Determining Acoustical Directionality in an Impedance Tube Using Multiple Fixed Microphones

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***Abstract*—Acoustic impedance of a port or object is a valuable piece of knowledge describing how well sound is transmitted or reflected. The commonly used slotted-line method is labourious and time consuming, requiring manual movement to find the maxima and minima at each frequency. This paper outlines a technique to computationally determine the magnitude and phase of the constituent travelling waves from the standing plane sound wave measurements in an impedance tube. Measured magnitude and phase data from multiple fixed microphones carefully spaced along the length of the impedance tube is numerically fitted to incident and reflected wave models, which can then be used to calculate the complex acoustic impedance at each frequency of interest.**

I. Introduction

The ability to fully characterise the interaction of sound with objects or materials has a wide range of applications, including sound absorption materials, musical instruments [1], speaker enclosures, exhaust systems [2], and ear canals [3]. Acoustic impedance efficiently describes acoustic reflection, absorption, and transmission for an element of a closed linear system. These properties allow designers to use the most effective material for a particular environment, or to optimise a design to suit a specific purpose.

In the electrical domain, a vector network analyser (VNA) is used to measure the s-parameters with high precision and accuracy by mixing and down converting the signal and reference sources. In order to do this, the forward and reverse travelling waves must be separated. A VNA utilises directional couplers to physically separate these waves by intelligent design with constructive and destructive interference. In the acoustical domain this mechanism is difficult to achieve due to the physical characteristics of sound in air.

Prior to the VNA, the slotted-line method was commonly used to determine the directional waves. This was done by allowing a standing wave signal (caused by the superposition of the incident and reflected plane waves) to stabilise in a transmission line and measure specific points of the standing wave through a small slot to find the standing wave ratio (SWR). This same theory can be applied to the acoustic domain [4][5] using an acoustic impedance tube and a movable microphone. One of the limitations with this method is that it is laborious and time consuming as the microphone must be manually moved to find the maxima and minima for each frequency. This method can also produce small errors attributed to the physical interference of the movable microphone with the sound waves inside the tube.

This paper will explore the concept that the SWR can instead be determined by placing multiple fixed microphones at various spacings along the length of the impedance tube and numerically fitting the standing wave interference pattern to the measured data points.

II. Test Apparatus

The apparatus as shown in fig. 2 is a length of circular tube with a loudspeaker mounted to the left end and a flange to the other. Microphones are inserted so as their faces are flush with the inner surface of the tube and have o-ring seals to minimise any possible interference with the plane waves. [6] states the upper frequency limit f_u is limited by the inner diameter d of the tube such that

$$f_u = \frac{0.58c}{d} \tag{1}$$

where c is the speed of sound in air. Above f_u, higher-order modes take effect and the sound waves are no longer planar.

Fig. 1 shows the system diagram. A VNA is used to measure the magnitude and phase from each microphone. Each input is referenced from the RF out. The VNA mixes the received signal with the output signal in order to isolate and only measure the frequency of interest. The loudspeaker is driven by an audio amplifier with the signal from the RF out of the VNA. A dual channel pre-amplifier was built to enable the microphones have full scale deflection over the VNA input voltage range.

III. Measurement Method

The raw measured data is passed through two stages of processing as shown in fig. 3.

A. Gain Compensation

The low-cost microphones are purchased uncalibrated, so they inherently have slightly different gain profiles. The gain and phase profile of each microphone is measured in the same position along the impedance tube with a solid covered

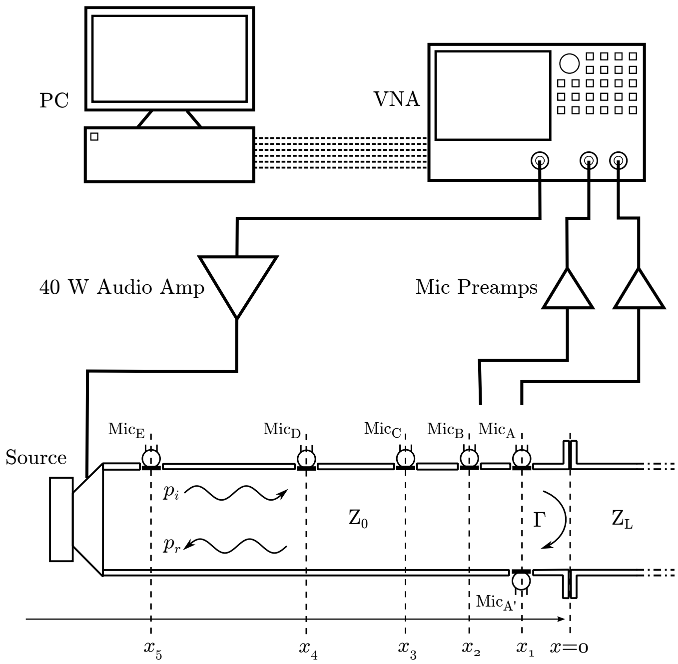

Figure 1. Multi-microphone apparatus.

Figure 2. Acoustic impedance tube.

end. This situation allows the uncompensated gains (shown in fig. 4) to be compared and normalised (shown in fig. 5) to one arbitrary microphone in a repeatable environment. A linear least squares fit of the normalisation matrix is then applied to raw measurement data sets. The effectiveness of the compensation matrix is shown in fig. 6. A high signal amplitude was used to ensure high signal to noise ratio across all of the frequencies. It should be noted that because of this there is clipping due to system resonance at 500 Hz.

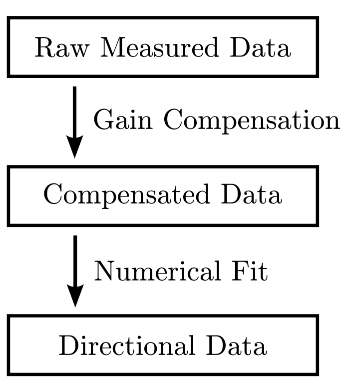

Figure 3. Data block diagram

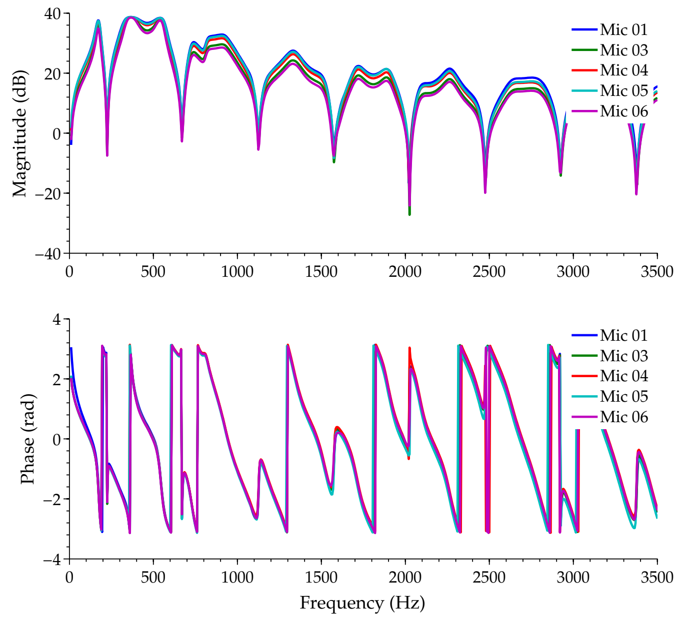

Figure 4. Uncompensated microphone measurements of a covered end.

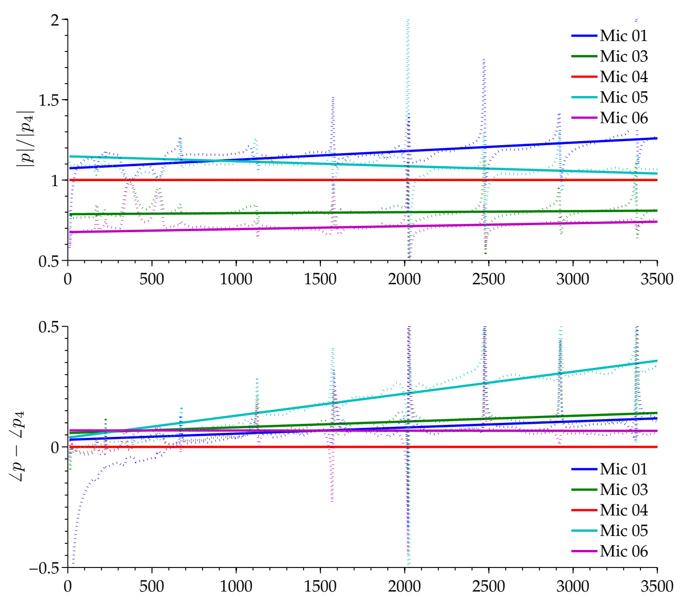

Figure 5. Magnitudes normalised to microphone 4 are shown with the dotted traces and the corresponding least-squares fit shown with a solid line.

B. Directionality

1) Standing Wave Model: An accurate model of the measured sound pressure must be developed in order to decompose the standing wave into the constituent travelling plane waves. The general form for the sound pressure of a travelling wave can be represented at any point on the x-axis and time t by

$$\boldsymbol{p}(x,t) = Ae^{j(\omega t - \boldsymbol{k}x + \phi)} \tag{2}$$

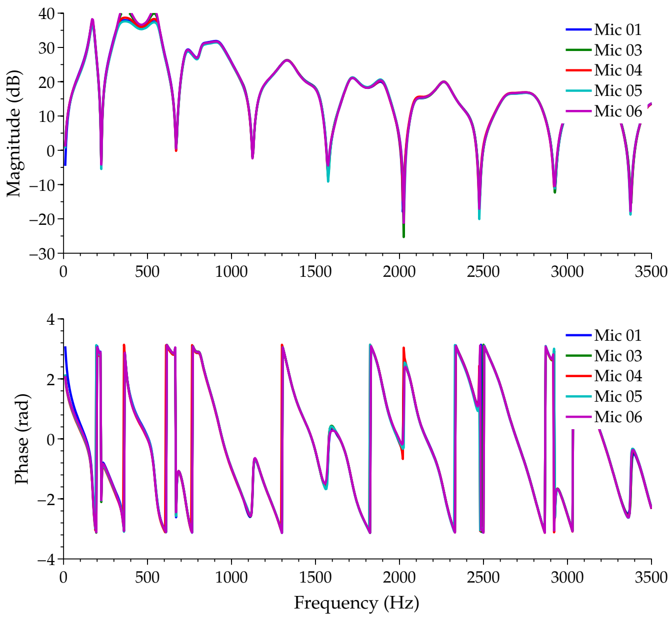

Figure 6. Compensated data of the covered end with the normalisation matrix applied.

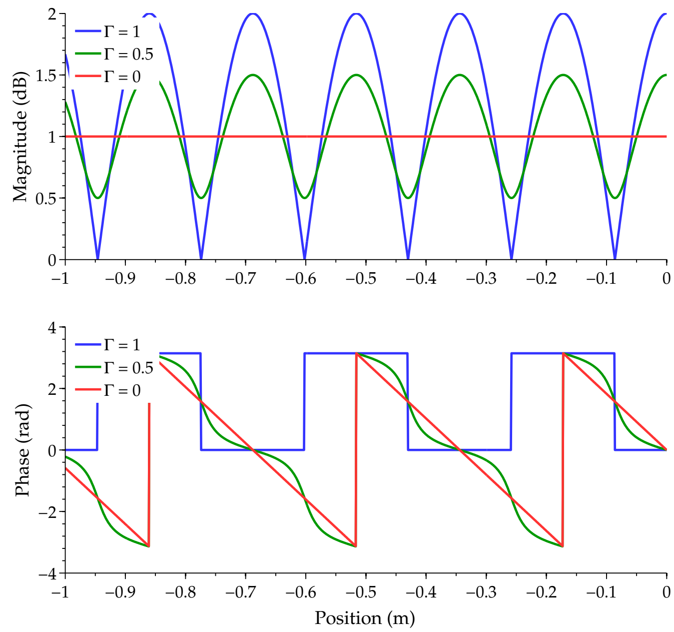

Figure 7. Various standing wave patterns at 1 kHz

$$\boldsymbol{k} = \frac{\omega}{c} + j\alpha \tag{3}$$

where:

$A =$ amplitude,
$\omega = 2\pi f =$ angular frequency,
$c =$ speed of sound in air,
$\phi =$ phase shift,
$\alpha =$ attenuation constant.

Letting $\boldsymbol{A} = Ae^{j\phi}$ and dropping ωt by using the phasor representation, the sound pressure of the incident (positive x) and reflected plane sound waves (negative x) can then be expressed as

$$\boldsymbol{p_i} = \boldsymbol{A_i} e^{-j\boldsymbol{k}x} \tag{4}$$

and

$$\boldsymbol{p_r} = \boldsymbol{A_r} e^{j\boldsymbol{k}x}, \tag{5}$$

respectively. Using the superposition theory, the actual measured magnitude and phase of the sound pressure by the microphones at any point x becomes:

$$\boldsymbol{p} = \boldsymbol{A_i} e^{-j\boldsymbol{k}x} + \boldsymbol{A_r} e^{j\boldsymbol{k}x} \tag{6}$$

The complex reflection coefficient of the load ($\boldsymbol{Z_L}$) at $x = 0$ is:

$$\boldsymbol{\Gamma} = \frac{\boldsymbol{A_r}}{\boldsymbol{A_i}} = \frac{\boldsymbol{Z_L} - \boldsymbol{Z_0}}{\boldsymbol{Z_L} + \boldsymbol{Z_0}} \tag{7}$$

The standing wave patterns are shown in fig. 7 for a range of reflection coefficients. If the impedance tube has a characteristic impedance $\boldsymbol{Z_0}$ then $\boldsymbol{\Gamma} = 0$, $\boldsymbol{\Gamma} = 0.5$, and $\boldsymbol{\Gamma} = 1$

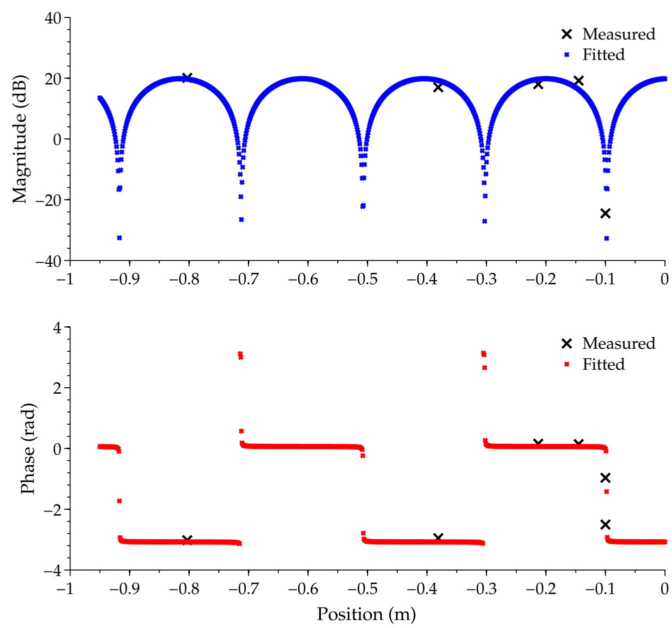

Figure 8. Fitted and measured data of a covered end at 840 Hz.

would be produced in the following conditions $\boldsymbol{Z_L} = \boldsymbol{Z_0}$, $\boldsymbol{Z_L} = 3\boldsymbol{Z_0}$, and $\boldsymbol{Z_L} = \infty$, respectively. The corresponding standing wave pattern expected in the impedance tube is shown in fig. 7. An excellent fit can be achieved due to the strong functions of both the magnitude and phase and by selecting the microphone positions carefully so that they lie across the features of the wave.

2) Implementation: MATLAB is used to fit the standing wave pattern (shown in fig. 8) to the measured data by allowing the non-linear solver to vary the magnitude and phase of the

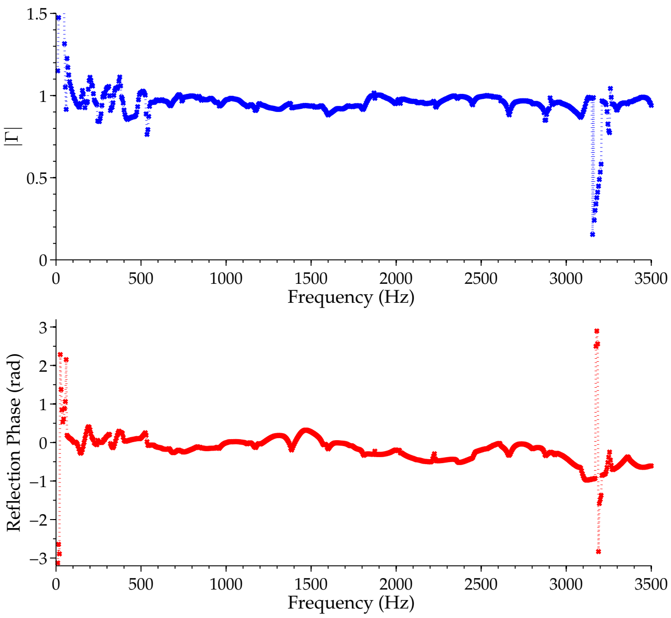

Figure 9. Reflection coefficient and phase of a covered end.

forward and reverse waves while attempting to minimise the least-squares error between the measured ($\boldsymbol{p_{meas}}$) and fitted data ($\boldsymbol{p_{fit}}$). The error term is calculated by:

$$\boldsymbol{p_{fit}} = \boldsymbol{A_i} e^{-j\boldsymbol{k}x} + \boldsymbol{A_r} e^{j\boldsymbol{k}x} \tag{8}$$

$$err_{real} = \Re(\boldsymbol{p_{fit}} - \boldsymbol{p_{meas}})$$

$$err_{imag} = \Im(\boldsymbol{p_{fit}} - \boldsymbol{p_{meas}})$$

$$err_{total} = \frac{err_{real}^2 + err_{imag}^2}{\frac{|\boldsymbol{p_{fit}}| + |\boldsymbol{p_{meas}}|}{2}}. \tag{9}$$

The fitting algorithm is run individually for each step in the frequency sweep to determine the acoustic impedance. Fig. 9 shows the acoustic impedance of the covered end which theoretically should have a reflection coefficient of 1 and a constant phase. The erroneous results below 500 Hz is caused by the microphone voltage clipping due to impedance tube and loudspeaker resonating at those frequencies. This effect can be corrected by lowering the amplitude of the driving signal but was chosen not to so to increase the accuracy of the majority mid-range frequencies. The disparity at 3155 Hz is caused by the 5 microphone measurements coincidentally falling on the same position of the waveform so not providing enough data for an accurate fit.

IV. Conclusions

The acoustic impedance measurements show that this method is a viable alternative to the slotted line method, although there are multiple issues that need to be addressed to improve reliability and accuracy. The precision of the magnitude and phase measurements has a large influence on the resulting fit, which means for this method to be improved, either higher quality microphones should be used or a more detailed calibration routine implemented. Extra elements also need to be included in the standing wave model such as amplitude decay from the tube material and gas losses. The speed of sound should also be intelligently measured precisely in order to further reduce errors rather than an estimate from room temperature.

The next stage of this work should include measuring various known physical elements to compare with the theoretical values and provide more calibration data.

References

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