the teacher-researcher who generates the sequence of pieces of data that becomes the researcher's data (Lankshear & Knobel, 2005). As the meaning of the research enquiry is a function of how the different pieces of writing are seen as interrelating, the process of building a research enquiry is closely linked with the process of generating pieces of writing. Absolute meaning is not sought; rather meanings or themes evolve as new contributions are introduced. The teacher-researcher's analysis of pieces of writing offers an instrument for monitoring practice and an approach to unite thinking with action through reflection (Brown, 2003). Writing provides a tangible product that enables the researcher to account for the reality to which he\she attends.

The main limitation of journaling is that it is time-consuming, requires skill, effort and commitment on the part of the practitioner researcher to give meaning to the applicability of experience in the development of professional knowledge (Harris, 2008; Scanlan & Chernomas, 1997). Another disadvantage with journals is that, if they are not the researcher's journals, they may be difficult to locate and\or obtain. It may also take time and sometimes be expensive to obtain someone's journal information.

How practitioner-researchers use journal extracts

Practitioner researchers generate journal extracts through writing and reflection and present them as data (Borg, 2001; Dana & Yendol-Hoppey, 2009; Mertler, 2012). By presenting journal extracts, practitioner researchers provide a record of events (actions and dialogues) or incidents, remarks, documented existing thoughts and ideas generated and explored, and discoveries made. Thus, practitioner-researchers use journal extracts to provide documentation of information related to the practitioner-researcher's experiences (Borg, 2006).

In practitioner research, journal extracts combine the objective and subjective aspects of daily experiences. The extracts can be used to report on both objective data (record of information) and personal interpretations and expressions of experiences, as well as intentional, personal and professional reflections, analysis, and planning and evaluation (Holly, 1989). Objective aspects are concerned with what actually happened – who said what and the actions involved, while subjective aspects deal with the practitioner-researcher's reflections at the time the event happened.

Practitioner-researchers present journal extracts (product) as evidence which provides a record of the researcher's personal experience during the research period. Thus, journal extracts provide an "evidential store" (Thomas, 1995, p. 5) which is used by the practitioner-researcher for interpretive and analytic purposes.

Not only do practitioner researchers present journal extracts, they reflect on them (e.g. Borg, 2001; Sakui & Gaies, 2003). Practitioner-researchers analyse journal extracts to get the bigger meaning of student and teacher actions. They present and analyse student behaviour, their actions, and their thoughts contained in journal extracts and presented as data.

In practitioner research, journaling is usually complemented by other data collection methods, such as interviewing.

Interviewing

Interviewing is one of the most commonly used data collection methods in practitioner research (Tashakkori & Teddlie, 1998). A research interview is a dialogue aimed at eliciting information or opinions and understandings, from the participants, "on a certain topic or topics of interest to a research enquiry" (Menter, 2011, p. 127). Interviews are, however, de-contextualized in that the focus of the activity is the interview itself, rather than the everyday life that occurs on a moment-by-moment basis that prevails, for example, in the mathematics classroom.

Interview data is generated deliberately and systematically by the researcher (Menter et al., 2011). Depending on purpose of the interview, the questions can be structured, semi-structured, or open-ended. The researcher's familiarity with and knowledge about the exploratory or confirmatory nature of the study determines the structure of

the interview questions. Important aspects in interviews are establishing trust , maintaining a relaxed manner, asking clear questions, note-taking, appropriate use of follow-up question or probes, and keeping track of responses (Cohen et al., 2000; Polkinghorne, 2005). When designing an interview, the relationship between the research, the interview protocol and the respondents should always be kept in mind (Creswell, 2009). Although participants will inevitably have preconceptions about any researcher, practitioner-researchers "must not publicise their own opinions about their research topic, nor contribute their own stories in interviews" (Mercer, 2007, p. 13).

A variety of methods can be used to record interview data. One way is to take copious notes. The advantage of note-taking over tape-recording is that it is less obtrusive. In spite of being obtrusive, audio-tapes are a common method of capturing interview data (Lankshear & Knobel, 2005). They are accurate and indisputable, but, this method can be intimidating to some respondents. Since voices can be recognized there is a need to assure respondents that tapes will be destroyed when the study is finished. Respondents should be made aware of the purpose of the interview, and given assurances of confidentiality (Creswell, 2009). If confidentiality (with respect to the researcher) is not possible, respondents should be made aware, and their permission to go ahead with the interview sought. It is also necessary that respondents feel confident that the researcher will not distort the information they provide.

Advantages and disadvantages of the interview

One of the advantages of using the interview is that it has the potential to provide the researcher with direct access to what the participant thinks (Tuckman, 1999), and hence capture interpretations directly from the participants themselves. When used with care and skill, an interview can be a rich source of qualitative data (Anderson, 1990; Drew et al., 2008). Another advantage is that an interview is flexible (Creswell, 2009; Menter et al., 2011). The interactive nature of an interview allows the researcher to adapt the questions to suit participants' responses. He or she has control over the line of questioning. An interview can be detailed in order to accommodate the information that is required, giving interviews the potential to

provide greater depth of information. In an interview, personal perspectives of the respondent are provided, and meanings and feelings can be quite detailed. It is possible to get the participants' actual thoughts around the topic of interest to the research (Anderson et al., 2007). Clarification of questions is possible, and the practitioner researcher has the opportunity to probe what is being said by the participant by asking for clarifications and/or examples of vague or unclear statements. By using an interview, trust and good rapport with students can be built, making it possible to obtain information that participants would probably not have revealed in class or by other data collection methods. Interviews can contain surprises that may enrich the study.

Through interviews, the practitioner-researcher is able to discover experiences that may have taken place in students' lives which might have a bearing on their learning of mathematics in the present (Cohen & Manion, 1997; Cohen et al., 2007). The researcher can use this knowledge and information to test hypotheses or to suggest new ones. Mason (2002) contends that one of the purposes of using interviews is for searching for theories-in-action in which students display behaviours that suggest, or are consistent with, the theories the researcher already knows.

The practitioner-researcher must be aware of the limitations or disadvantages associated with the interview method (Drew et al., 2008). The researcher can then plan strategies aimed at minimizing or eliminating them. One of the disadvantages of the using interviews is that interviews can be very expensive if extensive travel is involved. Another disadvantage is that interviews can be time consuming in terms of travel time and time required for transcribing and interpreting information (Anderson et al., 2007; Tashakkori & Teddlie, 1998). Interviews require great skill and expertise of the interviewer. One other limitation of the interview method is that the researcher's presence may bias interviewee's responses (Creswell, 2009). Another is that people, particularly second language speakers of the language in which the interview is being conducted, are not equally articulate and perceptive. This situation can adversely affect the quality of data collected by interviewing participants. Another limitation is that the information gathered through an interview is filtered through the views of the interviewee. So, it may not be accurate. An interview is

susceptible to manipulation by the respondent, which can result in collecting false or distorted information, which, in turn leads to false findings and conclusions.

Practitioner-researchers often want to augment spoken or observed data with written responses from participants. One of the ways of achieving this is to use participants' written work.

Students' work

Students' texts or written work can be used as data (Altrichter, Feldman, Posch & Somekh, 2008; Lankshear & Knobel, 2005). Students' work belongs to the category of documents and artefacts. It is a naturally occurring form of data that can be very powerful (Dana & Yendol-Silva, 2003). Systematically collecting students' work provides the researcher with the opportunity to analyse students' thinking in new and different ways. By examining students' work over a period of time, the practitioner-researcher can make claims that could not be made by viewing a single piece of student work in isolation. While collecting students' work, some researchers note the context in which it was produced (Burton & Bartlett, 2005).

Documents, including students' work, can stimulate further discussion by those involved (e.g. teacher and students). In the mathematics classroom, the teacher can, for example, ask why a student decided to use a certain strategy to solve a particular problem. Students' work or documents can provide stimulus for interview or observation or "it may provide useful contextual or explanatory data for something a researcher has found through questionnaires, observations, and so on" (Burton & Bartlett, 2005, p. 162).

To sum up, the main data collection methods in practitioner research that are relevant to this study are audio-taping classroom dialogue, journaling, interviews with groups of students, and collecting some students' work. After data collection, the next major step in practitioner research is data analysis.

4.2.4 Data analysis in practitioner research

In practitioner research, data analysis tends to move through at least four main steps (Creswell, 2009; Dana & Yendol-Silva, 2003): description, sense-making,

interpretation, and implication drawing. The steps are interactive and interrelated. After assembling all data (audio-\video-tapes, student work, personal journals, interviews, etc.), and having reread the research question, the first step in data analysis involves reading and rereading the data set with the intention of developing a descriptive sense (Anderson et al., 2007; Holly, Arhar & Kasten, 2005). Developing a descriptive sense requires the practitioner-researcher to be able to describe what he or she noticed during the inquiry, what was happening, and their initial impressions or insights into the data.

The second step is to look for parts (Holly et al., 2005). This step can be referred to as sense-making (Dana & Yendol-Silva, 2003). The practitioner-researcher reads to make sense of the data, and gain an understanding of what is happening. The researcher identifies and organizes units (e.g. issues, strategies, episodes) of analysis that emerge from the data. Sense-making units may lead to a process of putting ideas together, or grouping or sorting data by categories (e.g. ideas, issues, themes, or dilemmas) (Holly et al., 2005). To support sense-making, data is coded. *Coding* involves detailed analysis (Creswell, 2009) or constantly matching, comparing, and contrasting pieces of data and subsequently categorizing them (Dana & Yendol-Silva, 2003). While categorizing data, the practitioner researcher also engages in *memoing*. That is, writing notes in the margin of data commenting on meaning of coded category, explaining patterns developing among categories, and\or describing some aspect of a phenomenon. As findings emerge, the researcher may regroup, name, expand or condense, combine the original organizing units or categories (Anderson et al., 2007). So, categories are subject to change during data analysis.

The third step involves getting a general idea of the important units of data and an idea of the emerging story: the interpretive stage of data analysis (Dana & Yendol-Silva, 2003). In this step, the researcher constructs statements that express what the researcher has learned and what that learning means (Creswell, 2009). This can be done by examining the patterns and asking questions such as What is happening in each pattern and across patterns? What was my initial wondering and how do the patterns inform it? How are the patterns connected to …my practice? ...my students? How do patterns relate to each other? Unlike the second step that focused on taking things apart, this step involves synthesis or a process of combining parts to make a

whole (Holly et al., 2005). Findings in the interpretive step can be illustrated in a number of ways: themes, patterns, categories, labels, naming, claims or assertions, or vignettes.

After the interpretive step comes the implications phase of data analysis. In this final step of data analysis, the practitioner researcher asks and answers questions articulating implications or So what? In practitioner research, interpretation of data tends to involve stating the larger meaning of findings (Creswell, 2008). The results may include themes or broad categories that represent findings. A rich, complex picture emerges from the description and identification of themes. The researcher enunciates what she\he has learned concerning self, students, larger context of professional site, and the implications of that learning for the researcher's teaching or practice. In addition, the researcher's new wonderings and possible changes to practice are spelt out.

Data analysis is inductive; that is, involves moving from particular or detailed data (field notes, journal entries, interviews) to general codes and themes. The first step in data analysis is to assemble all data (field notes, journal entries, etc.) and transcribe any audio and\or video tapes. Then, the practitioner researcher reads the data to get a descriptive sense of the material. Next, the researcher engages in sense-making through reading and memoing, and coding into categories and themes. The researcher then describes patterns and themes from the perspective of the participant(s), before interpreting them. In the final step, the researcher articulates implications of the findings.

As with all research, practitioner research is not exempt from ethical issues, but rather it is bound by particular ethical issues which need to be addressed.

4.2.5 Ethical issues in practitioner research

There are ethical issues when practitioners study their own work (Cochran-Smith & Donnell, 2006). The researcher must address these before embarking upon an inquiry, and also take them into account during the inquiry (Anderson et al., 2007; Burton & Bartlett, 2005). The researcher must not only anticipate ethical conundrums, but be committed to addressing them both before the research begins and as they arise. As

the research develops, the teacher-researcher must assume that he\she will continuously make ethical decisions. It is equally important to think of ways to minimize any ethical dilemmas that may be faced as a researcher. The teacher-researcher must consider ethical issues to do with informed consent of those involved, deception, confidentiality and anonymity of participants, consideration of possible harm, conflict of interest, and access to findings (Burton & Bartlett, 2005).

Informed consent of those involved

The researcher must seek informed consent of all participants before the research takes place (Burton & Bartlett, 2005). Consent of management and colleagues involved is important. Sometimes seeking consent of parents and students is appropriate. The practitioner-researcher has an obligation to ensure that each participant has a complete understanding of the purpose and methods used in the study, the risks involved, and the demands of the study (Drew et al., 2008). The purpose of the research and procedures must be clearly explained to potential participants, in the language that is understandable to prospective participants. How much participants are to be involved in the research needs to be considered by the researcher, and enough information regarding the research, and the participant's role in it, must be revealed to prospective participants. Voluntary consent is important but this may be compromised if potential participants feel coerced to become part of the research (Anderson et al., 2007) and so coercion must be avoided by the researcher (see Appendices A & C). Participants need to know that they may voluntarily withdraw from the study at any time during the research process.

Deception

A practitioner researcher should avoid deception or deliberately misleading participants into giving their consent to participate, or at any other time during the research period (Drew et al., 2008; Lankshear & Knobel, 2005). He or she should always be honest and open with the participants. On-site relationships between the teacher and the students may change once the teacher changes role from teacher to teacher-researcher (Anderson et al., 2007). Therefore the teacher-researcher needs to consider how open they make the research process without influencing the behaviours of the participants (Burton & Bartlett, 2005). Besides being morally wrong, deception

can prove counterproductive in the long run. Burton and Bartlett claim that much of the research done by teachers in their classrooms has benefited from being open and from involving other people.

Confidentiality and anonymity of those involved

Confidentiality refers to keeping the identity of a respondent from being known by any person other than the researcher (Drew et al., 2008) and ensuring the anonymity of those taking part in the research (Burton & Bartlett, 2005; Anderson et al., 2007) because participants are likely to feel more at ease when assured that they will remain masked as much as possible in any report that results from the study (Lankshear & Knobel, 2005). In addition, the practitioner-researcher must guarantee confidentiality of data collected (see Appendix D). The researcher must explain how the records relating to the research will be kept confidential. This further puts participants at ease. Participants should be given the names and contact details of people to whom questions and\or concerns could be directed, immediately or later.

Consideration of possible harm

The researcher must anticipate possible effects of or harm from carrying out the research on those involved, both in terms of the actual research process and any future actions that may result from the study, and work towards minimizing it (Anderson et al., 2007; Burton & Bartlett, 2005; Lankshear & Knobel, 2005). All participants need protection from exploitation or exposure to risks associated with participation in the research. Possible harm could be physical, psychological, legal, social, or economic. The researcher must describe physical, emotional or any other risks that may occur as a result of participating in the research (Drew et al., 2008). In the mathematics classroom, the practitioner should ensure that the audio recorder or video camera does not interfere with students' learning. In addition, the researcher must avoid instances where students participating in the research have preferential treatment or vice versa.

Conflict of interest

The practitioner-researcher must be aware of the possible conflict of interest that may arise due to the dual role of researcher and teacher (Lankshear & Knobel, 2005). The

teacher-researcher needs to carry out their normal duties of teaching as if no research were taking place. The researcher must not use his\her position as teacher to coerce or manipulate students. At no time should students feel coerced by their teacher into participating in the study or manipulated to respond or act in certain ways during the course of the study (Burton & Bartlett, 2005). The teacher-researcher needs to be aware that power relations between them as teacher and the participating students may have effects on the interview responses. Students may produce responses that they feel will please the researcher as teacher or act in ways they think the researcher wants them to act. So, lack of careful attention to the effects of the teacher as an authority in the classroom may render findings of the study invalid.

Access to findings

The final research report should be presented to the participants or made accessible to them (Burton & Bartlett, 2005). It is the responsibility of the practitioner-researcher to make the findings of the research known by participants. In this study, the final report will be in the university library to enable access to it by all participants and others.

Other key issues in practitioner research are trustworthiness and generalizability. They are discussed next.

4.2.6 Trustworthiness and generalizability in practitioner research

This section discusses trustworthiness and generalizability in practitioner research. Some of the strategies for establishing trustworthiness are identified and discussed, briefly.

Trustworthiness

Trustworthiness and validity are used to describe the quality of findings for qualitative and quantitative academic research, respectively (Anderson et al., 2007; Cochran-Smith & Lytle, 2009). Qualitative researchers favour trustworthiness instead of internal and external validity which is preferred by quantitative researchers. The notion of validity in quantitative research is similar to trustworthiness in practitioner research (Anderson et al., 2007; Lincoln & Guba, 1990; Zeichner & Noffke, 2001). In

all forms of qualitative research, practitioner research included, trustworthiness is based on "determining whether the findings are accurate from the standpoint of the researcher, the participant, or the readers of an account" (Creswell, 2009, p. 191). The authority of practitioner research can be judged by four general criteria: plausibility, credibility, relevance and importance of the topic. In practitioner research, the term *valid* should be seen in its usual meaning of well-grounded and supportable.

Strategies with which to establish trustworthiness

To achieve trustworthiness, practitioner-researchers engage in one or more strategies. These can include the use of thick descriptions of the process of data collection and analysis, member checking, clarifying the bias the researcher might bring to the study (Creswell, 2008; Lincoln & Guba, 1985). Other strategies to attain trustworthiness in practitioner research are the use of negative cases, prolonged engagement in data gathering or field experience, triangulation, and peer reviews (Krefting, 1991; Morse, Barrett, Maya, Olson, & Spiers, 2002; Tashakkori & Teddlie, 1998). The main research methods, such as observation and interview, complement each other and help to increase the trustworthiness in practitioner research.

In order to ensure trustworthiness of the study, a practitioner-researcher can use *thick or rich descriptions* to convey findings (Drew et al., 2005; Creswell, 2009). Detailed, accurate, vivid descriptions of the setting, participants, interactions, process of data collection and analysis, for example, make the results become more realistic and rich, giving rise to the trustworthiness of the study.

Another strategy to improve trustworthiness of the study involves *member checking* to verify the accuracy of the findings (Anderson et al., 2007; Richards, 2009; Tashakkori, 1998). Member checking is done by taking the research report or specific descriptions or themes to the participants so that they verify the accuracy of the account. Only parts of the report, such as themes or descriptions, not raw transcripts, are checked for accuracy by the participants (Creswell, 2009). This process may trigger follow up interview with participants to provide opportunity for them to make comments about the findings. This action increases trustworthiness of research findings.

The strategy of *clarifying the practitioner-researcher's bias and assumptions* can be used to increase trustworthiness of findings (Creswell, 2008). The credibility of practitioner research requires assurances that the researcher articulates his\her role or position in the research and reflects on his biases, values, experiences and assumptions. The researcher needs to declare these personal and intellectual characteristics in the research, and discuss how they may affect data collection and analysis. By doing this, trustworthiness can be increased.

To increase the trustworthiness of the study, a practitioner-researcher can present *negative or discrepant information* that contradicts the themes (Creswell, 2009; Tashakkori & Teddlie, 1998). Since real life involves perspectives that do not always agree or come together to form a perfect whole or system, discussing contradicting information may improve the credibility of the research report. A practitioner-researcher can build up a case for the theme and make the researcher's account more realistic and trustworthy by presenting information that contradicts general perspective of the theme (Creswell, 2008).

Prolonged engagement in data gathering is another strategy for increasing trustworthiness of research findings (Anderson et al., 2007; Creswell, 2009; Drew et al., 2005). Long periods of data collection allow the researcher sufficient time to capture important events and details of different situations and combinations. These extended periods help to minimize distortions of findings that may occur when insufficient time for data collection is allowed (Lincoln & Guba, 1985). Prolonged engagements help to prevent unrepresentative events getting unwarranted attention and significance in data analysis and findings. This action may enhance the researcher's deeper understanding of the phenomenon being studied, making it possible to convey details about the site and participants' views, and lending credibility to the findings. The longer the experience and time spent by the researcher in the natural setting, the more likely that the findings will be accurate and credible.

Trustworthiness in practitioner research is reinforced by *triangulation*. Triangulation is a process of using a variety of different sources, collection methods, or perspectives to check the consistency or accuracy of research findings (Altrichter et al., 2008; Anderson et al., 2007; Drew et al., 2008). So, to ensure trustworthiness, a

practitioner-researcher can seek data from a variety of sources, using different data collection methods, and possibly different perspectives (Menter et al., 2011; van Zee, 2005). A practitioner-researcher can use the multiple data sources to give greater depth to the analysis, corroborating findings, or leading to discussion of variation in findings (Creswell, 2008). What is needed in triangulation is not a combination of different kinds of data *per se*, rather an attempt to relate different "sorts of data in such a way as to counteract various possible threats to the validity" (Burton & Bartlett, 2005, p. 28) of analysis. Thick, rich, and supportive data from various sources help to increase the likelihood that the practitioner-researcher will be able to create a credible description and analysis of results, and a trustworthy account of findings.

Another strategy to enhance the accuracy of the study, and hence, trustworthiness of the findings involves the use of *peer review* (Creswell, 2008; Tashakkori & Teddlie, 1998). Quality and rigour can be achieved in practitioner research if the researcher considers his\her work as belonging to the community and makes it available for public scrutiny and critique (Borko et al., 2007). Making work available for public examination supports the improvement of the researcher's practice and increases the chances that the work will become useful to other mathematics educators. Borko et al. suggest that trustworthiness can be achieved by subjecting the practitioner-researcher's methods to scrutiny and critique by colleagues in the field. The researcher should seek to see "beyond self." (p. 9). Shulman (2000) asserts that practitioner research becomes useful and credible if the research work becomes public property and available for scrutiny by others. Also, in self-study, a form of practitioner research, "it is widely acknowledged that inquiry cannot be solely individual lest it become simply rationalizing one's own perspectives rather than genuinely grappling with the contradictions involved in improving practice by better understanding personal experience" (Cochran-Smith & Donnell, 2006, p.511). Peer reviewers appraise and ask questions about the study. This action is used to determine whether the account resonates with people other than the researcher. Interpretation of the research account beyond the practitioner-researcher adds to the trustworthiness of the study.

Generalizability

Practitioner research, like other forms of qualitative research, does not aim to generalize from one setting to the next, but to understand a specific situation (Pritchard, 2002), and to promote professional development of the participant (Elliott, 1991). Attention is on the local setting and unique context, rather than on generalizing the findings. Generalizations sought are not the lateral mould of being *across*, rather they are vertical (Corey, 1953).

Since practitioner research is concerned with guiding self-understanding of a particular population through the dynamics of its own changes into its own future, generalizations are "only meaningful relative to the specific contexts in which they occur" (Huberman 1996, p. 132). In practitioner research, generalization resides with the reader (Brown, 2001; Mason, 2002). The practitioner-researcher is interested in whether the research makes sense or is credible, rather than whether what was measured fits well with what was initially asked (validity). Rather than being replicated in other settings, practitioner research can be extended if others test it in their own settings (Lyons & LaBoskey, 2002). Practitioner research's strength "lies in its relatability to similar situations" (Bassey, 1990, cited in Bartlett & Burton, 2006, p. 398). In view of the fact that qualitative methods are appropriate for researchers seeking to understand how people make sense of their surroundings and the circumstances that shape their lives, that is, their cultural context (Denzin & Lincoln, 2005), the main features of practitioner research make it generalizable to theory, rather than to different research populations (Drew et al., 2008).

The issues about methodology and methods I have discussed so far relate to practitioner research in general. They helped me to better understand how qualitative research is done, and to think about how best I could examine my own practice. Furthermore, a close look at these issues assisted me to think about ways I could use to investigate the constitution of classroom norms and mathematics practices in this study. I became aware that, to be able to examine my practice in a systematic and coherent manner, I needed to identify a research design for this study.

4.3 Research design for this study

This section describes practitioner research design, data collection methods and thematic data analysis for this study. In addition, it discusses trustworthiness and generalizability in relation to this research.

4.3.1 Practitioner research design

In qualitative research, investigations that involve the practitioner studying their own practice are often referred to as practitioner research. Practitioner research design was chosen for this study because it best serves the orientation of the research question. I was simultaneously the practitioner and the researcher, so had the dual role of teaching and researching my own teaching practice. I was both an insider and an outsider during the research period. Practitioner research design best suits this situation. Since the investigation was about my teaching practice and NESB students' mathematics learning, it was done in my mathematics classrooms. Although some of my students participated in this study, I was the main participant.

Focus of the study

The major focus of this study was my teaching practice and NESB students' learning. In view of this, the aim of this research was to investigate this teaching practice through self-refection and evaluation, and further develop my effectiveness as a mathematics teacher of students whose first language of communication is not English. In the light of this aim, the study was guided by the following key question: *What can I do to my teaching practice to enhance my students' mathematics learning?* My expectation was to gain new understanding of myself and my practice, with special focus on NESB students' mathematics learning. The study focused on producing a sequence of writings (or a story) about my personal development through the research period, in the hope that this could be of benefit to other mathematics teachers of NESB students.

In particular, the study was focused on the strategies that could help to enhance NESB students' mathematics learning. That is, the negotiation of classroom social norms, sociomathematical norms and classroom mathematical practices, and the effects of this on NESB students' mathematics learning. In other words, the study

was focused on how I elicited students' ideas or solution methods, supported their conceptual understanding during class discussions, and extended their thinking, as well as students' reactions to my actions. I used different teaching strategies, the most common being whole-class and/or small-group discussions in which students were encouraged to engage in collaborative problem-solving, share ideas and solution methods, and justify their thinking. In addition to this, I insisted that NESB students explain and justify their solutions.

Research setting and participants

The setting for this study is two Statistics for Foundation Studies classes (Classes 1 & 2 in semester B, July to December, 2005) and one Calculus for Foundation Studies plus one Statistics for Foundation Studies class (Classes 3 & 4 in semester A, February to June, 2006) at a university in New Zealand. So, I worked with four mathematics classes at a tertiary institution. These papers are bridging mathematics papers for NESB students intending to do undergraduate studies at the university, and are equivalent to year 13 mathematics papers. Statistics classes, in 2005, had 14 NESB students in each. In 2006, the Calculus class had 18 NESB students and the Statistics class had 15. Students attended five, one-hour mathematics (statistics or calculus) lessons per week. Four of the five one-hour statistics lessons were conducted in a normal classroom setting and one lesson was conducted in the computer laboratory where students used computers to complete the tasks. In calculus, the teaching took place in small groups of 15 -20 students per group, and all five one-hour sessions per week are taught under normal classroom conditions.

4.3.2 Data collection methods used in this study

Data collection covered the period of one year, from July, 2005 to June, 2006, and was done using journaling, audio-taping of classroom dialogue, interviews and collection of students' work.

Journaling

I kept a journal of some of the events or episodes that took place during the data collection period. These events were to do with what I noticed, regarding my own actions and\or students' actions, as different lessons progressed. Written accounts of

episodes were not meant to capture the episode as such, but to record what struck me during that event. These accounts afforded me the opportunity to re-enter memories of the situation and others like it later, in order to inform my practice. The technique of journaling was used because in addition to making my otherwise invisible thoughts visible, conscious and open to scrutiny, journaling provided a way of describing my practice and identifying and clarifying my beliefs, perspectives, challenges and hopes. Journaling helped me to clarify my thinking. Journal entries assisted me to identify and keep track of which strategies seemed to work well for my students and which were less successful. I used this information to help me make decisions about *what* to teach and *how* to teach it, in later lessons. In short, the benefits of journaling included expanding awareness, understanding, and insights about my practice.

By journaling, I could also make connections between theory and practice and generate new hypotheses for future action. I took the role of a teacher-researcher who reflects and acts upon the context in which he practises. In other words, I engaged in conscious reflection and evaluation of my situation as it unfolded. More specifically, I documented, reflected on and interpreted events that occurred during the research period, and reflected on my actions during particular events in my mathematics classes. Additionally, I reflected on my students' actions during the same events in my mathematics classes.

In my role as practitioner-researcher, I brought about changes in my situation through my actions in the classroom and reflection on journal entries commenting on my practice. Descriptions of my classroom practice were meant to effect some changes to the situation I was in. Each journal entry was seen as a response to past action as well as a guide to my future actions. In total, I made forty-five journal entries. In addition to journaling, I used audio-taping as a data collection method.

Audio-taping classroom dialogue

During the data collection period, I audio-taped myself while teaching and later transcribed the data and reflected on it. Mathematical activities and classroom talk were audio-taped as they unfolded in my mathematics classrooms. My aim was to capture everyday slices-of-life that would help to address the research questions guiding my study. I simply switched on the tape recorder at the start of a scheduled

lesson, then let the tape roll and record while going about the business of teaching the students as usual. Lessons were conducted in as natural and normal context as possible, as if it were any other day and a recording device was not operating. Audio-taping enabled me to freeze-frame part of the event or activity as it occurred in context, and capture the speech *in situ*. This helped me to reflect and focus on what I had said in my mathematics classrooms. Audio-taping multiple mathematics lessons complemented my journal reflections and enhanced the likelihood of addressing questions raised in my study.

Throughout the data collection period, I was aware of the limitations of audio-taping. For example, I knew that sometimes it was impossible to record all talk that took place in the classroom because of the size of the room. However, because the recorder was attached to me, everything I said in class was recorded. I recorded 60 classroom dialogues/discussions during the research period. Using more than one research method is highly recommended in practitioner research. So, I used various productive ways to observe my practice, and one of these was interviewing.

Focus group interviews

I decided to use the focus group interview as a tool for data collection because student group interviews had the potential to capture a diverse range of thoughts directly from the students themselves. Focus group interviews enabled me to learn about the ideas and opinions of my students, regarding mathematics and its teaching and learning. Interviews were semi-structured (See Appendix B). I used pre-prepared questions to elicit student views and opinions, and to encourage interaction among participants in discussion. Since the interview is flexible, I followed up students' responses for more information. It was possible to clarify vague statements made by students and myself. I had the opportunity to probe what was being said by the student by asking for clarifications and/or examples. By using an interview, trust and good rapport with students could be built, making it possible to obtain information that students would probably not have revealed in class or by other data collection methods.

Through interviews I was able to discover what experiences had taken place in students' lives that may have had a bearing on their learning of mathematics at the time of data collection. I could use this knowledge and information to address my key

research question. One of the purposes of using interviews was to enable me to search for theories-in-action in which my students displayed behaviours that suggested, or were consistent with, the theories I already knew.

Interviews were done in groups of two, three or four students at a time. Students were not allocated to particular groups for the interviews. They chose their own group. In total, I interviewed 16 students in 2005. Thirteen of these were interviewed twice. The other three only once. In Semester A, 2006, I interviewed a total of 11 students. Two were involved in just one interview. The other nine were interviewed on two separate occasions. The data from these interviews were audio-taped, and later transcribed and subjected to thematic data analysis. It complemented data obtained by audio-taping classroom dialogue, journaling, and students' work samples. Tables 4.1 and 4.2 below, show the students who participated in the group-interviews in 2005 Semester B and in 2006 Semester A.

Table 4.1   Interviews (Semester B, 2005)

Name of Student	Interview Date(s)
Khoula	13/09/2005
Laila	13/09/2005 and 31/10/2005
Ahmed	13/09/2005 and 31/10/2005
Fiona	13/09/2005 and 26/10/2005
Lily	13/09/2005 and 26/10/2005
Miriam	13/09/2005
Carol	13/09/2005 and 26/10/2005
Susan	14/09/2005 and 26/10/2005
Rita	14/09/2005 and 26/10/2005
Em	15/09/2005 and 31/10/2005
Yosuke	15/09/2005 and 31/10/2005
Chen	15/09/2005 and 31/10/2005
David	15/09/2005 and 26/10/2005
Dong	15/09/2005 and 26/10/2005
Raymond	15/09/2005 and 26/10/2005
Derek	26/10/2005

Table 4.2   Interviews (Semester A, 2006)

Name of Student	Interview Date(s)
Honda	02/06/2006 and 26/06/2006
Sayuri	02/06/2006 and 26/06/2006
Najilla	02/06/2006 and 26/06/2006
Omar	02/06/2006 and 26/06/2006
Thomas	06/06/2006 and 30/06/2006
Muhannnad	02/06/2006 and 30/06/2006
Wataru	31/05/2006 and 30/06/2006
Deependra	31/05/2006 and 30/06/2006
Ray	31/05/2006 and 30/06/2006
Max	06/06/2006
Yuichi	31/05/2006

The main interview questions were the same for all groups, but follow-up questions differed. Each follow-up question was based on one or more students' responses to the initial question. Main questions were not directed to each student but to the whole group. Follow-up questions were, however, sometimes directed at particular students. Some students, in each group volunteered to answer various questions. One or more students in a particular group could respond to the same question.

Some of the interview questions were:

1) Why do you study mathematics?

2) What problems, if any, do you have studying mathematics here in New Zealand?

3) Is there any difference between the way mathematics is taught in New Zealand and in your home country?

4) Who do you prefer to ask when you do not understand something in class? Why?

5) Who should do most of the talking/explaining in class? Is it the teacher or students? Why?

6) Are story or word problems good for learning mathematics? Explain why?

7) What are your views about students solving mathematics problems in small groups before discussing their solutions as a class?

Students' work

In this study I used my students' texts or written work as data. This captured students' thinking on paper. By systematically collecting students' work, I had the opportunity to analyse students' thinking in new and different ways. Every two to

three days during the research period, I collected work from students who participated in the study. While collecting students' work, I noted the context in which it was produced. As I went through their work, I documented the strategies students used to solve particular mathematics problems. Sometimes I followed up this action, in the next lesson, by initiating and guiding whole-class discussion focusing on the strategies that stood out in terms of uniqueness, popularity (or lack of it), being unusual, etc. So, students' work could initiate further discussion involving my students and me. It provided insight into the life of my mathematics classrooms, which complemented data from other collecting methods, e.g. interviewing, audio-taping classroom dialogue. Also, sometimes students' work provided stimulus for journaling.

To achieve depth in my study, data were analysed in detail by means of thematic data analysis.

4.3.3 Thematic data analysis in this research

In order to develop an understanding of my practice and address the study research question, I engaged in detailed thematic analysis of transcripts of audio-recordings of classroom dialogue, my journal entries, transcripts of interviews with my students and copies of student work. The term *thematic analysis* is used, in this study, to refer to a qualitative analytic method for "identifying, analysing, and reporting patterns (themes) within data" (Braun & Clarke, 2006, p. 82). Thematic data analysis method can be described as a "form of pattern detection within data, where emerging themes become the categories of analysis" (Fereday & Muir-Cochrane, 2006, p. 82). In this practitioner research, I identified themes through careful reading and re-reading of the data (Rice & Ezzy, 1999). I went beyond merely organising and describing the data set in (rich) detail, I interpreted the patterns and themes, and articulated implications of the findings. The themes captured something important about the data in relation to the research question, and represented some level of patterned response or meaning within the data set (Braun & Clarke, 2006).

After assembling all data (audio recordings of classroom dialogue and student group interviews and my journal entries), thematic data analysis involved my going through the following interactive and interrelated phases: (1) transcribing audio tapes of

interviews and classroom data; (2) reading and re-reading the data to make sense of the material; (3) sense-making through reading and memoing, and systematically coding interesting features of the data into categories and themes; (4) describing first level patterns/themes (broad categories); (5) analysing and refining each first level theme and identifying second level themes (sub-categories/themes); and (6) interpreting themes and articulating implications of the findings.

Thematic data analysis - Classroom dialogue

Reading and scrutinising classroom dialogue data, I categorised it into the three major levels of focus developed by Cobb et al. (1997, 2001) (classroom social norms, sociomathematical norms and classroom mathematical practices) and described in detail in Chapter 3. Evidence of expectations relating to students being able to explain and justify their thinking or contribution through clarifying or elaborating on it, for example, was classified at the level of social norm. Evidence of expectations related to students being able to provide explanations and justifications based on mathematical reasons or mathematical interpretations was categorised at the level of a sociomathematical norm. In the study, as in Cobb et al. (2001), I identified data related to classroom mathematical practices by discerning student-student and teacher-student interactions concerned with specific ways of reasoning, acting and symbolising for the mathematics topics or ideas that were my particular focus for a lesson.

Having developed the three broad categories, further analysis of data in each of the three main categories led to the identification of a number of sub-categories. For social norms these were students volunteering to share ideas, explaining and justifying, and asking questions. These sub-categories or social norms were distinguished by identifying the focus of a particular classroom event or episode for classroom participation. An episode comprised a sequence of classroom interactions on a topic, so episodes varied in length. Episodes whose focus was about the expectation of students willingly sharing their thinking with others were linked to the social norm of students volunteering to share ideas, while those focusing on clarifying, defending and elaborating on contributions were associated with the norm of explaining and justifying. Episodes that were concerned with the expectation of

students asking questions for clarification or assistance with doing a classroom activity were associated with the norm of students asking questions.

Sub-categories for sociomathematical mathematical norms were mathematical problem-analysis, explaining and justifying mathematically, communicating mathematically, and mathematical questioning. Episodes that focused on reading a mathematics problem for understanding were linked to the sociomathematical norm of mathematical problem-analysis. Those that were concerned with students providing mathematical reasons or mathematical interpretations were associated with the sociomathematical norm of explaining and justifying mathematically. When the focus of classroom dialogue was using mathematical language and symbols to communicate thinking, the related sociomathematical norm was communicating mathematically. The sociomathematical norm of mathematical questioning was distinguished in classroom episodes that focused on students asking questions related to the mathematical idea of the problem.

Under the mathematical practices category, I identified the mathematical practices associated with a topic of counting the number of arrows, multiplying the number of choices (at each stage of the process), and using ratio to distinguish the graph of a geometric sequence, for example. Altogether I identified eight mathematical practices.

Data focusing on classroom social norms, sociomathematical norms and classroom mathematical practices are presented in three separate chapters. Chapter 5, focusing on social norms, deepened my understanding as a teacher and researcher of the ways NESB students can be assisted to know *when* and *how* to participate in classroom activity, and hence, develop social autonomy. Chapter 6 allowed me to better understand how NESB students can be helped to learn mathematics with conceptual understanding, and to develop intellectual autonomy in mathematics. Analysis of the data associated with mathematical practices (Chapter 7) was done separately to enable me to develop deeper understanding of the ways NESB students can be assisted to solve the types of mathematics problems associated with a topic, and contextualised mathematics problems particularly, associated with a mathematics

topic or idea covered by the lesson. Figure 4.1 presents an example of a classroom dialogue data extract, its associated code, and levels 1 and 2 themes.

Data extract	Me: Okay. If no group is volunteering to share their ideas, we will hear what each group has agreed on, starting with Ahmed's group.
Code	Encourage students to actively participate.
Level 1 theme (broad category)	Expectation and obligation regarding classroom participation (Classroom social norm).
Level 2 theme (sub-category)	Classroom social norm of sharing ideas.

Figure 4.1   Classroom dialogue data extract (to do with a classroom social norm), the associated code, and levels 1 and 2 themes

Figure 4.2, below, shows a second example of a classroom dialogue data extract, associated code, and levels 1 and 2 themes.

Data extract	Lily: Common ratio. In graph C we have 4 over 8; or 2 over 4, or 1 over 2. So its geometric sequence. Raymond: …. We multiply by common ratio …. Geometric sequence has common ratio. Its $\dfrac{t_{n+1}}{t_n} = r.$
Code	Describing the process of solving a problem
First level theme (broad category)	Specific way of acting, reasoning, and symbolising (Classroom mathematical practice)
Second level theme (sub-category)	Classroom mathematical practice of using ratio to distinguish a graph of a geometric sequence.

Figure 4.2   Classroom dialogue data extract (involving a classroom mathematical practice), the associated code, and first and second level themes

Next, I worked through transcripts of interviews with students, my journal entries, and copies of student work, to help me add and clarify evidence. Analysing data from multiple sources allowed greater depth to data analysis, making it possible for me to better understand my practice through amalgamating different insights.

Thematic data analysis - Transcripts of interviews with students

Through analysing data from interviews with students, for example, I was able to see what the story is from my students' perspectives. I examined students' views

regarding mathematics teaching and learning. Analysis of these perspectives helped to explain my situation by providing relevant information hitherto unknown to me, the researcher. For example, I identified students' opinions about using contextualised mathematics problems to support student learning. Analysing interview data also enabled me to develop better understanding of some of my students' classroom actions and my practice. Figure 4.3, below, shows an example of a student interview data extract, associated code, and levels 1 and 2 themes.

Data extract	Fiona: The language used in story problems is hard to understand… The teacher should show how the problem is analysed. Khoula: It is difficult to understand the language, sometimes. Ahmed: I agree that sometimes we fail to get the meaning of the question. They [contextualised mathematics problems] are tricky questions because they confuse students… they are difficult to understand.
Code	Contextualised mathematics problems are challenging and hard to understand.
Level 1 theme (broad category)	What constitutes analysing a mathematics problem for understanding (sociomathematical norm)?
Level 2 theme (sub-category)	Sociomathematical norm of mathematical problem-analysis

Figure 4.3   Student interview data extract, associated code, and levels 1 and 2 themes

Thematic data analysis - My journal entries

I analysed my journal extracts retrospectively (Borg, 2001) to get understanding of the events or issues being investigated – to get the bigger meaning. In addition to making it possible to identify and distinguish data relating to establishment of social norms, sociomathematical norms, and classroom mathematical practices, analysis of journal entries allowed reflection on past experiences and helped to build deeper insight into specific aspects of events or the research process. So data analysis evolved in focus and over time. I interweaved descriptions of events and analysis or interpretations. This process enabled me to keep a balance between description and

interpretation. I was able to trace the development of my perspectives and insights across different stages of the research process.

Journal extracts that were eventually selected for inclusion in the research report were based on the themes that emerged from detailed content analysis of the research journal, using analytic methods such as reading and re-reading the journal. I presented those extracts that highlighted significant aspects of issues related to the research. Figure 4.4 displays an example of my journal data extract, related code, and levels 1 and 2 themes.

Data extract:	In reply David said that for part (a) it was 'times because the events are independent' and in part (b) 'times for independent events followed by plus because its event A or B or C.'
Code:	Students seek and provide mathematical reasons to clarify.
Level 1 theme (broad category)	Normative specific to mathematical activity (Sociomathematical norm).
Level 2 theme (sub-category)	Sociomathematical norm of explaining and justifying mathematically.

Figure 4.4   Journal data extract, related code, and levels 1 and 2 themes

Thematic data analysis - Student work

By analysing student work over a period of time, I was able to make claims that I could not have made by viewing a single piece of student work in isolation. For example, I was able to claim that over time students shifted from using the mathematical practice of drawing a diagram and counting the number of arrows, to applying the multiplication principle, when solving problems involving arrangements and selections, individually. Data analysis in this study enabled me to confidently document findings of this study.

My ultimate goal in this study is to achieve disciplined subjectivity rather than objective knowledge (Cochran-Smith & Lytle, 2004; Creswell, 2009; Lincoln & Guba, 1985; Loughran, 2004) of the issues associated with initiating and guiding the development of classroom norms and practices in classrooms with NESB mathematics students. Certainty was not the goal of this study. I consider attaining

trustworthiness of this study as of vital importance. Generalizability, in this study, needs to be clarified.

4.3.4 Trustworthiness and generalizability in this study

I used various strategies to ensure the trustworthiness of my findings: the use of prolonged engagement in data gathering, comprehensive data analysis, triangulation, recognising and taking steps to minimise possible bias, and peer review.

In order to establish the trustworthiness of this study I engaged in prolonged data gathering lasting one year and involving two full mathematics courses. The extended period allowed me enough time to gather sufficient data to enable me to trace changes in my teaching practice and my students' responses and actions, from the beginning to the end of the research period. From this data it was possible for me to identify, select and pay attention to representative events, and avoid distortions that could have arisen from giving unwarranted attention and importance to striking but unrepresentative events if data collection had been done over only part of a course or for a short period. Conclusions were not based on one or two events only, but on instances covering the whole research period. Also, the conclusion that students were changing their classroom tactics was based on me, as teacher, observing that several students were demonstrating change, not just one or two.

To increase the trustworthiness of my study, I also developed and provided a detailed and comprehensive analysis of my actions and those of my students. Clear, thorough descriptions of the setting, participants, classroom dialogue, and the process of data collection and analysis have been used to convey the research account. My students' and my own words, during classroom interactions and in interviews, are extensively quoted to ensure credibility of my findings.

To enhance the trustworthiness of the study, I depended on triangulation through recording the opinions of different students, and using multiple data collection methods. The need and value of viewing events from a number of different perspectives and to establish trustworthiness motivated me to use different techniques to collect data. Furthermore, I wanted to accommodate the changing, complex, unplanned, and ephemeral qualities of lived experience of mathematics classrooms.

So, triangulation involved data sets, gathered over time, using each of the different data collection methods. Through triangulation data gathered by various methods complemented each other, allowing me to create a holistic picture of my teaching practice.   Data from my journal entries, students' work, audio-recording, and interviews were analysed (using known, valid approaches) and interpreted.

Trustworthiness, in this study, was increased through recognizing and taking steps to minimise the influence of possible bias (Menter et al., 2011) I brought to the study. The term *reflexivity* refers to the researcher being aware of and openly assessing the influence of the researcher's own background, preconceptions and interests on the research process (Ruby, 1980, cited in Krefting, 1991), as well as discussing his\her role in a way that respects the participants (Creswell, 2008). Being *reflexive* I was able to reflect on my biases, values and assumptions I could, as a researcher, bring to the study. Early in the research process (see Chapter 1) I described my background, disclosing my personal beliefs about mathematics learning that could shape my interpretations of events, information and the study. Throughout the research period I recognized that my personal preconceptions could affect the research process, and so I prevented this from happening by consciously avoiding them. This action allowed me to produce an open and honest account.

I was reflective about my role and position in the research, that is, teacher and researcher. Self-reflection helped me, in my role as a teacher, to avoid bringing my own biases to the study. I made the dual roles of teacher and researcher explicit, kept them separate, and continued my normal teaching duties as if no research was taking place. Being reflexive meant that my interpretations of events and students' actions remained tentative or inconclusive, allowing room for modification, alternative interpretations or new questions to address, but no room for my own biases.

When it was time for interviews with some students, I openly assumed the role of researcher. My questioning and probing could have been influenced by my preconceived ideas regarding mathematics and how it should be taught and learned. I recognized these issues and made every effort to shun my own bias. To prevent my preconceptions dominating, I, as much as possible, behaved and acted as if I was any other person rather than their teacher, allowing my students to openly share their

perspectives about mathematics teaching and learning, in ways they could have responded had it been any other researcher. I was aware of and openly discussed my role in the study in a way that respected my students as participants.

To avoid my prejudices and improve the trustworthiness of my study, I always recognized that my biases and assumptions could shape my interpretations of events and students' responses. In order to monitor this process, I kept a reflective journal in which I recorded my experiences during the research period. In it I kept a record of my experiences and decisions, my reflections, insights, and challenges, and my interpretations of events, at various stages of the research. A reflective journal enabled me to keep track of the research process, make ongoing decisions, and trace my own reorienting and refocusing in light of the evolution of the research. Being self-reflective enabled me to avoid biases or preconceptions that could affect the research process. This action enabled me to present a reflective account of the research.

Peer review was one of the strategies used in this study to establish trustworthiness of the findings. To be of significance beyond being an idiosyncratic story, my story needed to have a clear and significant face value that is reflected in and resonates with the experience of others in the community (Jaworski, 1999; Mattingly, 1991). My work was peer reviewed during the entire research period by my colleagues who also teach NESB students. It was scrutinized and critiqued in order to ensure that it was credible and resonated with people other than me, the researcher. This process helped to increase the trustworthiness of the study.

Generalizability is interpreted in this study "as generalizability to identifiable, specific settings and subjects rather than universally" (Cohen et al., 2011, p. 220). This practitioner research does not aim to generalize from one setting to the next. Rather, its aim is to develop knowledge and understanding about my teaching practice and my NESB students' mathematics learning, for specific mathematics classes. Rather than being replicated in other settings, I hope my study will be extended by others trialing it in their own local contexts. Its goal is to make the study meaningful relative to my specific situation, that is, *that* particular group of NESB mathematics students. My aim was to have the study relate to other similar situations.

This study is generalizable to theory (Drew et al., 2008) rather than to different research settings. In this study, I provided detailed descriptive data hoping that readers recognize similarities with their own situations.

4.4 Chapter summary

This chapter has outlined the main theoretical influences on the methodology of the study. Quantitative and qualitative research were identified as the two major research orientations in mathematics education; and then compared and contrasted. The key assumptions of qualitative research methodologies were deliberated on. This helped me to appreciate the suitability of applying qualitative research in this study. Naturalistic inquiry, interpretive inquiry and practitioner research are versions of qualitative research that were discussed in relation to their ability to guide the investigation of classroom interactions, in natural settings. Ethical issues associated with practitioner research were identified and addressed. The practitioner research design was adopted in order to investigate joint mathematical activity over a number of lessons. Data collection and analysis methods for practitioner research were discussed. Audio-taping classroom dialogue, journaling, interviews and collection of students' work were the main data collection methods chosen for this study. To help facilitate a better understanding of the ways classroom norms and practices are initiated and guided in mathematics classrooms, an interpretive approach was selected as the method of analysing data. Finally, issues related to trustworthiness and generalizability were clarified and discussed.

Now that I have outlined and discussed the research design for the study, data is presented and analysed in the next three chapters. Data from classroom social norms domain, sociomathematical norms domain and classroom mathematical practices domain are examined for actions associated with initiating and guiding the development of classroom norms and mathematical practices. Separation into social norms, sociomathematical norms, and mathematical practices, is made purely for convenience. It is an academic technique or artificial device I am using in order to highlight particular points. In practice, the three aspects are inextricably interconnected.

CHAPTER 5

DEVELOPING CLASSROOM SOCIAL NORMS

5.1 Introduction

This chapter presents and analyses data associated with the emergence of classroom social norms for student participation in a classroom culture that supports the social construction of mathematics as a social and cultural process. The sociocultural lens led me to conceptualize the constitution of classroom social norms as a process of negotiation and renegotiation. The social norms that were identified include normative aspects of student participation to do with (i) volunteering to share ideas, (ii) explaining and justifying their contributions, and (iii) asking questions to clarify actions and\or ideas made in classroom activities. Quotations from students and my diary entries are used extensively to illustrate what happened in my classroom and ground my interpretation of events relating to the development of my teaching practice. Evidence is presented of shifts that occurred in the ways my NESB students participated in the mathematics classroom that gave rise to the development of student social autonomy in mathematical activity. Student social autonomy is being used to refer to students being able to make independent decisions about *when* and *how* to participate in class. Student views of my mathematics teaching and learning are examined in detail because students co-determine how classroom social norms are constituted.

Although the main focus of this study was contextualised mathematics problems, examples involving both contextualised and non-contextualised problems are used to illustrate actions associated with establishing different social norms in different mathematics contexts and topics.

5.2 Volunteering to share ideas

This section presents data that demonstrates my desire and actions to involve all students in the development of and participation in the practice of volunteering to share ideas. I anticipated that when this became the norm, student participation would move from their being passive to more active participants in classroom teaching and learning, which would enhance their learning of mathematics. In this study, the social

norm of volunteering to share ideas is used to encompass students wanting and being able to share their ideas or thinking with other members of the classroom community in small-group and whole-class settings. The main strategies I used to develop this norm were organizing a mix of small-group and whole-class discussion, nominating students at random to answer questions, and allowing students longer wait-time to reply. Data on this norm is presented and discussed under the headings: my experiences, and student perspectives.

5.2.1 My experiences

In this section I detail seven episodes that illustrate the way I went about establishing the social norm of students volunteering to share ideas during classroom activities. Additionally, the episodes illustrate student actions to develop this norm, as well as the shifts in student participation, from non-active to more active, that occurred during the development process.

Example 5.2.1a: Students resist sharing ideas

This episode demonstrates students resisting the sharing of ideas in small groups and whole class. It also demonstrates my actions to encourage students to learn mathematics through sharing their thinking. The episode happened in the second week of the semester, in the first year of study. Students worked in small groups before joining whole-class discussion on a contextualised arrangements and selections problem, which was displayed on an overhead projector.

The arrangements and selections problem:

> A class of 12 students and the teacher are to go on a class trip. A car will take four of the party, and the rest will go by bus. In how many ways can they all go on the trip if:
> a)   the teacher must go by car?
> b)   the teacher must go by bus?
> c)   teacher can go by either car or bus? (Barton, 2001, p. 53)

While in small groups very few students interacted with each other. Most of the students worked on their own. Instead of asking other students in their group how to solve the problem, some students, for example Carol, chose to consult me. We enter the episode at the start of whole-class discussion.

1.	Me:	How did you solve the problem? Anyone to tell us how their group solved the problem?
2.		[*No response*]
3.	Me:	Can any group share their ideas with us?
4.		[*Silence*]
5.	Me:	Okay. If no group is volunteering to share their ideas, we will hear what each group has agreed on, starting with Ahmed's group.
6.	Ahmed:	We could not solve the problem.
7.	Me:	Just tell us the little you have agreed on, even if you have not completed the solution.
8.		[*No response*]
9.	Me:	Any group to tell us how they solved the problem [*Silence*]. Chen's group. I saw your group's solution. Can you share how you found the solution, please? You can write your ideas on the board.
10.		[*Chen said very little as he wrote his group's solution on the board*]
11.	Me:	Let us have more ideas from other groups [*Silence*]. Yosuki's group, how did you solve the problem? Can you write your solution on the board, as well? [*Yosuki just smiled*]
12.	Me:	Let's hear from Susan's group. Who did you work with, Susan?
13.	Susan:	I solved the problem alone.
14.	Me:	Can you tell us your ideas, please?
15.		[*Susan wrote her solution on the board*]
		(Episode 5.2.1 (22\07\2005) (Class 1): Sharing ideas)

My actions detailed above illustrate my desire to develop the social norm of students wanting and being able to share ideas in whole-class discussion. In contrast, students' actions demonstrate their reluctance to share ideas. Ahmed waited until after I had asked him to share his group's ideas, to indicate that their group had difficulties solving the problem. Even though Chen's group and Susan, working alone, had solutions, they did not volunteer to publicly share their ideas, reflecting an unwillingness to share ideas. Susan's actions, in particular, indicate that she had chosen to work alone rather than learn through sharing ideas with others. The episode indicates that students only shared their ideas after I had nominated them to.

Example 5.2.1b: Nominating students at random

 The journal extract that follows demonstrates my actions to guide the development of the social norm of volunteering to share ideas by nominating students at random to answer questions and suggest ideas. I nominated students to contribute an idea or answer, whether or not they indicated they were willing and or had something to contribute. I did this because on the whole, they were not volunteering ideas or responding to my questions. In my journal entry, which I wrote four weeks into the study in 2005 I noted that in both of the year 1 research study classes very few students were answering my questions.

The journal extract:

> In both classes the majority of students are not showing, by raising their hands, that they want to respond to my questions. So this week, I nominated students at 'random' - making sure that I distributed evenly the opportunity for every student to say something. I did not establish a predictable trend by following a particular order. By nominating students at random, I hoped to avoid a situation where students would think about an answer to the question only if it was their turn to respond.
>
> I noticed that in spite of my consistently prompting students to contribute their ideas, some students just shrugged their shoulders without saying a word when I nominated them to answer. Some students who responded to my questions gave very brief responses, such as "Yes" or "No". In future lessons I plan to continue pressing students to actively participate, asking them "What do you think? Any suggestions?" (Journal Extract (JE) 5.2.1 (01/08/2005) (Classes 1 & 2): Nominating students at random)

Nominating students at random to suggest ideas rather than my telling them the answer was integral to my attempt to promote students' active participation in my class. I considered that asking for their ideas positioned students as valued contributors of knowledge. This journal extract however indicates that some students were unwilling to participate and contribute in classroom activities by sharing their ideas. The extract also expresses my intention to persist in pressing and supporting my students to actively participate by sharing their ideas. By nominating students at random, I wanted to signal that I intended to value every student's contribution to the class. Having worked with this group of students for three weeks, I saw this action as initiating a staged process of sharing with my students the authority to contribute knowledge.

Example 5.2.1c: Planning to encourage students to share ideas with peers

The next journal extract comes from the next day, after two lessons in which students continued to resist participating. It details the further actions I was planning to take to try and achieve a classroom culture where volunteering to share ideas was an expectation and obligation, that is viewed as a norm.

In my journal I wrote:

> Student participation did not meet my expectation in today's two lessons. Although I was nominating both volunteering and non-volunteering students, and encouraging all students to share their thinking, the ones I nominated said very little, if anything. My initial thoughts for the future are:
> - Spend more time pre-preparing mathematics tasks or questions, which students can use for brainstorming in small groups, before whole-class discussion.
> - During class discussions allow longer wait-time to allow students to process their answers.
> -Use mathematics tasks with potential to attract students to want to contribute, e.g. tasks that are new to the students or that require solution strategies that are unfamiliar to the students.
> - Only ask students to contribute to whole-class discussion after they have brainstormed their ideas in small groups for a few minutes.
> - Encourage students to agree on negotiated solutions that are arrived at collectively and collaboratively.
> - Encourage students, in small-group and whole-class discussions, to voluntarily share their thinking and discuss openly what they think with their peers.
> - In these discussions, encourage students to listen to and comment on their peers' answers.
> - Accept and use both correct and incorrect answers from students' oral and written responses (during class discussion) as discussion points for the whole class.
> -Use follow-up questions to challenge misconceptions, and keep discussion alive.   (JE 5.2.2 (02/08/2005) (Classes 1 & 2): Plans to encourage students to share ideas with peers)

This journal extract indicates that I was well aware that there was a mismatch between my students' and my expectations regarding student participation, and that my expectations were not being met. I have noted that the contributions students made, after I had pushed them, lacked detail yet, for me, students elaborating on their mathematical thinking was an important aspect of learning and of sharing ideas. The extract sets out the range of strategies I planned to use in future lessons to encourage and support students volunteering their ideas in support of their own and each other's

learning. These strategies included encouraging students to brainstorm or discuss among themselves in a small group before articulating a considered response and taking part in whole-class discussions, and allowing students longer wait-time to process their ideas. Taken together, they can be seen as aimed at fostering an environment where learning was experienced as a social process involving discussion and debate in a variety of social settings.

Example 5.2.1d: The value of wait-time

This next journal extract shows that when I used strategies like those suggested on 2 August, 2005 above some of my students began to volunteer to share their ideas. In particular, when I used wait-time or waited after I posed a question and allowed time for the students to talk together in small groups before sharing their thoughts as a class, their responses were more fulsome. In the lesson on which this episode is based small-group discussion, involving Class 1 mathematics students, was followed by whole-class discussion of a problem to do with binomial distribution.

Part of the journal read:

> Hoping that students would eventually participate and contribute voluntarily, if I push them a little bit, I continued to nominate volunteering and non-volunteering students and allow longer wait-time so that students could process their ideas before responding. This strategy worked fairly well today in Class 1. In addition to allowing students longer thinking time after posing a question and before expecting them to respond, I also allowed more time for students to work on exercises before asking them to participate in whole-class discussion. This probably made it possible for some of my students to voluntarily offer to contribute ideas and for whole-class discussion to be reasonably lively today. Some students, who in previous lessons did not volunteer to share their thoughts, were responding even when I had not nominated them, for example, Carol, David, Raymond, and Derek. I was able to identify Derek's misconception about how to use the binomial probability formula $^{n}C_{x}p^{x}q^{n-x}$ to solve problems after he volunteered to share his ideas. I was able to identify that Derek's solution was based on the idea that, for a binomial random variable X, $1 \leq x \leq n$, instead of $0 \leq x \leq n$.
>
> I found it hard to generate, on the spot, more challenging- high order questions that entice student response. Next time I should encourage students to participate by probing and asking follow-up questions such as "What if?" or "Why?" I need to generate and find questions that prompt students to want to respond voluntarily. (JE 5.2.3 (03/08/2005) (Class 1): Students willingly share ideas)

There is evidence in the journal extract above that suggests that whole-class discussion following small-group conversation on the same problem can be productive since students are more likely to share ideas during whole-class discussion. In addition, the journal suggests that when the teacher allows longer wait-time and continually encourages the students to participate and contribute, an increasing number of them do so. I note that students sharing their ideas was good for NESB student learning and my teaching because I could identify misconceptions (e.g. Derek's misconception), or any gaps in their knowledge. The extract indicates that at least some of my students were starting to voluntarily contribute ideas and that they were beginning to transition from being non-active to active participants in classroom activities. It also indicates that the way forward for me was to continue to develop my students' willingness and ability to volunteer to share ideas, through prompting and probing questions.

Example 5.2.1e: Students share ideas in small-group and whole-class discussions

Evidence corroborating students' changing attitudes towards greater participation in general, and towards voluntarily sharing ideas in particular, was documented in my journal entry written in the eleventh week of the semester, and a few minutes after a lesson, involving a seedling problem. The problem was displayed on an overhead projector.

The seedling problem:

When planting grass seed, the number of seedlings that sprout is related to the weight of the seed thrown over a large area.

200 grams produce 600 seedlings per m^2
800 grams produce 792 seedlings per m^2
1200 grams produce 858 seedlings per m^2

Draw suitable graphs to investigate the relationship between *x*, the weight of seed thrown, and *y*, the number of seedlings that sprout per m^2.
Hence decide which type of model is most appropriate for this relationship.
Choose between: A) $y = mx + c$; B) $y = ae^{kx}$; C) $y = ax^n$.
(Barton, 2001, p. 323)

My comments in the journal read:

> While working in small groups, some students engaged in lively discussions to reach an agreement on the kinds of graphs to draw and the most appropriate model for the relationship. The pre-prepared logarithm modelling problem (seedling problem) I used today seems to have aroused students' interest for the discussion. However, during the first few minutes of the whole-class discussion that followed small group discussion, I unintentionally allowed the conversation to be dominated by David and myself. On realizing this shortcoming, I encouraged more students to contribute to the discussion by reiterating the value of learning from others' ideas. During the remainder of the lesson a number of students actively participated in whole-class discussion. For example, Fiona, Em, and Chen along with David commented on the model most appropriate for the relationship described in the problem.
>
> Student contributions today enabled me to gain more insight into their thinking, and decide on the next course of action. I need, however, to continue to look for ways of dealing with situations involving non-responses from students, as was sometimes the case in today's lesson. There was a moment's silence, for example, when I challenged my students to explain why $y = mx + c$ or option A, was not the most appropriate model for the data.
>
> Following whole-class discussion, most students continued to share ideas with their peers and were able to solve similar problems without my assistance.
>
> Today's events have encouraged me to continue using more contextualised and open-ended mathematics problems coupled with small-group and whole-class discussions to allow students the opportunity to share ideas and learn. Judging from student work following the small-group and whole-class discussions, it seemed both volunteers and non-volunteers benefited from the discussions that occurred today. (JE 5.2.3 (10/10/2005) (Class 1): More students share ideas in small-group and whole-class discussions)

In this journal extract I note that the contextualised seedling problem used in this lesson generated interest among students who voluntarily engaged in lively mathematical conversations and debates, in their small-group. The entry indicates that over the course of the lesson my students acted in line with the classroom social norm of volunteering to share ideas with each other that I was aiming to establish: the social norm of students volunteering to share ideas was emerging within the class. The extract also reflects that I considered my students were gradually being positioned as debaters and contributors of mathematical ideas. I recorded that once I indicated I was interested in a range of views more students voluntarily provided answers. In addition, my concluding comments demonstrate my commitment and actions to reflect on and continue to pursue changes in my teaching strategies.

Example 5.2.1f: Multiple teaching strategies to develop students' ability to share ideas

The next extract comes from the second year of study and was recorded during the first week of the semester. In the second round of the study I was able to use what I had learned the previous year to more effectively support the development of the norm of students volunteering ideas. This episode is focused on the arrangements and selections problem. I asked students to solve the problem in small groups, later followed by whole-class discussion.

The arrangements and selections problem:

> A woman could run, cycle, sail, or row from Waitangi to Paihia; then swim, row, or sail from Paihia to Russell.
> a) How many different ways are there for her to get from Waitangi to Russell via Paihia?
> b) How many different ways are there for her to get from Waitangi to Russell and back, travelling via Paihia in both directions? (Barton, 2001, p. 47)

After the lesson I wrote the following in my journal:

> Students attempted the solution in small groups for some minutes before I invited them to participate in a whole-class discussion. While working in small groups, very few students shared ideas and asked each other questions. On noticing that not many students in groups were sharing ideas, I made a public announcement highlighting the value of sharing ideas with others when learning mathematics, hoping this would encourage them to also share ideas.

> The idea of writing the problem on the board was a good one. I could see students reading and re-reading the problem and trying to figure out what the problem required. I would like to think that by displaying the problem on the board I encouraged students to read and reflect on the problem, their responses, and ideas.

> During whole-class discussion I allowed 'wait-time' before expecting students to give responses. After posing a question, I allowed a reasonable period of silence to allow students to process their ideas. Then, I would repeat or rephrase the question, and allow more wait-time, if no response was forthcoming. I continued to encourage students to participate and contribute. A few of the students responded by offering their ideas.

> At some stage in whole-class discussion I drew a diagram on the board to assist students' thinking. This diagram appears to have supported whole-class discussion because many students referred to the diagram on a number of occasions during whole-class conversation. For example, Omar said "If we count the number of arrows on the diagram that end at R.", and Muhannad

said "On the diagram we see that we cannot start with three lines..." During this discussion four students (Omar, Muhannad, Thomas and Najilla) students contributed to the solution of the problem. (JE: 5.2.5 (27/02/2006) (Class 3): Multiple teaching strategies to develop students' ability to share ideas)

This journal extract demonstrates my use of a variety of teaching strategies. Being the second year of the study, and a new group of students, the journal extract indicates that I had consolidated my teaching approach to include a number of the strategies I had found to be productive the previous year. The contextualised mathematics problem created a context for productive sharing of ideas, whilst displaying the question on the board allowed students re-read the problem and remain focused on answering the question. Allowing wait-time created room for students to think deeply about their responses. Students could process, revise and refine their ideas before sharing them with classmates. Small-group discussions provided a safe environment for sharing ideas with peers. I acted to encourage students to share the ideas developed in their groups with the whole class anticipating that they would be more confident to share ideas after group discussion. The journal extract identifies that I used a diagram to support student thinking and talk and that I noticed that some students were only able to solve the problem after the diagram had been drawn on the board. These students based their description of their solution on the diagram. The strategy of drawing a diagram for this purpose was an old one revisited because I had found it productive during the first year of study. In sum, this journal extract indicates that in the first week in year 2 I was using multiple strategies to help create a classroom environment that provided opportunities and encouraged students to engage in sharing and processing ideas. The norm of students being able to voluntarily share ideas to support their learning was beginning to emerge.

Example 5.2.1g: Students share ideas after being nominated

This episode, written in week 3 in year 2 of the study demonstrates that students shared their ideas after being nominated by the teacher. It illustrates my ongoing actions to promote students' willingness and ability to share ideas. Like the previous example, this episode is based on Class 3. The episode demonstrates once again that it may take time and effort for students to change from being non-active to active participants in classroom discussions. In this episode whole-class discussion followed small-group work on a contextualised probability problem.

The probability problem:

> Two marbles are drawn without replacement from an urn containing five red and four green marbles. If the random variable X is the number of green marbles drawn, copy and complete the probability table to show the value of P(X = x) for each value of x.

X	0	1	2
P(X = x)	$\dfrac{5}{18}$		

In small groups there was no substantial sharing of ideas among the students. Most students worked individually and only a few worked in pairs. We enter the episode during whole-class discussion.

1.	Me:	In the probability distribution table, why are there only values of x = 0, 1 and 2, but not 3, 4, and so on? [*Silence*]
2.	Me:	Any ideas? What do you think? Thomas.
3.	Thomas:	I don't know.
4.	Me:	Any suggestions? Why do we have only 0, 1, and 2 as the values of x? Omar, what do you think?
5.	Omar:	Number of green marbles can only be 0 or 1 or 2.
6.	Me:	Why only these values? [*Silence*] Sayuri what do you think? [*No response*] Muhannad, any ideas?
7.	Muhannad:	Only two marbles are drawn. So green marbles can only be 0, or 1 or 2, not 3 or more.
8.	Me:	Good. How do we find P(X = 0) $= \dfrac{5}{18}$? [*Silence*] Any suggestions?
9.	Me:	Karen, what do you think? [*No response*] What about Omar?
10.	Omar:	Use a (probability) tree diagram.
11.	Me:	How? Show us on the board.
12.		[*Omar drew a probability tree diagram (with associated probabilities for each branch) and then wrote* $\dfrac{5}{9} \times \dfrac{4}{8} = \dfrac{20}{72} = \dfrac{5}{18}$]
13.	Me:	Now, how do we find the probability when x = 1? [*Silence*] Honda what do you think? [*Honda shrugged his shoulders*]
14.	Me:	Any suggestions? Anyone? Muhannad, what did you agree on in

your group?

15. Muhannad: We say '$\frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{5}{8} = 2 \times \frac{20}{72} = 2 \times \frac{5}{18} = \frac{10}{18}$' [*I wrote what*

Muhannad said on the board, as he spoke]

16. Me: Did you use a tree diagram?

17. Muhannad: No.

18. Me: What do you think about Muhannad and his group's solution?

(Episode 5.2.2 (10/03/2006) (Class 3): Students share ideas after being pressured)

The episode indicates that, on this occasion, questioning (prompting and probing), nominating non-volunteering students, and a contextualised probability problem, together with small-group and whole-class discussion, were the main strategies I employed to continue the development of students' experience of sharing their ideas. Like Class 1 students in the first year of study, this episode indicates that Class 3 students only shared their ideas after I had pressed them to. The episode suggests that, with some encouragement from me as teacher, some of Class 3 students were able to share ideas with others.

Example 5.2.1h: Students volunteer to share ideas

This episode happened in week 10, in year 2 of study, and is based on Class 3, the class in the previous example. It demonstrates students volunteering to share their ideas, without being pressured. In the lesson preceding the one on which this episode is based, the *Central Limit Theorem* and its associated formulae were introduced and discussed. In this episode, the main strategy I used to continue developing the norm of sharing ideas was prompting and probing questioning in a whole-class setting.

The central limit problem:

A sample of 16 is taken from a population X with mean $\mu = 34$ and standard deviation $\sigma = 4$. Calculate the probability that the total, T, of the 16 items exceeds 530. (Barton, 2001, p. 183)

I began by asking the students to read the problem carefully and identify any unfamiliar words, terms and phrases, and write down all the key facts. After a time I asked them to join in a whole-class discussion. We enter the episode at this point.

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1.	Me:	Is there anyone who identified an unfamiliar word or statement? [*No response*] Does this mean that we all know the meanings of all the words and phrases in this problem?
2.	Najilla:	Yes.
3.	Me:	Okay. Let's have the key information, then.
4.	Sayuri	Mean is 34.
5.	Me:	34 is the mean of what?
6.	Sayuri:	Mean of the population.
7.	Me:	How do we know that?
8.	Omar:	Mean μ means population mean.
9.	Muhannad:	μ stands for population mean.
10.	Me:	[*I wrote $\mu = 34$ on the board*] What other important information is given in the problem?
11.	Karen:	$\sigma = 4$. So, population standard deviation is 4.
12.	Me:	Yes. What else is important?
13.	Jessica:	$n = 16$.
14.	Me:	What does *n* stand for?
15.	Najilla:	*n* is the number of items in the sample.
16.	Me:	Now, what are we required to find?
17.	Thomas:	Probability.
18.	Omar:	Probability of T greater than 530.
19.	Me:	[*I wrote P(T > 530) on the board*] So, how do we find this probability?
20.	Muhannad:	Probability of Z greater than… [*I interrupted him*]
21.	Me:	Why do we use Z when we have X in the problem?
22.	Omar:	We use Z so that we can use tables. Tables have Z values.
23.	Me:	So what should I write?
24.	Jessica:	$P(Z > \dfrac{530 - 34}{4})$ [*Muhannad shook his head*]
25.	Me:	Muhannad, why are you shaking your head?
26.	Muhannad:	Mean should be *n* times μ
27.	Me:	Why *n* times μ?
28.	Muhannad:	Because it is the mean of total, T. So it's $n \times \mu$.

| 29. | Thomas: | So we write $P(Z > \dfrac{530 - 544}{\sqrt{16}})$ [*I wrote what Thomas said on the board*] |

(Episode 5.2.3 (18/05/2006) (Class 3): Students voluntarily share ideas)

In this episode there is considerable evidence of the students building on each other's contributions. The dialogue pattern is no longer mainly one of a question or prompt by me followed by a student answer and another question or prompt by me (lines, 8 & 9; 17 & 18; 28 & 29). Student actions indicate that they wanted and were able to share ideas with their classmates. The episode demonstrates that students' willingness and ability to share ideas allowed smooth classroom conversation to take place, in a manner that supported learning. I sustained classroom discussion by prompting for further elaboration (e.g. line 14), by noticing student reactions such as Muhannad shaking his head (line 24) and by recording student ideas on the board (line 29) so that other students could easily refer to them. In this short sequence seven students offered ideas suggesting that the social norm of volunteering ideas was well established – there were fifteen students in the class. This episode contrasts with the previous one at the beginning of the semester when I could hardly get students to voluntarily contribute ideas.

My students' views about volunteering to share ideas were captured during student group-interviews. These opinions are presented in the next section.

5.2.2 Student perspectives

In this section I present data from student group-interviews which demonstrate that my students had mixed views about students volunteering to share ideas. Data illustrate that while some students were opposed to the idea of the teacher prompting students to share ideas by nominating students at random, others supported it. In addition, data is presented that indicate that while some of my students thought that sharing ideas with classmates is not important when learning, others disagreed. Also, this section presents data reflecting that my students valued wait-time. Furthermore, student perspectives demonstrate the need for the teacher to initiate and guide the development of the social norm of volunteering to share ideas.

The teacher should not prompt students to share ideas by nominating them at random

During the formal interviews some students were of the view that the teacher should not prompt students to share ideas by nominating them at random. For instance, in October, 2005, Rita said that she did not support the idea of the teacher asking a student to make a contribution when he or she has not shown, by raising up their hand, that they wanted to contribute. She said "Bad idea because if the student cannot answer he\she feels embarrassed when all the class is quiet and listening. Some students will only prepare for their question and will not listen to questions being directed at other students." In a separate interview, Fiona and Lily agreed with Rita. Fiona was of the view that it did not necessarily follow that students who did not raise their hands did not know the answer "sometimes I just think the answer in my head and I need to translate it into English (before sharing with others) but do know the answer." Lily agreed and added "We did not have that habit (of teacher nominating students at random) in China… it is better for all students to think about the answer." Derek (October, 2005), Deependra (May, 2006), Honda (June, 2006), and Sayuri (June, 2006) supported Fiona, Lily and Rita's idea that only volunteers should be nominated. Derek stated that "In China, the teacher asks those whose hands are up." Sayuri said "No. The teacher should not force everyone to say something. He should ask only those who raise their hands. If my hand is not up it means I don't know." Student views demonstrate their disapproval of the teacher prompting students to share ideas through nominating them at random. Instead, their comments suggest that, from their point of view, students should voluntarily share ideas.

All students should be nominated to share ideas

It also emerged from interviews that some of my students were of the view that all students should have to share ideas, regardless of whether they volunteer to or not. Susan's (October, 2005) view, for example, was that students should take turns to answer oral questions because "It is fair for everybody to answer the questions." David supported Susan by declaring, "The teacher should ask everyone (to share their ideas), not just some students." Like Susan, David preferred taking turns "because everyone has a chance to answer. It's fair to everyone." Carol added, "The teacher should push us to answer the questions…In Taiwan we rarely raised our hands or

answered questions." Statements by students indicate that rather than focus on the value, or lack, of sharing ideas in mathematics problem solving, some of my students were more concerned about the fairness of being nominated to say what they think while others are not.

In another interview in October, 2005, Ahmed and Laila supported the idea that the teacher should nominate both volunteering and non-volunteering students because, according to Ahmed, "Everyone should think about the answer and participate actively." Ahmed and Laila did not support the idea of going along rows or columns when nominating who should respond to a question because according to Laila "…only the person answering the question will think about the answer". Although Raymond and Dong believed that "the teacher has a right to do it [nominate non-volunteering students]," they felt that nominating students in a particular order was not right because, according to Raymond, "some students will prepare for their question and not listen to others." In separate group-interviews, Thomas, Wataru and Max (May & June, 2006) wanted all students to take turns to respond to the teacher's questions or to contribute ideas. Thomas, for example, stated "We must all take turns to contribute". Student comments suggest that they believed that all students should be pressed to contribute by sharing ideas.

During the interviews, some students referred to their prior experiences in other countries like China, Taiwan and Oman, where the teaching\learning styles differed from the one I was proposing. Such references suggest that learning mathematics is a social and cultural activity that varies from one country to the next. These students' views suggest that the classroom culture they had experienced in their home countries had a strong influence on their current mathematics learning. In addition, the views indicate that previous classroom cultures could influence NESB students' willingness to volunteer to share ideas in mathematical activity.

Sharing ideas with classmates is not important

In interviews some of my students indicated that they believed that sharing ideas with classmates is not important. On 14\09\05, Susan declared that she preferred working individually. She said "For me, I want to work on my own.... I don't want someone interrupting me or ask me some question or when his answer is different from my

answer to discuss." In a later interview (26\10\05), Susan reiterated that she did not believe in allowing students to discuss and come up with agreed solutions or shared and negotiated solutions. Susan thought "Some of the students will not be discussing the question, but other things (not related to the question)." During another interview (15\09\05), Raymond claimed that there was no need to discuss questions with peers because the questions were easy, "We can do it by ourselves [without sharing ideas]." Dong agreed with him and added, "We must be able to do it on our own." Susan, Raymond and Dong's comments show reluctance on the part of these students to engage in sharing ideas. These students' comments suggest that they believed that sharing ideas with classmates is not helpful in learning. These students' comments are indicative of the challenge to initiating and guiding the development of the social norm of students volunteering to share ideas. None of the students I interviewed in 2006 indicated that sharing ideas with classmates is not important.

Sharing ideas with classmates is important

In contrast to those students who thought that sharing ideas with peers was not necessary, it came out from the interviews that some students believe that sharing ideas with classmates is important for learning. Rita (26\10\05), for instance, believed in working in pairs because "sometimes questions can be solved by people working together.... Sometimes we can help each other." Yosuke said "Yes, we should discuss ideas in class. We can share [ideas]. Like, if I don't know something, I just ask. Someone may know something I do not know." Rita's and Yosuki's comments suggest that they thought the idea of sharing ideas with peers was helpful for learning.

In another interview (15\09\05), Chen suggested that small group discussion be followed by displaying solutions on the board so that different solution methods could be compared. Chen said "It is better to write students' solutions on the board, with explanations. We can learn by comparing answers. Some students may have different methods." Chen's views suggest that he believed that sharing ideas was useful. David (26\10\05) agreed with those who supported brainstorming, and shared and negotiated solutions. He justified this position by declaring that "Yes, it is good because you can check your ideas in a small group before you give the answer to the

whole class." David's comments indicate that he believed in sharing ideas in a small group and providing negotiated responses or solutions. Lily (26\10\05) also agreed. She added, "Maybe we have different ways to answer the question." Lily's response indicated that she was aware that problems could be solved using different methods, so students could benefit from sharing solution methods. On 31\10\05, Ahmed and Laila supported David's and Lily's views. The views were again supported by Wataru, Omar, Muhannad, and Deependra (May, 2006). Wataru asserted "Yes, good idea. Students learn by sharing ideas." These views partly describe the academic culture I was hoping all my students would eventually transition into – one that values volunteering to share ideas.

Longer wait-time should be allowed to enable students to volunteer sharing ideas

During small-group interviews, some of my students indicated that they needed wait-time to allow them to process their ideas and be able to share them with classmates. Yosuke and Em (15\09\05; 31\10\05), and Yuichi (31/05/06) and Max (06/06/06), for example, asserted that in their mathematics class more time to think could have been allowed, particularly when oral questions were involved. They claimed that absence of adequate wait-time hindered them from volunteering to share ideas. Em declared, "Actually, it is not enough. For Chinese students, they have done the work before, but for us who have not done it before, it is not enough. We need time to think." Similar views were echoed by Honda (May, 2006), Max (May, 2006) and Yuichi (June, 2006). These student comments suggest that they recognised the value of wait-time when sharing ideas. The comments also suggest that with longer wait-time, more students might volunteer to share their thinking with others.

While agreeing with those student views suggesting that wait-time was needed to enable them to share ideas, in October, 2005, Fiona, Lily and Carol, opposed the opinion that not enough wait-time was allowed in their mathematic class. In separate interviews, Chen (15\09\05), Rita and Susan (22\10\05), and Raymond and Dong (26\10\05), also made this point. This data reflects that although there was agreement among my students that wait-time was essential when sharing ideas in mathematics problem solving, there was no consensus on the length of time allowed.

Lack of consensus among students, as reflected by interview data, highlights the challenges in initiating and guiding a process that allows students develop a normative focus on volunteering to share ideas. The dilemma for me as teacher involved determining how much wait-time to allow is one example of this. This predicament was evidence of the diversity in students' social, cultural and mathematical backgrounds. It illustrates some of the challenges of teaching NESB students who have experienced different education systems and classroom cultures. Student comments suggest they were aware some students may have a stronger mathematical background than others; hence need less wait-time because the mathematics involved is familiar. Even though NESB students, in my mathematics classes, agreed with the view that allowing wait-time is necessary, problems may arise when it comes to implementation because of disagreement regarding the length of wait-time allowed.

Student participation also involves explaining and justifying solutions or answers. In this study, my wish was to involve students in the construction of classroom norms that stress explaining and justifying solutions, methods or answers, anticipating this would enhance NESB students' mathematics learning.

5.3 Explaining and justifying

This section presents data that illustrate how I guided the construction of social norms of explaining and justifying actions and contributions made in classroom activities. In this study, the social norm of explaining and justifying was understood to mean students being willing and able to give a reason(s) to clarify and justify a contribution or an idea. I detail the process of developing the norm of explaining and justifying through outlining and interpreting my experiences as portrayed by classroom episodes and my journal. Next, I present and interpret data from student group-interviews.

5.3.1 My experiences

In this section I present journal extracts and a classroom episode that illustrate my aspiration and actions to develop the classroom social norm of explaining and justifying, as a strategy that supports learning. The journal extracts and episode

demonstrate student actions to establish this norm in small-group and whole-class situations, in the mathematics topics of normal distribution, sequences, and logarithm modelling.

Example 5.3.1a: Encouraging students to give reasons

This example demonstrates the way I encouraged students to give reasons during small-group and whole-class discussions. A few minutes before the example started, whole-class discussion focused on the relation between the normal distribution and the standard normal distribution, and the formula, $Z = \dfrac{X - \mu}{\sigma}$. In this example, students first worked in small groups on the normal distribution problem which was displayed on an overhead projector. After group work they discussed the problem as a class.

The normal distribution problem:

> A business woman takes her car to work each morning. The time it takes is normally distributed with mean 28 minutes and standard deviation 3 minutes. She leaves for work at 8.00 am each day.
> a)   Find the probability that she takes less than 24 minutes to travel to work.
> b)   Find the probability that she arrives at work after 8.20 am.
> c)   Find the probability that she arrives at work between 8.25 am and 8.30 am.
> d)   If she starts work at 8.30 am and works, on average, 240 days a year, how many days would you expect her to be late for work in a year? (Lakeland & Nugent, 2004, p. 286)

In my journal I wrote:

> In small groups students shared ideas but I heard only few students providing or asking for reasons to clarify and justify thinking. So, I immediately made a public announcement to the class: "Remember to give and\or request the reasons for contributions that are not clear to you." I repeated this reminder a number of times. During whole-class discussion, Susan, Raymond, Miriam, and David were among those who asked "Why?" I supported students who appeared to struggle to explain, for example, Lily, by probing her. I continued to request and encourage students to give reasons throughout the lesson. At times I had to clarify some students' explanations or provide reasons myself to justify their contributions. Ahmed wanted to know why μ = 28 and σ = 3. David gave the reasons. He said "μ is the population mean which is 28 minutes, and σ is the population standard deviation, which is 3." I asked the class why in part (b) in their solution displayed on the board, Raymond's

group wrote $P(X > 20)$ instead of $P(X \prec 8.20)$. Raymond gave the reasons which I thought required clarification, so I did that.

There was a heated debate when the class was discussing the solution for part (d). I took this opportunity to ask students for reasons to justify their ideas. For instance, I asked Susan "Why do you calculate $P(X > 30)$?" Susan' response was: "To find the probability that she is late (for work)." I also asked Chen "Why multiply $P(X > 30)$ by 240?", and Chen responded by saying "Because she goes to work 240 days." I followed this up with another question: "So why multiply by $P(X > 30)$?" Chen replied "Because it is the probability of being late for work." In addition to me asking questions, I encouraged students to request reasons when necessary. On a number of occasions I said "Are there any questions?" As the lesson progressed and students worked on similar problems, they asked each other the reasons to justify their ideas. (JE 5.3.1 (26/08/2005) (Class 1): Encouraging students to give reasons)

The journal extract indicates my efforts to guide the development of the social norm of explaining and justifying through the use of small-group and whole-class discussions of a contextualised normal distribution problem. It shows that I made use of the strategy of posing questions, such as 'Why multiply $P(X > 30)$ by 240?', to prompt students to give reasons for their ideas. In doing this, I modelled asking for reasons to justify an idea, hoping my students would provide reasons to explain and justify, develop an expectation that their peers provide reasons to clarify and justify, and ask for reasons if they did not. In other words, the extract shows that I encouraged students to ask for or give reasons to justify their or others' ideas.

The next episode focuses on students providing reasons for contributions made. It further illustrates my wish and ongoing actions to encourage students to give reasons to justify their ideas.

Example 5.3.1b: Students provide reasons

In this example I guided students to give reasons while discussing a sequence problem. The lesson began with whole-class discussion focusing on the graphs and limits of sequences. Students then worked on the problem for a few minutes.

The sequence problem:

Decide whether or not $t_n = (-1)^n$ is an increasing or a decreasing sequence. Justify your answer. (Barton, 2004, p. 271)

While working in small groups, students debated the validity of each other's ideas. Some students demanded reasons to clarify and justify their peers' thinking. We enter the episode below when students had finished working in small groups and the whole class was discussing what they had agreed in these groups.

1	Me:	Is the sequence increasing or decreasing?
2	Khoula:	Decreasing.
3	Laila	Increasing.
4	Me:	Khoula, why do you think the sequence is decreasing?
		[*No response*].
5	Me:	What about Laila, why increasing?
		[*No response*]
6	Me	Khoula said decreasing while Laila thinks the sequence is increasing, what do others think?
7	Susan:	Not decreasing, not increasing.
8	Me:	Why? What do you mean by that?
9	Chen:	Because it is sometimes one and sometimes negative one, and on and on …. It keeps going up and down.
10	Me:	And so what is the limit of this sequence?
11	Em:	No limit.
12	Chen:	We can't have one and negative one.
13	Me:	Why not?
14	Susan:	There is only one limit. Not two. So here there is no limit.

(Episode 5.3.1 (06\10\2005) (Class 1): Students provide reasons to clarify and justify)

The statement "Justify your answer" in the task above, indicated to students the need to be able to explain and justify their ideas. My actions, in this episode, showed that contributions could be questioned, clarifications could be sought, and explanations could be demanded in this class. In this episode the sequence problem and whole-class discussion provided an opportunity for me to develop and students to experience the social norm of explaining and justifying. Through questioning, I prompted students to give reasons clarifying their contributions (lines 4, 5, & 13). I went

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beyond students' initial responses to ask Khoula "…why do you think the sequence is decreasing?" (line 4) and Laila "…why increasing?" (line 5), probing for reasons and clarification. My actions illustrate that explanations and justifications were expected in this class - whether the answer is right or wrong.

Chen's (lines 9 & 12) and Susan's (line 14) actions demonstrate that these students were willing and able to give reasons clarifying responses. Chen (Line 12) demonstrated the use of the social norm of volunteering to share ideas to explain and justify responses. In contributing explanations, these students demonstrated that, unlike at the beginning of the semester when I could hardly get any student to voluntarily justify or give a reason for an answer, let alone an extended response, some students were prepared to voluntarily explain and justify, without my persuading them. The data suggests that my wish to induct NESB students into a classroom culture of not just volunteering to share ideas but explaining and justifying them as well was beginning to be enacted.

The next example occurred towards the end of the semester. It is focused on students explaining and justifying in a whole-class discussion of a contextualised logarithm modelling problem.

Example 5.3.1c: Students seek and provide explanations and justifications

This example illustrates how my students participated by seeking and providing explanations and justifications, in a whole-class discussion focusing on a contextualised logarithm modelling problem, displayed on an overhead projector. Students first worked in small groups before taking part in whole-class discussion.

The logarithm modelling problem:

> The resale value in dollars, of a Laptop computer depreciates over time. It can be approximated by $V = 1200 \times e^{-0.015t}$, t weeks after it is purchased.
> a)   What is the resale value of the Laptop computer at the time it is purchased?
> b)   At what percentage rate is the resale value decreasing each week?
> c)   When does the model predict that the Laptop computer would resell for $600?

Part of the journal read:

> As students worked in small groups, on a contextualized logarithm modelling problem (displayed on an overhead projector), they discussed their ideas and some sought reasons for their classmates' contributions. During whole-class discussion, and at different times, Ahmed said "Why is the value $1 200 (in part (a))?", Herb said "Why is percentage rate (in part (b)) 1.5%?", and Fiona said "Why are we finding t (in part (c))?" While responding to a request for an explanation and justification Derek said "Because when new, t = 0, so $V = 1200 \times e^0 = 1200 \times 1.$ Therefore V = $1200." Also responding to a request for an explanation, Raymond said "0.015 is the growth factor or x, in the formula $y = ar^x$. Minus because value is going down. So rate is 1.5 when we change 0.015 to a percentage", and Lily said "Because 'when' means after what time." At various stages of whole-class discussion I reminded students to ask their classmates to provide reasons to justify their ideas. Laila asked for clarification on a number of occasions. Each time she made a request, an explanation was provided by one of the students or by me. Whenever one of the students requested a reason I would say "Can anyone explain 'Why'?" I also encouraged students to give reasons and clarifications for their ideas. When no student would explain, I did. Also, when I thought that a student's explanation was not clear enough or needed emphasis, I clarified or repeated important points. (JE 5.3.2 (18/10/2005) (Class 1): Students seek and provide reasons to clarify and justify responses)

This journal extract indicates that I used the strategy of questioning (prompting and probing) to urge students to explain and justify their ideas. It shows that at least Derek, Raymond and Lily provided reasons to justify contributions. Also, it suggests that, like in previous lessons, the use of a contextualized mathematics problem, and small-group and whole-class discussions, provided an opportunity for students to seek and provide explanations and justifications in a way that supported learning. Fiona, Herb and Ahmed's actions show that they were aware that demanding (in a good way) explanations was expected and acceptable in this class. On the other hand, Derek, Raymond and Lily's actions demonstrated that they could give reasons for ideas associated with a logarithm modelling problem.

Together, the three examples in this section suggest that some of my students progressively became able to give reasons for their own or their classmates' ideas, reflecting that the social norm of explaining and justifying evolved over time.

To sum up, the episodes and journal extracts, in this section, demonstrate that NESB students are prepared to participate and contribute in the development of the social

norm of explaining and justifying when the teacher put strategies in place that create opportunities for students and the teacher to ask for, or provide, reasons to explain and justify their own or others' contributions.

Some of my students' views about explaining and justifying in classroom activities were disclosed to me during the interviews. These views are presented in the next section.

5.3.2 Student perspectives

Interview data presented in this section demonstrates that my students had mixed views about whether it is the responsibility of the teacher, or the students, or both the teacher and students, to provide explanations and justifications of contributions, during classroom activities. In addition, the data indicates that most students expected explanations and justifications, and recognised their importance in learning mathematics.

The teacher or the students or both should provide explanations and justifications

During the interviews I had with students in both 2005 and 2006, students who commented about whose responsibility it is to provide explanations and justifications had varying views. On 13\09\05, Laila, Ahmed and Khoula, indicated that they expected the teacher to explain "step-by-step and work out examples on the board." They, however, supported the idea of students collaborating and sharing ideas in small groups. Also, these three students were in favour of their peers giving explanations, instead of relying on the teacher's explanations all the time. These students' views suggest that they only wanted the teacher's explanation when their peers cannot provide them. They expected the teacher to help, sometimes "It's hard for us to find out on our own, if you have not studied before…. If you cannot understand something, the teacher should explain." These comments suggest that these students recognized the importance of providing explanations in learning.

In a separate interview (13\09\05), with another group of students, Fiona stated that the teacher should "explain words in the question and in the text book…He should explain." Fiona's comments indicate that she expected the teacher to explain. In contrast, as the teacher, I expected students to take a more active role in explaining

while I took the role of facilitator. During the same interview, Lily declared "I understand the teacher's explanations … Teacher's explanations are better." Lily's comments reflected her preference for the explanations and justifications provided by the teacher because of the clarity. In another interview (14\09\05), Rita had a different view from Fiona and Lily's. She stated that she did not believe in the teacher "telling" the students everything "I think students should also explain." She added that she preferred classroom discussions to involve explanations and justifications because "Students should say more, not just yes or no." Rita's comments indicate that, in addition to wanting students to provide explanations and justifications, like Fiona and Lily, she recognised the importance of explanations and justification when learning mathematics.

In a later interview on 26 September, 2005, Fiona reiterated her views about the teacher explaining and justifying. She declared "If we do not have an idea the teacher should explain." Fiona's comment suggests that she wanted the teacher to explain only when she does not know. Em (15\09\05), however, expected the teacher to provide "model answers and explanations so that we can use that for our revision." Rather than emphasize the value of explanations in aiding understanding, Em seemed to be more concerned with using the same method or memorizing the method in order to use it later and get correct answers in future tests and examinations. In contrast to Em's view, Chen (15\09\05) did not want the teacher to write model answers on the board and explain them. He stated that "It is better if we do it together, with clear explanations and clear answer on the board." I note that both students highlight the significance of explanations but differ on how explanations are to be handled. I also note that Chen highlights "do it together." Chen's comment indicates that he valued cooperative learning. Additionally, Chen's comment reflects that he recognized mathematics learning as a social activity, involving explanations and justifications.

Similar student views to those referred to in the previous paragraphs came out when I interviewed students in May and June, 2006. Sayuri, Honda, Thomas, Max, and Najilla all stated that they were in favour of explanations and justifications being provided when they are learning. In May, 2006, Wataru, for example declared "The teacher must explain to us…. Sometimes we get confused. It's important that the teacher explains." These comments suggest that, although Wataru was in favour of

explanations and justifications, he preferred those provided by the teacher. Omar, Muhannad and Thomas (May and June, 2006) did not mind who (teacher or student) provided explanations and justifications, as long as they were "clear explanations." This comment reflects that these students were aware that explanations and justifications were of different quality and clarity.

To sum up, on the whole, student comments suggest that they all agreed that providing explanations and justifications in classroom discussions is important and necessary. However, they had mixed views about who should provide them. Interview data in both years indicated that while some of my students expected me, as the teacher, to provide all explanations, others expected me to help with explanations and they preferred the involvement of all students in providing explanations. Remarks by students reflect a challenge for establishing the social norm of students developing a focus on explaining and justifying. Finally, student perspectives suggest that, for them, clarity of an explanation is vital.

More evidence of my desire and actions to encourage students to shift from being non-active to active participants in classroom activities, through establishing the social norm of students being able to ask questions, is found in the next section.

5.4 Asking questions

Classroom episodes and journal extracts are used in this section to demonstrate my actions to guide the development of the classroom social norm of students asking questions. Also, episodes and journal extracts illustrate student actions to develop this norm in small-group and whole-class discussions, and individual situations, in the mathematics topics of probability, binomial distribution, and sequences. In this study, the social norm of asking questions refers to when a student asks a question or when he\she seeks assistance relating to a classroom learning activity.

Students developing a normative focus on asking questions was an aspect of the mathematics classroom culture I was eager to develop, anticipating this would support their mathematics problem solving. I was hoping that by encouraging NESB students to ask questions I could initiate them into an academic culture that recognizes the importance of asking questions to develop understanding and enhance

mathematics learning. This section presents data that demonstrate the ways my students and I developed the social norm of asking questions of the teacher and peers. This is done under the headings, my experiences and students' perspectives. My experiences around the development of the norm of asking questions are presented first.

5.4.1 My experiences

In this section, I present classroom episodes and journal extracts that illustrate the way I guided the development of the classroom social norm of students asking questions in a way that supported learning. Apart from this, the episodes and extracts demonstrate student actions to establish this norm in situations involving the mathematics topics of probability, binomial distribution, binomial theorem, and differentiation.

Example 5.4.1a: Posing questions in a small group and whole class

The purpose of this example, which happened 3 weeks into the semester, is to illustrate the way I posed questions in a small group and whole class, hoping my actions would encourage my students to ask questions in classroom activities. The topic of probability was introduced to the students in a lesson before the current one. In that lesson, students solved non-contextualised problems involving probability. In this example, students first worked on a contextualised probability problem, which was displayed on an overhead projector, before engaging in whole-class discussion.

 The probability problem:

> Three students are working independently on a probability problem. The probabilities that they will solve the problem are 0.2, 0.25 and 0.4. What is the probability that the problem will be solved? (Lakeland & Nugent, 2003, p. 109).

Part of my journal read:

> As students solved the probability problem in groups I noticed that there was very little progress and talking going on. I made a public announcement requesting students to work together, discuss and ask each other questions relating to the classroom activity they were focusing on. Even after the announcement there seemed to be nothing much going on in Ahmed, Laila and Khoula's group, so I joined them and posed some questions: "What is

My journal notes indicate that I paid close attention to asking questions in a small group and in a whole class. When Ahmed asked me how to solve the problem, I took this as an opportunity to model questioning that would help understand the problem. I again modelled questioning during whole-class discussion. The journal entry shows that Ahmed only asked me a question after I moved to his group and enquired about the group's progress. The journal indicates that during whole-class discussion, my students were responsive to my questioning.

The next example occurred about a month after the previous one and focuses on the way an individual student asked a question involving a binomial distribution problem. It also shows how I responded to the student's request.

Example 5.4.1b: Individual student seeks assistance/asks a question

This example is used to demonstrate the way I encouraged a single student, Susan, to take time to think about how a problem can be solved before rushing to ask me for assistance. In addition, it illustrates my actions, involving one student, to guide the development of the social norm of students asking questions. Furthermore, the example illustrates that, over time, some students can understand the importance of asking questions.

Part of my journal read:

> Towards the end (about 3 minutes from end) of today's lesson I assigned some homework to students but almost immediately Susan said: "Sir, how do we solve this problem?"
>
> The binomial distribution problem was:

'A person completes 35% of the crosswords she attempts. If she takes 9 away with her on holiday, find the probability that she does not manage to complete half of them.'(Barton, 2001, p. 69).

My immediate response was: "Go and try to work out the solution at home. Then bring your solution to class tomorrow and we will work together on deciding whether your solution is valid." (JE 5.4.2 (13/09/2005) (Class 1): Susan asks for assistance)

This journal extract indicates that, although I wanted to position NESB students as questioners, I wanted them to engage in thinking about how they could solve the problem, before rushing to me, as the teacher, for the solution. Susan's actions indicate that she wanted to immediately 'see' a solution to the given problem. When this did not happen, she quickly sought my assistance. In contrast, my actions indicate that I wanted Susan to spend more time attempting the solution on her own first, and only then ask for help. I wanted to encourage her to develop the habit of thinking about how she could solve the problem on her own, rather than expect to be told by me. I wanted students to ask more than the general, 'how do I do this?', type of question and ask more analytical questions. My suggestion to think overnight was to allow wait-time, to enable Susan to do some thinking. I did not mean to discourage Susan from seeking assistance. After she had attempted the solution herself, we would work together on deciding whether her solution was valid.

This journal extract is an example of how the teacher can be faced with the difficulty of maintaining a balance between assisting a student who asks for help, and encouraging a student to think before they ask questions.

The next example occurred during the second year of the study. It focuses on a student who wanted only the teacher to address her question.

Example 5.4.1c: A Class 3 student wanting to direct her question only at the teacher

The purpose of this example is to demonstrate a Class 3 student wanting to direct her question only at the teacher. Additionally, it illustrates that challenges for the teacher can arise when this happens. The episode illustrates that the use of a binomial problem, and small-group and whole-class discussions created a situation requiring students to ask questions.

In an earlier part of the lesson, I had guided a whole-class discussion focusing on what Pascal's triangle is, its use in mathematics, and its relationship with the binomial theorem. In this episode students solved a binomial problem, written on the board, in small groups before discussing their solutions as a class.

The binomial problem: Use Pascal's triangle to expand $(x+y)^4$.

We enter the episode during whole-class discussion of the binomial problem. Several students made contributions and I wrote them on the board. I had just written the statement $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$ when Najilla interjected.

1. Najilla: How do we find this?
2. Me: What do you mean, Najilla? Can you be more specific?
3. Najilla: Where does the expression come from?
4. Me: Right, Najilla is asking: "How do we find this expansion?" Omar, can you answer Najilla's question, please? How do we get this expression?
5. Najilla: No. I want you to explain.
6. Me: Why me, Najilla? Your friends can explain. Your classmates have just told me to write this expression on the board, so I think they can answer your question.
7. Najilla: No. I want you to answer because you are the teacher.
8. Me: [*After discussing with Najilla for a little while, I provided a detailed explanation*]
(Episode 5.4.1 (16/03/2006) (Class 3): A student requests to ask only the teacher)

Data suggests that Najilla recognized and was prepared to use the norm that attaches importance to asking for help. She asked questions (lines 1 & 3). Najilla's other actions, however, demonstrated that she valued teacher explanations more than those of her classmates or that she thought this was a teacher responsibility alone. This situation suggests that Najilla might lack confidence to ask her classmates questions when solving a mathematics problem. Data indicate that I wanted to involve other students in addressing Najilla's question. My hope was that this would encourage other students to ask each other questions. I gave in to Najilla's demand, albeit

reluctantly, since my wish had always been to encourage students to ask each other questions and share ideas, during problem solving.

For mathematics teachers trying to change class norms, the data demonstrates that unanticipated challenges can occur while guiding the development of the norm of students asking questions of each other. The dilemma for me was whether or not to give in to Najilla's demands and offer the explanation myself. Such situations may force a teacher to behave in ways they might not believe in. I offered an explanation myself, in spite of having, in previous lessons, encouraged students to ask each other questions and also explain their solutions for the benefit of other students, and themselves.

The next example happened in the second year of study and focuses on students asking each other questions in a whole-class discussion.

Example 5.4.1d: Students ask each other questions in whole-class discussion

In this example I demonstrate how students asked each other questions in whole-class discussion involving a differentiation problem. It demonstrates more evidence of my actions to promote students' ability to ask questions in a manner that supports learning. After the lesson I wrote my reflections in a journal.

The journal extract:

> In today's lesson I asked my students to work in small groups and answer the question: 'Find the turning points for the function $4x^3$-12x-8, and determine the nature of each point'. The question was displayed on an overhead projector.

> After a while Wataru said: "Sir, I can't find the turning points. How do we find them?" I immediately interrupted the whole class and invited all students to assist Wataru. During whole-class discussion students debated several suggestions before agreeing that the intercepts were at points (0,3), (-1,0) and (3,0). When one of the groups gave (2,-27) and (-1,0) as the turning points, other students disagreed. During the debate students asked each other questions. For instance, (1) Deependra said "How did you get (2, -27)"; (2) Ray said "Why do you say $4x^3$-12x-8 = 0?"; (3) James said "Why do we use the remainder theorem?" and (4) Snowy said "How did you get (0,3)?". I also asked questions. On probing, I found out that some students had wrongly factorized $4x^3$-12x-8. (JE 5.4.4 (15\05\2006) (Class 4): Students ask questions)

The journal extract suggests that Wataru demonstrated that he could use the social norm of asking questions. It indicates that he felt comfortable to openly declare that he could not "find the turning points."   Given this episode took place late in the course, the journal extract indicates that I had created a safe environment in which students were not afraid of being ridiculed for not knowing. Instead of engaging in a discussion with Wataru, I asked class help for him. In doing this, I redirected Wataru's question to other students, to keep the whole class involved. This way, I conveyed that students had ideas that were valued and that every student was expected to help to answer questions. In addition, I distributed the authority to ask and answer questions to my students. Wataru's decision to seek assistance in a public way benefited not only him, but other students as well. Wataru's question led to a situation where students shared ideas to find a solution. Consensus on the intercepts was arrived at by negotiation with the whole group. Further monitoring of students' ideas enabled me to discover that students' difficulties were to do with failure to factorize the expression $4x^3$-12x-8. The journal extract indicates that when the lesson happened (towards the end of the semester) students demonstrated the use of the social norm of asking questions, in a manner that supported learning. Furthermore, it indicates that the students who contributed by asking questions recognised and used the social norm of volunteering to share ideas, to ask each other questions.

When I asked my students how the expression $4x^3$-12x-8 could be factorised, Deependra was the first to volunteer a response.

1.	Me:	How do we factorise this expression? [*Pointing at the expression on the screen*]
2.	Deependra:	Remainder Theorem.
3.	Me:	Yes. Remainder Theorem. How do we use it? How do we use the remainder theorem to factorise $4x^3$-12x-8?

(Episode 5.4.2 (15\05\2006) (Class 4) : Asking probing questions)

This episode shows that as soon as Deependra offered his idea (line 2), I supported him by providing an instant replay of what he had said. Instant replay was intended to support Deependra and other listeners' thinking. I then used Deependra's response to advance discussion by asking "How do we use it? How do we use the remainder

theorem to factorise $4x^3$-12x-8?" (line 3). The data indicates that Wataru's question and Deependra's contribution created an opportunity for me, as teacher, to model asking questions.

Student generated questions were elements of the social and cultural environment which I was hoping to create with my students. I believed that students asking questions influenced both the construction of norms that value student participation and NESB students' mathematics learning. It was, therefore, important for me to know what NESB students thought about my aim for them to ask questions. During student group-interviews I was able to glean student perspectives. Some of my students' views about students asking questions are presented next.

5.4.2 Student perspectives

This section presents interview data regarding students asking questions in classroom activities. The data demonstrates that my students were of the view that the teacher should create opportunities for students to ask questions. Additionally, it demonstrates that although students agreed on the importance of asking questions when learning, they disagreed on whether student questions should be responded to by the teacher or by other students. Also, the data indicates that some of my students preferred to direct their question at a friend before the teacher. Finally, it demonstrates that some students felt more confident to ask questions in a small group or after class.

The teacher should create opportunities for students to ask questions

During small group-interviews, there was consensus on the importance of questions in classroom activity, and the need for the mathematics teacher to create opportunities for NESB students to ask questions. Raymond (15/09/05), for example, disclosed that he expected the mathematics teacher to allow students to ask questions. He declared "Give us more time to ask questions. It's important. We ask questions because we do not know. We have questions... We learn and ask... We also learn by asking questions...." Dong and David, who were in the same group-interview as Raymond, agreed with him. David added, "Yes, it's a good idea. Sometimes the students do not know how to do some questions so they should be allowed to ask questions." The

same ideas were echoed in separate interviews, for example, by Fiona and Carol (26/10/05); Susan and Rita (26/10/05); Em, Yosuke and Chen (31/10/05); Honda and Sayuri (02/06/06); and by Max (06/06/06). The data suggests that, these students recognised the value of asking questions in mathematics learning. It further suggests that, my students expected the teacher to create opportunities for students to ask questions.

Student questions should be responded to by the teacher/by other students

Participating students had differing opinions about whether student questions should be responded to by the teacher or by other students. Raymond, David and Fiona, for example, were not much concerned about who answered student questions. They expected, first and foremost, to get answers from the teacher but, according to Raymond, did not mind if "other students answer because they may know the answer to the question." Raymond's comment suggests that he valued his classmates' ideas. Rita and Susan disagreed with this idea. They believed that students should direct their questions to the teacher because, according to Rita, "If you ask a friend, maybe he also does not know the answer." Rita's view was supported by Em (31/10/05) who declared that he preferred to direct his questions to the teacher and get the answer from him "because the teacher can express exactly." Em's remark reflects that he believed that the teacher's explanations were clearer, and hence, better than those of his friends. Susan was more candid in her response. She stressed that she did not expect the teacher to request other students to provide an answer to her question. In a separate interview, Derek agreed with Susan's view. His argument was "The teacher should answer student questions, not other students, because he gives you a better answer." Rita, Susan, Em and Derek's views indicate that some of my NESB students had confidence in the teacher's answers and explanations but not those of their peers. For them, the only reliable source of knowledge was the teacher, who had the ability to help them learn mathematics.

Prefer to direct a question at a friend before the teacher

Some of my NESB students stated that they preferred to ask their friends first before they ask the teacher. Others would only ask the teacher if their peer(s) could not answer their question or offer assistance. Yosuke (31/10/05), for example, would

consult the teacher only when his friend failed to help him "because it is easy to talk to him." Yosuke believed that other students could explain in simpler, easier to understand language than the teacher who he believed would use specialised, not easy to understand language. He claimed "Teachers use specialised language, which is not easy to understand." Chen, who attended the same interview, preferred to have his questions addressed by his Chinese friends because "it is easier to communicate in Chinese." Chen was, however, in agreement with the idea of inviting the whole class to deal with a student's question because, according to him, some students could be struggling with the same problem "but were shy to tell you [the teacher]." On 26\10\05, Fiona, like Yosuke and Chen, stated that she preferred to seek assistance from Lily (her friend) first before asking the teacher. She declared "My friend because I think she is better in mathematics and we can talk in Chinese. I think Lily can understand my problem better." Lily agreed and added, "If we can do it we will. If not, I ask the teacher...We want to talk to a friend in Chinese." Student comments demonstrate language issues experienced by my students that may prevent them from asking questions.

During a separate interview (26\10\05), Raymond agreed "if he [friend] cannot answer, I ask the teacher." This view was supported by Susan (26\10\05) and by Ahmed (31\10\05) who said "My friend first because if he knows something he will tell me. Same education level, same thinking, so I can understand." In the same interview, Laila said that she felt more confident interacting with her peers. Laila preferred to "ask anyone in class" but Khoula thought that the teacher's assistance was also essential. Carol (26\10\05) had other ideas. She said "I do not want to ask someone. I want to try to solve the problem myself. If I still do not have an idea I will ask my friend before asking the teacher." Carol's response indicates that she wanted to achieve a balance between asking questions and, exploring and relying on her own thinking. A similar spread of comments emerged when I interviewed other students in 2006. Ray, Muhannad, Najilla, Omar, Yuichi, and Max, for example, stated that they preferred to direct their question at a friend before the teacher, reflecting their confidence in their peers' ability to address their questions.

More confident to ask questions in a small-group setting or after class

There was a group of my NESB students who were more confident to ask questions when students were working in small groups or after normal lessons, rather than during whole-class discussions. For instance, during a student group-interview in 2006 Honda stated "We can't ask questions (during normal lessons) but in the extra time we can ask questions like, 'How could you do this?' and we can ask many times." Sayuri, who was attending the same interview, emphasized that extra time, separate from normal class time, should be created to allow NESB students to ask questions. Both, Honda and Sayuri, believed that time for student questions should be timetabled to allow NESB students to ask questions, in small groups. Honda and Sayuri's remarks indicate even though they recognized the importance of student questions in their mathematics learning, they lacked the confidence to ask questions during whole-class discussion. The comments also reflected a need to establish the norm of students asking questions.

In summary, student comments, during interviews, suggest that, for them it was important to be given the opportunity to ask questions and for their questions to be responded to by the teacher or other students. Additionally, student comments suggest that some of my students may be discouraged or prevented from directing their questions to people other than those who speak the same language as them, by limitations in their English language proficiency. The comments also reflect that some students may lack the confidence to ask questions in whole-class discussions. Overall, student comments indicate the challenge to develop the social norm of students asking questions in classroom activity.

5.5 Chapter summary

This chapter has presented and analysed data that demonstrate my desire and actions to initiate and guide the construction and development of classroom social norms that value and provide a safe environment for student participation. The social norms for participation I initiated and promoted were volunteering to share ideas, explaining and justifying, and asking questions.

Analysis of student interview data suggests that there is no consensus among NESB students regarding their expectations of how mathematics should be taught and learned as a social process, nor about their obligations to volunteer to share ideas, explain and justify contributions, and ask questions.

Data indicated that a focus on developing the social norm of students volunteering to share ideas, gave rise to more effective small-group and whole-class discussions and greater communication of student ideas. It indicated that there was greater student participation and contribution of ideas. Students experienced learning mathematics as a social process.

Regarding the social norm of students explaining and justifying, data showed that the development of this norm allowed clarification of ideas to be expected and sought by students. Reasons or explanations and justifications were provided publicly by the students. Data showed that my students sought and learned from explanations and justifications, were held accountable for their contributions by classmates and me, as the teacher (see Examples 5.3.2, 5.3.3 & 5.3.4). It indicated that, progressively, my students offered explanations and justifications without being pressured, and experienced and shared authority (see Example 5.2.8).

As for the social norm of students asking questions, the data indicated that its development became a stimulus for engagement in small-group and whole-class discussions, fostered active exploration and learning (of mathematics), and made it possible for student questions to be used as a tool for teaching and learning. In addition, the data indicated that, over time, my NESB students developed the ability to formulate and ask questions and learned to use student questions as a learning tool and developed the ability to formulate and ask questions.

Different student perspectives, as reflected by interview data, underscored the challenges in initiating and guiding a process that allows NESB students to develop normative focus on sharing ideas, explaining and justifying, and asking questions (see sections 5.2.2, 5.3.2 & 5.4.2). Interview data indicated that, in spite of having mixed views about whether it was the teacher's, or the students', or both the teacher and students' responsibility to explain and justify contributions, students expected explanations and justifications, and recognised their importance. In relation to the

norms in this chapter, the data also reflected language demands experienced by NESB mathematics students as needing to be addressed (see section 5.4.2). In addition, data reflected some NESB students' gradual acceptance of a classroom academic culture that values sharing ideas, explaining and justifying, and asking each other questions, in classroom learning activities.

Regarding teacher change, data indicated that I changed my teaching strategies as time progressed. Changes in teacher practice, as reflected by the data, included spending more time pre-preparing mathematics tasks and focus/key questions (section 5.2.1; section 5.3.1 & section 5.4.1), displaying focus questions and student work on the board or overhead projector (e.g. examples 5.2.1g; 5.3.1c; & 5.4.1a), and making use of multiple teaching strategies to develop students' ability to share ideas (see journal extract JE 5.2.5; examples 5.2.1c & 5.3.1c). Other changes in my teaching practice, portrayed by the data are teacher move to (1) creating a classroom environment where students felt safe to articulate their thinking through organising small-group to whole-class discussions (e.g. example 5.2.1b; example 5.2.1e); (2) allowing students longer wait-time (e.g. examples 5.2.1b; 5.2.1c; 5.2.1d; 5.2.1f); (3) making use of more prompting and probing questions to encourage students to share their thinking and to explain and justify their thinking (e.g. example 5.3.1a; example 5.3.1b); and (4) creating more opportunities for students to ask questions (e.g. JE 5.4.1 in example 5.4.1a). The data presented illustrated that I moved from first nominating all students at random at the beginning of the semester and then only volunteers (see examples 5.2.1c; 5.2.1g & 5.2.1h), and when I began to consciously work to overcome challenges, for example, striking balance between assisting and avoiding 'telling' (see example 5.4.1b).

Classroom social norms alone do not, however, provide adequate support for NESB students' mathematics learning. They are concerned with the general social aspects of the classroom, yet NESB students need support on specifically mathematical aspects of the classroom. Furthermore, based on the sociocultural perspective adopted for this study, social norms and sociomathematical norms are complementary. It is therefore logical that while developing social norms, attention should be given to the development of sociomathematical norms. Chapter 6 presents and analyses data

relating to the initiation and development of sociomathematical norms for promoting conceptual understanding.

CHAPTER 6

DEVELOPING PRODUCTIVE SOCIOMATHEMATICAL NORMS

6.1 Introduction

This chapter focuses on the development of sociomathematical norms to do with mathematical problem-analysis, mathematical explanation and justification, and mathematical communication. I demonstrate the ways by which the students and I, as the teacher, collectively negotiated these norms, during ongoing classroom interactions. The episodes used are not meant to be typical or to reflect an ideal classroom practice. Rather, they have been chosen for their power to clarify and explain important aspects of how productive sociomathematical norms were negotiated in my classroom across a range of mathematics content areas.

Since sociomathematical norms are concerned with normative aspects of classroom discussions specific to mathematical activity and the evolving criteria for what counts as acceptable mathematically, I have presented results that show the ways I guided students to develop taken-as-shared understandings of what constitutes mathematical problem-analysis, explaining and justifying mathematically, and communicating mathematically. Data are provided that demonstrate the development of these sociomathematical norms as enabling coherent mathematical discussions to occur and to create mathematical learning opportunities for the students. I argue that productive sociomathematical norms are emergent and communal phenomena, rather than individual. In addition, the data demonstrate the shifts that occurred in students' mathematical beliefs and values as they participated in the negotiation of sociomathematical norms. I illustrate the ways in which NESB students' social and individual mathematics learning can be supported through the development of these norms.

6.2 Mathematical problem-analysis

In this section, I present five episodes that demonstrate the ways I initiated and guided the development of the sociomathematical norm of mathematical problem-analysis. The episodes illustrate that in this study, mathematical problem-analysis came to mean students developing (i) a normative focus on reading the problem

carefully, (ii) identifying and finding the meaning of unfamiliar words, terms and phrases, (iii) identifying key information and (iv) then not just summarizing but thinking through how the information related to the problem to be solved. In addition, the mathematical problem-analysis norm came to mean (v) students asking themselves what the problem required them to find. I document the evolution process of the norm of mathematical problem-analysis by describing and interpreting my experiences, as depicted by classroom episodes. I triangulate my experiences from classroom episodes, based on audiotapes of classroom discussion and my journal with data from student group-interviews.

6.2.1 My experiences

In this section, I detail classroom episodes and associated journal extracts that demonstrate my desire and actions to develop the sociomathematical norm of mathematical problem-analysis, in a manner that supported my students' learning. The episodes also illustrate student actions to develop this norm in whole-class, small-group and individual student settings in the mathematics topics of normal distribution, binomial distribution, arrangements and selections, confidence intervals, and modelling problems. The classroom episodes and journal extracts demonstrate how students learned what constitutes a useful process of a mathematical problem-analysis. That is, a problem-analysis as a sociomathematical norm involved identifying the vital information embedded in a mathematical problem description; finding the meaning of unfamiliar words, terms and phrases; thinking through how the key information related to the mathematics problem being posed; and identifying what the problem requires students to find.

Example 6.2.1a: Whole-class analysis of a contextualised normal distribution problem

The purpose of this episode is to demonstrate the way I guided the whole class to collectively analyse a contextualised normal distribution problem. The episode occurred a few minutes into the lesson on 25 August, 2005. In previous lessons, students had studied basic concepts of the normal distribution and the standard normal distribution. That is, main properties of a normal distribution, parameters of a normal distribution, notation used, standard normal tables, and the relationship

between the normal and standard normal distributions. During these lessons, I always encouraged my students to analyse a mathematics problem to understand what was required by the problem. On this occasion the problem was about a phone company.

The phone company problem:

> A company is considering setting up a long distance phone company. From the investigations it has been found that the length of long distance phone calls is approximately normally distributed with μ = 12.5 minutes and $\sigma = 5.4$ minutes.
>
> Find the probability that a randomly selected long distance call is between 10 and 15 minutes. (Lakeland & Nugent, 2004, p. 265)

The episode opens when the whole class was analysing the problem to understand what was required.

1.	Me:	Looking at the problem on the board, what key information can you extract or identify?
2.	Hao:	Normal distribution.
3.	Me:	What do we know about the normal distribution? What important information about the normal distribution is in this problem?
4.	Rita:	Mean is 12.5.
5.	Me:	Mean of this normal distribution is 12.5. Anything else?
6.	Ahmed:	Standard deviation is 5.4.
7.	Me:	Yes. What other information can we deduce?
8.		[*Silence. No response*]
9.	Me:	Someone mentioned normal distribution. In this case, is there anything we know about this normal distribution?
10.	Chen:	X is a variable, I think.
11.	Me:	I do not see X in the question. Where is X coming from? Or can we use X to represent something? What does X stand for?
12.	Rita:	Length of calls. X = length of calls. X is normally distributed.
13	Chen:	So we want probability of X between 10 and 15
14.	Me:	What do you mean by "X between 10 and 15"?
15.	Chen:	X = 10, 11, 12, 13, 14, 15. Length of calls

16. Me: Good. Let us write down a summary of the information we have
                identified. What about fractions? X = 11.25, for example?

This episode demonstrates the way I guided the establishment of the sociomathematical norm of analyzing a mathematical problem to identify key information, and then thinking through how the information related to the problem that students were required to solve. My action in asking the opening question (line 1) was to prompt my students to read the problem carefully to understand what was involved in the problem. I continued to probe students for more ideas about the focus of the problem – the nature of the normal distribution, the variables, and what the problem required students to find (lines 3, 5, 7, 9). The episode shows that at least those students who contributed to the discussion, could read the problem to identify key information and relate it to the problem and decide what they needed to calculate. Chen's contributions in lines 13 and 15, for example, suggest that he had analysed what the problem required him to do.

The next example focuses more closely on how an individual student went about the process of problem analysis involving a contextualised binomial problem.

Example 6.2.1b: Individual student analysis of a contextualised binomial problem

This example demonstrates the way I guided a single student, Fiona, to analyse a contextualised binomial problem. This episode happened a few minutes after whole-class discussion of the four conditions for a random variable X to have a binomial distribution. These conditions were that there are only two possible outcomes for each trial; there is a fixed number, n, of identical trials; the probability of a success, p, at each individual trial is a constant; and each trial is independent (Barton, 2001, p. 65). The class discussed the meaning of each condition and the need for students to be able to identify and apply it appropriately when it was directly or indirectly referred to in a mathematics problem. The binomial probability formula, i.e. $P(X = x) = {}^{n}C_{x}p^{x}q^{n-x}$ for $0 \leq x \leq n$ (Barton, 2001, p. 65) was discussed and its legitimacy was collectively explained and justified, using mathematical reasoning.

We enter the episode as students are finding the solution of the soccer striker problem, some individually and others in small groups.

The soccer striker problem:

> A soccer striker scores in one game in three, on average. Find the probability that he scores only once out of his next seven games. (Barton, 2001, p. 67)

The students had been working on the task for a few minutes when Fiona called me over and asked me a question.

1.   Fiona: What does this mean? [*Pointing at the statement: 'Find the probability that he scores only once out of his next seven games.'*]

2.   Me: What do you think?

3.   Fiona: It means probability X = 7.

4.   Me: Look at the phrase 'scores only once.' What does this phrase mean?

5.   Fiona: One.

6.   Me: What about the one?

7.   Fiona: Probability X = 1. … [*Pause*] ….in seven games.

8.   Me: So what do we want to find?

9.   Fiona: Probability X = 1.

10.   Me: What about 'seven games'? Do we need this information?

11.   Fiona   Yes, number of trials.

12.   Me: Can you do the rest on your own now?

13.   Fiona: Yes.

(Episode 6.2.2 (14/09/2005): Soccer striker problem)

In the episode, Fiona's action in asking a question suggests that she could not find the solution because she did not understand the question. Fiona did not appear to be expecting me to find the solution for her, rather she seemed to want me to explain what the question meant so that she could find the solution herself (lines 1, 12, 13). Instead of telling Fiona the solution, I supported her by asking a sequence of questions to help her analyse the question. When I said "What do you think?" (line 2) I was encouraging Fiona to think and contribute to the problem-analysis. Her reply

suggested she had read the problem but incorrectly attached 7 with the probability of scoring a goal. By prompting her to "Look at the phrase 'scores only once'" (line 4), I focused Fiona's attention on an important part of the question to support her analysis of the problem. I continued to encourage her by giving her clues to identify key information, relate it to the problem and ask herself what the problem required her to find (lines 6, 8 &10). Fiona's responses indicate that she had read the question. As we collectively analysed the problem, Fiona made attempts to relate key information to the problem. After getting some support from me to analyse the problem, Fiona appeared more confident to complete the solution on her own.

Example 6.2.1c: Student public analysis of a contextualised arrangements and selections problem

This episode illustrates the way I guided four students through a public (in front of the whole class) analysis of a contextualised arrangements and selections problem. The episode occurred during a tutorial class at the beginning of Semester A, 2006 in the second year of my study. Tutorial lessons, in the second year, were organized so that students attempted solutions to problems I gave them, either individually or in small groups. They could also ask me for assistance, when they needed it. This arrangement was new to some students and based on what I learned about my teaching in my first year of study. Its aim was to support my students' mathematics learning by adding a dimension that allowed students to consolidate what they learned in a particular topic.

In the lesson prior to the tutorial, we had covered the meanings of the terms 'permutation' and 'combination' and the formulae: $^nP_r = \dfrac{n!}{(n-r)!}$ and

$^nC_r = \dfrac{n!}{r!(n-r)!}$.

A few minutes before this episode, students, in small-groups, had solved the arrangements and selections (number of valid votes) problem which read:

A graduate of a university is allowed to vote for up to three candidates for three vacancies on the University Council. If there are seven candidates altogether, in how many ways can the graduate cast a valid vote? (Barton, 2001, p.53)

As was usual when they were working in small groups, most students worked quietly or just listened and watched the few who did the talking and writing down solutions. We enter the episode when I had interrupted group work and asked all students to participate in a whole-class discussion of the problem. I took this action after one group of students told me that they could not solve the problem. I began by focusing students on the phrase I had heard students in a small group debating the meaning of.

1.	Me:	What do you understand by "up to three"?
2.	Thomas:	Can be one?
3.	Me:	Is that all?
4.	Muhannad:	One or two.
5.	Me:	Someone has said one or two, what do others think about this answer?
6.	Omar:	I think it means one, two or three.
7.	Me:	Now, when do we stop? One, or two, or three, or four … How far should we go?
8.	Thomas:	It is one or two or three only, not four.
9.	Me:	Why do we stop at three?
10.	Omar:	Because three candidates only.
11.	Me:	So, one or two or three. Is there anything else that is given in this problem?
12.	Omar:	$n = 7$. Number of candidates is 7. And $r = 3$.
13.	Me:	Anything else? What else is important in this problem?
14.	Omar:	Nothing.
15.	Me:	We cannot have all the answers coming from one person. What do others think? [*Long silence*]
16.	Me:	So, what do we want to find in this problem?
17.	Sayuri:	Number of ways of casting a valid vote.
		(Episode 6.2.3 (03/03/06) (Class 3): Number of valid votes a graduate of a university can cast)

In contrast to the previous example where only two people, Fiona and me, were involved in problem analysis, in this example, the discussion involved four students and was conducted in front of the whole class. In this episode, my questions were

directed at the class as a whole - I wanted all my students to read the question carefully (line 1), identify important information (lines 3, 5, 11, 13 & 15), think through how the information related to the problem (lines 7 & 9), and ask themselves what the problem required them to find (line 16). The data shows students, belonging to various small groups, actively participating in the problem-analysis process with me. It also shows that I reminded my students that I expected all of them, not just one or two, to contribute to problem analysis by sharing their thinking regarding, among other things, key information in the problem (line 15). Based on my actions and student responses, this episode shows that I guided the establishment of the sociomathematical norm whereby students had strategies to analyse mathematical problems by reading carefully, finding the meanings of unfamiliar phrases, identifying key information and relating it to the problem, and asking themselves what the problem required them to find.

Example 6.2.1d: Small-group work following whole-class analysis of a contextualised confidence intervals problem

This example demonstrates the way I guided whole-class analysis of a contextualised confidence intervals problem, which was followed by small-group work. It illustrates students undertaking careful analysis of a mathematics problem through reading, checking the words, identifying key information and what the problem is asking, as a class, and then practising problem analysis in small groups. This episode happened a few days after the previous one, (example 6.2.1c) had occurred. The mathematics ideas of confidence interval, confidence level, sample mean, sample size, and population standard deviation, and their associated notation, were introduced and discussed in the lesson preceding that on which this example is based. In this episode, I began the lesson by organising a whole-class discussion focused on analysing a contextualised confidence intervals (market gardener) problem. The students then solved the problem in small groups.

The market gardener problem:
> A market gardener is preparing a report for her seed supplier. She wants to include information about the mean weight of the pumpkins she grows. A 95% confidence interval for the mean weight when 40 pumpkins are sampled gives 4.71 kg $< \mu <$ 5.93 kg.

a) What is the mean weight of the 40 pumpkins?

b) What is the margin of error in this confidence interval?

c) The seed supplier complains that the confidence interval is too wide to give useful information about the mean weight. Suggest two ways in which the confidence interval could be made narrower. (Barton, 2001, p. 231)

After the lesson, I wrote a journal entry reflecting on the events that took place in class. My journal extract read:

In today's lesson whole-class discussion focused on analysing the problem, which was displayed on the overhead projector. This session preceded small-group work. I began the whole-class discussion by asking students to read the problem carefully, identify key information, think through this information and relate it to the problem, and then ask themselves what the question required them to find. I allowed students wait-time so that they could process their ideas. After a little while I asked them whether there was anything in the problem that needed clarification. Students indicated that there was nothing needing explanation. In spite of this, I reiterated the importance of knowing the meanings of mathematical words, terms and symbols when analysing a mathematics problem. Then, through asking a sequence of questions I was able to elicit students' ideas about key information in the problem and what the problem required them to find. Several students contributed this information and I summarised it on the board. Thomas, Najilla, Muhannad, Omar and Jessica were the main contributors of ideas. Many students used mathematical language and symbols that had been introduced and utilized in previous lessons. For example, today some students used the terms 'margin of error', 'confidence level', and 'mean weight', appropriately. Students' ability to correctly use these terms allowed coherent communication of mathematics ideas to take place. There were, however, students who struggled with pronouncing some of the words. I helped these students with pronouncing the words by verbalising them myself, hoping this would encourage them to continue to participate and contribute. I used these words and terms while clarifying some students' contributions.

To further support students analyse the problem, at one point I drew a diagram or line segment on the board, and then marked the midpoint and two endpoints. I was conscious, though, that my actions could possibly block some students' ideas. Through questioning and probing, I was able to guide students relate key points (midpoint, two equal parts of the line segment, and the two endpoints) on the diagram (or line) and the key information in the problem. As a class we labelled sample mean $(\bar{x})$, margin of error (e), confidence interval (C.I.), and the lower and upper limits of the confidence interval, on the diagram (or line segment). Students and I exchanged ideas during this process. It appears students used this labelled line segment to link key information in the problem with what they were required to find.

> After analysing the problem as a class, students completed the solution in small-groups. Then they continued to solve similar problems in their groups. (JE 6.2.2 (16\05\06) (Class 3): Market gardener problem)

This journal extract details how I helped students to analyse the market gardener problem by encouraging them to read the problem carefully, identify and find meanings of unfamiliar words, identify key information and then relate it to the problem, and ask themselves what the problem required them to find. I also noted that I supported student thinking by asking a sequence of questions that prompted them to provide key information in the problem. I further supported student thinking by using diagrams to help them think through what was involved and required.

The journal extract illustrates that I emphasized the importance of correctly using mathematical language to aid problem-analysis and support learning. This is evident in my comment that "I reiterated the importance of knowing the meanings of mathematical words, terms and symbols when analysing a mathematics problem." I also encouraged students to use mathematical words, terms and phrases, both verbally and in written form. In my journal entry, I noted that I modelled pronouncing and using mathematical words, terms, phrases and symbols to support students in collectively analysing the problem. For students, knowing key words is important when undertaking mathematical problem-analysis because this allows them to think through and relate key information to the problem and identify what the problem requires.

This journal extract indicates that I observed that a number of my students willingly participated in analysing the problem as a group. I took these actions to reflect that these students were aware of and valued the social norm of sharing ideas, and its vital role in supporting collective problem analysis. These student actions can be seen to reflect student social and intellectual autonomy. The students knew *when* and *how* to actively participate. They freely contributed mathematical ideas. The actions also suggest student adoption of an academic culture that valued sharing thinking. Students publicly shared their ideas with everyone else in class. This journal extract suggests that some of my students had adopted the norm of carrying out careful reading, finding the meanings of words and phrases, identifying key information and relating it to the problem, and identifying what the problem was asking.

The next example happened towards the end of the semester. It is focused on students collectively analysing a contextualised modelling problem.

Example 6.2.1e: Whole-class analysis of a contextualised modelling problem

This example illustrates the way my students contributed to whole-class analysis of a contextualised modelling problem. Students worked on a modelling (dehumidifier) problem in small groups before participating in whole-class discussion.

The dehumidifier problem:

> A dehumidifier is installed in a damp room to remove moisture from the air. It is claimed that t hours after installation, the humidity level (percentage of moisture in the air) will be modelled by the function:
>
> $$h = 100 - 20e^{0.05t} \text{ for } 0 \leq t \leq 24$$
>
> Calculate the claimed humidity level 12 hours after installation.
> How long will it take for the claimed humidity level to halve? (Barton, 2001, p. 311)

In my journal I wrote:

> As in earlier lessons, I allowed students time to discuss their ideas and find a solution to the problem, in groups. In addition, I encouraged them to identify and agree on the meanings of unfamiliar words, terms and phrases. I opened whole-class discussion by asking students to report-back on what they had agreed in their group. No student or group indicated that they could not reach agreement on the meanings of words, terms and phrases in the problem. I, however, once again emphasised and explained why it was important that they know the meanings of the words in a problem. I also reminded students that understanding contextualised mathematics problem increased their chance of finding its solution. Then, I asked students to say what key information was and what they were required to find. Students took turns to provide key facts they had identified in small groups, and I summarised them on the board.
>
> When Karen responded to one of my questions saying "$h = \frac{1}{2} \times 64$", I wrote what she had said on the board. Immediately Davy challenged Karen's idea, demanding to know where her numbers came from and why she multiplied $\frac{1}{2}$ and 64. Some students supported Davy. Further whole-class discussion first focused on Karen's suggested solution method. After a few exchanges, during which the class had another look at what information was given in the problem and what they were required to find, Karen's suggestion was dismissed. Instead, students agreed that for part (a), time t = 12, hence humidity after 12 hours could be found by replacing t by 12 in the formula.

Secondly, for part (b) the phrase "claimed humidity to halve" became a topic of conversation. Some students proposed finding $\frac{1}{2}$ of 100 while others suggested $\frac{1}{2}$ of the original humidity level, i.e. when t = 0. Both camps made attempts to justify their claims. After some debate, collective understanding was arrived at. That is, finding initial humidity level first, when t = 0, halving it, and then using this value to find the time. While answering similar problems after whole-class discussion, many students, working individually or in pairs, summarised key information on paper before writing the remainder of their solution. Most students produced acceptable solutions. (JE 6.2.3 (08/06/2006) (Class 3): Dehumidifier problem)

This journal extract demonstrates the ways I continued to support the establishment of the sociomathematical norm of students undertaking mathematical problem-analysis of a contextualised modelling problem. It suggests that the students negotiated the meaning of "claimed humidity to halve", as part of finding the meaning of unfamiliar words and phrases. It also indicates that at this time, near the end of the semester, students had become accustomed to reading the problem carefully, could identify key information and relate it to the problem, and identify what the problem was asking. The journal suggests that for this group of students, mathematical problem-analysis had become part of their solution process. It suggests that the discussion, focusing on Karen's idea, may have reinforced students' ability to analyse a contextualised modelling problem by allowing students another opportunity to collectively carry out problem-analysis. Davy's challenge created a stimulus for further problem-analysis. It triggered communication of mathematics ideas in a way that encouraged other students to take part in analysing the problem in a way that supported learning. Davy's actions demonstrated student authority and autonomy. He, and other students, acted volitionally and made independent mathematical decisions about the validity of Karen's solution method. In addition, the students acted autonomously when they made decisions to suggest an alternative approach solution which turned out to be the acceptable one.

In this subsection, I have presented five episodes that demonstrate the ways I initiated and guided the establishment of students undertaking careful mathematical problem analysis through careful reading of the problem, finding meanings of unfamiliar words and phrases, identifying and relating key information to the problem, and

asking what the problem required. The next section presents student perspectives related to mathematical problem-analysis.

6.2.2 Student perspectives

This section presents interview data that demonstrate that my students found contextualised problems challenging and hard to understand because English is not their first language, and that students could not undertake effective mathematical problem-analysis. Additionally, the section presents student perspectives that demonstrate the need for the teacher to initiate and guide the development of the sociomathematical norm of analysis of a mathematics problem through reading the problem carefully, checking the meanings of unfamiliar words, terms and phrases, identifying key information and relating it to the problem, and identifying what the problem requires them to find.

Contextualised mathematics problems are challenging and hard to understand

During separate interview sessions with groups of students, most of the students who commented about contextualised mathematics problems said that these are challenging and hard to understand. On 13 September, 2005, for example, I had a formal interview with a group of four students: Fiona, Lily, Miriam and Carol. We enter the dialogue when we were discussing the role of contextualised problems in learning mathematics.

1.	Me:	What is your opinion on story problems? Is it a good idea to have questions involving 'stories' in mathematics?
2.	Carol, Lily & Fiona:	I prefer questions with no stories.
3.	Me:	Why? Which mathematics problem is more interesting, one with a story or one with no story?
4.	Fiona:	Stories are more interesting but more harder. The language used in story problems is hard to understand.
5.		If we don't understand the meaning of the story we get more confused. Stories are not important.
6.	Me:	So do stories make mathematics questions easier or more difficult?
7.	Carol,	

Lily & Fiona:	More difficult.

8. Carol: We must understand the question first. Then we do it.

(Interview 6.2.2 (13\09\2005) (Class 1): Fiona, Lily, Miriam and Carol on their difficulties with understanding the problem)

This data suggests that for these four students contextualised mathematics problems were challenging and hard to understand and, in their view, hindered rather than helped them to learn mathematics. Despite apparent agreement among the four students about their preference for questions with no stories, Carol acknowledged the importance of understanding the question in line 8. In a separate interview, (14 September, 2005) Rita expressed the same view as Carol. Across these two interviews, student comments indicate that they recognised the value of understanding a problem in mathematics learning. Given they found contextualised story problems hard to understand, these data indicate the need for the teacher to guide the development of students' problem-analysis strategies.

In another interview, also in September 2005, Ahmed, Khoula and Laila acknowledged that they had problems with the language of questions and asserted that this affected their being able to understand and solve contextualised questions. Khoula said, "It is difficult to understand the language, sometimes." Ahmed added, "I agree that sometimes we fail to get the meaning [of the question]." Laila, Ahmed and Khoula advised that the mathematics teacher should assist them to understand what Ahmed referred to as "tricky questions." Ahmed declared that he disliked "tricky questions because they confuse students… they are difficult to understand", and so, he preferred "questions with no stories." Together, comments by students in both Fiona's and Ahmed's groups along with Rita reflect that these students did not have the ability to engage in productive problem-analysis.

When I interviewed Ahmed the second time, on 31 October, 2005, he had changed his mind. He preferred to learn mathematics using contextualised rather than non-contextualised problems. Ahmed said "...although they are hard to understand, it's better with stories than just figures." Ahmed's statements indicated that, over time, he had shifted his views about mathematics. During the same interview, Laila proposed that students should ask if they did not understand something "some words you did

not understand. If you share with other people it helps to understand the problem. This is important." Laila's comments reflected that she believed in analysing a problem for understanding. Furthermore, the comments suggest that Laila recognised the social norm of asking questions to support mathematical problem-analysis. In spite of their difficulties with contextualised problems, these students recognised the value of using them for learning mathematics.

In contrast, Raymond openly dismissed the value of using contextualised mathematics problems. On 15 September, 2005, Raymond's view was that "In a question we are looking for numbers... many words are useless and hard to understand." Raymond's comments suggest that he could not engage in effective problem analysis, and also that he had an abstract view of mathematics in that he did not recognise mathematics as a human activity involving context. About one month after the first interview, Raymond had this to say about contextualised problems, "…some words are difficult. We do not know what they mean." In the same interview (26 October, 2005), Dong supported him and added, "…some words in the textbook we cannot understand." These students' views again confirmed that they lacked the strategies or vocabulary, or both, needed to undertake problem analysis, indicating, once more, the need for the teacher to guide the development of the norm of problem-analysis.

For Susan, who I interviewed on 14 September, 2005, understanding the question before solving the problem was important. She asserted that "You need to understand the question and then work out the answer." This view was supported by Chen, in a different interview, who claimed that his drawback was with understanding the problems. He stated "If I can read the mathematics problems and understand them, then I can do the problems." In the same interview, Yosuke supported Chen by stating "If I read and understand the question, then I say, 'So I can make it.'" Like the data in previous paragraphs, this illustrates the need for the teacher to help students develop the ability to carry out productive problem-analysis.

In a second interview with Susan on 26 October, 2005, she pointed out she usually asked her peers or the teacher for assistance when she could not understand the question. Susan's comments demonstrated that she could use the social norm of

asking questions to support mathematical problem-analysis. She also recognized the value of careful problem analysis. In a separate, also second, interview (26 October, 2005), Fiona, Lily and Carol were of the view that even with assignment questions their major problem was "understanding the question." Carol's advice to mathematics teachers was "Questions should have simple language which can be understood by international students." Fiona's was "In class the teacher should write the question on the board so that we can see and read it many times. We need to understand it." These comments further demonstrate these students needed the teacher's support to develop the ability to undertake productive problem analysis.

Like those in 2005, eight of eleven students interviewed in 2006 indicated that contextualised mathematics problems were challenging and hard to understand. Sayuri, for example, agreed with those NESB students interviewed in 2005 that when solving contextualised problems, the important thing was to understand the question first. She said "Yes. First I have to understand the question." Honda, who attended the same interview, acknowledged that he found it difficult to understand "the mathematical terms because of English… the words are difficult." He went on to suggest that NESB students could be helped understand contextualised problems by affording them more time with the teacher. "Yes, if the teacher had time to help students who couldn't understand... teach them in extra lessons." Rather than ask their peers, Honda and Sayuri preferred to ask the teacher to explain the words they did not understand. Honda said "Because if not sure I can make sure the meaning is the correct one by asking the teacher." Honda and Sayuri's views were echoed by Max, Thomas, Ray, Wataru and Yuichi, all interviewed in 2006. As in previous paragraphs in this section, the data in this paragraph indicate that these students had difficulties understanding contextualised mathematics problems. These data demonstrate, once more, the need for the teacher to guide the development of the norm of mathematical problem-analysis.

Suggestions for carrying out problem-analysis

Regarding how problem-analysis should be done, the students made a number of suggestions. Two typical student views become apparent from interviews with groups of students. One suggestion was that students could analyse contextualised problems

in groups and then, during whole-class discussion, the teacher should "ask someone in the group to show the working on the board and then we compare, not just discuss in (small) groups, but as a class as well." This student, Chen, added "It is better if we do it together [rather than the teacher doing it] with clear explanations and clear answers on the board." Em suggested that if teachers give more examples involving contextualised mathematics problems that would be more helpful for the students. He claimed that the number of examples that were being used in class was not enough to be of significant benefit to students' mathematics learning. Instead of analysing questions in small groups, he preferred that the teacher "should do more examples, explain the meaning of the question and show us the working on the board." Em added "the teacher should explain how the problem is analysed, so we can follow his approach." Em's comments suggest that he wanted the teacher to model the strategies students could use to analyse a problem. Although these two students (Chen & Em) seemed to agree on the value of analysing contextualised problems, their comments suggest major differences in their views about mathematics. Chen's advice is illustrative of a view of mathematics as a social activity whereas Em's is consistent with a view of mathematics as a product that could be transmitted from the teacher to students. As in the previous section, these data demonstrate that students considered their teacher has an important role to play in supporting the analysis of contextualised mathematics problems.

To sum up, it emerged during interviews detailed in this section, that, for NESB students, their ability to understand the language used to express a contextualised or story problem was often a challenge and their ability to carry out mathematical problem-analysis was closely linked with their understanding (or lack of understanding) of the language of the problem. A summary of student perspectives on issues associated with contextualised mathematics problems and mathematical problem-analysis, including the number and percentage of students who expressed each view, is presented in Table 6.1. Not every student, attending a particular focus group interview, was expected to answer every question, or to make a verbal contribution on every issue being discussed during that interview. Each student made his/her own decision regarding which question to answer and on which issue to contribute an opinion.

Table 6.1   Student perspectives on issues relating to contextualised problems and mathematical problem-analysis

Student perspective	Number of students who expressed the view (n = 27)	
	First interview	Second interview
Contextualised mathematics problems are challenging and hard to understand.	24 (89%)	8 (30%)
It is important to understand the problem.	12 (44%)	25 (93%)
The teacher should help students understand the problem.	16 (59%)	7 (26%)
The teacher should explain the meaning of the question and how to answer it.	15 (56%)	5 (19%)
The teacher should demonstrate problem analysis.	15 (56%)	6 (22%)
Class should collectively analyse a contextualised problem.	8 (30%)	24 (89%)

This table demonstrates that, in each semester, changes occurred in students' perspectives on contextualised problems and mathematical problem-analysis. At the time of the second interview, fewer students stated that they found contextualised mathematics problems challenging and hard to understand. The table indicates that, in the second interview, a smaller number of students expected only the teacher to explain the meaning of a question and how to answer it, and to demonstrate problem-analysis. At the same time, more students recognised the importance of understanding the problem and the value of collectively analysing a contextualised problem in small groups or as a class.

In this study, mathematics learning, in general, and solving contextualised mathematics problems, in particular, were often accompanied by the normative aspects of explaining and justifying mathematically. The next section focuses on these aspects.

6.3 Explaining and justifying mathematically

This section presents data that demonstrate the ways I initiated and guided the constitution of the norm of students expecting and being able to explain and justify mathematically. In this study, this norm included students developing a normative focus on clarifying their own or their classmates' contributions or solutions with the use of mathematical language, reasons and or mathematical interpretations. My practices associated with this were (1) the use of contextualised and non-contextualised mathematics problems, (2) organizing small-group and whole-class discussions, (3) teacher modelling mathematical explanation and justification, (4) revoicing, (5) questioning, and (6) allowing students the opportunity to explain and justify mathematically. In addition, the section outlines students' perspectives on explaining and justifying one's own or others' contributions, mathematically.

6.3.1 My experiences

In this section, I present examples (journal extracts and episodes) to illustrate my actions to establish the norm of explaining and justifying mathematically. These examples also describe students' actions and how they learned what constituted an acceptable mathematical explanation and justification. In addition, the examples demonstrate that students came to understand that an explanation or a justification was acceptable, as a sociomathematical norm, if it comprised a mathematical reason or a currently taken-as-shared mathematical interpretation.

Example 6.3.1a: Encouraging students to request/provide mathematical reasons

The purpose of this journal extract is to demonstrate the way I encouraged my students to request and provide mathematical reasons. This example comes from a series of lessons on probability. In lessons prior to this episode, students had learned about complementary events, mutually exclusive events and independent events, and their associated probability rules and terminology.

The lesson in this example began with my introducing the idea of a probability tree diagram, and discussing how this diagram could be used to solve problems involving probability. Following this introduction, students worked in small groups on a probability problem involving passing university examinations.

The probability problem:

> A student sitting three university papers has the probability of 0.6 of passing her first paper. If she passes the paper, her probability of passing the next one increases by 0.1. If, however, she fails the paper her probability of passing the next one drops by 0.1. Assuming this pattern continues for all three papers, find the probability that:
> a) she passes all three papers.
> b) she passes only one paper. (Lakeland & Nugent, 2004, p. 114)

My journal extract read:

> As I moved from group to group I questioned students about their diagrams. For example, I asked Yosuki and Chen to explain why they started by drawing three and two branches on their tree diagrams, respectively. Yosuki's response was 'because she writes three papers' while Chen's was that 'because she can pass or fail (a paper). So two branches.' I let Chen's group continue with their solution but joined Yosuki's group and asked them to analyse the problem with me. Together we identified key information and what the problem required. As students in the small group responded to my sequence of questions, including those requiring them to provide reasons for their ideas, Yosuki, used the information agreed by the group to draw branches representing outcomes for the first two papers. The group then completed the solution without me.
>
> During a whole class debrief on the problem I asked David, Raymond and Dong to explain their thinking. To do this they drew a fully labelled probability tree diagram on the board, with all possible outcomes and associated probabilities for each event clearly indicated on the branches. They also displayed their calculations leading to the final answers, on the board. This display prompted Em to ask why in part (a) they had multiplied probabilities but in part (b) they had multiplied three sets of probabilities before adding them up. In reply David said that for part (a) it was 'times because the events are independent' and in part (b) 'times for independent events followed by plus because its event A or B or C.' As he continued to elaborate on his explanation, I supported him by revoicing some of what he said, such as, 'for independent events we say times but when we say event A or B or C we add the probabilities'. (JE 6.3.1 (14/09/2005) (Class 1): Probability of a student passing three university papers)

This journal extract indicates that, my setting a contextualised problem and encouraging small-group discussion, followed by whole-class discussion created conditions that made it possible for the students and me to request and\or provide mathematical reasons that clarified and justified student problem analysis and contributions towards a solution. The combination of a small group and whole class interactions allowed me to prompt students for reasons and students to encourage peers to do the same. By revoicing David's explanation, I aimed to encourage and

affirm his provision of mathematical reasons. Em's request for an explanation of the mathematical thinking underpinning David's explanation indicates that he was aware of the need to understand the mathematical rationale behind the decision to multiply or add probabilities. This example from the second half of the semester demonstrates the progress students had made in understanding and being able to explain and justify their ideas.

Example 6.3.1b: A student clarifies (in front of the whole class) the difference between sum to infinity and sum of n *terms*

This example is used to demonstrate a student making use of mathematical reasons and interpretations to clarify the difference between sum to infinity and sum of *n* terms. Posing probing questions was the main strategy I used in this example. The mathematics idea of sum to infinity and the notation $\sum$, S_n, and S_∞, was introduced to students in previous lessons involving the topic sequences and series. The previous example involved using a 'tree diagram' to solve probability problems but in this example the focus was on consolidating the work learned during the previous few lessons.

We enter the episode when I had written the statements $\sum_{r=1}^{\infty} a_r = S_\infty$ and $\sum_{i=1}^{\infty} a_i = S_n$, on the board.

1. Me: Look at the two statements on the board and decide whether they are correct/true or wrong/false, mathematically. Work in small groups and justify your answer.

2.   *[After a little while I asked for students' contributions].*

3. Susan: The first statement is correct and second is wrong.

4. Me: Why?

5. Susan: It's sum to infinity.

6. Me: What do you mean by 'It's sum to infinity?'

7. Susan:
           Left side of the statement, $[\sum_{r=1}^{\infty} a_r = S_\infty]$, is sum to infinity

           because $\sum_{r=1}^{\infty} a_r$ is a shorter way of writing the sum in the

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form $a_1 + a_2 + a_3 + ...$

8. Susan: $\displaystyle\sum_{r=1}^{\infty} a_r = a_1 + a_2 + a_3 + ... =$ sum to infinity or S_∞ , in short.

9. [*Susan wrote:*

$\displaystyle\sum_{r=1}^{\infty} a_r = a_1 + a_2 + a_3 + ... = sum\ to\ infinity = S_\infty$, *on the board*]

10. Susan: But right side, $[\displaystyle\sum_{i=1}^{\infty} a_i = S_n]$,

[*of the second statement*] says adding values when *i* goes from 1 to sum of *n* terms, which is wrong. So the second statement is wrong.

(Episode 6.3.1 (10/10/2005) (Class 1): Susan gives reasons to distinguish $\displaystyle\sum_{r=1}^{\infty} a_r = S_\infty$ and $\displaystyle\sum_{i=1}^{\infty} a_i = S_n$)

The episode shows that I used questioning (lines 3 & 6) to prompt Susan to explain and justify her claim that the first statement was correct and the second was wrong. In lines 5, 7, 8 and 9 Susan gives a mathematical reason to justify her statement $\displaystyle\sum_{r=1}^{\infty} a_r = S_\infty$ is valid, mathematically, but $\displaystyle\sum_{i=1}^{\infty} a_i = S_n$ is not. Additionally, it indicates that Susan was willing and able to justify her contribution to the whole class with the use of mathematical reasons. Her extended explanation and use of mathematical terms indicated her understanding of the mathematical ideas involved. In this instance, Susan used writing to communicate her thinking and support her explanation. This strategy helped her communicate her mathematical interpretation. This extract suggests that Susan was aware that mathematical explanations and justifications become acceptable when backed by authentic mathematical reasons or interpretations.

Example 6.3.1c: Four students explain and justify (in front of the whole class) why y = e^x is an exponential function

This example, which happened two days after the previous one, illustrates that my actions to develop normative class use of mathematical reasons and\or interpretations

to clarify and\or defend mathematical thinking were continuous. The main strategy I used was questioning (posing prompting and probing questions). In the example, four students explained why $y = e^x$ is an exponential function. I began the lesson by writing y = e^x on the board. We join the episode at this point.

1.   Me: We used this statement [*Referring to y = e^x on the board*] yesterday. What did we say it is called? What can we say about this statement?

2.   Raymond: Exponential function.

3.   Me: Why do we call it an exponential function?

4.   Raymond: Because we have…

                [*I interrupted Raymond*]

5.   Me: We cannot have all answers coming from Raymond. Let us have answers from others. [*Raymond had answered the previous questions*]

6.   David: Because we have e to the power of x.

7.   Me: What is e? What is x? Can someone add or subtract from what David has said? Do we always say an exponential function when the statement contains e and x?

8.   Dong: e is the base and x the power. It is a function.

9.   Me: What else? Why do we say it's a function?

10.   Susan: Function because it's a rule associating x and y.

                (Episode 6.3.1 (12/10/2005) (Class 1): The function $y = e^x$)

In this episode, my line of questioning indicates that I wanted students to give mathematical reasons for their contributions. Raymond, David, Dong and Susan in lines 2, 6, 8 and 10 used mathematical reasons appropriately, indicating that they had some understanding of the statement $y = e^x$. My actions in this episode illustrate the way I used questioning (prompting and probing) to elicit mathematical reasons from the students. Data suggest that I probed for the reasoning behind the response (line 3). It shows that I asked follow-up questions after a student's response. My actions in lines 7 and 9 suggest that I was looking for a detailed mathematical explanation and justification. I was not fully satisfied with David's explanation (line 6) so I challenged my students to add to David's explanation (lines 7). When Dong

responded, I was again not completely satisfied with what he said, so I probed for more details. My follow-up questions were intended to prompt students to provide detailed mathematical reasons.

In responding to my regular requests for elaboration of their explanation and justification, but in other episodes, I consider my students learned what was expected of them regarding an acceptable explanation and justification, that is, their being able to provide mathematical reasons to clarify and\or defend thinking. It further illustrates that, unlike the situation at the beginning of the semester when few students volunteered to offer mathematics reasons, by the time of this episode (near the end of the semester) more students were willing to do this. Also, they made substantial use of mathematical language.

Example 6.3.1d Students explain and justify the difference between $y = e^x$ and

$$y = ae^{kx}$$

In contrast to the previous episode that focused on one function, this one involves two functions. It took place as soon as the previous one ended. This episode demonstrates students explaining and justifying the difference between the functions *$y = e^x$ and $y = ae^{kx}$*.

1.	Me:	I have another statement on the board.
		[*I wrote $y = ae^{kx}$*]
2.		What is similar and what is different between the two statements [$y = e^x$ *and* $y = ae^{kx}$]?
3.	Raymond:	They are similar.
4.	Me:	What are your reasons for saying that they are similar?
5.	Derek:	Both are exponential functions.
6.	Me:	How do we know that?
7.	Chen:	Both have a base, *e*, and the variable x is in the power.
8.	Me:	Good. Anything else?
9.	Lily:	*x* and *y* are variables in both.
10.	Me:	Yes. Anything else?

11.		[*Silence*]
12.	Me:	Are there any differences?
13.	Raymond:	In the first one $a = 1$ and k is also 1. In the second it's different.
14.	Me:	What is the difference? Fiona what do you think?
15.		[*Silence*]
16.	Fiona:	I think first one is a particular function and second one is a general one.
17.	Me:	Why? What do you mean by a particular function or a general function?
18.	Fiona:	Because in the second statement a and k stand for any number. In the first one $a = 1$ and $k = 1$. Therefore different.

(Episode 6.3.2 (12/10/2005) (Class 1): Comparing and contrasting $y = e^x$ and $y = ae^{kx}$)

This episode shows how I pushed my students to give detailed mathematical reasons to back their contributions (e.g. lines 4, 6 & 17). It also shows that a number of students responded to my requests for reasons (lines 5, 7, 9 & 18). A moment of silence (lines 11 & 15), allowed students to think through their responses to my questions. Students appeared to have benefited from wait-time because after each period of silence, one of the students gave an extended explanation and justification. The student actions indicate that this group of students was willing and able to give mathematical reasons to explain and justify the difference and similarity between the functions $y = e^x$ and $y = ae^{kx}$.

By being able to give mathematical reasons to clarify and defend their ideas, students in this episode demonstrated that they knew the criteria for an acceptable explanation and justification. Data suggest that these students recognized that to explain and justify mathematically, they needed to provide mathematical reasons or interpretations.

My students had views regarding explaining and justifying mathematically. Some of these perspectives were disclosed to me during the formal interviews. These are presented next.

6.3.2 Student perspectives

This section presents student perspectives that demonstrate that while some students expected both the teacher and their peers to explain and justify mathematically, others expected only the teacher would do this. Also, the section presents data that demonstrate the need for the teacher to support students to develop a normative focus on mathematical explanation and justification.

Teacher and peers should explain and justify mathematically

Some of my students were of the view that both the teacher and peers should be expected to explain and justify mathematically. During an interview in September, 2005, Susan asserted that both the teacher and students should explain and justify their ideas when students are learning mathematics. She said:

> I think students should think about the question and how to answer it. If the teacher tells them everything, they cannot think about this (explaining and justifying). Students should also give reasons or explain and justify their answers.

In a separate interview (September, 2005), Chen stressed that the teacher should not work out and explain problems for students who then just copied this, because "if you can't do it by yourself it is useless… It is better if we do it together with clear explanations and clear answers on the board." These views were supported by Ahmed, Lily and Fiona (September, 2005) and by Omar, Muhannad and Wataru (June, 2006). Lily, for example, said "Students or teacher should do it [give mathematical reasons]…If a student cannot do it, the teacher should do it." Their responses suggested that the students expected both the teacher and the students to undertake mathematical explanations and justifications in mathematics learning. Lily's comment, in particular, suggests that she expected me, as teacher, to explain and justify if a student is unable to. Together, the comments reflect the need for teachers to focus on developing students' ability to provide mathematical reasons and\or interpretations.

Teacher should provide all mathematical reasons and\or interpretations

In contrast to the eight out of twenty-seven students mentioned in the previous paragraph, whose view was that the teacher and students should share the

responsibility for explaining and justifying mathematically, nine out of twenty-seven students said that this was the teacher's responsibility and not the student's. David, Dong and Raymond (September, 2005) stated that they were used to a system (in their home country) where the teacher did most of the talking, so they saw no problem if the teacher in New Zealand provided all mathematical reasons and justifications. However, when I interviewed them a second time (October, 2005) they had changed their minds. Raymond said, "We should all [teacher and students] explain our answers, with reasons." David and Dong agreed. In June, 2006, Sayuri had this to say, "In Japan, the teacher tells us how to do it… He explains to us… but here the answers, reasons and explanations come from the students. It is different. It is better if teacher explains, gives reasons." Honda (June, 2006), who came from the same country, agreed, and added that the teacher would "give us the question and we do it and the teacher would give us an answer… and explain everything clearly. If teacher speaks and explains, it's helpful." We can see that these students recognized the different approach I was taking but did not support it. Similarly, Thomas (June, 2006) declared "[It is] very different [here]. The difference is that in Fiji they [mathematics teachers] give more explanations, the teacher talks more. The teacher does most of the talking and explains more to the students." Thomas added that he expected the same to happen in New Zealand, but did not specifically say that he disliked the approach I was using. Max, Yuichi and Najilla, who were also interviewed in June 2006, agreed. These students' comments, however, suggested that they considered explanations and justifications to be essential for mathematics learning, reflecting a need for developing the norm of explaining and justifying mathematically.

In summary, student perspectives in this section demonstrated that my students differed on whether the teacher and the students, or the teacher only, should carry out mathematical explanation and justification through providing mathematical reasons and\or interpretations. They, however, seemed to agree that explaining and justifying mathematically is important. Table 6.2 provides a summary of student perspectives about issues related to explaining and justifying mathematically, together with the numbers and percentages of students who articulated each perspective, during the

first and second focus group interviews. Note: Not every student responded to all questions or made public their opinion of each issue discussed.

Table 6.2　　　Student perspectives on matters associated with explaining and justifying mathematically

| Student perspective | Number of students articulating the view (n = 27) | |
	First interview	Second interview
Both the teacher and students should explain and justify mathematically	10 (37%)	21 (78%)
Students need to explain and justify mathematically	7 (26%)	20 (74%)
The teacher should provide all mathematical interpretations and\or reasons	18 (67%)	6 (33%)
Mathematical reasoning and interpretation is essential when learning mathematics	14 (52%)	25 (93%)

This table indicates that, during the second interview there was an increase in the number of students whose view was that both the teacher and students should provide mathematical explanations and justifications, that students need to explain and justify mathematically, and that mathematical interpretation and reasoning is essential when learning mathematics. At the same time, there was a decrease in the number of students who expected the teacher to provide all mathematical interpretations and reasons. Thus, data reflect a shift in student perspectives on explaining and justifying mathematically, as the semester progressed. The table also indicates that students recognised the value and importance of mathematical explanations and justifications when learning mathematics. These data indicate that, between the first and second interview, shifts may have occurred in student beliefs and values about the way mathematics should be learned.

It emerged, however, that when students engage in explaining and justifying their thinking using mathematical reasons and\or interpretations, they invariably engage in communicating mathematically.

6.4 Communicating mathematically

This section presents episodes that illustrate how I guided the establishment of the sociomathematical norm of students communicating mathematically. In this study, communicating mathematically came to mean students developing a normative focus on using mathematical language (i.e. terminology and symbols) appropriately to communicate mathematics ideas. This entailed students being willing and able to use mathematical terms and symbols and knowing that they were expected to. In the first part of this section, the episodes demonstrate that the development of this norm was done through (i) the use of contextualised and non-contextualised mathematics problems, (ii) a mix of small-group and whole-class discussion to allow students the opportunity to use mathematical language, (iii) the use of a diagram, (iv) teacher modelling conventional mathematical language, and (v) questioning (focus question displayed on the board, then asking prompting, probing and extending questions). The episodes show how this norm regulated classroom conversation and influenced learning opportunities for NESB students. They illustrate the emergent nature of the norm of communicating mathematically. As in earlier sections, these examples have been chosen for their clarifying and explanatory power. In the second part of the section, I present student perspectives regarding communicating mathematically in classroom discussions.

6.4.1 My experiences

This section demonstrates my actions to initiate and guide the establishment of the norm of using mathematical language and symbols to communicate ideas during classroom interactions. It shows these across three different mathematics topics and concepts. The examples illustrate how students came to learn and appreciate the criteria for what counts, in their class, as communicating mathematically. That is, examples demonstrate the way students learned that a communication was acceptable, as a sociomathematical norm, if it encompassed conventional mathematical terminology and symbols.

Example 6.4.1a: Students use mathematical language to communicate conditional probability ideas

This example demonstrates the way I guided students in developing a normative focus on communicating their ideas using the appropriate mathematics terms, in this case conditional probability. It is typical of my approach that the problem is set in a context and the solution process included revision of work done previously and small-group and whole-class discussions. The meaning of the term *probability* and how to calculate it for various types of events, including the use of probability tree diagrams, had been covered in previous lessons. The lesson on which this example was based began with whole-class discussion of the meaning of conditional probability and how to find conditional probability. In this example, students first worked in small groups on a contextualised conditional probability (pie) problem before discussing their solution as a class.

The pie problem:

> At a bakery 12% of customers who enter the shop buy a pie while 25% buy a filled roll. 8% of customers buy both a pie and a filled roll.
> Find the probability that a randomly selected customer
> a)   buys a filled roll given that they have also bought a pie.
> b)   buys a pie given that they also buy a roll.
> c)   buys a pie or a filled roll. (Lakeland & Nugent, 2004, p. 120)

Part of my journal read:

> While working in small groups some students communicated using mathematical language, others just listened. This is what some students said in their different small groups: David: 'The percentages are the probabilities;' Raymond: 'This is conditional probability'; Susan: 'How do we find conditional probability?' and Fiona: 'Buys a pie or a filled roll means union.'

> During whole-class discussion Carol used the notation '$\cap$' in her solution which was displayed on the board. Ahmed demanded an explanation of what Carol meant by this notation. Carol struggled with the explanation, so I assisted her by revising, with the class, the meaning of the notation '$\cup$' and '$\cap$'. During the remainder of the lesson, more students used mathematical words or terms and symbols that had been used in the previous lessons. (JE 6.4.2 (8\2005) (Class 1): The pie)

My journal entry indicates that the problem created an opportunity for students to use mathematical language to communicate their ideas and to experience communicating

mathematically. The actions of David, Raymond and Susan demonstrate that they were able to communicate about conditional probability ideas using mathematical terminology. Carol's use of the notation, $\cap$, and Ahmed's reaction to this, prompted me to undertake revision of work done previously. The journal suggests that Carol's, Ahmed's and my actions created another opportunity for students to communicate about conditional probability ideas. Also, it indicates that, following this opportunity, more students used mathematical terminology and symbols, reflecting these students' increased ability to use mathematical language to converse about conditional probability ideas.

The next example occurred in the second year of study. The class consisted of students I taught in the first year of this research, while they were studying a different mathematics paper, and those I started teaching during the second year of the study. So, with reference to this study, the class comprised old and new students.

Example 6.4.1b: Students use mathematical language to communicate ideas about a perpendicular bisector of a straight line

In this episode, students use the language of mathematics to communicate their ideas about the meaning of perpendicular bisector. It demonstrates that I used questioning (prompting, probing and extending) and a diagram to guide the development of the norm of communicating mathematically. Students first worked in pairs solving a coordinate geometry (perpendicular bisector of a straight line) problem and then came together as a class to discuss their solution processes.

Perpendicular bisector of a straight line problem:

> A landscaper is drawing up a plan of a triangular garden she proposes to use as part of a reserve development. To make sure that the angles and lengths work out, she decides to first sketch it on a Cartesian graph and then use straight line coordinate geometry to check her calculations. She begins by drawing the line L and marks out the points A, B, C and D. Study the diagram drawn below. All lengths are in metres.
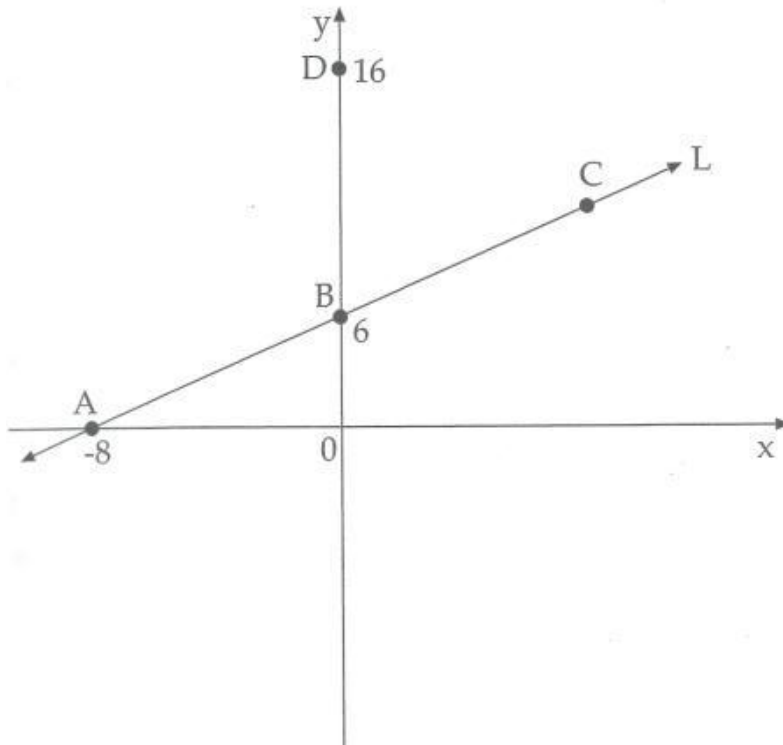
Calculate the gradient of the line AD

B is the midpoint of AC. What are the coordinates of C?

Find the equation of the perpendicular bisector of the line segment AC. Write the answer in the form ax + by + c = 0. (Lakeland & Nugent, 2004, p. 195)

As students discussed their solution processes as a class, Harries asked me the meaning of 'perpendicular bisector.' We enter the episode at this point.

1.	Harries:	What is the meaning of $\perp$ bisector?
2.	Me:	What do you think?
3.	Harries:	I have no idea.
4.	Me:	[*To the whole class*] Let us all look at part (c). What is the meaning of perpendicular bisector?
5.	Wataru:	It means divide into two equal parts.
6.	Me:	From which word does bisector come from?
7.	Ray:	Bisect.
8.	Me:	What does bisect mean?
9.	Wataru:	Divide into two equal parts.

10. Me: Yes. Here we have a ⊥ bisector. What does it mean? Does it mean this?

[*I drew a diagram on the board*]

11.
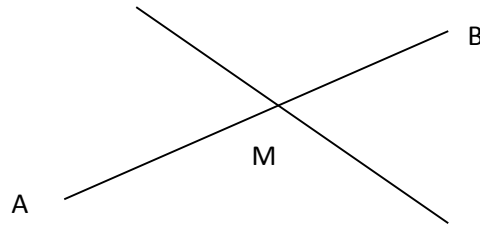
12. [*For some time, there was silence. Then, after a while, Tiffany responded*]

13. Tiffany: No. Lines must cross at 90º. [*After a pause*] And divide AB into two equal parts.

(Episode 6.4.2 (14/03/2006) (Class 4): Perpendicular bisector)

Harries' action (line1) implies that he was aware of the importance of understanding the language in the problem, in order to solve it successfully. His question provided me as teacher with an opportunity to support the development of the sociomathematical norm of communicating mathematically as it encompassed students being able to apply mathematical language appropriately. The data indicate that I redirected Harries' question to other students. In contrast to Harries who asked for an explanation, the actions of Wataru, Ray and Tiffany suggest that these three students had some understanding of the meaning of perpendicular bisector and that they were willing to share their knowledge with the rest of the class. The episode shows that I supported students' thinking by asking extending and probing questions and using a diagram. By legitimizing Wataru's explanation of the term bisect (line 10), I accepted the role of mathematics authority and representative of the wider mathematical community. Tiffany's contribution indicates that not only was she able to use mathematical language extensively, she also had a deep understanding of the phrase 'perpendicular bisector.' These students can be seen as making use of the social norm of sharing ideas. It is noteworthy that they used a number of other mathematical terms, such as bisect, equal parts, and 90º, as part of explaining the term

perpendicular bisector, thereby demonstrating their wider ability to communicate mathematically.

The next example happened about eight weeks later.

Example 6.4.1c: Class 4 students use mathematical language to communicate application of differentiation ideas

This example illustrates my continuing actions to develop class 4 students' ability to use the language of mathematics in spoken and written text, in this case to communicate about the application of differentiation ideas. It shows that I made use of whole-class discussion, questioning (prompting, probing and extending questions), teacher modelling mathematical communication, and teacher revoicing some students' statements, to support the development of the norm of communicating mathematically. In previous lessons, students had studied differentiation, including the differentiation of polynomials and composite functions. This episode begins a few minutes into the lesson. Its purpose was to prepare students for group work in which they solved a problem involving the application of differentiation.

We enter the episode at the introduction stage of the lesson.

1. Me: Today we are looking at application of differentiation.

2. [*I write the topic 'Application of differentiation' on the board*]

3. Me: What does application of differentiation mean?

4. Wataru: Applying rules of differentiation in different situations.

5. Me: Applying rules of differentiation in different situations. What does 'different situations' mean?

6. [*Silence. No response*]

7. Me: Right. Can we have examples of these different situations? Those doing Physics, can you help us out?

8. Wataru: Velocity. When velocity is involved.

9. Me: Yes. Velocity. So what is the link between velocity and differentiation?

10. Wataru: Velocity is rate of change. Differentiation is rate of change

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of y with respect to x.

11.	Me:	Velocity is rate of change of what?
12.	Deependra:	Displacement with respect to time.
13.	Me:	Deependra brings in another term. Displacement. What does displacement mean?
14.		[*Silence. No response*]
15.	Me:	What other simpler word can we use in place of displacement?
16.	Ray:	Distance.
17.	Deependra:	Distance travelled.
18.	Me:	So, if distance covered is s we can say velocity is rate of change of what?
19.	Ss:	*ds, dt.*
20.		[*I write* V=ds/dt *on the board*]

(Episode 6.4.3 (15/05/2006) (Class 4): Application of differentiation)

My actions of eliciting student ideas and encouraging their active participation throughout this episode (e.g. lines 5, 7, 11 & 15) demonstrate the way I pursued the development of the norm of communicating using mathematical terms through whole-class discussion. The episode shows that, to initiate whole-class discussion, I posed a prompting question (line 3). Extending and probing questions sustained whole-class discussion which created an opportunity for my students to communicate ideas about application of differentiation, using the terminology of mathematics. As teacher, I modelled communicating mathematically (e.g. lines 3 & 9) and provided instant replays of some students' statements (lines 5, 9, 11 & 13). Through revoicing students' contributions, I legitimised the language they used. In doing this, I intended to encourage students to continue using mathematical language. By allowing a moment of silence in line 6, I provided time for students to consider their responses before articulating their ideas.

Granting students the opportunity to use an example as an alternative way of explaining what 'different situations' meant, in line 7, turned out to be helpful

because Wataru (line 8) then introduced another important mathematical term, that is, velocity.

The actions of Wataru (lines 4 & 9), Deependra (line 11) and Ray\Deependra (lines 16 & 17) illustrated that these students demonstrated that they could use the norm of communicating using mathematical terms. That is, they demonstrated their willingness to participate and contribute using mathematical language. In doing this, they demonstrated what this norm could look like when enacted by students. This had the potential to support the norm's ongoing evolution as something more and more students did. Taken together, these student actions illustrate the students embracing the sociomathematical norm of using mathematical terms as part of routine classroom dialogue. As the lesson progressed, the students continued to use the language of differentiation when discussing their ideas and solutions.

My students had their opinion about students communicating mathematically. These perspectives are presented next.

6.4.2 Student perspectives

In this section, interview data are reported that indicate that my students found it hard to communicate mathematically during mathematics activities. Also, the data suggest that students need the teacher's support to develop a normative focus on using mathematical terminology and symbols to communicate mathematics ideas.

Communicating mathematically is hard

In separate interviews, my students disclosed to me that they found communicating mathematically hard. When I held an interview with Ahmed, Laila and Khoula in September, 2005, Ahmed preferred to "mix Arabic [his first language] with English" when discussing mathematics with his peers. He added "…because we don't know the correct language to use in English. Discussing mathematics in English is hard." He, however, felt that Arabic lacked some terms used to describe certain things so meaning could sometimes be lost if they discussed using Arabic only. Laila and Khoula were of the view that discussing mathematics with peers was better done in Arabic, rather than English, because they understood each other better when talking in their first language. In a separate interview, Susan stated that she preferred to use

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her first language (Chinese) rather than English when discussing mathematics with peers because "it is easier to understand and to speak. It is hard to discuss mathematics in English." Rita agreed, and added that students were not always willing to communicate mathematically with peers because "they are shy or have problems with the language." Similar views emerged when I interviewed different students in 2006, the general view being that communicating mathematically is hard for NESB students, in particular, due to language issues. Najilla (June, 2006) preferred to "communicate in Arabic with her friends, because it is easy." For Thomas, "Some mathematical words are unfamiliar. We do not know their meaning. We cannot use them." In 2005 and 2006, a total of nineteen students stated that they found communicating mathematically hard, particularly at the beginning of the semester. Student comments suggest that my students found it hard to use mathematical language, in English, to communicate mathematics ideas.

Teachers should support student language learning and use

In both years, there were students who spoke of the need for teachers to support student language learning and use. For instance, in 2005, Fiona suggested that mathematics teachers "use simple language when teaching mathematics because it is easier to understand." Yosuke, also in 2005, stated the teacher should focus on teaching NESB students vocabulary, particularly mathematical vocabulary. In 2006, Honda said "It's important for the teacher to teach us [how to communicate mathematically]." Sayuri, who attended the same interview, agreed. Similar views were repeated in other interviews, for example, by Em (2005), Laila (2005), Yuichi (2006) and James (2006). In total, fifteen of my students (in 2005 & 2006) confirmed during interviews that they expected the teacher to assist them learn and develop the ability to use English language to communicate their ideas.

The interview data in this section suggest that these students were sensitive to the role their understanding of, and ability to use, language played in their communicating and learning mathematics. Most did not, however, talk about the importance of students developing the ability to use mathematical language. They were more concerned with their ability to use English language in general, not the language of mathematics.

My students' comments, in both 2005 and 2006, reflect that communicating mathematically was challenging for them. Additionally, some student comments indicate that there is need for teacher care in developing the sociomathematical norm of communicating mathematically where this encompasses students being willing and able to use mathematical words and symbols when discussing mathematics ideas. Table 6.3 presents a summary of student perspectives on matters to do with communicating mathematically, and the numbers and percentages of students who publicly voiced the perspective.

Table 6.3   Student perspectives on issues around communicating mathematically

Student perspective	Number of students voicing the view (n = 27)	
	First interview	Second interview
Communicating mathematically is hard	22 (81%)	10 (37%)
Teachers should support student language learning and use	15 (56%)	15 (56%)
The teacher and students need to communicate mathematically	10 (37%)	20 (74%)
Communicating mathematically is essential	11 (41%)	23 (85%)

This table indicates that, some students, who had indicated, during the first interview, that communicating mathematically was hard for them, changed their opinion as the semester progressed. It also indicates that some students' views about the need for both the teacher and students to communicate mathematically and the value of communicating mathematically, shifted between the interviews. At the time of the second interview, more students than during the first interview, wanted both the teacher and students to communicate mathematically and the number of students who expected mathematical communication increased. Apart from this, more students were of the view that communicating mathematically is essential when learning mathematics. During both the first and second interviews, the majority of students wanted the teacher to support their language learning and use. Overall, the table lends support to the notion that there is value in a teacher helping students to communicate mathematically.

6.5 Chapter summary

This chapter has presented data that illustrate and clarify the ways I initiated and guided the construction and development of the sociomathematical norms of mathematical problem-analysis, explaining and justifying mathematically, and communicating mathematically. Data showed that the main strategies to do this were organising a mix of small-group and whole-class discussions, the use of contextualised and non-contextualised mathematics problems, allowing wait-time, the use of prompting and probing questions, and teacher modelling. Other strategies were displaying the problem on the board/overhead projector, revoicing and the use of diagrams.

For each sociomathematical norm, illustrative data was separated into my experiences and student perspectives. Examples were used to illustrate that students participated by contributing their ideas during the development and enactment of each of the three sociomathematical norms. They showed the ways I guided students to develop an understanding of what constituted mathematical problem-analysis, explaining and justifying mathematically, and communicating mathematically. That is, they illustrated the criteria for these activities in my classroom. The examples provided illustrate that, over time, students took a more active part in mathematical problem-analysis, explaining and justifying mathematically, and communicating mathematically.

The focus on establishing the sociomathematical norm of mathematical problem-analysis was accompanied by a focus on understanding the mathematical question embedded in a problem (see Example 6.2.1b) and by classroom interactions focused on negotiating mathematical meaning (e.g. Example 6.2.1c). Over time, as my NESB students learned how to analyse a mathematical problem, they undertook mathematical problem-analysis by themselves and within their small group (e.g. Example 6.2.1e). Gradually, as students' analysis of contextualised mathematics problems strengthened, they were able to produce acceptable solutions (see Example 6.2.1e). Student interview data reflected that, progressively, fewer students found contextualised mathematics problems challenging and hard to understand, and fewer expected me as teacher to help them understand the problem, explain the meaning of

the question and how to answer it, and to demonstrate problem-analysis. At the same time, a majority of students came to recognise the importance of mathematical problem-analysis in mathematics problem solving and the value of collectively analysing a contextualised problem in small groups or as a class (see Table 6.1).

Classroom episodes and journal extracts indicated that guiding the development of the sociomathematical norm of explaining and justifying mathematically permitted clarifications and justifications to become the norm, an expectation that students had of each other. More explanations and justifications, based on mathematics reasons and interpretations, were offered publicly over the course of a semester (see Examples 6.3.1a and 6.3.1d, with Class 1, at the beginning and near the end of semester respectively). In Example 6.3.1a, the students sought and learned from the mathematical explanations and justifications of others, and their mathematical reasoning was made public and open to reflection and challenge by classmates. Examples 6.3.1c and 6.3.1d show that, later in the semester students offered mathematical explanations and justifications that were of better quality. They defended their own and their classmates' thinking using mathematical reasons, and rejected or refined some of their original mathematical thinking without being embarrassed or pressured by others (e.g. Example 6.3.1c). Student interview data indicated that between the two interviews most of the students who were interviewed came to understand the need for both the teacher and students to explain and justify mathematically (Table 6.2). This suggests that some students gradually recognised that mathematical reasons and interpretations are essential when learning mathematics. At the same time, fewer students expected only the teacher to provide mathematical interpretations and reasons.

Finally, in this chapter, examples have been presented that indicate that developing the sociomathematical norm of communicating mathematically led to an expectation that conventional mathematical language was used to communicate mathematics ideas (e.g. Example 6.4.1b). This enabled improved NESB student communication of ideas as in Example 6.4.1c.

Information from interview data suggested that as time went by, fewer students found communicating mathematically difficult. Students who had indicated in the first interview that it was hard to communicate mathematically had changed their mind at the time of the second interview. Data also indicated that some students wanted the teacher to support them learn and use language that would enable them to communicate mathematically (see Table 6.3).

Overall, examples illustrate that, progressively, some of my NESB students used each of the sociomathematical norms of mathematical problem-analysis, explaining and justifying mathematically, and communicating mathematically. The examples suggested that my students accepted and experienced authority and intellectual autonomy in mathematics. An example was Example 6.3.1d, when, during a short episode, five students explained and justified (in front of the whole-class) the difference between $y = e^x$ and $y = ae^{kx}$.

Examples also illustrate that NESB students' mathematics learning can be supported by involving them in the negotiation of normative aspects specific to mathematical activity. From the sociocultural perspective, I have illustrated that the act of constructing sociomathematical norms is a social, cultural and emergent activity.

In addition to demonstrating that, gradually, students used various sociomathematical norms, this chapter has illustrated how progressively I changed my teaching style. The changes to my teaching practices were similar to those associated with the establishment of classroom social norms. Some of these changes were a move to (1) allow students longer wait-time (Examples 6.3.1d & 6.4.1c), (2) the use of writing on the board as a form of communicating mathematics ideas (Example 6.2.1d), (3) making use of small-group to whole-class discussion (Examples 6.2.1c & 6.4.1a); and (4) making use of more prompting and probing questions to elicit student thinking and to encourage them to explain and justify mathematically (Example 6.3.1b), and to communicate mathematically (Example 6.2.1d).

Data presented so far have been concerned mainly with classroom social norms and sociomathematical norms. This provides partial illustration and clarification of the support that was given to NESB students' mathematics learning in this study. In order to fully understand my teaching practice and what I did to enhance my students'

mathematics learning, it is necessary to present classroom-based analyses of classroom mathematical practices. These analyses are presented in Chapter 7.

CHAPTER 7

CLASSROOM MATHEMATICAL PRACTICES

7.1 Introduction

This chapter is concerned with the initiation and development of classroom mathematical practices. The data demonstrates the ways I initiated and guided the development of specific mathematical practices and at the same time illustrates the practices that emerged. The analysis is centred on teacher and student actions, reasoning, and the use of symbols and tools in different mathematical activities. Guided by sociocultural theory, the analysis locates students in the social environment described earlier and demonstrates the collective mathematical learning that accompanies the initiation and communal development of topic specific classroom mathematical practices. The chapter's focus is on the immediate and local situation of student mathematical development. I document the emergence of negotiated ways of reasoning and acting mathematically that arose as students participated in the constitution of classroom mathematical practices. By doing this, I provide evidence of mathematical practices that were established in the course of classroom conversations, and account for the shifts in the quality of NESB students' mathematical reasoning and understanding.

Eight classroom mathematical practices are described and discussed. They include arrangement and selection practices, sequence practices, equation practice, logarithm modelling practice, and limit practice. These practices were selected because they cover processes associated with key mathematics content areas. Different examples highlight particular aspects of the processes involved in the development of classroom mathematical practices. While discussing the development of mathematical practices, I acknowledge the diverse ways in which individual students revised their arguments as they participated in collective mathematical practices and contributed to their development. The individual and social nature of mathematics learning is highlighted. Student perspectives concerning classroom mathematical practices are presented and discussed, and a case is made that learning mathematics is, in part, a process of negotiating classroom mathematical practices. Finally, I present the chapter summary.

7.2 Arrangements and selections practices

My analysis in this subsection is based on an episode that occurred in my mathematics class on the first day of semester A, 2006. We enter the episode when students had solved the following task:

Arrangements and selections problem:

> Consider a group of friends who decide to go to the beach during the day, then have a meal at the restaurant before going to watch a movie at the movie house. If there are 2 beaches, 3 movie houses and one restaurant, in how many different ways could the group of friends spend the day? (Barton, 2001, p. 44)

7.2.1 Mathematical practice 1: Counting the number of arrows

While students were solving the task, in the text book, in small groups, I moved from group to group monitoring their work and supporting their thinking through questioning. After a few minutes, I opened a whole class discussion by asking the students to explain how they had solved the task. When no one offered a response, I suggested that we use a diagram to help us solve the problem. With further probing some students started to respond.

1. Me: How many beaches should they choose from?
2. Omar: Two
3. Me: How many do they want to go to?
4. Muhannad: One
5. Me: Suppose they choose Beach A, how many choices do they have for the restaurant?
6. Thomas   Three choices.
7. Me: So what is their first choice?
8. Sayuri: A, C, F

> (Episode 7.2.1 (27/02/06) (Class 3): Number of ways of choosing from different items)

As the whole-class conversation progressed, I summarised students' responses by writing the following on the board:

Beach	Restaurant	Cinema
A	C	F
B	D	
	E	

I then asked the students to work in pairs and find the total number of ways. I could see students reading and re-reading the question in an effort to figure out what the question required. Eventually, many students solved the task by drawing and counting the number of arrows on their diagrams. Figure 7.1 below is a typical example of the students' working.
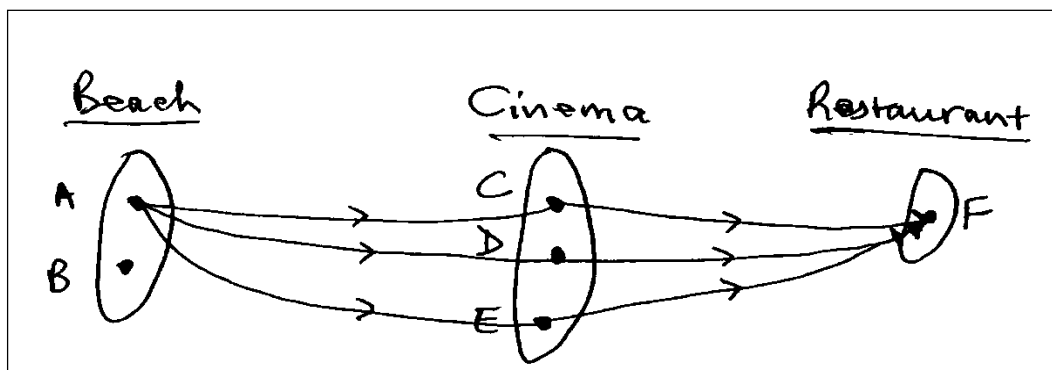
Figure 7.1: A student strategy to find the number of possible choices

After a few minutes, I again invited the students to explain how they had found their answers. The discussion led to the identification of six different ways of choosing a beach, cinema, and restaurant. There was an almost total agreement that the possible number was 6. This answer was justified by counting the number of arrows at the final stage of the diagram, that is, arrows ending at F.

I then changed the problem from 2 beaches, 3 movie houses and 1 restaurant to 3 beaches, 4 movie houses and 2 restaurants, and asked students to find the number of ways of spending the day. Again some students solved the task by drawing a diagram and counting the number of choices as shown in Figure 7.2.
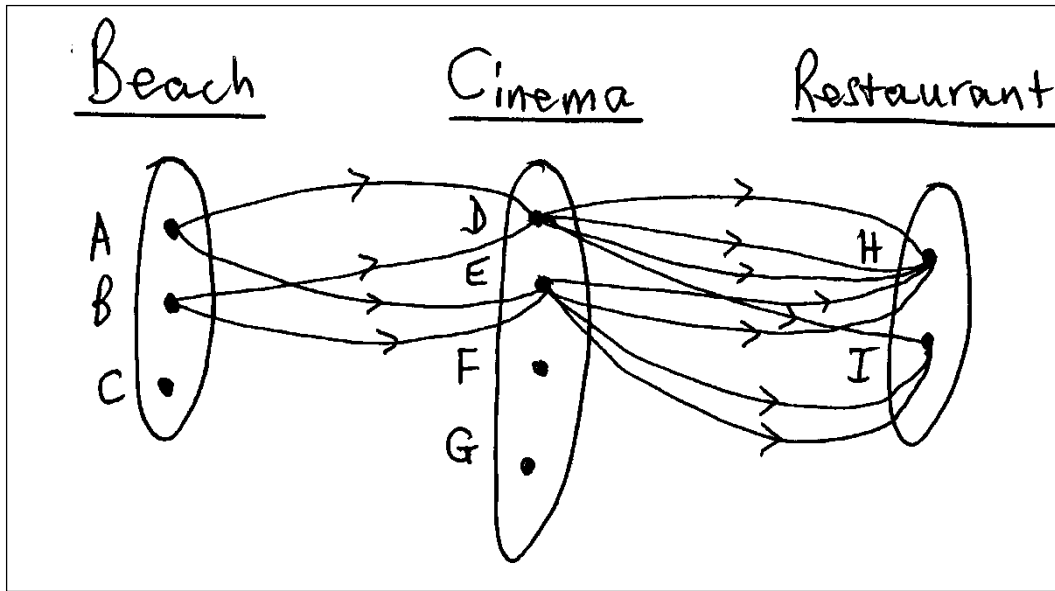
Figure 7.2: Finding the number of possible choices

During whole-class discussion, following this small-group activity, some students reported that they had found the answer of 24 by counting the number of arrows that end at H and at I. Other students said they had not used the diagram with arrows but were able to get 24 by multiplying 3, 4 and 2. The students who had used a diagram with arrows were not asked to justify their answer indicating that the mathematical practice of counting the number of arrows had been established for this classroom community. In contrast, the group that had solved the task by multiplying was asked by their classmates to explain how and why. These students were able to describe how they had found the answer (i.e. multiplying) but struggled with explaining why their method was valid, mathematically. Students' difficulties in articulating their thinking demonstrated language demands on NESB students learning mathematics. At this point of the discussion, I intervened and made use of a diagram on the board and different colours of whiteboard pens to help support the students' explanations and justifications of their actions of multiplying the number of choices at each stage. I supported students' explanations by asking questions such as: "How many ways can they choose the beach? For each way of choosing a beach, in how many ways can they choose a restaurant?" and making clarifying statements such as "Each choice of a beach goes with four choices of a movie house and two choices of a restaurant." This conversation provided a transition from the mathematical practice of counting arrows to that of multiplying.

7.2.2 Mathematical practice 2: Multiplying the number of choices at each stage

I next asked "Suppose we increase the number of beaches to 7, cinemas to 12 and restaurants to 9, how best can we find the number of choices available?" Almost all students agreed that drawing a diagram with arrows would be inconvenient. Instead of drawing a diagram and counting the number of arrows, it was agreed that the quicker and more convenient way of determining the number of choices was to "multiply 12, 7 and 9." On the basis of this observation, I consider that the mathematical practice that was being developed was *Multiply the number of choices at each stage*. Prior participation, in a previous lesson, by students in the negotiation of the first mathematical practice of *counting the number of arrows* may have influenced the rapid emergence of the *multiplication principle* as taken-as-shared.

In subsequent tasks, students used the multiplication principle to solve problems involving the possible number of arrangements. Students were able to transfer the mathematical practice of multiplying to new contexts. For example, students worked on the following two tasks, in small groups:

Task 1: In how many different ways can four students be arranged in a row?

Task 2: Three books are to be arranged in a bookshelf. In how many ways can this be done if there are nine books?

As I was moving round while students worked on the tasks, I noticed no students relied on drawing diagrams with arrows. Instead, some students created four blank spaces while solving the first task and three blank spaces when doing the second, that is, $"\_\times\_\times\_\times\_"$ and $"\_\times\_\times\_"$ before writing $"\underline{4}\times\underline{3}\times\underline{2}\times\underline{1}=24"$ and $"\underline{9}\times\underline{8}\times\underline{7}=504"$. During class discussion it became apparent that students preferred to use the multiplication principle, rather than drawing a diagram and counting the number of arrows. It was apparent that, for most students, the actions taken to solve the tasks were self-evident. The actions of multiplying were accepted without any objections and no one sought any clarification.

On the basis of these events, it is reasonable to conclude that the mathematical practice of multiplying the number of choices at each stage of an arrangement had been established for this classroom community. Events suggested that the mathematical practice of multiplying numbers evolved naturally from the mathematical practice of counting arrows. In addition, events suggested that when a mathematical practice is taken-as-shared in one context it can be transferred and applied, without need for further justification, in another context.

In contrast to the two mathematical practices I have just described and discussed, in the next two examples that involve sequences, it is whole-class discussion that had a huge impact on the development of the mathematical practices. Additionally, managing multiple mathematical practices is illustrated.

7.3 Sequence practices

In this section, I use three examples that demonstrate how I initiated and guided the establishment and development of three classroom mathematical practices associated with the graph of a geometric sequence and the graph of a sequence. Major strategies to do this were small-group and whole-class discussions, prompting and probing questions, and managing multiple mathematical practices at the same time.

7.3.1 Mathematical practice 3: Using ratio (i.e. $\dfrac{t_{n+1}}{t_n}$) to distinguish a graph of a geometric sequence

The purpose for presenting this episode is to demonstrate the way I organised small-group and whole-class discussions to develop the mathematical practice of using ratio to distinguish a graph of a geometric sequence. Working individually or in small groups, students solved a sequence problem before a whole-class discussion.

The sequence problem:

Determine whether or not each of the following graphs shows a geometric sequence.
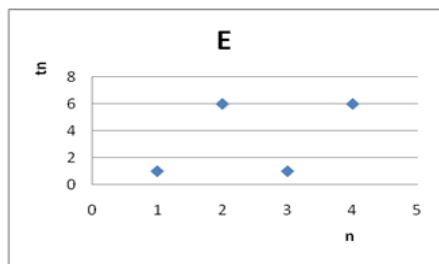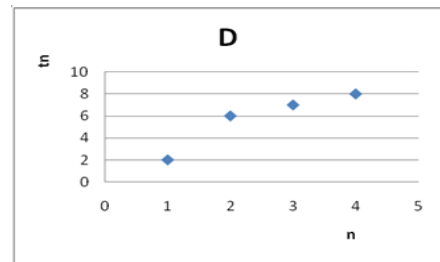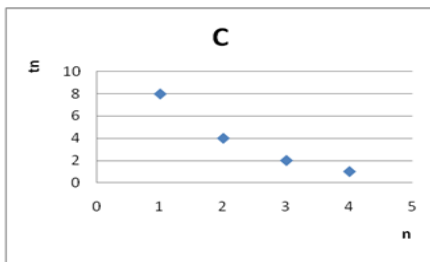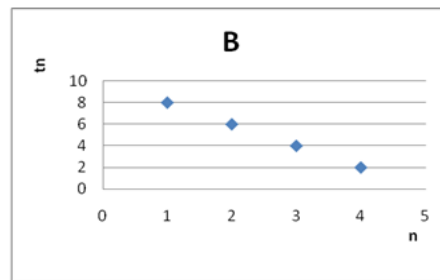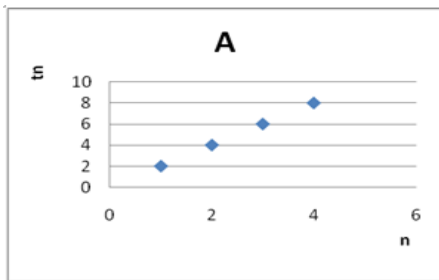
Figure 7.3: Identifying the graph of a geometric sequence (Dividing t_{n+1} by t_n) (Source: Barton, 2001, p. 285)

While working on this task in small groups, many students referred to their notes from an earlier whole-class discussion and applied a number of tests in order to establish a rule for getting from one term to the next. Some students read off consecutive terms on the graph and used them to find the ratio (i.e. a fraction); others worked out the difference (subtracting one term from the other). Their exchanges were centred on convincing other group members that the graph represented either a geometric sequence or an arithmetic sequence. During whole-class discussion, a

number of ideas were put forward and discussed. We enter the following episode when graph A was being discussed.

1.	Me:	Is A the graph of a geometric sequence?
2.	Ss:	Yes/No.
3.	Me:	Some are saying yes while others are saying no.
4.		Why do you think it's 'yes' and why do you think it's 'no'?
5.	Carol:	There is first term and common ratio.
6.	Me:	What is the common ratio? How did you find the common ratio?
7.	Carol:	We subtract 2 from 4.
8.	Me:	What do we mean by common ratio?
9.	Raymond:	4 over 2; or 6 over 4; or 8 over 6. So, it's (i.e. graph A) not a geometric sequence. There is no common ratio in graph A.
10.	Me:	Now when we subtract 2 from 4 what do we get?
11.	Lily:	Common difference.
12.	Me:	So, now in this case are we looking for common ratio or common difference?
13.	Lily:	Common ratio. In graph C we have 4 over 8; or 2 over 4; or 1 over 2. Common ratio is 1 over 2. So it's geometric sequence.
14.	Me:	Why common ratio and not common difference?
15.	Raymond:	Because we want to see if it is a geometric sequence. Geometric sequence has common ratio. Its $\frac{t_{n+1}}{t_n} = r$. We multiply by the common ratio.
16.	Me:	We multiply by the common ratio. When do we need a common difference? Carol?
17.	Carol:	To see if its arithmetic sequence.
		(Episode 7.3.1 (10/10/2005) (Class 1): Graph of a geometric sequence)

During small-group work, students had used their prior mathematical practices of adding a specific number to a term in order to get the next one when dealing with an arithmetic sequence, and multiplying each term by a particular number to find its successor (geometric sequence). I opened this discussion by eliciting student ideas (line 1), and followed up by asking students to give justification for their "Yes" or

"No" (lines 3, 4). In doing this, I initiated the negotiation of a mathematical practice involving the determination of a sequence as geometric through the calculation of a constant ratio. The ensuing class discussion led to the establishment of a mathematical practice of using ratio to distinguish between different sequences. Common ratio and common difference became explicit issues of whole-class conversation. Lily's comment in line 13 shifted the conversation from graph A to graph C. She used graph C to describe the actions she and other members of her group had taken during small-group work, to prove that C was a geometric sequence, by describing a sequence of actions involved in finding a ratio that can be used to get from one term to the next.

Significant differences in students' reasoning were apparent as they participated in the constitution of the mathematical practice of using ratio to distinguish between different sequences. For example, although both Carol (line 7) and Raymond (line 9) used the values $t_1 = 2$ and $t_2 = 4$ in their calculations and focused on making whole-class discussion calculational, Carol based her argument on the difference between consecutive terms while Raymond's was based on the ratio. Carol's statement in line 7 shows that she had subtracted a term from its predecessor, whereas Raymond had divided a term by its predecessor. In addition, Carol did only one calculation and used it to defend her viewpoint while Raymond did several which he used to justify his answer. In contrast to both students, the question I posed in line 14 initiated a conceptual discussion that focused on how arithmetic and geometric sequences were defined by common difference and common ratio, respectively.

Students did not object when Raymond (line 9) used an example as a substitute for an explanation. This indicates that giving an example was accepted, in this class, as a way of explaining. Changes in forms of argument are apparent when Carol realizes that common difference refers to an arithmetic sequence, and not a geometric sequence (line 17). Raymond gave an extended explanation in line 15, indicating that the sociomathematical norm of explaining mathematically was being used to support the development of the mathematical practice. Gradually, Carol's reasoning shifted, and ultimately there was a taken-as-shared understanding of dividing a term by its predecessor, and using this ratio to pick out the geometric sequence. Throughout the negotiation of the mathematical practice, I insisted that students give reasons or

justify their thinking. For example, in lines 4, 8, and 14, I encouraged students to justify their solutions and explain how they had arrived at the answer. However, in line 13, Lily provided an explanation without being prompted, indicating that she was willing and able to use the sociomathematical norm of explaining mathematically.

After further exchanges, the taken-as-shared understanding appeared to be that one could identify the nature of the sequence by looking at the relationship between consecutive terms. The mathematical practice the students negotiated was 'Take any consecutive terms, divide the second by its predecessor and repeat this process a number of times and examine the pattern. A common ratio indicates a multiplicative relation of getting from one term to the next. This is associated with a geometric sequence'. And if that relationship is a common difference, the sequence is arithmetic. If the relationship is neither of these two, the conclusion would be neither geometric nor arithmetic.

Eventually, students agreed that the mathematical practice of dividing a term by its predecessor ($i.e.\ \dfrac{t_{n+1}}{t_n}$) and repeating the process a number of times in order to discover a common ratio, makes it possible to distinguish a geometric sequence from other sequences. This taken-as-shared understanding was achieved to a point where it was no longer necessary for actions to be justified. Students' actions in this episode demonstrated communication of mathematics ideas to support learning.

The journal entry I wrote after the lesson further illustrates my eagerness to have students provide justifications during the development of mathematical practices.

This is what I wrote:

> Discussion resulted in students agreeing that the common ratio is found only in graph C where $\dfrac{t_2}{t_1} = \dfrac{t_3}{t_2} = \dfrac{t_4}{t_3} = \dfrac{1}{2}$. It was further agreed that this is the number that can be used to multiply a term in order to get the next one. The collective understanding was that for a multiplicative relation, multiply a term by $\dfrac{t_{n+1}}{t_n}$ in order to get the next term. In contrast, for an additive relation, add $a_{n+1} - a_n$ to a term to get to the next one. The backing was 'a number can be found (common ratio), in C only, that multiplies each term to get its

successor.' (JE 7.3.1 (10\10\05) (Class 1): Multiplying term by $\dfrac{t_{n+1}}{t_n}$ to get successive terms)

The agreement that C was a geometric sequence was arrived at through exchanges based on conceptual rather than calculational understanding. Like Alagic (2003), by conceptual understanding I mean understanding a concept or topic of study in a manner that enables one to carry out a variety of actions or performances with the topic by ways of critical thinking: explaining, applying, generalizing, and so on. I use the term calculational understanding to refer to having the ability to simply use a memorised skill that is going to be useful for that type of task, nothing more. It is about having the ability to get the correct answer for that particular task. A conceptual approach was thus used to enable students to solve other problems involving sequences by working from their own understanding. Both the episode and the journal extract indicate that whole-class discussion of a sequence problem created an opportunity for students to negotiate the classroom mathematical practice of using ratio to distinguish different sequences.

The collective understanding of how a common ratio or common difference is used to move from one term to the next, and the mathematical practice of using a common ratio to distinguish between different sequences, provided a basis for the development of the classroom mathematical practice of plotting sequence terms on the graph without joining them.

7.3.2 Mathematical practice 4: Plotting sequence terms on a graph without joining them

This episode demonstrates the way I guided the development of the mathematical practice of plotting sequence terms on a graph without joining them. Additionally, it demonstrates how I used small-group and whole-class discussions, and questioning (prompting and probing) to do this.

In small groups, students solved a sequence problem before taking part in whole-class discussion.

The sequence problem:

Draw the graph of the sequence, $t_n = \dfrac{12}{n}$.

Many students started by drawing up a table of values (see Figure 7.4) and sketching the horizontal and vertical axes before plotting the points (1, 12), (2, 6), (3, 4), etc.

n	1	2	3	4	5	6
t_n	12	6	4	3	2.4	2

Figure 7.4: Example of student's table of values

Students' tables of values varied because not all students used values of n from 1 to 6. Some had more numbers, while others had fewer. After marking the points representing sequence terms on the graph, some students joined the points and produced a curve, (e.g. Najilla and Thomas). Others did not join the points, (e.g. Omar and Muhannad). Whether or not to join the points became the focus of conversation during whole-class discussion. The issue that emerged was what to do with the space between the points.

1. Me: Right. What do we do with these points?

2. Muhannad: Just leave them like that.

3. Me: Just leave them like this, why? Can we say we have a graph if we leave the points like this?

4. Najilla: No. We join the points.

5. Omar: No, we don't join because we are using natural numbers, so we can only use whole numbers.

6. Me: Let us find the value of t_n when n = 5.3

7. Davy: No. We cannot because 5.3 is not a whole number. For a sequence   we can only use whole numbers for n.

(Episode 7.3.2 (29\05\06) (Class 3): Joining points on a graph of a sequence)

In this episode, I started by eliciting student ideas on the issue at hand (line 1) and got an immediate, voluntary response (line 2). I extended the discussion by seeking a justification for Muhannad's answer. The question "Just leave them like this, why?" (line 3) shows that a justification was expected and suggests that the mathematical

practice of plotting the points is in the process of being negotiated. My question "Can we say we have a graph if we leave the points like this?" sought students' understanding of what a graph is. This question also distributed authority to evaluate the graph to the students. Najilla, Omar and Davy did this. Najilla's understanding was that points on the graph must be joined but Omar and Davy disagreed. Omar's extended justification (line 5) indicates that he demonstrated use of the social norm for volunteering responses and the need to explain and justify responses as part of the constitution of the mathematical practice. My suggestion in line 6 that we find the point for 5.3, as a means of challenging Omar's idea was met by a definite response from Davy (line 7).

As the discussion progressed, I continued to elicit students' ideas, hoping to maintain a conceptual discourse leading to the establishment of a classroom mathematical practice of plotting sequence terms on a graph without joining the points. As the exchange continued, Thomas and Najilla revised their reasoning for joining the points and, later, while working on similar tasks, plotted sequence points without joining them.

From the exchanges, it came to be taken-as-shared among students that on a sequence graph the points (n, t_n) could not be joined up, because n took on natural numbers only. In the later part of the lesson, students drew sequence graphs by marking the points (n, t_n) without joining them. On the basis of this, I can claim that it became self-evident that the graph of a sequence involves plotting the sequence terms but not to join the points. Omar's (line 5) and Davy's (line 7) comments indicate that they understood that only natural numbers are the only possible first coordinates of a point. Davy was aware that fractional and negative numbers cannot be first coordinates of a point in a sequence graph.

The next mathematical practice emerged as the class continued to discuss the interpretation of the sequence graph. The example focuses on the mathematical practice of increasing the size of the natural number, n, indefinitely and describing the behaviour of the graph.

7.3.3 Mathematical practice 5: Progressively increasing the size of n and monitoring the behaviour of the sequence, t_n

As the lesson progressed, in small-groups students calculated more sequence terms, extended their table of values, graphed them and observed the behaviour of their graph as n becomes larger. In addition, using their table of values, they analysed the pattern of sequence terms as n increased in value. Several students had their table of values (See Figure 7.5).

n	1	2	3	4	5	6	7	8	9	10	...
t_n	12	6	4	3	2.4	2	1.7	1.5	1.3	1.2	...

Figure 7.5: Student table of values for increasing value of n.

When whole-class discussion resumed, it focused on both the table of values and the graph. That is, the students and I, as teacher, continued with the conversation of the behaviour of both the graph and the sequence terms as n becomes bigger and bigger without limit. The graph and the table of values were dealt with separately. For each, students interpreted the behaviour of the sequence when n approaches infinity. Part of whole-class conservation was:

1	Me:	What can we say about this graph as *n* becomes large? When n gets big [*I wrote* $n \rightarrow \infty$ *on the board*], how does the graph behave?
2	James:	The graph goes down.
3	Me:	What do you mean by goes down? Can you explain?
4	Kitty:	The graph goes towards the x-axis. Like this. (*Kitty demonstrates using gestures*)
5.	Me:	Where does the graph cross the x-axis?
6.	Ss:	Does not cross the x-axis.
7	Me:	Now looking at the table of values, what can we say about the behaviour of t_n when n increases? How can we describe the behaviour of the terms?
8.	Thomas:	The terms are getting smaller and smaller.
9.	Me:	What do you mean by that? By smaller and smaller?
10.	Najilla:	The numbers are becoming very small.

11.	Me:	What value(s) of n will give us a negative value for t_n?
12.	Omar:	No value.
13.	Me:	What about zero? What value of n will give us zero?
14.	Omar:	No value.
15.	Me:	Okay. Is there anyone with a question? Anything that is not clear so far?

(Episode 7.3.3 (29\05\06) (Class 3): Increasing n indefinitely)

This is an example of the way I managed and supported the constitution of multiple mathematical practices. Specifically, two mathematical practices, involving the same sequence, were simultaneously collectively negotiated by the classroom community. The first one using the sequence graph and the second using a table of values. In this episode I prompted for an initial response (lines 1 & 5), and probed to get the reasoning behind the response (lines 3 & 9). In addition, I modelled using writing to communicate mathematical ideas when I wrote $n \to \infty$ on the board. Simultaneously, I modelled using conventional mathematical language and symbols (line 1). My questions were intended to support fine-tuning of both students' mathematical thinking and the mathematical practices being developed. Student actions show that they were responsive to my questions. They used the social norm of sharing ideas to support the development of the mathematical practice of progressively increasing the size of n (number of terms in a sequence) and monitoring the behaviour of the sequence.

After this class I wrote the following in my journal:

> For the majority of students, it was self-evident that for this example when n gets bigger and bigger, t_n becomes smaller and smaller. It did not matter whether one used the graph or the table of values. The likelihood that t_n equals zero or becomes negative was discussed. An agreement was reached quickly that in this particular sequence, t_n will not equal zero and is never negative. Increasing n indefinitely was collectively understood to mean there is no largest natural number. That is, there is an unbounded input of values of n. In the course of the conversation, it was agreed that the alternative description of this is that the natural numbers increase to infinity, written, $n \to \infty$. The concept infinity was discussed, eventually collectively agreeing that ∞ is numerically indefinable and hence cannot be used as a number in calculations. (JE 7.3.1 (29\05\06): Increasing n without limit)

As the class continued to discuss the behaviour of t_n when n increases without bound, it was collectively understood that the sequence $t_n = \dfrac{12}{n}$ decreases and becomes close to zero. Eventually, the classroom community's taken-as-shared mathematical meaning seemed to be that 'By selecting sufficiently large values of *n*, we can obtain values of $\dfrac{12}{n}$ as close to zero as we wish.' This can be expressed in the following way: as n becomes very, very large, $\dfrac{12}{n}$ becomes very, very close to zero. More succinctly, this means as n approaches infinity, $\dfrac{12}{n}$ approaches zero. After some conversation during which some students, for example Davy, sought some explanation, it became taken-as-shared that, here, zero is the limit of $\dfrac{12}{n}$ when n approaches infinity.

The teacher clarified that in symbolic form, this is written $\lim\limits_{n \to \infty}\left(\dfrac{12}{n}\right) = 0.$ In later discussion, it was agreed that the limit of a sequence does not always exist and is not always equal to zero.

There was, however, a difference of opinion among students regarding the most efficient way of determining the behaviour of the sequence when n increases indefinitely. Some students (e.g. Jessica) thought that using a graph was a more efficient way of monitoring the behaviour of a sequence, while others (e.g. Sayuri) felt using the table of values was better.

(JE 7.3.2 (29\05\06) (Class 3): Increasing natural number, n, indefinitely)

The journal further illustrates how I initiated and managed the development of the classroom mathematical practice of increasing the value of n and monitoring the behaviour of the sequence. In addition, it shows that some of the students used mathematical language to communicate mathematics ideas. By asking for an explanation, Davy showed that he knew the importance of explanations and asking questions in problem solving. He used the social norm of asking questions to support development of the mathematical practice of progressively increasing the size of n and monitoring the behaviour of the sequence, t_n.

The journal also suggests that the mathematical practice was established rather quickly. This could be because its development was supported by students' knowledge of prior mathematical practices involving sequences in which they had participated. Students' active participation suggests that they were willing and able to

use the social norm of sharing ideas to support learning. In addition, they could justify the reasoning behind the behaviour of the sequence, t_n, as n approaches infinity.

Unlike the previous example that highlights the value of whole-class discussion in the development of classroom mathematical practices as well as the management of multiple practices, the next example demonstrates how some practices can originate from student contributions. It focuses on the constitution of the classroom mathematical practice of finding the point of intersection of perpendicular bisectors of the sides of a triangle and the use of distance formula.

7.4 Equation of a circle practice

The classroom mathematical practice of finding the point of intersection of perpendicular bisectors of sides of a triangle and using the distance formula is associated with the equation of a circle. Its development was supported by small-group and whole-class discussions of a coordinate geometry problem. In addition, it was supported by students' prior knowledge of perpendicular bisectors, distance formula and the general equation of the circle.

7.4.1 Mathematical practice 6: Finding the point of intersection of perpendicular bisectors of a triangle and using distance formula

Students solved a coordinate geometry problem in small groups and then joined whole-class discussion.

Coordinate geometry problem:

> Find the equation of a circle passing through the points (-1, 0), (3, -2), (7, 6). (Barton & Laird, 2003, p. 358)

As they worked in small groups, many students attempted to make use of the formulae (i.e. $(x-a)^2 + (y-b)^2 = r^2$ and $x^2 + y^2 + 2gx + 2fy + c = 0$) that had been discussed and applied in the previous lesson. When I noticed that some students were not making any progress, I made a public announcement discouraging them from relying on substituting given numbers directly into the general equation of a circle. I encouraged them to create their own approaches or strategies instead - their own ways of finding the equation of a circle. As I was moving around while students were busy

with the task, I had noticed Carrie and Winnie making progress. Their solution is shown in Figure 7.6 below.

$$A(-1,0)$$ $$B(3,-2)$$

$$C(7,6)$$

9. a) $A(-1,0)$ $B(3,-2)$

middle — point $P(1,-1)$

$m_{AB} = \frac{2}{-4} = -2$

$m' = \frac{1}{2}$

$y = \frac{1}{2}x + c$ pass $(1,-1)$

$\frac{1}{2} + c = -1$

$c = -\frac{3}{2}$

$\therefore$ $y = \frac{1}{2}x - \frac{3}{2}$ --- ①

$B(3,-2)$ $C(7,6)$

middle point $Q(5,2)$

$m_{BC} = \frac{8}{4} = 2$

$m' = -\frac{1}{2}$

$y = -\frac{1}{2}x + c$.   pass $(5,2)$

$-\frac{5}{2} + c = 2$

$c = \frac{9}{2}$

$\therefore$ $y = -\frac{1}{2}x + \frac{9}{2}$ --- ②

$\Rightarrow$ ① and ②, $\frac{1}{2}x - \frac{3}{2} = -\frac{1}{2}x + \frac{9}{2}$

$x = 3$.

$\Rightarrow$ $y = \frac{3}{2} - \frac{3}{2} = 0$.

Centre at $O(3,0)$.

$d_{OA} = \sqrt{4^2 + 0} = 4 = r$.
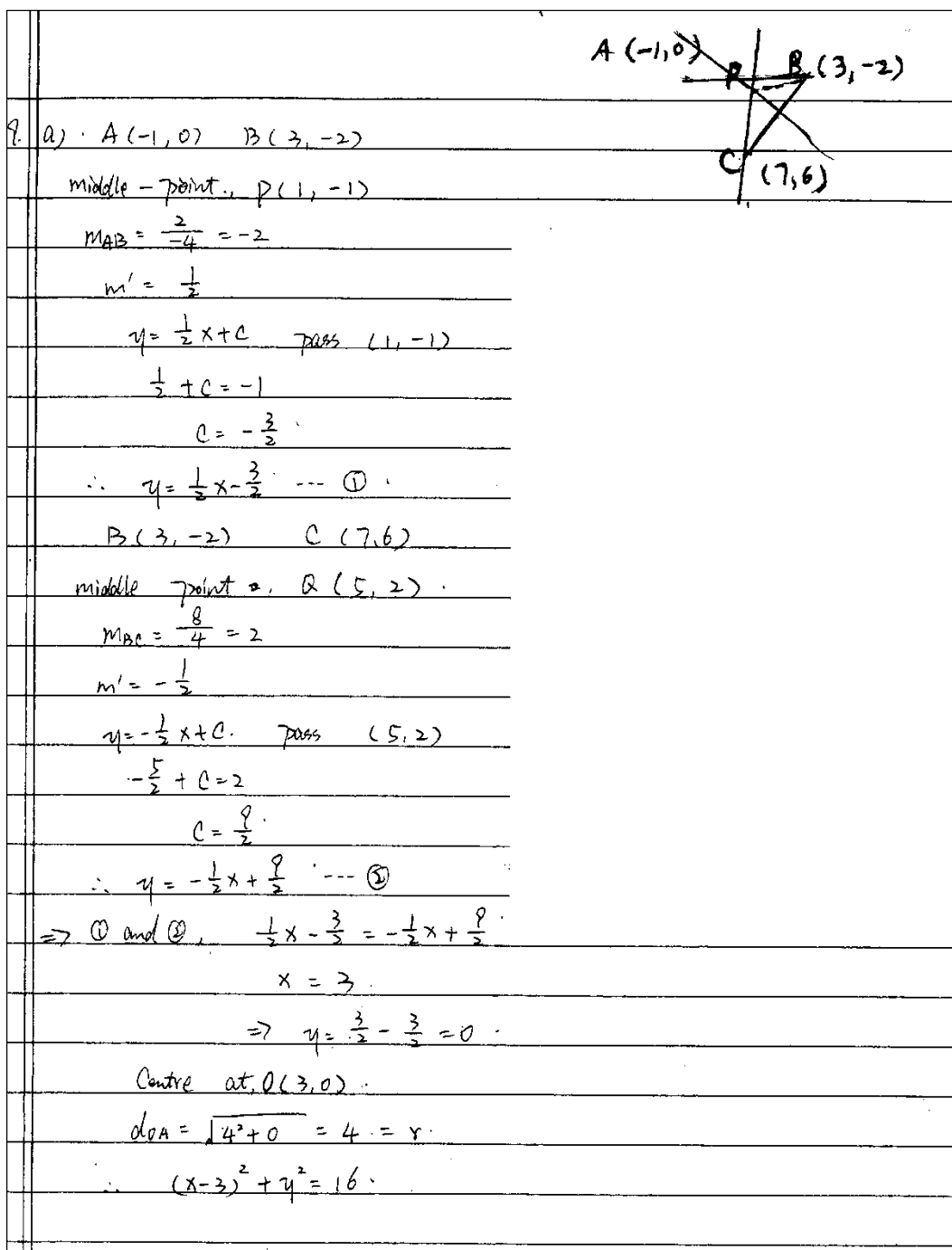
$\therefore$ $(x-3)^2 + y^2 = 16$.

Figure 7.6: Carrie and Winnie's solution (16\03\06): Equation of a circle

Carrie and Winnie's solution was presented on the board and became the topic of whole-class discussion. The issues that were debated included legitimizing Carrie and Winnie's approach. When some students demanded some justification for the

approach, Carrie provided reasons to explain why their method was conceptually valid, and should be accepted as authoritative. Part of the conversation was:

1	Harris:	Where is the centre of the circle?
2	Carrie:	Centre is on this line (*Point at the line on the board*) passing through the midpoint of AB. It has same distance to A and B.
3	Harris:	So how do we know the point? What about C?
4	Carrie:	We need another equation of the line through the midpoint of AC.
5	Tiffany:	How do we get the centre, then?
6	Winnie:	The two lines meet at the centre.
7	Harris:	So do we draw the lines?
8	Chen:	Solve 2 simultaneous equations.
9	Yuichi:	Where is equation 3?
10	Carrie:	We do not need it because the line will pass through the same centre (as the other 2).
11	Me:	What about the radius? How do we find it? Remember we need it in our equation.
12	Carrie:	Use this formula (*Pointing at the distance formula on the board*) to find distance from O to A.
13.	Me:	Is this clear to everyone? Any more questions?

(Episode 7.4.1 (16\03\06) (Class 4): Point of intersection of perpendicular bisectors and distance formula)

Coordinate geometry and Carrie and Winnie's solution created an opportunity for students to use the social norms of sharing ideas and asking questions to support their learning and the development of the mathematical practice of finding the point of intersection of perpendicular bisectors of a triangle and using distance formula. In the course of this discussion, taken-as-shared ways of reasoning with tools or diagram and symbols emerged. The episode shows that I allowed students to share ideas and ask each other questions without my interfering. My actions indicate that I probed for clarification (line 11) and prompted (line 13) to ensure that all students understood the mathematical practice. Student actions indicate that they demonstrated use of the social norms of volunteering to share ideas and ask questions to support their earning. During the negotiation, some students sought clarification on the perpendicular

bisector concept. Carrie exhibited authority to create and judge mathematical knowledge. Other students first challenged and only accepted Carrie's solution as authentic after the explanation. By agreeing to Carrie's approach and explanation, students legitimized the reasoning with tools and symbols that were used. I can claim that the arguments put forward by members of the classroom community during the conversation proved decisive because in later discussions no one objected the use of this normative method of finding the equation of a circle.

It is apparent from this example that I did not view learning to find the equation of a circle as merely a matter of using socially acceptable ways such as substituting numbers in the general equation of a circle. Rather, I organised for the conceptual discourse which led to negotiation of the mathematical practice of finding the point of intersection of perpendicular bisectors of a triangle and using distance formula.

While solving subsequent tasks, I saw a number of students using Carrie and Winnie's method. This indicates that the classroom mathematical practice of finding the point of intersection of perpendicular bisectors of a triangle and using distance formula had been taken-as-shared and as an authoritative practice. The example shows that some mathematical practices can originate from students' contributions or solutions.

In my journal I wrote the following reflection on this episode:

> As I moved around while students worked on the task I noticed that some students were trying to use the general formula of a circle without success. I intervened and advised them that it was possible to find the equation of the circle with or without using either of the two general formula of the circle. Even after my intervention some students persevered with their idea of using the formula, forming three equations and solving them simultaneously. I was, however, particularly impressed by two students (Carrie and Winnie) who used the idea that the perpendicular bisectors of the sides of an inscribed triangle pass through the centre of the circle. In addition, they used distance formula to find the radius. A whole-class discussion that followed small-group discussion focused on verifying the legitimacy of the Carrie and Winnie's approach. Some students wanted Carrie and Winnie to explain and justify their method. To assist her in both her own working while solving the task with Winnie and in explaining to the rest of the class, Carrie used a diagram. To find equations of circles later in this lesson, a number of students used Carrie and Winnie's approach rather than substituting the given numbers directly into the general equation of the circle and solving the equations simultaneously. (JE 7.4.1 (16\03\06): Finding equation of a circle)

This journal further demonstrates the way I guided the development of mathematical practices originating from some students' contributions. In addition, the journal shows that using student work as focus for developing a mathematical practice can benefit other students in the class. It is important, though, for the teacher to look out for unique and exciting approaches used by some students, and utilise them to advance whole-class discussion. With little guidance from the teacher, students can learn from their classmates. Carrie utilized a diagram to support her explanation. Although this mathematical task is non-contextualised, it generated a substantial interest and debate among the students, and in the end they seemed to have gained conceptual understanding of the topic being studied.

Whereas the mathematical practice involving the equation of a circle illustrates how a mathematical practice can originate from students' contributions, the next example demonstrates how comparison of students' solutions can support the development of classroom mathematical practice.

7.5 Logarithm modelling practice

Logarithm modelling practices evolved as students participated in small-group and whole-class discussions where comparing students' solutions and whole-class discussion were valued. The next example focuses specifically on the classroom mathematical practice of multiplying the original amount by increasing powers of a constant greater than one, in this case, three.

7.5.1 Mathematical practice 7: Multiplying the original amount by increasing powers of 3

The classroom mathematical practice of multiplying the original amount by increasing powers of three was negotiated as the class discussed the solutions to the logarithm modelling problem.

The logarithm modelling problem:

> The number of snails in a garden, under ideal conditions, trebles every 5 days. If there were 100 snails at the start of September, how many would there be at the end of the month? (Barton, 2001, p. 310)

While working in small groups, students used different ways to solve the logarithm modelling problem. Some tried to establish a pattern or generalization based on their own thinking, while others tried to find an already-established formula from their text book and use it. Then there were those who spent some time trying to understand the problem. I assisted those students who asked for help in understanding the problem. During whole-class discussion, different solutions were displayed on the board and compared. Figure 7.7 shows the solutions written on the board by several students.

5 Sept = 100×3

10 Sept = $100 \times 3 \times 3 = 100 \times 3^2$

15 Sept = $100 \times 3^2 \times 3 = 100 \times 3^3$

20 Sept = $100 \times 3^3 \times 3 = 100 \times 3^4$

25 Sept = $100 \times 3^4 \times 3 = 100 \times 3^5$

30 Sept = $100 \times 3^5 \times 3 = 100 \times 3^6$

Omar's solution

$100 \times 3^{30} = 2.058911321 \times 10^{16}$

Backing: Trebles means x3 and September has 30 days

Muhannad's solution

$100 \times 3 \times 30 = 9000$

Backing: Trebles means 3 times and September has 30 days

Najilla's solution

$100 \times 3 \times 5 \times 30 = 45000$

Backing: Trebles means x3, September has 30 days, and every 5 days means multiply by 5.

Honda's solution

Figure 7.7: Episode 7.5.1 (31\05\06) (Class 3): Finding the number of snails in a garden

These solutions became the centre of whole-class discussion. Arguments in support of or against different solutions were offered by many students. Students who had displayed their solutions on the board wrote brief explanations below their solutions as the discussion progressed. For some students, Omar's solution was mathematically valid and needed no further explanation. But for others, Omar's solution needed further clarification. Part of the conversation was:

1. Jessica: Why are the powers of three increased after every five days?

2. Omar: Trebles every five days means that after each period of five days we multiply the original number by three. Trebles means times three.

3. Max: What happened to the five? Why is five not appearing in every line?

4. Omar: Because first five days from beginning of the month to 5 September, next 5 days to 10 September, and so on.

5. Thomas: What does treble mean?

6. Muhannad: Means times three.

Episode 7.5.2 (31\05\06) (Class 3): Students challenge Omar to justify his reasoning

Forms of arguments in the justifications focused on valid mathematical reason(s) for each action or step. As the teacher, I insisted that students give justifications involving explanations of how they found the number of snails at the end of September. By demanding explanations and justifications, Jessica, Max and Thomas indicated that they recognised the value of mathematical explanations and justifications in mathematics learning. Unlike the beginning of the semester, when students relied heavily on me to instigate responses, student actions in this episode, indicate that there was no reliance on me prompting and sustaining discussion. Students' actions suggest that they wanted to develop conceptual understanding. Omar's solution or explanation, however, eventually emerged as the legitimate taken-as-shared way of solving the task. The practice that emerged, therefore, involved multiplying the original amount by increasing powers of three, with powers starting at one and ending at six. The other three solutions were rejected by members of the classroom community on the grounds of lacking acceptable mathematical reasons.

In my journal that refers to this episode, I wrote:

> Solutions displayed on the board prompted a lively debate in which some students demanded further clarification of the reasons that were used to justify the authenticity of different solutions. Some students (e.g. Max and Thomas) demanded explanations and justifications for each step. 'Trebles' and 'every five days' were subjects of fierce debate. After an exchange of ideas Omar's solution emerged as the agreed, acceptable answer. Jointly, students were able to 'show' why the other solutions were not mathematically valid. It would appear some students were interested in conceptual understanding, rather than merely getting the final answer. (JE 7.5.1 (31\05\06): Finding the number of snails in a garden)

The evidence in this example suggests that when students' solutions are displayed and used as focus for classroom activity, they can promote discussion that may facilitate the development of a mathematical practice and support student learning. In addition, it is apparent in this episode that writing on the board as a way of communicating mathematics ideas is productive. The value of comparing students' contributions in teaching is illustrated. This example also demonstrates that the taken-as-shared meanings that emerge during joint development of a mathematical practice can be assisted, in part, by communicating mathematical thinking through written form. The next section focuses on limit practices.

7.6 Limit practice

This section discusses the development of the mathematical practice of dividing the numerator and the denominator of a rational function, $f(x)$, by x^n, where n is the largest power of x in the denominator.

7.6.1 Mathematical practice 8: Dividing the numerator and the denominator by x^n, where n is the largest power of x in the denominator

The purpose of this example is to demonstrate that reorganization of established practices can lead to the establishment of other classroom mathematical practices by the class. In addition, the example illustrates finding the limit of a rational function, algebraically. Its other purpose is to demonstrate the role of writing for communication in the mathematics context of limit and rational function. The

example also illustrates that students can exercise autonomy in the mathematics context of limit of a function.

Earlier in the lesson, students worked in small groups where they used a calculator to explore the behaviour of function $f(x) = \dfrac{1}{x}$, when x gets bigger and bigger without limit. Classroom discussion following this small group reached agreement that $\lim\limits_{x\to\infty}\left(\dfrac{1}{x}\right) = 0$. As the lesson progressed, students worked in small groups on a limit of a function problem.

The limit problem:

Find $\lim\limits_{x\to\infty}\dfrac{x-1}{x^2-1}$, if possible.

While students were solving the limit problem, I noticed discrepancies among students' solutions, and interrupted the whole class.

1.	Me:	Some of you are saying limit = 0, while others are saying limit is…
2.		[*Snowy interrupted me*]
3.	Snowy:	Limit is 0.
4.	Deependra:	Limit is 1.
5.	Me:	We have heard two different answers. Is there any other?
6.		[Silence]
7.	Me:	Okay, Snowy and Deependra, can you write your solutions on the board, please? Then we can all look at them.
8.		Snowy wrote: Deependra wrote:

Snowy wrote:

$$\lim_{x\to\infty}\frac{x-1}{x^2-1}$$
$$\frac{x-1}{(x+1)(x-1)} = \frac{1}{x+1}$$
$$\lim_{x\to\infty}\frac{x-1}{x^2-1} = 0$$

Deependra wrote:

$$\frac{x-1}{x^2-1} \Rightarrow \frac{1-\dfrac{1}{x}}{1-\dfrac{1}{x^2}}$$
$$when\ \lim_{x\to\infty}$$
$$\left(\frac{1-0}{1-0}\right) = \frac{1}{1} \Rightarrow 1$$

(Episode 7.6.2 (06\04\06) (Class 4): Snowy's and Deependra's solutions (Limit as $x \to \infty$))

During whole class discussion following the brief episode above, students compared Snowy's and Deependra's solutions. I asked both Snowy and Deependra to justify their solution. The two students took varying positions on the solution and each one defended his position. I take this as evidence of knowing and exercising authority. The mathematical practice that was established as students discussed these solutions involved dividing the numerator and the denominator by the largest power of x in the denominator. Arguments for and against each of the two solutions were put forward and debated by the classroom community. During the conversation, the class noted that both students made use of ideas from the earlier part of the lesson. In particular, determining the behaviour of $\frac{1}{x}$ as x approaches infinity, as well as factorising and simplifying a fractional function were ideas that had been referred to while negotiating other mathematical practices.

Deependra's solution was dismissed by the classroom community because he divided the numerator in the function $\frac{x-1}{x^2-1}$ by x but the denominator by x^2. His classmates argued that the function he was working on was no longer equivalent to the original one, hence the solution was wrong. However, his classmates could not convince him that his reasoning was not valid. On one hand, by questioning Deependra's solution, my students demonstrated authority. On the other hand, by contesting his peers' thinking, Deependra demonstrated he had developed autonomy. Regarding this incident, I wrote the following in my journal:

> Both Snowy and Deependra exercised autonomy. Deependra's misconception was quickly identified by his classmates. However, his colleagues' explanations failed to convince him that he had a misconception. It is at that point that I intervened and suggest that we use an ordinary fraction, $\frac{4}{8}$ to help Deependra to understand how to get the equivalent fraction. I then asked Deependra a sequence of questions which led him to write the following on the board:

$$\frac{x-1}{x^2-1} = \frac{\dfrac{x}{x^2} - \dfrac{1}{x^2}}{\dfrac{x^2}{x^2} - \dfrac{1}{x^2}} = \frac{\dfrac{1}{x} - \dfrac{1}{x^2}}{1 - \dfrac{1}{x^2}}$$

$As\ x \to \infty,$

$\lim = 0.$

(JE 7.6.1 (06\04\06) (Class 4): Finding limit of rational function, f(x), as x approaches infinity)

Students noted that Snowy's argument was based on factorizing and simplifying the function while Deependra relied on dividing the numerator by x and the denominator by x^2. In the course of the negotiation, it emerged that Snowy had visualized that when x was very big, $\frac{x-1}{x^2-1}$ was equivalent to $\frac{1}{x}$, hence $\lim_{x \to \infty}\left(\frac{x-1}{x^2-1}\right) = 0$. Snowy did not use a calculator to determine the behaviour of the function since the mathematical practice of computing with increasing values of x had been established earlier. He used a mental picture. Alternatively, he knew, from an earlier lesson, that $\lim_{x \to \infty}\left(\frac{1}{x}\right) = 0$. The discussion focused on how the original function could be transformed to an equivalent function involving $\frac{1}{x}$. As this discussion progressed, the classroom mathematical practice of dividing the numerator and the denominator of a fractional function by the highest power of x in the denominator was negotiated. These ways of acting and reasoning became long-standing since in later work in this lesson, all students first changed their fractional functions to equivalent forms involving $\frac{1}{x}$, before visualizing the behaviour of the function as $x \to \infty$. In this session, no one demanded a justification for why $\lim_{x \to \infty}\left(\frac{1}{x}\right) = 0$ since this had been established in the earlier part of the lesson. The classroom mathematical practice that emerged from this activity did not rely on the use of the calculator. Rather, the focus was on algebraically finding the limit as x approached infinity. This mathematical

practice emerged as a reorganization of the two earlier practices involving limits that had been established earlier by this class.

Student perspectives of classroom mathematical practices that are effective for solving specific mathematics problems are presented next.

7.7 Student perspectives of classroom mathematical practices

In this section, I present student perspectives that demonstrate that my students recognised the value of having the knowledge of specific ways of reasoning and acting associated with a particular mathematics idea. Additionally, the section presents student perspectives that demonstrate that while some of my students expected me, as teacher, to provide specific ways of reasoning and acting about a mathematics idea, others expected all students to be involved in doing this. Student perspectives demonstrate that my views about how mathematical practices could be developed did not always coincide with my students' views. They also demonstrate the need for the teacher to guide the development of students' knowledge of specific ways of acting and reasoning about certain mathematics ideas.

It is important to know the specific ways of reasoning and acting associated with a mathematics idea

In different group interviews, students who made comments about being able to solve problems involving various mathematics topics said that it was important for students to know specific ways of reasoning and acting associated with a mathematics idea. During a group interview on 13 September, 2005, Lily had the view that mathematics students should "understand how to do it." By this, I inferred that she meant that the students should have the knowledge of specific ways of reasoning, acting and symbolizing when faced with problems involving a particular mathematical idea.

In a separate interview, Susan (14\09\05) concurred with Lily's point of view when she declared that knowing "how to do it" was important. Susan added that it was important for the teacher to "check the answer so that everyone knows how to do it. Knowing the process is important." The 'how' was highlighted by both Lily and Susan. I took this how to mean the calculation process as well as the normative ways of reasoning, arguing, and symbolizing.

In yet another interview (15 September, 2005), Dong expressed the same view as Lily and Susan in asserting the importance of both the correct method and the final answer. His justification for this was "you need full marks." Rather than highlighting the need for conceptual understanding, Dong declared that for him it was about getting correct answers and earning high marks. When Dong said "Both method and answer are important" I interpreted his use of the word 'method' to mean the normative ways of reasoning about, arguing about, and symbolizing a particular mathematical idea. These students' views overlapped with my views about classroom mathematical practices. I agreed that normative ways of reasoning, acting and symbolizing were important, but I did not support the idea of highlighting getting full marks.

Ahmed, Khoula and Laila (31\10\05) were of the view that knowing the method was more important than knowing the answer. They claimed that "If you don't know the method, you cannot get the correct answer." To some extent, Ahmed, Khoula and Laila agreed with other students and with me that it was essential for students to grasp normative ways of reasoning, arguing, and symbolizing. However, my interpretation of their views was that they recognized the importance of conceptual understanding.

Students in 2006 had similar views. In separate interviews, Omar and Muhannad (31\05\06 & 30\06\06), Wataru, Deependra and Harris (31\05\06 & 30\06\06) all emphasised the need to have the knowledge of normative ways of acting and reasoning about a mathematics idea. Omar said "Yes. Students must know how to do it… and the reasons for doing it that way." Wataru asserted "Knowing the process is important… and the reason why." These comments demonstrate that students recognized the significance of having the knowledge of specific ways of reasoning and acting to do with a mathematics idea.

Students should provide specific ways of reasoning and acting associated with a particular mathematics idea

Some of the students I interviewed were of the opinion that when discussing mathematics problems in class, students should provide specific ways of reasoning and acting associated with a particular mathematics idea. Susan (14\09\05), for

example, did not believe in the mathematics teacher telling students everything, she preferred students to provide specific ways of acting and reasoning about a mathematics idea. According to Susan, students must provide justifications for their actions and reasoning, because they should "think of something more, not just yes or no. Students must provide the steps and reasons [to justify their actions or reasoning]. What they did and why." On 15 September, 2005, Chen supported Susan's view but added "It is better if we do it together with clear explanations and clear answer on the board. We need explanations of all actions and all the steps." I took 'do it together with clear explanations and clear answer', to imply specific ways of acting, reasoning, arguing, and symbolizing. Chen's comments suggest that he understood the value of knowing specific ways of acting and reasoning about a mathematics idea, as well as being able to provide the justification for the actions and reasoning. His statements suggest that he understood the significance of conceptual understanding in mathematics learning. In his second interview, Chen (31\05\06) believed that students should be allowed time to brainstorm, in small groups, before they give an agreed response. He said "If they discuss first, they may have an idea. They must explain and justify their actions." This comment by Chen indicates that he supported the idea of students engaging in discussions about normative ways of acting and reasoning related to mathematics ideas, rather than being spoon-fed by the teacher. In separate interviews, in 2006, Muhannad, Thomas, and Wataru agreed with Chen's view.

The teacher should provide specific ways of reasoning and acting related to a mathematics idea

There were students who disclosed to me during separate group-interviews that they expected the teacher to provide specific ways of reasoning and acting to do with a mathematics topic. Em (15\09\05) and Fiona (26\10\05), for example, had this view. Em expected the teacher to "give us model answers so that we can use that for revision. The teacher should justify every step for us so that we understand." Fiona also expected the teacher to give model answers, but "If we do not have an idea the teacher should tell us the ways to do it, and justify each step. Tell us why he did this and that…." These requests show that Em and Fiona, and maybe others, expected me, as teacher, to tell students specific ways of reasoning, arguing, and symbolizing, and to provide justification for the actions. Through their comments, Em and Fiona

demonstrated a product view of mathematics, and hence, the need for the teacher to guide development of classroom mathematical practices that are effective for solving certain mathematics problems. In another interview, Sayuri (02\06\06) claimed that "In Japan the teacher tells us how to do it… but here the answers come from students." In the same interview, Honda suggested that the teacher "do the talking… If the teacher speaks, it is helpful. The teacher must tell us how to do it… give reasons for each step." Honda's comments indicate that he expected me, as teacher, to provide specific ways of reasoning, acting, and symbolizing about a mathematics idea.

In a separate interview, Deependra and Harris (31\05\06) expected the mathematics teacher to "explain slowly [his actions and reasoning]." On 6 June, 2006, Max was in agreement with Deependra and Harris. He stressed that the mathematics teacher "must explain from the beginning how to do it." Max's comments indicate that he expected me to tell students the normative ways of acting, reasoning, and symbolizing. From these interviews, I concluded that some of my students had a product view of mathematics rather than the social, process and cultural view adopted for this study. Student statements confirm the need for the teacher to initiate and guide the development of classroom mathematical practices that emphasise the use of normative ways of acting and reasoning to do with particular mathematics ideas.

7.8 Chapter summary

Through the use of eight examples, this chapter has presented and analysed data that illustrate the ways I initiated and guided the development of classroom mathematical practices. While discussing each example, I highlighted the value of whole-class discussion, negotiation, communicating mathematical thinking through various ways including writing student solutions on the board, and exercising autonomy in the development of mathematical practices.

The first two examples involve arrangements and selections. Not only do the examples show the ways I initiated and guided the development of the mathematical practice of counting the number of arrows and that of multiplying the number of choices at each stage, they demonstrate the evolution of mathematical practices. Data showed that a focus on developing the mathematical practice of counting the number

of arrows made possible the occurrence of class discussions around ways of reasoning about arrangements and selections. In addition, students drew and used arrow diagrams to solve problems involving arrangements and selections. A focus on establishing the mathematical practice of multiplying the number of choices allowed more discussion around ways of reasoning about selections and arrangements. Following these discussions, my students used the multiplication principle to find the number of possible choices, arrangements or selections. Data suggested that students' ability to solve arrangements and selections problems may have been reinforced.

The third, fourth and fifth examples focus on sequences. The third example describes how using ratio to distinguish different sequences was jointly negotiated by the classroom community. Developing this mathematical practice enabled more focus on the reasoning and symbolising about arithmetic and geometric sequences. In addition to this, there was greater attention on the differences between the arithmetic and geometric sequence. It was possible to manage more than one mathematical practice at the same time. Apart from this, data indicated that constituting this mathematical practice enabled my NESB students to use ratio to distinguish a geometric sequence. Data suggested that students may develop better understanding of geometric and arithmetic sequence when they participate in the development of the mathematical practice of using ratio to distinguish a graph of a geometric sequence.

The fourth example involves the mathematical practice of plotting sequence terms on a graph without joining them. My data has shown that establishing this mathematical practice allowed more attention on ways of reasoning about graphs of sequences to happen. Simultaneously, debate about joining and not joining plotted sequence terms occurred. When agreement was reached, some students revised their reasoning about sequence graphs.

The fifth example focused on the mathematical practice of increasing the size of n (where n is a natural number) indefinitely and monitoring the behaviour of the sequence. It emerged that jointly developing this mathematical practice allowed more focus on ways of reasoning and symbolising concerning sequences when sequence terms are gradually increased. There was greater attention on both the table of values and the graph. Additionally, multiple mathematical practices were managed

simultaneously and discussions around limit of a sequence occurred. Regarding NESB student learning, establishing this mathematical practice allowed my students to experience drawing sequence graphs and to use them to determine limit of a sequence. Students learned to describe the behaviour of a sequence when *n* (the number of terms) approaches infinity. Data also suggested that students developed better understanding of limit of a sequence. It showed that my students used mathematical language and symbols to communicate their thinking about limit of a sequence.

Example six is associated with the mathematical practice of finding the point of intersection of perpendicular bisectors of a triangle and using distance formula. The data demonstrates how mathematical practices can originate from students' work. In addition, it demonstrates that students can exercise authority during the establishment of a mathematical practice. A focus on establishing this mathematical practice allowed greater focus on ways of reasoning about coordinate geometry in general, and equation of circle, particularly. Data suggests that when this mathematical practice is collectively developed by the classroom community, students may develop better understanding of the equation of a circle, their reliance on the textbook formula for equation of a circle may decrease, and their ability to solve problems involving equation of a circle may increase.

In the seventh example, I emphasize the value of comparing student solutions when establishing classroom mathematical practices. The example demonstrates the usefulness of writing on the board as a way of supporting the development of a mathematical practice. Establishing this mathematical practice gave rise to debates focusing on the reasoning behind the actions to solve problems involving logarithm modelling. Student solutions became the focus of whole-class discussions, and there was greater attention on comparing them. The data also showed that different students used different ways to solve the same problem. It suggests students may develop better understanding of the topic logarithm modelling when they participate in the establishment of the mathematical practice of multiplying the original amount by increasing powers of a constant, such as three. Additionally, students can develop the ability to establish a generalisation for amounts that increase at a steady rate and their reliance on using textbook formula may decrease.

The eighth and last example is associated with the limit of a function. It shows how reorganization of one classroom mathematical practice can be used to develop another mathematical practice. In addition, the example demonstrates finding the limit of a rational function, algebraically. It illustrates the way students can exercise autonomy in the mathematics context of the limit of a function. Constituting this mathematical practice allowed more focus on ways of reasoning around limit of a rational function. Debate arose around the reasons to justify transforming the rational function. My students experienced finding limit of a rational function. This suggests that students may develop better understanding of the limit of a rational function when they participate in developing the mathematical practice of dividing the numerator and denominator of a rational function, $f(x)$, by x^n, where n is the largest power of x in the denominator.

As I analysed the eight examples, I described the ways of acting and reasoning mathematically among members of the classroom community that gradually became institutionalized and therefore needed no further justification. By describing the development of classroom mathematical practices that emerged, I accounted for what occurred as my students learned different mathematics topics.

While analysing data regarding student perspectives on classroom mathematical practices, student comments suggested that my students recognized the value of having the knowledge of normative ways of acting and reasoning about a particular mathematics idea. However, it came to light that not everyone had the same opinion concerning classroom mathematical practices. One group of students was of the view that, in class, all students should think of, and provide, specific ways of acting and reasoning associated with different mathematics topics. Another group indicated, at the beginning of the semester, that they expected only the teacher to provide normative ways of acting and reasoning about a mathematics topic. They only bought into the idea of collectively developing mathematical practices on being continually encouraged by me to do so.

The findings of the study and key lessons emerging from this practitioner research are discussed next, in Chapter 8.

CHAPTER 8

DISCUSSION

8.1 Introduction

The research question that guided this study was: What can I do to my teaching practice to enhance NESB students' mathematics learning? In line with this question, the main objective of this study therefore was:

> To interrogate my experience; to record, analyse and reflect on episodes or events that occurred as I taught and to report a story of my professional learning as it enabled and constrained the learning of my NESB students.

More specifically, the objectives of the study were:

1. To document, reflect on and interpret teaching and learning events that occur during the research period;

2. To reflect on my own actions during selected events in my mathematics classes;

3. To reflect on my students' actions during the same events in my mathematics classes;

4. To build new understandings of myself and my practice, with special focus on the implications for NESB student mathematics learning;

5. To identify and explore the impact of new teaching practices on students' mathematics learning; and

6. To produce a sequence of writings (or story) about my professional learning as my practices evolve during the research period, in the hope that this will be of benefit to other mathematics teachers of NESB students.

In order to answer the research question and achieve my objectives, I set out to understand my teaching practice by studying it. The practitioner research process led to some important lessons that are discussed in this chapter along the three levels Cobb (2001) set out in his emergent perspective: classroom social norms,

sociomathematical norms, and classroom mathematical practices. A focus on the three levels allowed mathematics learning and teaching to be considered as a social and cultural process. The findings from this approach are examined in relation to the mathematics education literature discussed in Chapters 2 and 3.

8.2 Lessons emerging from this practitioner research

Key lessons emerging from this practitioner research concern what can be done to enhance NESB students' mathematics learning. Findings to address this concern are detailed here. The findings are deliberated on under the headings: Constructive classroom social norms for NESB students' active participation, Productive sociomathematical norms to promote conceptual understanding, and Classroom mathematical practices that are effective for solving particular mathematics problems. The main pedagogical strategies I used to develop these norms and practices are outlined. Since some of these pedagogical strategies cut across the three levels of focus being used in this study, they are referred to and elaborated more than once and, in more than one way, during the discussion of research findings.

8.2.1 Constructive classroom social norms for NESB mathematics students' active participation

This section details insights that emerged when the classroom social norms of volunteering to share ideas, explaining and justifying, and asking questions, which emphasise student participation and negotiation of mathematics meaning were collectively developed by NESB students and me, as the teacher. Based on understanding from literature (see section 2.3), I set out to develop these three social norms, hoping this action would enable my NESB students to become more active in mathematics activities and their learning would be enhanced.

Students volunteering to share ideas

In this study, the social norm of volunteering to share ideas required that students explicitly share their thinking with their peers, without being pressed to do so by me as the teacher. My findings suggest that establishing this as a social norm takes time but it can give rise to more active ways for NESB students to learn mathematics. Verbalizing thoughts, giving group responses or solutions, and offering alternative

solutions can advance collaboration and sharing of ideas among NESB students, in a manner that supports their learning (section 5.2.1). The main pedagogical strategies I used to establish the social norm of students willingly sharing ideas with their classmates and the teacher were the use of both contextualised and non-contextualised mathematics problems, and small-group followed by whole-class discussions. In addition, I used the strategy of initially nominating students irrespective of whether or not they were willing to contribute (e.g. Example 5.2.1b) and then moving to picking only volunteers (see Example 5.2.1h). I consistently encouraged students to participate and contribute through the use of prompting and probing questions. Furthermore, I allowed wait-time after asking a question to allow students time to process their ideas (e.g. Example 5.2.1d), and wrote mathematics problems and solutions on the board in order to encourage and allow students to refer back to the problem if necessary, and to reflect on contributions made (e.g. Example 5.2.1e).

My findings suggest that a focus on establishing the social norm of students willingly sharing their ideas makes it possible for smooth classroom discussions to take place that are good for NESB student learning (section 5.2.1). Discussion is important in the sociocultural view of learning (Cobb et al., 2001; Turner, Gutiérrez & Sutton, 2011) whereby social interactions are thought to enable higher mental processes to occur within an individual (Wertsch, 1985 cited in Dixon et al., 2009). Findings indicate that collectively developing the norm of students volunteering to share ideas enables small-group and whole-class interactions around mathematical ideas with more ideas available for meaning making (see Example 5.2.1e). Progressively, some students more willingly share ideas through engaging in classroom discussions and debates, which may benefit their learning (Examples 5.2.1g & 5.2.1h). This gradual shift by students from non-volunteering to willingly volunteering was reflected in my students' increased public participation in lively mathematical conversations in which they shared ideas and elaborated on their thinking when I as teacher legitimized their ideas. Changes in students' ways of participation suggest that the teacher needs to continually monitor and modify their own teaching in response to movements in NESB students' classroom tactics. I did this by moving to nominate only those

students who showed, by raising their hands, that they wanted to make a contribution (e.g. Example 5.2.1h).

Findings show that establishing the social norm of students volunteering to share ideas can support student learning partly by making students' thinking public (e.g. Example 5.2.1g) so it can be guided in mathematically acceptable directions. Debates that arise when student ideas are publicly shared can stimulate and sustain class discussions that facilitate better communication of productive ideas because students have the opportunity to reflect on and to question the validity of their classmates' thinking. As this happens, the teacher gains more insight into students' thinking, making it possible to decide the next constructive course of action (e.g. Derek's contribution in Example 5.2.1d).

In this study, more effective discussions thrived over time because more student ideas gradually became readily available for public scrutiny and sense making. Some of the students' ideas were questioned or challenged by others (in a productive way) during class discussions. Challenges to solutions of some mathematics problems sometimes led to extended conversations that supported some students' learning in the course of class discussions (e.g. Muhannad's challenge of Jessica's solution in Episode 5.2.3). Other researchers, for example Kazemi and Stipek (2001), and Stein, Engel, Smith and Hughes (2008), in separate studies of mathematics students, stressed the importance of the teacher orchestrating productive mathematical discussions to support student learning. In this study, establishing the norm of students volunteering to share ideas gave rise to more lively classroom discussions that may have supported learning. My findings are compatible with Rasmussen and Kwon (2007), Walsh and Sattes (2005), and Wood, Williams and McNeal (2006) who highlight the significance of promoting the norm of volunteering to share ideas to support student learning as a social process.

Student interview data (section 5.2.2) indicate that some NESB students in my study came to support the view that students volunteering contributions can facilitate meaningful mathematics learning. Their comments suggest that when the establishment of the social norm of volunteering to share ideas is coupled with a situation in which all ideas are valued and treated with respect, students come to

realise that it is safe for them to contribute ideas, even when they are not completely sure these ideas will be right. Allowing students to work first in small groups and then contribute to class discussions can be an effective way of building student confidence and helping them refine their thinking before presenting it in a more public forum.

The comments by those students who came to appreciate the value of sharing ideas as part of mathematics problem solving suggest that they were moving towards a social view of mathematics. This was the case, in this study, in spite of some students having demonstrated a product view of mathematics, (e.g., Carol in Example 5.2.1a expected me, as teacher, to tell her how to solve a problem but later on willingly shared her ideas with peers in Example 5.2.1d) as described in chapter 2, section 2.2.1, at the beginning of the semester (sections 5.2.1 & 5.2.2). Interview data reinforce the view that as the semester progressed some of my students recognised a classroom academic culture that values and encourages students sharing ideas and learning mathematics as a social process.

Students explaining and justifying

The social norm of explaining and justifying, in this study, is about students being able to publicly offer clarifications or reasons or explanations to support their thinking and solutions. There was an expectation for students to publicly describe *why* their viewpoint or solution strategy was valid. The main strategies I used to develop this norm were the use of both contextualised and non-contextualised mathematics problems, small-group and whole-class discussions, allowing wait-time, teacher asking questions that prompt students to explain and justify their ideas, and teacher modelling explanations (section 5.3.1).

Findings suggest that a focus on establishing the social norm of explaining and justifying allows clarification of ideas to be expected by the students (e.g. Example 5.3.1c). Developing this norm permits students to seek and learn from explanations. Clarifications or explanations can be provided publicly, making it possible for some students to benefit from other students' explanations and justifications (see Example 5.3.1c). My findings suggest that collectively developing the social norm of students explaining and justifying encourages and provides a context that prompts NESB

students to think more deeply about their responses because they need to clarify and\or defend their thinking to others (section 5.3.1). This norm presses NESB students in small groups to pay attention to detail and understanding in case explanations and justifications are sought by classmates and\or the teacher during whole-class discussions. Collectively developing this social norm makes it possible for students to be held accountable for their contributions by their classmates and the teacher (e.g. Example 5.3.1b). Students are obliged to provide more well-thought-out explanations and justifications. Over time, NESB students in this study were able to elaborate on their thinking or solutions in small groups and during whole-class discussions (see Examples 5.3.1b & 5.3.1c). My findings demonstrate that as students explain and justify their thinking, they communicate mathematics ideas in a manner that may support their own learning and that of others. Both the student explaining and the listeners may benefit from this process. As a number of other researchers argue (e.g. Levenson, Tirosh & Tsamir, 2009; Perry, 2000; Rasmussen & Kwon, 2007; Yackel, 2001), my findings show that NESB student explanations and justifications aid their mathematics learning (section 5.3.1).

Establishing the social norm of explaining and justifying makes it possible for students to progressively offer explanations and justifications without being pressured (e.g. Examples 5.3.1b & 5.3.1c). While doing this, students practise and share authority. Over time, some NESB students in this study (section 5.3.1) willingly provided explanations and justifications for their own and their peers' contributions, indicating growing student authority, and their willingness to exercise and share it. As they act in this way, NESB students can refine their thinking, and begin to see the value of actively participating and voluntarily offering explanations and justifications in classroom discussions. This finding is consistent with studies that suggest that the development of social norms that encourage students to be actively involved and justify thinking during mathematical discussions gives rise to learning opportunities that benefit students' learning (Ellis, 2007; Cobb et al., 1992; McCrone, 2005).

Data from interviews suggest that as each semester progressed, students expected their peers to provide explanations and justifications during class discussions (e.g. Fiona and Chen in section 5.3.2). Some student comments indicate that, over time, they wanted the teacher to explain only when students could not, reflecting students'

confidence in their classmates' explanations. Student statements suggest that, gradually, they recognised the importance of explanations and justifications in mathematics learning (see section 5.3.2). In addition, they reflect that, over time, my students recognised mathematics learning as a social activity encompassing explanations and justifications. Interview data is compatible with a classroom academic culture that encourages substantial communication among members of the classroom community – in small-group or whole-class discussions.

Students asking questions

The social norm of students asking questions involved the normative expectation that students would ask questions whenever they needed clarification of contributions made in class, or to seek assistance related to solving a classroom task. The main pedagogical strategies that supported the establishment of this social norm included the use of both contextualised and non-contextualised mathematics problems (e.g. Examples 5.4.1a & 5.4.1c), organising small-group and whole-class discussions, teacher openly requesting students to ask questions related to the classroom activity, teacher modelling questioning, and displaying student work on the board (see Example 5.4.1a, in section 5.4.1).

My findings suggest that establishing the social norm of students being able to ask questions is a stimulus for effective small-group and whole-class discussions (Examples 5.4.1a & 5.4.1d). This norm fosters active exploration and learning. It allows student questions to be used as a tool for teaching and learning. The teacher benefits from student questions by being able to evaluate students' understanding of the work being learned through the questions they ask (e.g. Example 5.4.1d). Based on this, the teacher can decide which direction the lesson should take.

Findings suggest that establishing the norm of students asking questions has the potential to raise NESB students' mathematics learning and thinking to higher levels through the classmates' and the teacher's clarifications in responses to student questions (e.g. Example 5.4.1d in section 5.4.1). My findings indicate that student learning, in this study, was promoted by explanations and justifications contributed by students or the teacher during small-group and whole-class, in response to others' questions. They illustrate that NESB students learn to use student questions as a

learning tool to support their understanding of the concept and benefit from doing this (e.g. examples 5.4.1b, 5.4.1c & 5.4.1d in section 5.4.1). By experiencing environments in which questions are asked and responded to, NESB students learn not only how to respond, but how to formulate and ask their own questions. Findings show that when the social norm of asking questions is established, NESB students may recognise the importance of asking questions in problem solving. My findings satisfy the view of Elliott et al. (2009) and Walsh and Sattes (2005) who, in separate studies, emphasised the significance of establishing the social norm of students asking questions to enable multiple voices and ideas, and support learning.

During interviews, student comments suggested that they recognised the importance of asking questions in classroom activity (section 5.4.2). Statements by students suggest that although they wanted to ask questions, if the teacher allowed them that opportunity, they were more confident to do this in small-group setting than in whole-class discussions. Interview data reflect that small-group settings provided a safer environment for students to ask each other some questions.

Insights across the three classroom social norms

The social norms of students being able to volunteer to share ideas, to explain and justify their contributions, and to ask each other questions allow classroom discussions to happen in a safe, productive and orderly manner. Once established as mutual expectations and obligations, these norms sustain productive small-group and whole-class conversations in support of NESB students' mathematics learning (sections 5.2.1 to 5.4.2).

The findings of this study show that the classroom social norms of volunteering to share ideas, explaining and justifying, and asking questions, can develop amongst NESB students when the teacher gradually but explicitly initiates and guides their development. These norms evolve over time and, as emergent and fragile phenomena, they need to be continuously reinforced. In this study, the development process was sustained by NESB students and me, the teacher, through regular classroom interactions that included strategies such as small-group followed by whole-class discussions, asking prompting and probing questions, and allowing wait-time. These

findings reinforce Cobb and his team's (2001) finding that productive social norms need to be jointly established and will evolve over time.

Similar observations to mine were made by McCrone (2005) in a study of fifth-grade students in USA. In that study, students explored and shared ideas while solving mathematics problems in small groups before participating in whole-class discussions. In her findings, McCrone highlighted the significance of establishing social norms that promote discussion of mathematical ideas among students, such as, student active listening and use of classmates' ideas to develop novel conjectures, as a key component of student learning. There are, however, major differences between McCrone's and my students. One major difference is that my NESB students had to cope with more pressing language demands due to English not being their first language (see sections 5.3.2 & 5.4.2). Another key difference was that, unlike McCrone's, my students were working in an academic culture that was different from those they had previously experienced in their home countries (see section 5.2.2).

Findings suggest that a focus on establishing these norms allows students to progressively become more active participants in classroom activities. In this study, increased NESB student active participation is illustrated by episodes and journal extracts in sections 5.2.1 to 5.4.1 (e.g. few students were active in Example 5.2.1a but many were in Example 5.2.1e) which show that NESB students gradually shared ideas, explained and justified thinking, and asked each other questions, willingly, when I, as the teacher, initiated and guided the construction of these norms. These findings are consistent with Wood, Williams and McNeal's (2006) view of a theoretical link between social norms that are supportive of discussion and the nature of social interaction patterns that evolve.

The findings in Chapter 5 (sections 5.2.1 to 5.4.2) suggest that the development of social norms of students volunteering to share ideas, explaining and justifying, and asking questions are influenced by, and also influence, development of NESB students' individual beliefs about their roles, the roles of others and the nature of mathematics activity. Findings indicate that a focus on developing norms of volunteering to share ideas, to explain and justify, and to ask each other questions allows NESB students to progressively exercise and share authority during ongoing

classroom interactions. In addition, findings show that establishing these norms promotes NESB students' development of the social autonomy that is required for student learning. In this study, the term 'social autonomy' is used to mean the ability to make decisions for oneself, about what behaviour is acceptable and not acceptable (or right and wrong) in the social realm (Kamii, 2004). An example, in this study, is when my students sought and provided explanations and justifications (Examples 5.3.1b & 5.3.1c in section 5.3.1). Another one is when my students asked each other questions in a whole-class discussion (e.g. Example 5.4.1d in section 5.4.1). As they progressively become socially autonomous, NESB students gradually shift their participation habits to satisfy the classroom community's expectations and their obligations for participatory behaviour.

These findings are similar to Cobb et al.'s (1997) view that social and individual aspects of learning are reflexively related. That is, establishing each of the three social norms, in this study, enabled both social and individual actions and reasoning to occur. So, my findings conform to the view that learning (mathematics) is a social activity (Hershkowitz & Schwarz, 1999). Developing social norms that enable students to become socially autonomous is in line with Rasmussen and Kwon's (2007) idea of "social norms that empower students to be creators of mathematical ideas" (p.192). The finding is, however, in contrast to some previous research (e.g. Leung, 2001), which indicates that NESB students are inactive and learn by memorising information.

Challenges such as the teacher dilemma of avoiding telling whilst giving support were also highlighted, in this study. My journal extracts in section 5.4.1 illustrate this challenge. In addition, they show that complex moments arise when the teacher is not sure how and\or whether to intervene when students working on a task appear to be going in the wrong direction. In such situations, tensions can be experienced between particular values, such as those of allowing students time to work through a problem by themselves, and the desire to offer a supportive learning environment. This finding is consistent with Smith's (1996) arguments regarding the teacher's sense of loss of efficacy when he finds himself in a situation where he has no clear sense of his role.

A mathematics teacher needs to find ways to cope with inherent tensions and dilemmas such as these, as they interact with students on a daily basis. In this study, sections 5.2.1, 5.3.1, and 5.4.1 demonstrate that I addressed this challenge by using judicious or selective telling, rather than a general telling model (e.g. Example 5.2.1f). I intentionally introduced information to the discussion, such as productive ways of representing mathematical ideas (e.g. diagram, equation, and verbal statement) and useful terminology, to support students' thinking. Alongside this, I openly showed students that I valued their constructive attempts. Thus, results suggest that as teachers deal with the pressures and predicaments that arise during classroom interactions, their learning can productively involve developing instructional strategies that transcend dichotomies such as telling or not telling, and nominating volunteers or non-volunteers.

Overall, my findings in this section, concur with other researchers (e.g. Dixon et al., 2009; Lopez & Allal, 2007; Weber, Maher, Powell & Lee, 2008; Yackel & Rasmussen, 2002), who have shown that developing constructive social norms that encourage and provide opportunities for social interactions can promote more effective classroom discussions that can support students' mathematics learning.

In light of the findings discussed in this section, I propose that NESB students' mathematics learning can be supported by having a focus on establishing social norms that value and encourage student participation through willingly sharing ideas, explaining and justifying contributions, and asking questions during classroom interactions.

This said, social norms alone are not enough to advance NESB students' *mathematics* learning. As a teacher, I needed to consider and attend to both the social and the mathematical elements of the mathematics classroom environment. Findings related to sociomathematical norms appropriate for NESB students are detailed next.

8.2.2 Productive sociomathematical norms to promote conceptual understanding

This section discusses insights for mathematics teaching that came to light as I sought to initiate and guide joint development of productive sociomathematical norms of mathematical problem-analysis, explaining and justifying mathematically, and

communicating mathematically (Chapter 6). As with social norms discussed in the previous section, I set out to establish these particular sociomathematical norms for mathematics problem solving in the course of ongoing classroom interactions. I did this based on understanding from literature (Chapters 2 & 3). I deliberate on insights specific to each of these three sociomathematical norms separately, before focusing on those aspects that are shared by all three.

Mathematical problem-analysis

In this study, mathematical problem-analysis came to mean students developing a normative focus on reading a problem or task carefully for understanding to the extent that a student could identify vital information embedded in the problem, find the meaning of unfamiliar words, terms and symbols, think through how the key information relates to the mathematics of the problem, and identify what the problem requires students to find (Examples 6.2.1a to 6.2.1e in sections 6.2.1).

My findings suggest that greater focus on the capacity to understand a mathematical problem can occur when the teacher openly guides the establishment of the sociomathematical norm of mathematical problem-analysis, through the use of contextualised mathematics problems (e.g. Example 6.2.1a), small-group to whole-class discussions (e.g. Example 6.2.1c), allowing wait-time, and questioning (prompting and probing) (e.g. Example 6.2.1b). Other strategies to develop the norm are displaying mathematics problems on the board or overhead projector and using a diagram (e.g. Example 6.2.1d).

Findings indicate that as NESB students participate in collective development of the sociomathematical norm of mathematical problem-analysis, they learn not only what counts as an acceptable and effective mathematical problem-analysis, but how to analyse contextualised mathematics problems for understanding, as well (section 6.2.1). Specifically, students learn to read a mathematics problem for understanding, find the meanings of unfamiliar words, terms and symbols, pick out crucial information embedded in the problem, and think through how the key information relates to the problem (e.g. Example 6.2.1a). When this happens, findings suggest that, progressively, NESB students' analysis of contextualized mathematics problems

may be strengthened (see Example 6.2.1e). Journal extracts in section 6.2.1 suggest that after practising mathematical problem-analysis, individually, in small-groups and during whole-class discussions, my NESB students showed improvement in their ability to solve contextualized mathematics problems (e.g. Example 6.2.1d).

Episodes in section 6.2.1 show that NESB students working in small groups could negotiate collective understanding of the mathematics problem as part of the process of solving it, demonstrating adoption of the norm of problem-analysis by, on different occasions, reading a mathematics problem for understanding and, identifying and linking vital information in the problem to its solution (see Examples 6.2.1d & 6.2.1e). Additionally, these episodes demonstrate adoption of a classroom culture that values analysing a problem for understanding.

Interview data suggest that many students recognised the importance of mathematical problem-analysis in mathematics problem solving (see section 6.2.2). Student comments indicate that they expected teacher support to develop the ability to undertake mathematical problem-analysis. Data suggest that as the semester went on, more students expected the class to collectively analyse mathematics problems (see Table 6.1), reflecting that they recognised mathematics learning as a social process involving joint problem-analysis. Interview data also strengthen the view that NESB students experience language demands (Adler, 2001; Campbell et al., 2007; Moschkovich, 2002), discussed in section 2.4.2, that need addressing. My findings are in line with other researchers, for example, Bernado (1999), Chapman (2006), Verschaffel, Greer and De Corte (2000) who, in separate studies, emphasized the significance of teachers assisting students develop the ability to analyse mathematics word problems for understanding, through the use of small-group and whole-class discussions, that is, strategies similar to those I employed.

Explaining and justifying mathematically

Providing an explanation or justification, as discussed in the previous section, referred to aspects of a social norm of sharing ideas and reasons. When there is movement to explaining and justifying *mathematically*, then the focus is on the aspects of a sociomathematical norm. In this study, the sociomathematical norm of explaining and justifying mathematically required that students be able to provide

explanations and justifications based on mathematical reasons or interpretations. An explanation or a justification was acceptable as a sociomathematical norm, if it involved a mathematical reason or a currently taken-as-shared mathematical interpretation (section 6.3.1). In this study, the norm of explaining and justifying mathematically was established by the students and teacher in the course of ongoing classroom interactions in mathematical activity.

The main strategies to develop the norm of explaining and justifying mathematically, in this study, were the use of both contextualised and non-contextualised mathematics problems (e.g. Examples 6.3.1a & 6.3.1b), organizing small-group and whole-class discussions (e.g. Example 6.3.1a), posing prompting and probing questions (see Example 6.3.1c), allowing wait-time (Example 6.3.1d), and affording students the opportunity to explain and justify mathematically (e.g. Example 6.3.1b). Other strategies involved teacher modelling mathematical explanation and justification, revoicing student statements (Example 6.3.1a), and displaying mathematics problems on the board or overhead projector (e.g. Example 6.3.1c).

My findings suggest that when the teacher openly initiates and guides the establishment of the sociomathematical norm of explaining and justifying mathematically, by making use of one or more of the strategies in section 6.3, over time, clarifications and justifications of mathematical ideas are expected by students. Progressively, more explanations and justifications, based on mathematical reasons or interpretations, are offered publicly by students (section 6.3.1). A focus on developing the norm of mathematical explanation and justification encourages and allows conversations about explaining and justifying to happen (e.g. Example 6.3.1c). These deliberations can create more opportunities for students' mathematical reasoning to be made visible and open to reflection and questioning by others (e.g. Em questioned Raymond, David and Dong's ideas, in Example 6.3.1a).

Establishing the sociomathematical norm of students being able to explain and justify mathematically enables NESB students to learn not only what counts as acceptable mathematical explanation and justification, but also how to explain and justify (section 6.3.1). A taken-as-shared understanding of what counts as a mathematical

explanation and justification was reached through acts of communication within a classroom mathematical discussion, as students responded to my and other students' requests for reasons to justify different contributions. Students gradually develop a taken-as-shared understanding of the norm and willingly offer mathematical explanations and justifications (see Example 6.3.1c). Over time, the students can seek and learn from mathematical explanations and justifications of others (e.g. Example 6.3.1d). Giving attention to developing the norm of explaining and justifying mathematically presses NESB students working in groups to pay careful attention to mathematical ideas they legitimise, in case their thinking is questioned during subsequent whole-class discussions (Chapter 6).

A focus on establishing the norm of explaining and justifying mathematically encourages and makes it possible for NESB students to question their colleagues' mathematical thinking. At the same time, it allows students to provide mathematical explanations and justifications for their ideas which might not be clear or self-evident to others at first. The students defend their own and their classmates' thinking, using mathematical reasons or interpretations (e.g. Susan defended her thinking in Example 6.3.1b). While doing this, they practise explaining and justifying mathematically. Progressively, students may begin to offer mathematical explanations and justifications of better quality.

As with social norms, both a student making the explanation and justification, and the listeners, can benefit by rejecting or refining their original thinking. While explaining and justifying their ideas to others, students may clarify their own thinking as well, allowing them better understanding of the concepts involved, and improved chances to solve related mathematics problems. Episodes and journal extracts in section 6.3.1 demonstrate that, over time, my students provided extended or detailed explanations and justifications, using mathematical reasoning (see Example 6.3.1d). At the same time, they offered mathematical explanations and justifications of their own and\or their classmates' contributions or answers, when asked for mathematical reasons. In doing this, some NESB students demonstrated that they can offer mathematical explanations and justifications without being pressured by anyone. In addition, they demonstrated adoption of a classroom academic culture that attaches importance to explaining and justifying with the use of mathematical reasons or interpretations.

Students willingly explaining and justifying mathematically illustrate that some NESB students may embrace the sociomathematical norm of explaining and justifying mathematically when the teacher guides its establishment.

Student perspectives, emerging from the interviews, suggest that the number of students who expected only the teacher to give mathematical explanations and justifications decreased as the semester progressed (see section 6.3.2). At the same time, those who expected the teacher and students to explain and justify mathematically increased (see Table 6.2). This suggests that, over time, my students recognised mathematics learning as a process of social construction encompassing mathematical explanations and justifications. Student comments suggest that they gradually recognised that mathematical reasons and interpretations are essential in learning mathematics. Interview data reinforce the value of students being able to provide mathematical explanations and justifications, and communicate mathematically.

Similar conclusions to mine were arrived at by other researchers (e.g. Cobb et al., 2001; Gresalfi, 2009; Levenson et al., 2009; Rasmussen & Kwon, 2007; Stylianou & Blanton, 2002; Weber, Radu, Mueller, Powell, & Maher, 2010). These researchers worked in separate studies that involved students with different language and academic culture backgrounds from mine. Students they worked with ranged from primary (e.g. Cobb et al.) to undergraduate level (e.g. Rasmussen & Kwon). However, as I did, they highlighted the importance of establishing the norm of explaining and justifying mathematically.

My findings conform with the views held by Dixon and his colleagues who contend that when students participate in classroom discussions that encourage them to explain and justify mathematical thinking, they strengthen and extend not only their understanding of the concepts involved, but the understanding of other students in the class as they listen to the explanations (Dixon et al., 2009). Furthermore, they are in line with Walshaw and Anthony (2008) who, in their study, concluded that mathematical explanations stimulate, challenge and extend other students' mathematical thinking in ways that support learning.

Communicating mathematically

In this study, the sociomathematical norm of communicating mathematically was understood to mean students being able to communicate mathematics ideas using conventional mathematical language (i.e. terminology and symbols). To develop the norm of communicating mathematically, I organised students' mathematics learning around academic mathematical discourse and communication. My major strategies were the use of contextualised mathematics problems (e.g. Example 6.4.1a); managing small-group and whole-class discussions (e.g. Example 6.4.1b); explicitly teaching\discussing mathematical language (words or terms and symbols) (Example 6.4.1b); teacher modelling communicating mathematically (e.g. Example 6.4.1c); and teacher posing questions that oblige students to use mathematical language and symbols (e.g. Example 6.4.1c).

As the data indicates, establishing the norm of communicating mathematically allows students to develop a taken-as-shared understanding of what counts as communicating mathematically (section 6.4.1). Findings show that students learn and use mathematical language acceptable to the wider mathematics community in the course of collectively constructing the norm. Episodes and journal extracts in section 6.4.1 show that my NESB students discussed the meanings of unfamiliar mathematical words, terms and notation, and later used mathematical terminology and symbols when I, as teacher, asked them to (e.g. Ray, Wataru and Tiffany in Example 6.4.1b).

My findings suggest that when the teacher openly guides the establishment of the norm of communicating mathematically, progressively, conventional mathematical language is used to communicate mathematics ideas in classroom activities (see Example 6.4.1c). When this happens, students expect the use of mathematical language in classroom discussions involving mathematics. Findings indicate that communication of mathematics ideas between the students and the teacher and among the students can also improve when students use conventional mathematical language during classroom interactions (e.g. Examples 6.4.1b & 6.4.1c).

As my findings suggest, establishing the norm of communicating mathematically allows students to use mathematical language in small-group and whole-class discussions, in a way that supports their learning (e.g. Example 6.4.1a). Over time, some NESB students in this study communicated using mathematical language, in classroom discussions (e.g. Deependra and Wataru in Example 6.4.1c). Findings show that, gradually, some NESB students' ability to communicate their ideas using mathematical terminology and symbols grows. This growth can be accompanied by an increase in some students' ability to solve contextualised mathematics problems. Some NESB students in this study showed gradual improvement in their ability to use mathematical language to communicate their ideas during classroom discussions (Example 6.4.1c). My findings suggest that establishing the norm of students being able to communicate mathematics ideas using mathematical language allows NESB students the chance to embrace the norm of communicating mathematically.

Interview data suggest that some of my students found communicating mathematically hard and expected the teacher to support student language learning and use. This data further supports the view that NESB students have language demands that hinder their mathematics learning (see section 2.4.2). In addition, interview data support the view that NESB students need teacher support to develop the ability to communicate mathematically (Adler, 1997; Moschkovich, 2002). Data also indicate that, as the semester went by, fewer students found communicating mathematically hard. Some students who stated at the beginning of the semester that it was hard to communicate mathematically changed their minds (see Table 6.3).

My findings reflect that NESB students can develop an understanding of how, and the ability, to appropriately use mathematical language which they need in order to function effectively in mathematical activity. These conclusions are consistent with Enyedy et al. (2008), Gould (2008) and McKenzie (2001) who highlight the need to develop students' ability to communicate in ways that are acceptable to the wider mathematical community. In addition, my findings are in agreement with other researchers (e.g. Cobb, Yackel & Wood, 1992; Forman, 2003; Moschkovich, 2002; Tate, 2008; White, 2003) who found that an ability to communicate mathematically creates additional opportunities for students to explore and learn mathematics. Similar inferences were made by Adler (1998, 1999) and Campbell et al. (2007) who, through

separate research, concluded that language is a crucial resource in mathematics classrooms.

Insights across the three sociomathematical norms

Findings suggest that the social norms discussed in the previous section are both the foundation and the building blocks for the construction of the sociomathematical norms of mathematical problem-analysis, explaining and justifying mathematically, and communicating mathematically (Chapters 5 & 6). These sociomathematical norms are collectively developed by the students and the teacher, evolve over time and require ongoing renewal. As illustrated by episodes and journal extracts in sections 6.2.1 to 6.4.1, they are emergent phenomena.

Establishing the three sociomathematical norms, in this study, allows more coherent and productive mathematical discussions to occur giving rise to better classroom interactions. A focus on developing each of these norms permits NESB students to engage in constructive discussions that allow more meaningful mathematics learning to occur because students would know the criteria for acceptability (sections 6.2.1 to 6.4.1). Lively debates concerning mathematics can occur that generate more effective communication of mathematics ideas, benefiting NESB students' mathematics learning. Individually or collectively, these norms promote genuine communication about mathematics, negotiation of mathematical meaning and conceptual understanding. NESB students' ability to solve mathematics problems may improve as they participate in the development of each norm. Sections 6.2.1 to 6.4.1 illustrate this.

Findings suggest that constituting these three sociomathematical norms allows the development of an academic culture which values and encourages students communicating mathematics ideas (e.g. Example 6.4.1c). My findings indicate that while collectively establishing the sociomathematical norms, some NESB students may adopt the sociocultural view of learning, as described in Chapter 3. At the same time, some NESB students may recognise mathematical learning as a social and cultural process (sections 6.2.1 to 6.4.2).

A focus on establishing the sociomathematical norms of mathematical problem-analysis, explaining and justifying mathematically, communicating mathematically and mathematical questioning allows distribution of authority to NESB students, in a way that supports students' mathematics learning (sections 6.2.1 to 6.5.1). My findings suggest that these norms facilitate the development of NESB students' intellectual autonomy (e.g. Example 6.4.1b), making it possible for students to make what Kamii (1985; 2004) referred to as independent decisions and judgements about mathematical contributions. At the same time, jointly constituting these norms promotes NESB students' ability to exercise intellectual mathematical autonomy and to share authority (Chapter 6). My findings (e.g. Examples 6.2.1b, 6.3.1d & 6.4.1b) demonstrate that, progressively, the authority to analyse a mathematics problem for understanding, explain and justify mathematically, communicate mathematically, ask a mathematical question, as well as to judge the correctness of mathematical ideas can become the responsibility of all members of the classroom community, when the teacher openly initiates and guides the development of sociomathematical norms described in Chapter 6. Sections 6.2.1 to 6.4.1 show that, over time, mathematical ideas in this study were not offered only by me, but by NESB students as well, who willingly and publicly made independent decisions and judgments regarding mathematics. These actions reflect that authority and intellectual autonomy can be embraced by some NESB students, following their experience of an academic culture that emphasises and promotes students' public expression of their thinking.

The sociomathematical norms in this study shaped the way NESB students were expected, obligated and entitled (Greeno, 2007; Gresalfi & Cobb, 2006; Gresalfi & Williams, 2009) to engage with mathematical content in the classroom. Establishing these sociomathematical norms allowed this to happen in a way that assisted NESB students' learning, enabling them to participate more actively in mathematical activities. This situation fits with Cobb's (1999) view of gradual movement from being relatively non-active participants to substantial participants; in which students rely on their own judgements rather than the teacher's.

Based on findings in this section, I argue that if the teacher initiates and guides the development of sociomathematical norms, such as mathematical problem-analysis, explaining and justifying mathematically, and communicating mathematically, that

promote negotiation of mathematical meaning and conceptual understanding, NESB students' mathematics learning can be enhanced.

Social norms and sociomathematical norms do not fully support NESB students' mathematics learning. In this study, I needed to also attend to normative aspects specific to particular mathematics ideas. Findings associated with classroom mathematical practices that are effective for solving particular mathematics problems are discussed next.

8.2.3 Classroom mathematical practices that are effective for solving particular mathematics problems

In this section the focus is on insights for mathematics teaching and for NESB students' learning that became apparent when I initiated and guided the development of classroom mathematical practices that are effective for solving mathematics problems associated with particular mathematics topics. To develop these mathematical practices, my four main strategies were the use of both contextualised and non-contextualised mathematics problems (e.g. to develop mathematical practices 1 & 3); coordinating small-group to whole-class discussions (e.g. while developing mathematical practice 3); using a diagram (e.g. to develop mathematical practice 1); and displaying mathematics problems and student solutions on the board\overhead projector (e.g. to support development of mathematical practices 7 & 8). These practices have already been discussed but here I focus on taken-as-shared ways of reasoning and acting mathematically that were collectively established by the classroom community, and to do with particular mathematics topics. For the topic arrangements and selections, these mathematical practices were those of counting the number of arrows and of multiplying the number of choices (at each stage of a process) (sections 7.2.1 & 7.2.2). Sequence mathematical practice included using ratio to distinguish a graph of a geometric sequence, plotting sequence terms on a graph without joining them, and progressively increasing the size of n (number of terms) and monitoring the behaviour of the sequence, t_n (sections 7.3.1, 7.3.2, & 7.3.3). Apart from these practices, I guided the establishment of a mathematical practice concerned with the topic coordinate geometry (section 7.4.1) – finding the point of intersection of perpendicular bisectors of a triangle and using distance

formula. In relation to the mathematics topic logarithm modelling, my students and I constituted the mathematical practice of multiplying the original amount by increasing powers of a constant, 3 (section 7.5.1). For the topic limit of a function, NESB students and I collectively developed the mathematical practice of dividing the numerator and denominator of a rational function, $f(x)$, by x^n, where n is the largest power of x in the denominator (see section 7.6.1).

Similar to social and sociomathematical norms (sections 8.2.1 & 8.2.2), I set out to develop these classroom mathematical practices on the basis of understanding from literature (see sections 2.3 & 3.4), anticipating my actions would enable my students to engage in more meaningful mathematical problem solving and have their mathematics learning enhanced. Even though different mathematical practices were established for different mathematics topics, insights that cut across all mathematical practices were identifiable and are discussed next.

Classroom mathematical practices emerge

Firstly, the examples in Chapter 7 suggest that mathematical practices, just like the social and sociomathematical norms detailed in Chapters 5 and 6, are emergent phenomena. They evolve over time as students and the teacher discuss particular mathematics problems and solutions during regular classroom interactions. The data indicate that it is possible to build from one mathematical practice to another when the teacher creates conditions that encourage students to explore further an already established mathematical practice. This point is illustrated by episodes in sections 7.2.1 and 7.2.2 which show that mathematical practice 2 (or multiplication principle) evolved from mathematical practice 1 (or drawing a diagram and counting the arrows), during classroom interactions. Bowers et al. (1999) described the evolution of mathematical practices that emerged over a period of nine weeks of teaching primary school mathematics students in the USA. Rasmussen et al. (2004) had similar findings. These researchers documented the emergence of six classroom mathematical practices, during 22 days of teaching differential equations to undergraduate mathematics students. This study contributes an example of how this process can also take place over a shorter time scale of two to three days, and involving the emergence of two or three mathematical practices.

Establishing mathematical practices requires a focus on specific ways of reasoning appropriate for that mathematics idea or topic

A focus on establishing classroom mathematical practices supports attention being given to the ways of reasoning and symbolizing involved in a specific mathematics topic (see sections 7.2.1 to 7.6.1). My findings show that a focus on developing specific mathematical practices creates an intellectual need for students to explain the reasoning behind their actions, solutions and or use of symbols for a particular mathematics topic, particularly when the teacher encourages them to justify their actions. As they take part in ongoing development of a mathematical practice, students develop the ability to seek and offer the reasoning behind their actions (e.g. Episodes 7.4.1 & 7.5.1). Students' reasoning moves to be based on conceptual rather than procedural understanding, and specific to a particular mathematics idea as was the case with Snowy's reasoning in episode 7.6.2. Wood et al. (2006) stress the value of students being able to reason conceptually when they argue that it is generally believed that when students learn mathematics through thinking and reasoning they develop conceptual understanding.

A focus on classroom mathematical practices promotes coherent discussions

A focus on establishing a problem-specific classroom mathematical practice enables the coherent mathematical discussions that are needed for student learning (see, for example, mathematical practice 5 in section 7.3.3). This finding is consistent with research by Enyedy et al. (2008), McCrone (2005), and Stephan and Rasmussen (2002) who argue that a focus on classroom mathematical practices creates coherent mathematical discussions and models the academic discourse of mathematics that students need to learn in order to be academically successful.

Establishing mathematical practices, such as those in Chapter 7, provides opportunities for NESB students to engage in more reasoned mathematical discussions linked to a specific mathematics idea (e.g. in section 7.3.1). By participating in the development of different mathematical practices, some NESB students in this study became more able to work and contribute to logical discussion, in different mathematics situations (e.g. involving graphs, in section 7.3.2, and equations, in section 7.4.1).

My findings indicate that developing classroom mathematical practices in this study allows discussion to occur around students' ways of reasoning to do with specific mathematics ideas or topics. At the same time, students' actions or solutions may become the centre of class discussions from which other students can benefit. In this study, some students' ways of reasoning about specific mathematics ideas created opportunities for productive class discussions (e.g. Najilla and Thomas' ways of reasoning about graphs of sequences, in section 7.3.2). Other examples were when students discussed and legitimized Carrie and Winnie's solution (mathematical practice 6) and Omar's solution (mathematical practice 7).

Findings suggest that a focus on developing mathematical practices allows students to learn how to contribute to logical discussions, especially when the teacher elicits and values students' ideas about specific mathematics topics. As they respond to the teacher's request to be actively involved in their learning, NESB students contribute to joint development of mathematical practices (Chapter 7) that enable the conceptual understanding of specific mathematics ideas, through a combination of problem solving and negotiation of meaning. While negotiating meaning, students may engage in coherent mathematical discussions, as was the case in this study (see, for example, section 7.3.3). When NESB students negotiate meaning and solve problems linked to different mathematics topics, they may develop specific and linked understandings of different mathematics topics, which may enable them to engage in coherent discussions when, later, they collectively develop other mathematical practices. An example in this study was when sequence practices in sections 7.3.2 and 7.3.3 were being collectively established.

As students contribute to the constitution of each mathematical practice, their ideas can also stimulate, challenge and extend other students' thinking in a manner that supports the learning of all students in class, as well as promote further productive discussion. While doing this, NESB students' ability to make more effective contributions related to specific mathematics topics may gradually improve. Episodes 7.3.1, 7.4.1 and 7.5.1 demonstrate this.

In this study, coherent discussions following establishment of problem-specific mathematical practices created the opportunity for students to communicate mathematically (Moschkovich, 2002).

Mathematical practices are transferrable

Examples in Chapter 7 provide evidence that when a mathematical practice is taken-as-shared in one context it can be transferred and used to support learning in another context, without having to be justified again in whole-class interactions. For example, during the establishment of the mathematical practice of dividing the numerator and the denominator by x^n, where n is the largest power of x in the denominator (mathematical practice 8 in section 7.6.1), some students referred back to and used the practice of calculating values of a function, $f(x) = \dfrac{1}{x}$, as the value of the variable, x, gets bigger and bigger without limit (mathematical practice discussed earlier in the lesson in section 7.6.1), without needing to justify this process or students being questioned by classmates. This finding is similar to Rasmussen et al.'s (2004) who found that a gesture\argumentation combination that developed while constituting one mathematical practice could change role to support the development of ideas in other mathematical practices.

A focus on developing mathematical practices allows the use of different forms of representation to support students' mathematics knowledge building

My findings show that a focus on developing classroom mathematical practices allows and benefits from the use of what Rasmussen and Marrongelle (2006) refer to as multiple forms of representations of an idea (e.g. diagram, graph, table of values). This appears to support NESB students' mathematics knowledge growth. Scott, Mortimer and Ametller (2011) highlighted the importance of a teacher making pedagogical links to the use of different modes of representation to support student knowledge building. In this study, NESB students used arrow diagrams (mathematical practice 1, in section 7.2.1), graphs (mathematical practice 3, in section 7.3.1), tables (mathematical practice 4, in section 7.3.2), equations (mathematical practice 6, in section 7.4.1), and texts (mathematical practice 7, in section 7.5.1). For instance, by using a table of values and a graph separately,

students were able to negotiate a taken-as-shared understanding of limit of a sequence (Episodes 7.3.2 & 7.3.3). Following situations where various types of representations were used to support collective knowledge development, NESB students were better able to solve related mathematics problems (e.g. task 1 and 2, in section 7.2.2). Mathematical practice 1 and 2, in Chapter 7, demonstrate how I intentionally used, in aid of student learning, diagrams and verbal or text statements to connect to student thinking while pressing forward the mathematical agenda (e.g. section 7.2.1).

When writing is used to communicate mathematical thinking while collectively developing mathematical practices such as those in Chapter 7, it can play a crucial role in triggering and sustaining lively conversations that support student learning. NESB students, in this study, were gradually able to communicate their thinking through writing on the board (e.g. Carrie in section 7.4.1). Progressively, my students contributed to the development of several mathematical practices through writing on the board, when I first modelled this type of communication and encouraged them to do the same (see, for example, journal extract JE 7.3.1, and sections 7.2.1 & 7.2.2). Over time, my students were willing to display their solutions on the board allowing for comparison, scrutiny, reflection, refining or legitimisation by other members of the classroom community to occur (see section 7.5.1). These actions reflected a social view of mathematics that had been embraced by the classroom community. Writing as a form of communication facilitated not only the development of a mathematical practice, but students' understanding of the mathematics topics involved and their ability to solve related problems. This is demonstrated by Episode 7.6.2, and Figures 7.2 and 7.4. Writing on the board (e.g. Omar, Najilla & Honda in section 7.5.1, and Snowy & Deependra in section 7.6.1) may have alleviated some students' language demands (see section 2.4.2), and enabled them to communicate mathematically.

Multiple mathematical practices can be managed at the same time

Two or more mathematical practices can be managed simultaneously during ongoing classroom interactions when the teacher notices and utilises situations that allow this to happen. In this study, Episode 7.3.1 in section 7.3.1 illustrates how the mathematical practice for a geometric sequence and that for an arithmetic sequence were simultaneously managed as NESB students and I discussed ideas put forward

during whole-class discussion. This action enabled misconceptions some students had about the distinction between a geometric and an arithmetic sequence to be resolved and student learning supported. By resolving a misconception through negotiation of meaning, student action reflected their recognition of mathematics learning as a social process involving communicating mathematically.

A mathematical practice can originate from a student's contribution or work

My findings suggest that a classroom mathematical practice can originate from students' work when the teacher notices and uses an 'unusual or unique' student solution as a focus of whole class discussion. In this study, an example was mathematical practice 6, to do with finding the equation of a circle. NESB students legitimised the mathematical practice initiated by their classmates and later used it to solve similar problems (see section 7.4.1). As in Jilk's (2009) research, in this example my students were subjected to learner-centred learning in which they could exercise authority and experience a classroom academic culture where student ideas were recognised, valued, used and sometimes questioned. By legitimizing student-initiated mathematical practices and adjusting instructional strategies to accommodate students' new understandings and ways of reasoning specific to that topic, a teacher can promote distribution of authority among students in support of their learning.

A focus on classroom mathematical practices fosters development of student authority and autonomy in mathematics

The examples in Chapter 7 show that when NESB students participate in the development of mathematical practices, some of them come to embrace a classroom academic culture that values exploring mathematics ideas and engaging in reasoning during mathematical discussions. Over time, some students may revise their views about mathematics and how it is learned. They may begin to recognise mathematics as a process of social interaction. Episodes in sections 7.3.3 (mathematical practice 5), 7.4.1 (mathematical practice 6) and 7.5.1 (mathematical practice 7) support these claims.

The collective establishing of classroom specific mathematical practices allows for the distribution of authority to NESB students. In this study, Episodes 7.2.1 to 7.6.2

illustrate that students' intellectual autonomy grew as they participated in the establishment of a mathematical practice, because their growing knowledge of the practice they were helping to establish allowed them to make more effective contributions that supported their learning. A number of other researchers, including Cobb et al. (2001), Cobb et al. (2009) and Greeno (2006; 2007), have stressed the importance of mathematics students developing and exercising authority and intellectual autonomy in mathematics. My findings contribute a body of examples of how NESB student can move to adopt this orientation. Student interview data (section 7.7) further strengthen these researchers' viewpoint that establishing specific mathematical practices allows students to exercise agency and act authoritatively in mathematics activities.

In view of the findings elaborated in this section, it can be argued that when mathematical practices are collectively developed by the students and the teacher, NESB students may come to understand and use specific ways of acting and reasoning related to the mathematics topics arrangements and selections, sequences, equation of a circle, logarithm modelling, and limit of a function. Also, students' mathematics learning of these topics may be enhanced.

Taken together, findings suggest that social norms, sociomathematical norms and mathematical practices, as described in Chapters 5, 6 and 7, can each be identified and described when a teacher works with his NESB students to support their mathematics learning. The development of norms and practices is inextricably interlinked and can be initiated and guided by very similar pedagogical approaches. What distinguishes these approaches at each level is that at the classroom social norm level, they support development of general normative expectations and responsibilities that apply to any subject; at a sociomathematical norm level, they help to develop standards of actions and interactions specific to mathematical activity. At a mathematical practice level, the pedagogical strategies were used to assist students develop taken-as-shared ways of acting and reasoning associated with a mathematics idea.

8.3 Chapter summary

This chapter discussed lessons emerging from this practitioner research. The discussion was done at Cobb's (2001) three levels of focus adopted for this study: classroom social norms, sociomathematical norms and classroom mathematical practices. Main pedagogical strategies to develop the norms and mathematical practices were outlined. Insights were identified as findings were being discussed under the headings Constructive classroom social norms for NESB students' active participation, Productive sociomathematical norms to promote conceptual understanding, and Classroom mathematical practices that are effective for solving particular mathematics problems. Lessons learnt for mathematics teaching and for NESB students' mathematics learning that became apparent when I openly initiated and guided the establishment of three social norms, three sociomathematical norms, and eight classroom mathematical practices were discussed in relation to mathematics education literature discussed in Chapters 2 and 3. Insights specific to each social and sociomathematical norm were discussed separately first, before focusing on those aspects shared by all three social norms and all three sociomathematical norms, respectively. Although different mathematical practices were collectively constituted for different topics, insights that cut across all of them were identified and deliberated on.

Lessons learned from this practitioner research led me to make some conclusions that answer the research question posed in Chapter 1. That is, What can I do to my teaching practice to enhance my students' mathematics learning? The next chapter, Chapter 9, concludes this thesis by presenting the main conclusions and implications that emerge from my study.

CHAPTER 9

CONCLUSIONS AND IMPLICATIONS

9.1 Introduction

The impetus for this study came from my experiences working with NESB students. These experiences have extended over my whole mathematics teaching career (see Chapter 1). Through this study I have formally investigated my teaching practice. This chapter begins by presenting a summary of the main conclusions emerging from my research. These conclusions answer the research question posed in Chapter 1. After this, limitations of the methodology used in this study are discussed under the heading: Limitations in research design. Next, I discuss implications of the findings for teaching and outline areas for further research. The final section presents concluding remarks.

9.2 Conclusions

This section presents the main conclusions I arrived at in my study. The conclusions are based on evidence provided in Chapters 5, 6 and 7, and findings discussed in Chapter 8. One of the main conclusions emerging from this study is that NESB students' mathematics learning can be enhanced by, among other actions, openly focusing on guiding the establishment of social norms whereby students feel safe and are encouraged to contribute their ideas, sociomathematical norms that value, encourage and provide for negotiation of mathematical meaning and conceptual understanding, and classroom mathematical practices that are effective for solving particular mathematics problems. These norms and practices can be collectively constituted by the students and the teacher, using teaching strategies, such as contextualised and non-contextualised mathematics problems, a mix of small-group and whole-class discussions, the use of prompting and probing questions and allowing wait-time, to facilitate their ongoing development.

A focus on developing classroom social norms (Chapter 5) that promote and value NESB students' participation in classroom activity can lead to them being comfortable with freely discussing and sharing their ideas with their classmates and the teacher, to the benefit of their learning. Gradually, students can begin to expect

and provide explanations and justifications for contributions made during small-group and whole-class discussions. They ask each other and the teacher questions for clarification and to get assistance with a classroom task or activity. Some NESB students' uneasiness with collaboration during classroom activity can diminish over time. NESB students' learning can benefit when the teacher promotes sharing of ideas, students explaining and justifying thinking, and asking each other questions. Students can develop an understanding of *when* and *how* to behave in classroom activity, and become socially autonomous. When this happens, progressively, NESB students can shift from being non-active to active participants in mathematics activity, in ways that support their mathematics learning.

Another conclusion is that NESB students' apparent difficulties with solving contextualised mathematics problems can be addressed and NESB students' mathematics learning enhanced by openly guiding the establishment of sociomathematical norms for mathematical problem-analysis, explaining and justifying mathematically, and communicating mathematically (see Chapter 6). When NESB students and the teacher collectively establish normative shared criteria for what constitutes productive mathematical activity in the different phases of problem analysis, explaining and justifying, and communication, students can engage in more effective mathematical problem-analysis, and mathematical explanations and justifications of their thinking. They can better understand contextualised mathematics problems, be able to provide mathematical reasons and/or mathematical interpretations, and communicate ideas using conventional mathematical language and symbols, in ways that support their learning. NESB students can benefit from contextualised mathematics problems through linking mathematics with real-life situations. Gradually students can learn to make independent decisions regarding mathematical activity – they can develop intellectual autonomy in specific mathematics topics. The confusion, frustration and despair some NESB students can experience at the beginning of the mathematics course can gradually diminish as they blossom in class and successfully solve contextualised mathematics problems.

The other conclusion is that a focus on guiding the development of classroom mathematical practices associated with specific mathematics ideas (see Chapter 7) can promote NESB students' ability to solve related mathematics problems. Focusing

NESB students on the specific ways of acting, reasoning and symbolising associated with the mathematics ideas of a lesson allows them to develop conceptual understanding of these mathematics topics and their mathematics learning is reinforced.

One more conclusion is that NESB students' mathematics learning can be strengthened by guiding the development of norms and mathematical practices (see Chapters 5, 6 and 7) that make it possible for students to see and embrace mathematics as a human and social activity, and mathematics learning as a process involving exploration, collaboration and negotiation of meaning, not just the transmission of knowledge from the teacher to the students. NESB students' mathematics learning may be enhanced if, instead of learning mathematics by memorising information without understanding, they learn it through sharing ideas, explaining and justifying mathematically, asking questions, and communicating mathematically.

A further conclusion is that teaching strategies such as organising small-group and whole-class discussion, the use of prompting and probing questions, displaying problems and solution on the board/overhead projector, and contextualised and non-contextualised mathematics problems, as described in Chapters 5, 6 and 7 are useful for initiating and guiding the development of social and sociomathematical norms and classroom mathematical practices that enhance NESB students' mathematics learning.

Reasons other than those discussed so far may have impacted on my students' perceptions of the practices I was implementing. Students' rather negative perceptions of and attitude towards these practices, at first, but largely positive towards the end of each semester could be attributed to (1) students' pre-conceived ideas about contextualised problems at the beginning of each semester; (2) students' unfamiliarity with the type of questioning; (3) clarity of my questions at the beginning of the first year of study, in particular; and (4) unfamiliar teaching/learning style. It is also possible that the way my students were being taught in other subject areas could have had some impact on their perceptions of the teaching style I was implementing.

Although this study enabled me to gain a better understanding of my teaching practice and develop some fresh ways for supporting NESB students' mathematics learning through the joint development of particular social norms, sociomathematical norms and mathematical practices, I am aware that there are limitations in the research design. Some of these limitations are outlined and discussed next.

9.3 Limitations in the research design

The limitations in the research design relate to the technical and human aspects of practitioner research. On the technical side, because of the sizes of the classrooms and the single cassette tape-recorder attached to me it was impossible to capture everything that was said by the students. However, my supplementary notes and personal journals, written soon after the lesson, were used to fill in the gaps. Also, it was not possible to record, on an tape, some student or my actions, for example, shaking one's head. My journal entries were used to triangulate the classroom dialogue data. Another limitation was that in each semester I had a new group of students. Each time I began data collection with a particular group, some of the students' behaviour could have been influenced by the presence of the tape-recorder in the classroom to the extent that they may not have acted in ways they could otherwise have done. Some students may have withheld their responses because they did not want their ideas captured on tape.

On the human side, interview data is subjective, so has limitations associated with reliability. Students' views, during formal interviews in particular, may have been influenced by our unequal relationship. I was their teacher, with the power to grade their work. So, during the interviews, students may have said things they thought I wanted to hear. I tried to minimize the effect of the unbalanced power relationship by creating space for multiple perspectives. I triangulated data from multiple sources. Another human limitation relates to researcher prejudices and bias. Since I was both the practitioner and the researcher, data collection and analysis could have been affected by my predispositions and partiality. I was always conscious of the possible negative impact this could have on my research. Consequently, to offset my bias and prejudices I made my work public property, available to others (e.g. my colleagues in our department and supervisors) for review and critique (Shulman, 2000).

In order to highlight particular points, interpretive data analysis was done at three levels of focus linked to the emergent perspective (Cobb et al., 1997, 2001). I separated my data into three units of analysis: classroom social norms, sociomathematical norms, and classroom mathematical practices. In practice, these aspects are inextricably interlinked. By separating them, I may have created the impression that each one is independent of the others. However, focusing on classroom social norms, for example, served to delineate the classroom participation structure (Cobb et al., 2001). This allowed me to discern patterns or regularities in the teacher and students' ongoing interactions by inferring normative aspects of classroom activity. So, separating the aspects of the classroom micro-culture is a limitation I had to contend with.

During data collection, I inferred that some students did not want to give responses voluntarily. It is possible that some non-volunteers were willing to offer ideas but had nothing to offer. Therefore, it may have been erroneous for me to conclude that they were unwilling to contribute. Personal interpretations are subjective. They can be influenced by a person's understanding of the situation and how they saw it at the time. To counter these shortcomings and the influence of the researcher's perceptions of participants, conclusions in this study were not based on one or two incidents but on events covering a period of one year. In addition, various sources of data, such as episodes and journal entries, were used.

Finally, my analysis gave precedence to the social perspective in Cobb's emergent model, which foregrounds normative, taken-as-shared ways of talking and reasoning (Cobb et al., 2002). Some people may consider this to be a limitation since the social perspective is not independent of the individual perspective. However, I have provided examples that locate individual students' reasoning within an evolving classroom micro-culture and the interview data sections provide some insight into the individual aspects identified by Cobb such as a student's own role, others' roles and the general nature of mathematical activity.

9.4 Implications of findings

This study has important implications for mathematics educators, specifically in pre-service education of teachers and professional development of some in-service teachers as well as the work of professional development leaders and administrators.

9.4.1 Implications for pre-service teachers

In view of the fact that the development of particular social norms, sociomathematical norms and mathematical practices plays an important role in NESB students' mathematics learning, it is imperative that mathematics teacher educators orchestrate opportunities to ensure that pre-service teachers experience doing mathematics in an environment that promotes initiating and guiding the constitution of norms and practices. Pre-service teachers must not experience mathematics as a product waiting to be acquired, but as a social and cultural activity, a process. This suggestion is supported by other literature that acknowledges that mathematics learning is socially and culturally determined (Forman, 2003; Kumpulainen & Renshaw, 2007; Lerman, 1996; Yackel & Cobb, 1996). In addition, pre-service teachers need to experience activities that recognize mathematics as a human activity in which students need to take ownership of their own learning. They need to experience working in a classroom environment that values and encourages active participation, sharing ideas, and pressing peers to provide mathematical explanations and justifications. They also need to experience negotiating mathematical meaning (Cobb et al., 2001; McClain et al., 1999), as well as publicly justifying their mathematical reasoning, and listening to their classmates' justifications.

During the times they are in a mathematics methods class, pre-service teachers need to experience the development of a mathematics community that emphasizes understanding a problem and sense-making, encourages mathematical discourse, and underscores the importance of social and mathematical autonomy. Pre-service teachers need opportunities to experience and develop their ability in deciding the criteria of what counts as legitimate, mathematically. It is through these kinds of experiences and or teacher modelling of these pedagogical practices, that pre-service teachers will develop a foundation for understanding how they can use productive social norms and practices such as those described in this study to support their

students' mathematics learning. As novices to teaching, pre-service teachers are not ready to conceptualize the full complexity of mathematics teaching; however, they need opportunities to enact brief episodes of these practices. For instance, it would be helpful for pre-service teachers to analyse contextualized problems in small groups; observe mathematics lessons to practise listening for students' explanations and justifications; and practise selecting or forming and posing pre-prepared key questions, pressing for sense-making and for similarities and differences among strategies or solutions, as well as allowing wait-time.

9.4.2 Implications for in-service teachers

Training or retraining is required that is aimed at convincing in-service teachers of the importance of initiating and guiding the development of social norms, sociomathematical norms and classroom mathematical practices that can support NESB students' participation in the social construction of mathematics understanding. In addition, in-service teachers need opportunities to understand how they can use these norms and practices to support their students' mathematics learning. Much of what is important regarding the education of pre-service teachers would also be applicable for in-service teachers. Specifically, in-service teachers need to experience doing mathematics in a supportive, collaborative environment. They need the experience of struggling to understand their colleagues' mathematical thinking and demanding that peers explain and justify their reasoning. It is through such experiences that in-service teachers are likely to find the trigger that stirs their desire to bring the same kind of experiences to their own mathematics classrooms.

Although in-service teachers have more experience than pre-service teachers in dealing with complexities of the mathematics classrooms, changing one's practice occurs gradually, if it does occur. To have an impact on classroom practices, professional development for teachers needs to be a sustained activity. As part of professional development, in-service teachers need opportunities to trial new practices in their classrooms and then reflect on those experiences with peers. They can benefit from enacting many of the practices listed above for pre-service teachers. For instance, it could be useful for pre-service teachers to observe mathematics classrooms in order to practice identifying sharing of ideas during small-group and

whole-class discussion; use of pre-prepared key questions as sources of discussion; and sense-making and student attempts to pinpoint similarities and differences among strategies or solutions. In addition, in-service teachers can benefit from practising listening for student explanations and justifications. However, because of their experience, in-service teachers might be able to more quickly implement new practices in the context of their regular classes.

Teachers' practices are, to a large extent, determined by what they know about the situation and how they see it at the time. Since practices change all the time, in-service teachers should consciously monitor movements in their own and their students' practices. To do this, they need the ability to continually evaluate their practices. In particular, it is essential that teachers keep an eye on issues to do with the support they can provide to their mathematics students.

9.4.3 Implications for administrators and professional development leaders

The challenge for administrators and leaders of mathematics professional development is to arrange sustained and generative professional development in which teachers have the opportunity to both experience and practice mathematically productive teaching routines that involve initiating and guiding the development of particular social and sociomathematical norms and mathematical practices. Such opportunities need to build strong learning communities among colleagues within schools and across the country. Elliot, Kazemi, Lesseig, Mumme, Carroll and Kelley-Petersen (2009) stress the significance of a framework of social and sociomathematical norms in designing professional development activities. Ideally, administrators and professional development leaders should work hand-in-hand to develop and use such a frame.

9.5 Matters for further research

Very little has been published in New Zealand about the plight of NESB students studying mathematics in New Zealand. While the present study has been concerned with understanding my own teaching practice, it did not investigate the link between particular classroom norms and practices and NESB students' achievement in tests and examinations. Further research is needed to investigate the long term effects of

jointly developing certain social norms, sociomathematical norms and mathematical practices on the achievements of NESB students in mathematics tests and examinations at tertiary level.

Research could examine the similarities and differences in the development of social and sociomathematical norms and mathematical practices in primary, intermediate or high school classrooms that include or are comprised solely of NESB students. Since the English language is not NESB students' first language, the resources NESB students have access to for expressing their mathematical thinking play a crucial role in their mathematics learning. Further research could investigate the range of resources NESB students can best use to express their mathematical ideas.

The current study focused on helping NESB students analyse and understand contextualized problems, but not specifically how they interpret contextualized problems. Investigating how NESB students interpret contextualized mathematics problems is another area for further study. This could involve examining the cultural assumptions embedded in contextualized problems (Campbell et al., 2007), and the challenges NESB students face as a result of these assumptions. Such data, coming from mathematics students themselves, could be most valuable to mathematics teachers in classrooms with NESB students.

Considering that New Zealand is becoming more and more multicultural, the increase in linguistic diversity and the emphasis on communication in mathematics classrooms (NCTM, 2000) demands further research in mathematics education that pays closer attention to the complex issues arising from the presence of multiple languages in mathematics classrooms. Only one teacher, me, was a participant in this study. Further studies in this area could focus on extending this research to include more teachers and more classroom settings, and a cross-case analysis. Findings from a wide range of teachers and mathematics classrooms with NESB students could give a broader perspective of the topic. This kind of study would be useful to the wider mathematics education community. The results of my study make me wonder what professional development experiences would be most productive in helping mathematics teachers facilitate the negotiation of social and sociomathematical norms and mathematical practices. Further research can investigate this.

9.6 Reflections on changing my teaching practice

This section details reflections on my teaching practice, during the research period, under the headings deep thinking, heartaches/challenges, aspects of literature that really became real to me, and what I had to pay attention to in my practice.

Deep thinking

My students' reluctance to talk, to respond to my questions, to participate actively, or to share ideas with their classmates was the first major obstacle I had to deal with during the research period (e.g. example 5.2.1a). I needed different teaching strategies in response to this student behaviour. This problem made me think deeply about how to encourage my students to talk, to participate actively, to willingly share ideas with other members of the classroom community, and to instigate a discussion or a response. I engaged in detailed analysis and planning to make changes to some of my teaching strategies. For instance, I decided to (1) spend more time preparing focus questions and (2) first nominate all students at random then only those who volunteered.

Students' unwillingness to publicly share ideas made me think deeply about the time students may need to process their ideas before they can produce a well thought out response. I then decided to allow students longer wait-time. As students continued to resist participating actively, I also thought seriously about ways to make students feel safe to make contributions without fear of being ridiculed. On this issue, I decided to make small-group to whole-class discussions one of my main teaching strategies, hoping students would feel safer to discuss and share ideas in small groups, rather than in a whole-class discussion. I also thought intensely about the nature of my questioning, specifically, about how I could make use of this approach in a way that encourages students to participate actively and enhance their mathematics learning. To further support my NESB students' mathematics learning, I spent some of my preparation time thinking and making decisions about which teaching strategy or strategies to use for a particular mathematics topic. Such decisions required me to engage in deep thinking.

Heartaches/challenges

Many challenges occurred during the research period with some of them nearly causing me to give up. The main distress, during the initial stages, was the lack of tangible response from my students. Students were not talking or responding to my questions. How to encourage students to be more active was very challenging. I needed to think of different strategies in response to this student action. Being in the situation I was caused me a lot of heartache and anxiety.

Aspects of literature that really became real to me

Three aspects of literature became 'real' for me in this study. The first one was *social nature of mathematics learning*. This aspect was conspicuous when my NESB students worked first in groups and then during whole-class discussion. As soon as the students began to cooperate and share their ideas, I noted that collaboration around the exchange of ideas and negotiation of mathematical meaning that took place and mathematical understanding followed. These actions convinced me that students were learning mathematics as a social process and the social nature of mathematics learning became real, for me. The other two aspects are *emergent nature of new practices* and *wait-time*. These two aspects, like the first one, became strikingly visible over the course of a semester and hence the literature became real to me.

What I had to pay attention to in my practice

There was a need for me to pay special attention to creating a classroom environment where my students felt safe to participate actively. I did this through organising small-group to whole-class discussions. In small groups, students were able to benefit from using their first language and code switching, although often they needed to learn mathematics terms, in English, as well.

Since I was aware of the significance of maintaining a balance between offering assistance and 'telling' students, I had to focus on ensuring that there was always evenness between encouraging students to ask each other or me questions, and encouraging them to engage in their own thinking. I needed to keep an eye on how to provide assistance without 'telling'. Apart from this, I paid attention to being explicit

about the teaching and learning practices I was implementing, and my expectations of students. Overtly telling students about my teaching style was appropriate and necessary because of the differences the teaching students may have been used to and the style I was adopting.

9.7 Concluding remarks

The fundamental question that guided this study was: What can I do to my teaching practice to enhance my students' mathematics learning? In order to answer this question I used practitioner research into my teaching, over a period of two consecutive semesters, each lasting about six months. For theoretical guidance I drew on the sociocultural perspective (Cobb et al., 1997, 2001; Cobb & Yackel, 1996), and adopted Cobb and his colleagues' three level framework of classroom social norms, sociomathematical norms, and classroom mathematical practices. Unlike Cobb and his colleagues, who investigated primary school children working in mathematics classrooms with their regular teachers, my study focused on my teaching practice and some of my own NESB tertiary mathematics students.

During the research period, I initiated and guided the joint development of particular classroom social and sociomathematical norms and mathematical practices. Through examining data from classroom episodes, my journal, and student group interviews and students' work, I learned that a focus on establishing social and sociomathematical norms that attach importance to and encourage student participation and negotiation of meaning makes it possible for NESB students to shift to being more active participants in mathematics classrooms. I now better understand that collectively developing the social and sociomathematical norms for working together can enable NESB students to become creators and evaluators of mathematical knowledge. I learned that some NESB students' ability to communicate mathematics ideas may grow when these norms are constituted by the classroom community. In addition, I developed an understanding that NESB students' ability to solve problems on a specific mathematics topic may be reinforced when the associated mathematical practices are jointly established by the students and the teacher. When this happens, movements in the teacher's teaching practice occur progressively, and in tandem with those in new student practices.

My findings address a gap in the field of research around NESB students' mathematics learning and teaching. Studies by researchers who have actually explored the area of NESB mathematics students at tertiary level, as a teacher-researcher, are not readily available. This practitioner-research demonstrates strategies a teacher can use to position NESB students with mathematical authority and with accountability to mathematics and to each other, thereby enabling them to learn to act authoritatively and accountably in mathematical activity. Norms and mathematical practices that value and encourage negotiation of meaning help NESB students develop autonomy in mathematics classes and they begin to act with conceptual agency (Greeno, 2007). Teacher strategies that focus on meaningful learning and encourage negotiation of mathematical meaning and conceptual knowledge such as allowing wait-time, revoicing, prompting and probing questions, and small-group and whole-class discussions, can generate changes in some NESB students' views about mathematics and how it is learned. When that happens, NESB students can shift from viewing mathematics as a product to perceiving it as a social and cultural activity, and as a process.

Now that I was able to change my teaching in such a manner that my NESB students also started to change the way they learn mathematics, I propose that this finding has implications for the wider mathematics education community. In particular, there are wider implications for teacher education and teachers in relation to lecturers and teachers needing to be more explicit about the differences in practices between teaching style students may have experienced before and the style that promotes negotiation of mathematical meaning being adopted by the lecturer/teacher. Teachers and teacher educators would be wise to be open about the teaching methods they are using and explain to students why they are using them otherwise students may view the style as a waste of time and/or develop a negative perception of mathematics and its teaching. Also, if more mathematics teachers move to change their teaching in ways that encourage their students to learn mathematics as a social and cultural activity, then more students are likely to change the way they learn the subject, and begin to learn it as a process involving social construction of mathematical understanding.

My participation in the investigation of my own teaching enabled me to explore my actions, aspirations, fears, and motivations with respect to my teaching practice. Through this investigation, I now understand that there is a need to challenge stereotypical assumptions about NESB students' inability and unwillingness to take part in classroom activities. NESB students can and will actively participate in mathematical activity; and it is good for their mathematics learning. This is an important insight that has the potential to move forward ways of thinking about working with NESB students.

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APPENDICES

APPENDIX A: PARTICIPANT INFORMATION LETTER AND CONSENT FORM

To: Students enrolled in Mathematics with Calculus (CAFS004) and Mathematics with Statistics (CAFS005)

Dear student,

Besides being your mathematics teacher, I am also a student at the University of Waikato. For purposes of a Doctor of Philosophy (PhD) degree, I am carrying out a research study entitled: **Understanding teaching practice in support of Non-English-Speaking-Background (NESB) students' mathematics learning.** The aim of the study is to improve my teaching of mathematics as well as to try and enhance your learning of mathematics. I have permission from the director of Foundation Studies to conduct this research in class.

As part of the study I will keep a journal in which I will document my reflections on some of the activities that take place during the lessons. Journal entries will help me to keep track of these activities. I am aware that some activities may have worked well while others may have been less or not successful. These entries will not be about students, but rather about my role as a teacher.

If you choose to be an active participant in the research, your permission is sought for making copies of some of your work.

Also, I will interview you twice during the research period about your learning of Calculus or Statistics. Both interviews take place out-of-class time at a venue that is convenient to both of us. The first interview will be done about halfway through the semester and the second one towards the end of the semester. You can choose to be interviewed individually or in a group of about 3 or 4 students. You can opt out of a group or individual interview at any time. Each interview will last about one hour.

During the interview you can choose not to answer a question, and /or to withdraw from the interview at any time. Interviews will be audio-taped with your consent and a summary of the transcript of the interview will be made available for you to check, correct, and edit. The only people who will have access to the tape are me and a professional transcriber. The professional transcriber will sign a statement of confidentiality before having access to the tape for transcription of interview data. Only those present at the interview will have a copy of the summary transcript.

I need a maximum of ten students for this study because I may not be able to handle the data when more than ten participants take part. If more than 10 of you volunteer to be involved in the study, I will use random sampling to select only 10 participants from the group of volunteers.

This letter is to invite you to participate in the study. Your participation will enable me to understand better your experiences in learning mathematics as a second language speaker of English.

Participation in this research is entirely voluntary. There is no obligation for you to participate and if you choose not to do so it will have no effect on your results. In addition, a decision not to participate will **not** disadvantage you with respect to any Mathematics paper. You may withdraw from the research without explanation at any time until the end of the paper, even if you have previously given consent.

Your name will not be used in the final research report and all communication will remain confidential to me, and used for research purposes only. All data used for published research will be archived according to the University of Waikato Human Research Ethics Regulations.

When the research report is complete, copies of the report will be put in the University library so that you can read it. Information in the report is likely to be included in article(s) that will be sent for journal publications and may be presented at conferences.

If you have any questions or require more information, now or at any other time, please feel free to contact me (Martin Gwengo), personally (Office K2.10) or on (07) 838 4466 x8441 or email me (address: gwengom@waikato.ac.nz). Alternatively you can contact any of my supervisors, on (07) 838 4987 (Dr Bronwen Cowie), or on (07) 838 4500 ext 6298 (Dr Sashi Sharma).

If you are willing to participate in data collection of this study, please fill out and sign the consent form below and return it to me, in room TX.12, within two days from today.

Yours sincerely,

Martin Gwengo.

Informed Consent

I have read and understand the contents of the information letter above.

I am willing to participate in the research that is taking part within Mathematics with Calculus for Foundation Studies (CAFS004) and Mathematics with Statistics for Foundation Studies (CAFS005).

I consent to my selected assignments, exercises and tests being copied for the study.

I am aware that during the two interviews I can choose not to answer any question and/or to withdraw from the interviews at any time.

I give consent to be audio-taped during the interviews.

I realize that in any report, presentation or publication my name will be changed to ensure anonymity.

I understand that I may withdraw from the study at any time, up until the end of the paper, even if I have previously given consent.

I understand that my assessments in any of these papers will not be prejudiced in any way as a result of my involvement in this research.

Name: _____

Signature: _____

Date: _____

APPENDIX B: INTERVIEW STRUCTURE

The interviews will be semi-structured, and therefore the structure given below is meant to act as a guide. Whenever necessary, prompts will be used to follow-up respondents' answers and vague questions and/or statements will be clarified.

Broad categories
1. Student profile.
2. Student's mathematics background.
3. Views on teaching and learning mathematics in a New Zealand class.
4. Differences between studying in New Zealand and student's home country.
5. Ideas about learning in English language.
6. Advice to mathematics teachers of NESB students.

APPENDIX C: LETTER TO THE DIRECTOR FOR FOUNDATION STUDIES AND INFORMED CONSENT FORM

Martin Gwengo

269 Clarkin Road

Hamilton

(07) 855 9001

gwengom@waikato.ac.nz

The Director,

Centre for Foundation Studies

University of Waikato

Hamilton.

Dear Sir,

Besides being a mathematics teacher in Foundation Studies department, I am also a student at the University of Waikato. For purposes of a Doctor of Philosophy (PhD) degree, I am carrying out a research study entitled: **Understanding teaching practice in support of Non-English-Speaking-Background (NESB) students' mathematics learning.** The aim of the study is to improve my teaching of mathematics in general, and my teaching of mathematics to NESB students specifically, as well as to enhance students' learning of mathematics.

This letter is to request you to give me informed consent to involve some Foundation Studies students in the research mentioned above. I would like to work with my Semester B, 2005 and Semester A, 2006 Calculus and Statistics classes. I can assure you that neither my teaching responsibilities nor my students' learning will be compromised in any way. All class activities will be designed for the purpose of teaching and learning statistics or calculus. If anything, both the teacher and the students are likely to benefit directly since this study is meant to develop me

professionally. New understandings of my practice will be continuously implemented, hence the students may benefit immediately.

Since I am also a participant, my role in the research involves examining and reflecting on my normal teaching duties as well as interviewing participating students. I will document, reflect on, and interpret the teaching and learning during the research period. Although I will be reflecting on all activities (actions, interactions and talk) that take place in class, written work and interviews of only those who agree to participate in the study will be used as data. Any student referred to will remain anonymous. Journal entries will not be about students, but rather about me as a teacher.

For those students who choose to participate in the research, their permission to be interviewed and to make copies of some of their assignments, exercises and tests will be sought. I will interview participating students twice during the research period about their learning of Calculus or Statistics. One interview will be done about halfway through the semester and the second one towards the end of the semester.

During the interview students can choose not to answer a question, and /or to withdraw from the interview at any time. Interviews will be audio-taped with their consent. A summary of the transcript of their interview will be made available to the students involved to check, correct and edit. The only people to have access to the tape are me, and a professional transcriber who transcribes the interviews. The transcriber will be asked to sign a statement of confidentiality before having access to the tapes.

Data will be coded and used anonymously. Participants' names will not be used in the final research report. All communication will remain confidential to me and my supervisors, and will be used for research purposes only. When the research report is complete copies of it will be put in the University library so that students can read it. Information in the report is likely to be included in article(s) that will be sent for journal publications and may be presented at conferences. All data used for published

research will be archived according to the University of Waikato Human Research Ethics Regulations.

Students will be assured that participation in this research is entirely voluntary. There is no obligation for them to participate and if they choose not to do so this will have no effect on their results. In addition, a decision not to participate will **not** disadvantage them with respect to any Mathematics paper. Any student may withdraw from the research without explanation at any time, up till the end of the paper, even if they have previously given consent.

Students will be informed that if they have questions or concerns they may contact me or one of my supervisors.

Since the study has a bearing on the core business of the department of Foundation Studies, a copy of the finished report will be made available to you or to the department.

If you have any questions or require more information, now or at any other time, please feel free to contact me (Martin Gwengo) on (07) 838 4466 x8441 (email: gwengom@waikato.ac.nz) or any of my supervisors, on (07) 838 4987 (Dr Bronwen Cowie), on (07) 838 4500 (Dr Peter Grootenboer x7846, and Dr Sashi Sharma x6298).

Yours sincerely,

Martin Gwengo.

Informed Consent

I _____(Name), Director for Foundation Studies, consent to Martin Gwengo carrying out a research study that involves his participation and that of some students in the department of Foundation Studies, University of Waikato. Consent is granted provided all the conditions of the original Informed Consent form given to the students are met.

I understand that the study will be of direct benefit to all students including those who choose not to take part. I realise that students have a right to withdraw at any stage without having to explain why, and that those who feel compromised at any stage of the research can contact the researcher's principal supervisor.

Signed: _____

Date: _____

APPENDIX D: STATEMENT OF CONFIDENTIALITY FOR TRANSCRIBERS

Martin Gwengo
269 Clarkin Road
Hamilton.

(07) 855 9001
gwengom@waikato.ac.nz

STATEMENT OF CONFIDENTIALITY FOR TRANSCRIBERS

I _____ (PRINT NAME) of

_____

_____ (ADDRESS)

_____ (TELEPHONE NUMBER) do consent to transcribing from audiotape to a computer disk the contents of interviews supplied to me by Martin Gwengo. I consent that all the information that I hear on the tapes will remain confidential and I will not discuss the contents with anyone.

_____ (Signature)
_____ (Date)