Necessary Connections Between Distinct Existences: A Peircean Challenge to Humeanism

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0. Introduction

- My research aims to use Peircean ideas to revive contemporary mainstream philosophy in the analytic tradition, many of whose key debates – in my opinion – are 'stuck', and increasingly unproductive.

- An area where such revival is sorely needed is *epistemology*. I believe this field is uncritically wedded to a legacy from Hume which it currently cannot even see clearly enough to criticise.

- This legacy manifests in an absurdly sceptical treatment of *modality*, according to which nothing in our experience could ever teach us about this important dimension of truth. Any attempt to challenge this assumption is met with a scary charge of *anti-naturalism*: something no contemporary analytic philosopher wants to be.

Thus Crispin Wright has written, citing Simon Blackburn:

...'we do not understand our own must-detecting faculty.' Not only are we aware of no bodily mechanism attuned to modal aspects, it is unclear how such a mechanism could work even in principle... (1986, pp. 206-7)

- 'Truth-makers' for modal claims are even placed in other universes allegedly entirely spatiotemporally disconnected from this one **(Lewis, 1986).**

- Relatedly, in philosophy of mathematics Benacerraf has made a career out of invoking a crass fear of Platonism in the claim that the usual "semantics for mathematics" does not "fit an acceptable epistemology", since it: **"…will depict truth conditions in terms of… objects whose nature, as normally conceived, places them beyond the reach of the better understood means of human cognition (e.g. sense perception and the like)" (Benacerraf, 1973, p. 667).**

- Mathematics is in fact an ideal place to observe the limitations of the Humean legacy, and try to build a better view. For as Peirce observed, following his esteemed father, **"Mathematics is the science that draws necessary conclusions."**

- The materials Peirce offers us to do this will include a considerably more rich and plausible theory of *perception* than is found in Hume.

1. Examples of Diagrams in Which Necessity is Perceived:

[1] The sections of this presentation devoted to Hume owe much to previous joint work with Professor James Franklin, Department of Mathematics, University of New South Wales. The sections on Peirce I am entirely responsible for.

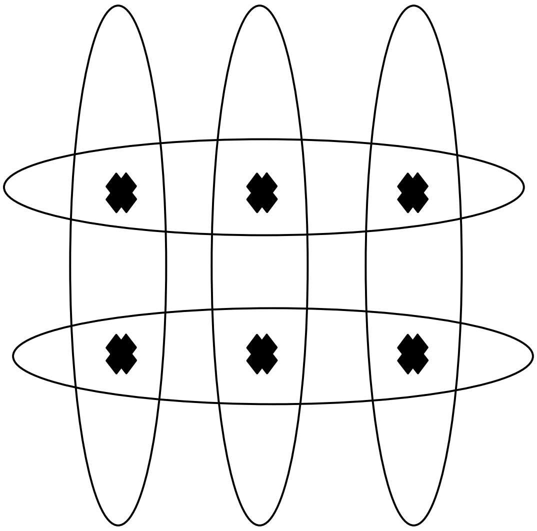


Fig 1. **2 × 3 = 3 × 2**

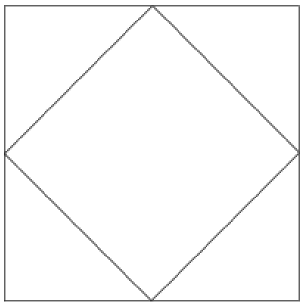


Fig. 2. **The inside square is half the area of the outside**

In each case it seems we can perceive that a mathematical truth *is* so, and also at the same time that it *must be* so – it is necessarily true. This is remarkable.

- There has been a recent upsurge of interest in the role of diagrams in *mathematical reasoning* (e.g. Brown 1999, Giaquinto 2007, Mumma 2010: many inspired by Manders 2008) and *logic* (Shin 2002).

- Peirce of course, with his profound understanding of iconic signs has much to offer this movement, since he understood that all necessary reasoning concerns **"A diagram of our own creation, the conditions of whose being we know all about"** **(*Philosophy of Mathematics*, p. 19).**

2. Hume's Denial that we May Perceive Necessity

The reason Humeans deny we may perceive necessity derives from Hume's particular brand of *empiricism*, and a supposedly common-sense maxim which he drew from it: **"there are no necessary connections between distinct existences"**.

- This maxim is widely taken for granted today – treated as something like an axiom of metaphysics.

Thus, Hume writes:

''There is no object, which implies the existence of any other if we consider these objects in themselves.'' (*Treatise*, 1, III, vi)

"Any thing may produce any thing. Creation, annihilation, motion, reason, volition; all these may arise from one another, or from any other object we can imagine…no objects are contrary to each other, but existence and non-existence." (*Treatise*, 1, III, xv) ***Quite remarkable!** ↑

- Consider Hume's most famous application of his maxim, to so-called *causal necessity*, in the famous passage about billiard balls:

"I consider, in what objects necessity is commonly suppos'd to lie; and finding that it is always ascrib'd to causes and effects, I turn my eye to two objects

suppos'd to be plac'd in that relation. . . . I immediately perceive, that they are contiguous in time and place, and that the object we call cause precedes the other we call effect. In no one instance can I go any farther...." (*Treatise*, 1, III, xiv)

i) Note that this is a phenomenological argument

Hume claims that we suppose that causes and effects are united by **"a necessary connexion of power, of force, of energy, and of efficacy..."**, but nothing in his immediate experience corresponds to that.

ii) What are the 'objects' Hume is referring to?

- Here Hume speaks as if the 'objects' are the balls themselves, but he is actually talking about their *motions*.

- Properties and events must therefore count as 'objects' or 'existences' for Hume. But then, what about the properties 'black' and 'white'? The events of 2 and 3 hours passing? Or, to return to mathematics, the 'objects' $\emptyset$ and $\{\emptyset\}$?

- In an attempt to understand what Hume means by 'distinct existences', I turn to his theory of perception and the epistemology he twines around it.

3a. Humean Theory of Perception → Epistemology: Passive

i) Ideas are simple copies of impressions

ii) Impressions of reflexion consist solely in combinations of impressions of sensation

- Sensory impressions are the building blocks of all thought.

iii) All mental activity is 'perception-like'

- Reflexion too is a form of perception – of ideas that are 'weaker' and 'less vivid'. In fact: **"To hate, to love, to think, to feel, to see; all this is nothing but to perceive"** (*Treatise* 1, II, vi)

iv) Denial of *abstract ideas*.

- Hume defines abstract ideas as ideas that are general in that at least some of their determinable properties lack determination. E.g. a 'general triangle': neither isosceles or scalene.

- He claims (following Berkeley) there are *no general ideas*, only *particular ideas used in a general way* (e.g. a proof about triangles to be valid might need to draw on the particular ideas of isosceles *and* scalene *and* equilateral triangles...)

- Allowing abstract ideas would render the mind active since it would need to choose *which* determinables to abstract from.

3b. Humean Theory of Perception → Epistemology : Atomist

i) Separate Imaginability Criterion of Distinctness

- When we distinguish shape from colour in an object such as a white globe, it is not that we examine the white globe and use reason to distinguish its whiteness and roundness *as abstract ideas*.

- Rather, what we do is imagine *black globes* and *white cubes.*
- Without such a literal, quasi-perceptual forcing apart of ideas we cannot distinguish them, though we might think we can, a cause of much confusion and wasted time in philosophy: **"...that distinction of reason, which is so much talked of, and is so little understood, in the schools."** (*Treatise*, 1, I, vii)
- Thus Hume denies that we can *prescind without separating*: **"...all ideas, which are different, are separable..."** (*Treatise*, 1, I, vii). This is crucial.
- To deny this is to allow philosophers to postulate *occult qualities* – a good example is the Aristotelian idea of substance ('prime matter'): **("…these philosophers carry their fictions still farther in their sentiments concerning occult qualities, and both suppose a substance supporting, which they do not understand, and an accident supported, of which they have as imperfect an idea. The whole system, therefore, is entirely incomprehensible…"** (*Treatise*, 1, IV, iii))

4. Modal Combinatorialism

- Recall Hume's remark: **"...no objects are contrary to each other, but existence and non-existence"**
- This implies that 'objects' as he understands them are all compossible – they have no *natures* which might constrain their combination in any way. Thus such necessities as do exist in the world may only consist in constant conjunction, nothing more 'binding'.
- This view has been very influential in C20th philosophy: e.g. Wittgenstein's *Tractatus*: §1.12: **"Each item can be the case or not the case while everything else remains the same"** → Carnap → David Lewis. (See also Armstrong, 1989.)

> ***'top-down' combinatorialism***: any whole can be decomposed into some given set of atomic parts. (*HUME DOESN'T MEAN THIS)
>
> ***'bottom-up' combinatorialism***: given some set of atomic parts, any permutation of them is possible. (*HE MEANS THIS)

This is essentially a denial of real universals. Real universals precisely consist in constraints (hopefully intelligible) on the happy combination of any possible thing with any possible thing. A physical example: the whole point of the concept of *force* is that it is **contrary** to certain (unforceful) *behaviors*.

5. Back to our Examples

- Reconsider *Fig. 1*:

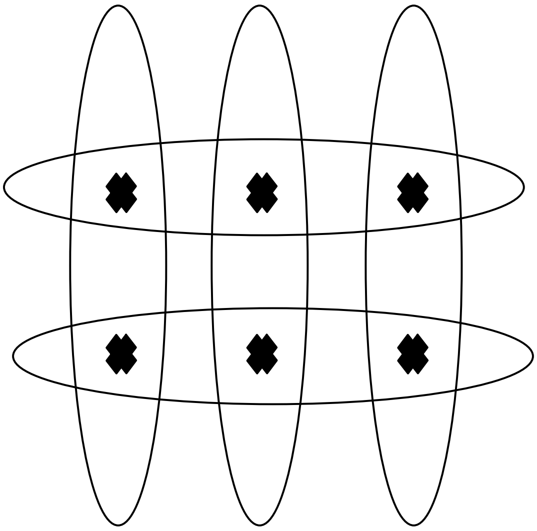
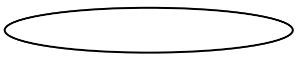
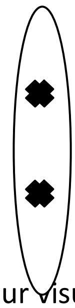


Fig 1. **2 × 3 = 3 × 2**

Does this constitute a necessary connection between distinct objects? Well, what are the objects here? It seems the Humean has a number of choices:

i) *Physical Mark View*: The relevant objects/existences are:
5 of these: 
6 of these: 
which go together to make 5 oval + star combinations, such as: 

- This appears to be a natural choice in terms of the organisation of our visual field when regarding the diagram. *Humean Separate Imaginability Criterion of Distinctness*: we can imagine each of these shapes existing on its own on the page.

- But then it is false that there are no necessary connections between these objects as positioned in *fig 1*. For instance, one cannot change the number of stars in the vertical ovals without changing the number of stars in the horizontal ovals. Interpreted thus, then, Hume's maxim is simply incorrect.

ii) *Abstract Object View*: On the other hand, one might claim that *fig.1* doesn't display a truth about physical marks but about something more purely mathematical or ideal – for instance the relevant objects are *three '2s'* and *two '3s'*.

- These objects are arguably not distinct. E.g. 2 is made up of 'two ones' and 3 is made up of 'three ones', so 2 is a proper part of 3.

- At this point, then, Hume might defend his maxim by stating that *fig 1* only expresses relations between ideas.

- But there is something unsatisfying here. It seems puzzling to claim that we can gain mathematical knowledge, as we clearly can, by examining this diagram, and yet that mathematical objects are *entirely* separate from perceived experience.

- Furthermore, now Hume's claim: **"There is no object, which implies the existence of any other if we consider these objects in themselves..."** seems to beg the question. He seems to be arbitrarily ruling out that we perceive the kinds of existences between which necessary connections hold, by labelling them as 'mere ideas'.

- His maxim then effectively becomes: **"there are no necessary connections between distinct existences, which are those existences between which there are no necessary connexions"**. This seems to rob it of all philosophical content.

iii) <u>*"Both" view*</u>: One might think of compromising by combining the two views as follows: the objects represented by *fig 1* are ovals and stars *and* '2s' and '3s'.

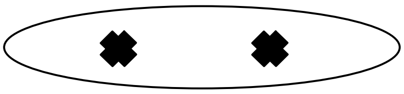
- However this raises tricky questions of the relationship between the physical marks and the numerical ideas. If they are all separate objects, why include the stars and ovals in the diagram at all....?

- This points the way to our preferred interpretation:

iv) <u>*"Hybrid...but not both"*</u>: Rather than understanding physical marks and numbers as separate objects, attribute to them *partial identities*.

- What does this mean?

- Just that 'twoness' is a property which may be *prescinded* from this:



while precisely *not* being separable from it. This is of course exactly what Hume's theory of perception (and associated epistemology) rules out as impossible.

- So what kind of theory of perception *could* do justice to what is clearly going on in these mathematical examples?

<u>6. Peirce's Theory of Perception</u>

- Enormously different to Hume's.
- Forster (*Peirce and the Threat of Nominalism*) writes that nominalists such as Hume take for granted **"….that knowledge is grounded immediately in experience of particulars."** (p. 109). However this confuses **"the question of the nature of empirical evidence in inquiry with psychological and metaphysical questions about the nature of the mind and its relation to the world"** (p. 109).
- Peirce suggests we need to give separate, though interlocking, accounts of:

i) immediate experience

ii) the truth of symbols derived from that experience.

- The first becomes Peirce's account of the *percept*, the second his account of the *perceptual judgment*.

<u>*i) The Percept*</u>

- comprises a *felt quality* and the *vividness* with which it is presented. Neither of these is 'cognitive'. The percept is not a Humean idea. Nor does it express truth-claims. Peirce writes that it **"…does not stand for anything. It obtrudes itself upon**

my gaze; but not as a deputy for anything else, not 'as' anything. It simply knocks at the portal of my soul and stands there in the doorway." (7.619).

- Forster: **"While the content of a percept is inherent in it apart from everything else, the content of a sign is not"** (p. 114).

<u>ii) The Perceptual Judgement</u>

- cannot be a copy of the percept, as they are too unlike one another: **…as unlike... as the printed letters in a book, where a Madonna of Murillo is described, are unlike the picture itself"** (5.54).
- The percept has an *integration* which cannot be possessed by the perceptual judgement, which requires a subject and a predicate. Consider the perception of a yellow chair:

"The judgement, 'This chair appears yellow', separates the color from the chair, making the one predicate and the other subject. The percept, on the other hand, presents the chair in its entirety and makes no analysis whatever" (7.631).

- The perceptual judgement expresses a proposition, which can be true or false. By the same token its interpretation is thrown open to the community of inquiry, to which each symbolic judgment properly belongs, namely: **"…an endless series of judgments, each member of which is logically related to prior members"** (p. 120)
- These inquirers may now develop the meaning of the terms *yellow* and *chair* in unanticipated ways.

<u>iii) The Relationship between Percept and Perceptual Judgement</u>

- But now Peirce has so convincingly separated the firstness / secondness of the percept from the thirdness of the perceptual judgement – how are we to bridge the two? Moreover, how are we to bridge from the *uncontrollable* in perception to the *controllable* in thought?
- Don't we now have a great mystery at the heart of perception?
- No we do not. The British empiricists (and their downstream followers) are too unimaginative in assuming that the only possible relation between percept and perceptual judgment is that the latter *copies* the former.

- Rather: percepts *cause* perceptual judgements, while not being the source of their content.
- How does this happen?
- The human mind is organised such that each percept causes **"direct and uncontrollable interpretations"**. (These are sometimes referred to by Peirce, after 1903, via a third term - the *percipuum*).
- This causal process cannot be *willed* but it can be trained and perfected via the cultivation of appropriate mental *habits* (also known as 'education').

iv) Abstract ideas

We're all familiar with how Peirce challenges early modern parodies of abstractions such as 'dormitive virtue'. Suffice it to say that prescinding without separating is not only possible, but crucial. (Hypostatic abstraction is **"...an essential part of almost every really helpful step in mathematics." (1903 lectures, p. 133)**.

7. Perceiving a Mathematical Diagram

- So how does all this work in the case of perceiving mathematical necessity? What is a *mathematical* percept? It's obviously not going to be quite like the percept of a yellow chair, so what is it going to be like?
- Suggest that we take seriously Peirce's repeated claims that mathematics is *as experimental a science as physics*: **"I have sometimes been tempted to think that mathematics differed from an ordinary inductive science hardly at all except for the circumstance that experimentation which in the positive sciences is so costly in money, time, and energy, is in mathematics performed with such facility that the highest inductive certainty is attained almost in the twinkling of an eye." (1903 Lectures, p.131)**
- The laboratory equipment of the mathematician is the *diagram*.
- Let us reconsider *fig. 1*, in which we perceived that 2 × 3 = 3 × 2 is necessarily true, and analyse it in terms of Peirce's theory of perception.

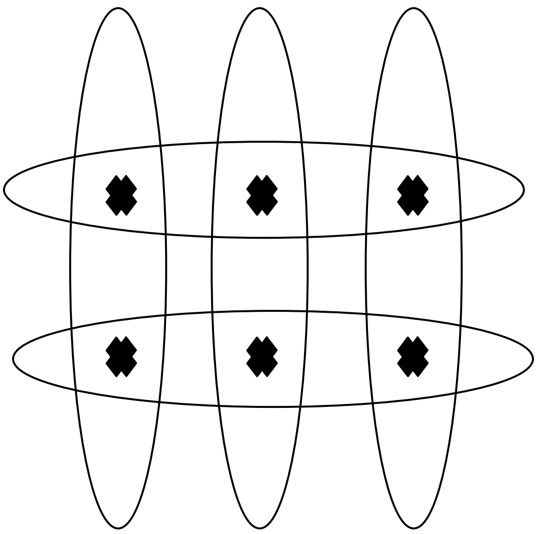


***Fig 1.* 2 × 3 = 3 × 2**

- *The Percept*: The percept for *fig. 1* is exceedingly difficult to describe in words, as are all percepts. However I will try to convey indirectly something of my phenomenological experience when I 'got' this proof.
- I had a sudden grasp of the horizontal and vertical arrangements of the stars *as one*, as if the same 5 ovals were 'holding together' both arrangements, although that 'holding together' is only a metaphor as those arrangements are not strictly *parts* of the whole diagram (they are not separable, only prescindable).
- At the same time, looking at the diagram and thinking about other possible arrangements of the stars that might be prescinded from it (such as three threes, or three fours), I perceived some kind of primitive blocking of those other options. It is as if I could *feel myself not being able to think of* those other options.
- We might call this primitive blocking or constraint, in homage to Wittgenstein, 'the hardness of the mathematical must'.

- *The Percipuum:* My *fig. 1* percept gives rise to an 'uncontrollable interpretation' that it shows the necessary truth of: 2 × 3 = 3 × 2.
- This might seem 'too easy to say'.
- But no matter how hard I might try to interpret *fig. 1* as, for instance, 2 × 3 = 3 × ***3*** – I just can't do it. That is a simple fact – I can't manage to think that way. (*Try it yourself*...)
- Thus Peirce writes: **"Although mathematics deals with ideas and not the world of sensory experience, its discoveries are not arbitrary dreams but something to which our minds are forced..." (*Philosophy of Mathematics*, p. 41)**.

- *Perceptual judgment*: The phenomenological 'hardness of the mathematical must' which was felt by my mind in viewing the diagram now becomes the necessary truth of: 2 × 3 = 3 × 2.
- Felt hardness in the **inner world** and necessity in the **outer world** are not the same thing. For insofar as the proposition 2 × 3 = 3 × 2 is a *symbol*, it can now be put to a multitude of general uses – for instance drawn on in practical tasks (e.g. food rationing), integrated into a broader theory of arithmetic (e.g. "multiplication is commutative")...and so on.

- *Occult-hood revisited.*
- We saw that Hume took as definitive of naturalism to avoid positing occult powers at all costs. Peirce on the other hand calmly evaluates the phenomena, admits that certain powers of the mind *are occult*, and characteristically, usefully clarifies the notion of occultness so that it can do real philosophical work: **"[The clustering of ideas] is either due to an outward occult power or to an inward one. That it is due to some occult power is plain from this, that the ideas although they are in our own**

minds and thus normally subject to our will. cluster in spite of our will, and that in certain regular ways....But it is occult in this sense, that nothing more about it can be learned by mere observation of these phenomena." (*Philosophy of Mathematics*, p. 50).

8. Conclusion

- Despite analytic philosophers' bafflement as to how it should be possible, we do of course perceive necessity. Perception is in fact the *only* way in which we gain knowledge of necessity, insofar as all necessary reasoning involves experimenting upon diagrams.
- It's time mainstream philosophy got past its crass horror of Platonism.
- Peirce's nuanced theory of perception allows us to see that Hume is interestingly right and wrong phenomenologically about perceiving necessity. We might say that Hume is *correct* in his devastating analysis of causal necessity that there is no external necessity in his immediate experience (i.e. in his *percept*). He is just incorrect that it follows from this that he is perceiving no such thing in reality (i.e. he has no *perceptual judgement* of it)
- Hume failed to see this because he failed to understand that *ideas are not copies of impressions*. This was arguably a phenomenological defect in his philosophising.

- I end with a quote by Peirce: a beautiful example of the gloriously enigmatic depth of which he was capable: **"It is self-evident that every truth of pure mathematics is self-evident if you regard it from a suitable point of view" (1903 Lectures, p. 128)**.

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