Plug flow versus mixed flow modelling of a pressure screen

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ABSTRACT
The plug flow and mixed flow methods of modelling a screen are analysed in terms of screen length, internal fibre concentration and screen separation performance. Experimental data for consistency changes in screens of narrow length are incorporated into the analysis and from this data an equation for determining the real thickening in the screen and the fibre concentration profile along the screen is derived. Lastly, thicken equations are related to common equations used for modelling overall screen performance. The work directly links screen performance equations to passage ratio measurements from narrow length screen sections.

INTRODUCTION

Pressure screens have been in wide spread use for over forty years in the pulp and paper industry, but are still not well understood at a fundamental level. Unsteady flow fields and non-uniform feed materials combine to make conditions inside a pressure screen very complex and difficult to analyse. Promising mathematical expressions to predict consistency changes and screen performance have been developed based on lumped parameter modeling, and some validation of these models has occurred (1-4). Single slot studies of fibre passage ratio have also been made and linked to screen performance (5-7). Experimental screening studies performed with narrow lengths of screen are providing further understanding to these models and some of this work is presented in this paper.

Solid-Solid screening

The separation of wood pulp fibre from other solids, such as shives, contaminants or longer fibres can be accomplished by solid-solid screening. In this process desired particles (usually fibre) are separated from undesirable particles (usually contaminants) by selective passage through apertures. Where particles are larger than the apertures in all dimensions they are retained on the screen. This is called barrier screening. Where the particles are smaller in at least one dimension, they pass through the apertures by their probability of passage. This is known as probability screening.

In probability screening particle passage is governed by the complex interaction of hydrodynamic forces, particle orientation, particle-particle interactions and the relative size of particles to the apertures (3,8). In barrier screening relative particle size is the main factor and hydrodynamic forces are less important. Wood fibres, which are small in at least two dimensions, are screened according to their probability of passage. Shives and contaminants, which span a considerable size range, are screened by either probability or barrier screening.

Pressure screens separate solids by passing a feed stream across a perforated plate or cylinder (see Figure 1). Material with a high probability of passage pass through the plate to form an exit stream (called accepts) and material with a low probability of passage are retained on the screen and form a second exit stream (called rejects). P represents the probability of a solid particle passing through a screen aperture, and can be used when modelled both barrier and probability screening since:

$P = 0$ is barrier screening
$P < 1$ is probability screening

Screens have also been known to dilute the rejects i.e. more solids pass through the screen than liquid as a proportion of the feed (9). This is know as reverse screening and for this situation $P > 1$.

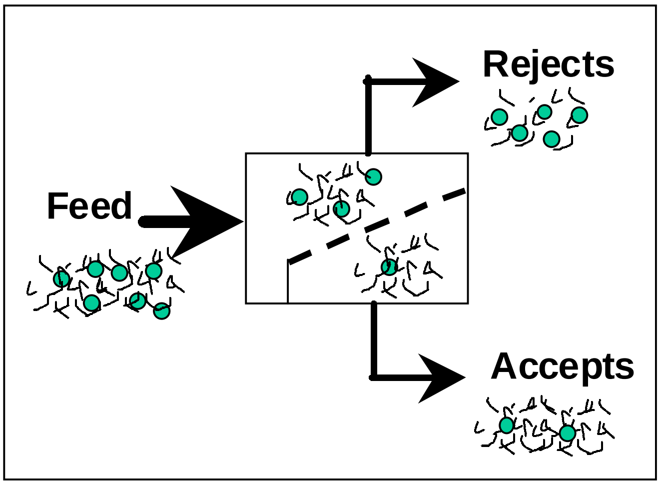


Figure 1. Solid-solid separation by screening

Predicting concentration changes with a screen

The thickening or concentration increase of pulp, shives or contaminants across a screen from the feed to the rejects is a well-known phenomena in the paper industry (1). In the case of shives or contaminants the concentration increase is desirable and key to good screen performance. In the case of pulp the concentration increase is undesirable and can results in loss of good fibre out the rejects and can cause blocking and shut-down of a screen. For these reasons modelling the thickening behaviour of pulp and other solids provides the foundation to predicting the overall performance of a screen.

When a pulp slurry passes through a screen the flow behaviour within the screen lies between to extreme conditions. One case is where pulp is well mixed and each aperture processes stock in parallel. The second case is where pulp flows as a small well-mixed plug and each aperture processes stock in series. Based on these two conditions equations for predicting the concentration increase of pulp or components across the screen can be derived.

MIXED FLOW MODELLING

In mixed flow modelling the pulp is assumed to flow into a well-mixed chamber which disperses the fibres and other components both axially and radially to all parts of the screen. This is shown in Figure 2 for a screen with an open foil rotor.

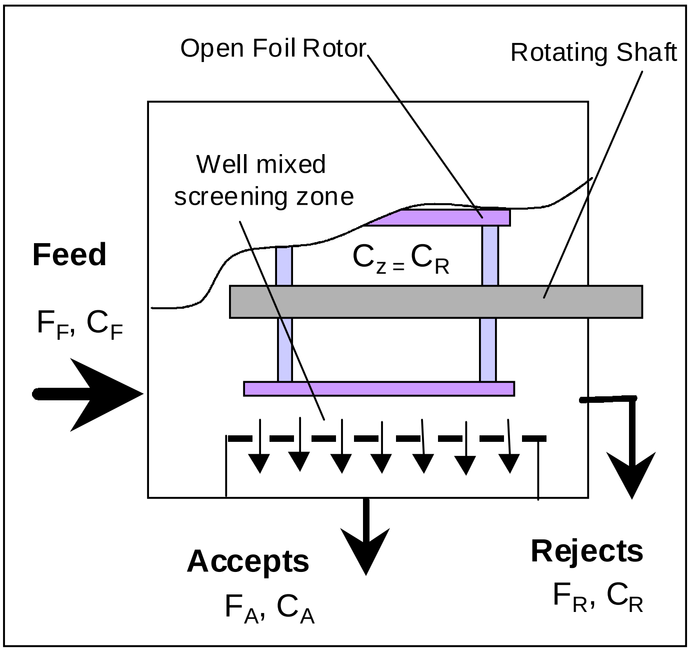


Figure 2. Mixed flow screening characterised by well-mixed screening zone, rotating open rotor and screening concentration C_Z equal to reject concentration C_R.

The feed enters with a pulp concentration of C_F and immediately mixes with the pulp in the screen chamber that is at a higher concentration C_Z. At any instant pulp can leave the chamber via the rejects outlet at a concentration of $C_R = C_Z$ or through the screen apertures at concentration C_A. All apertures process the pulp equally. The concentration change across each apertures is the same and the concentration profile along the length of the screen is constant. This is also know as *parallel screening*. An open foil rotor provides a large mixing chamber and is most likely to exhibit this behaviour .

By fibre mass balance around the overall screen, and assuming 1. constant accept flow, 2. the concentration within the screen C_Z is equals to the reject exit concentration C_R and 3. the probability of fibre passage though the apertures is $P = C_A/C_R$, then the pulp concentration increase (pulp thickening) is predicted by:

$$T = C_R/C_T = \frac{1}{P - R_v P + R_v} \qquad (1)$$

where $R_v = F_R / F_F$ the volumetric rejects rate.

Equation 1 is presented graphically in Figure 3 and shows that pulp thickening increases with decreasing reject flow and decreasing probability of fibre passage. At *P=1* the screening is acting as a flow splitter and there is no concentration changes occurring $T = 1$. At *P=0* the screen is acting as a filter and only water is passing through the screen. The screen is acting as a barrier to solids passing through the screen and represents the case of barrier screening. With *P* between 0 and 1 thickening increases with decreasing *P* and probability screening predominants.

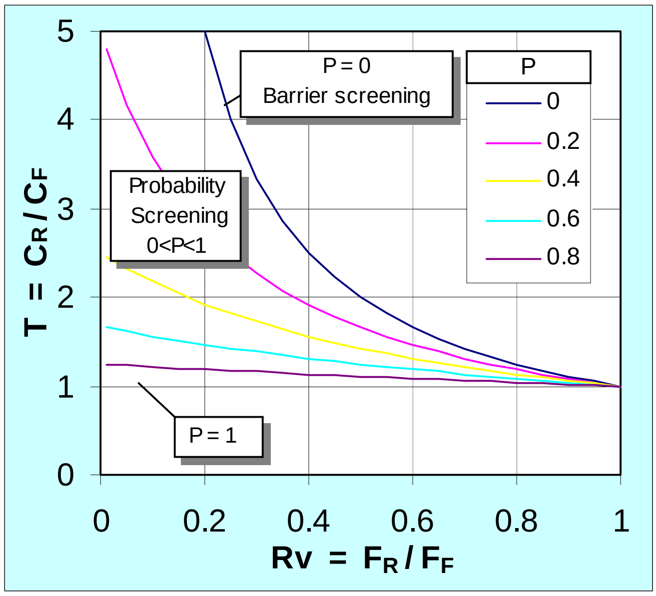


Figure 3. Pulp thickening behaviour in a screen predicted by the mixed flow model.

P is also known as the fibre passage ratio and is defined as the pulp concentration flowing through an aperture C_s, divided by the pulp concentration of the flow immediately upstream of the aperture C_u.

$$P = C_s / C_u \qquad (2)$$

Concentration profile in a screen

In the mixed flow or parallel screening situation the pulp concentration flowing through the aperture C_s is equal to the accept pulp concentration C_A, and the upstream pulp concentration C_u is equal to the concentration in the screen C_Z which is also equal to C_R, from the mixed flow assumption. Hence for mixed flow screening the concentration profile for all distances *z* along the screening zone will be:

$$C_z = C_R \qquad (3)$$

Mixed flow behaviour in an industrial screen is most likely to be present when the screen is of short length or low length to diameter ratio and an open rotor is used. Under these conditions pulp matting or blocking at the rejects end is less likely to occur as there is little concentration increase along the short length of the screen. This is illustrated in Figure 4 where the concentration profile is presented for the mixed flow and plug flow models of screening. The plug flow model is discussed in the next section. The concentration ratio for the mixed flow case is constant for a particular set of R_v and P values and stays well below the plug flow concentration profile at the rejects end of the screen *(z/L = 1)*.

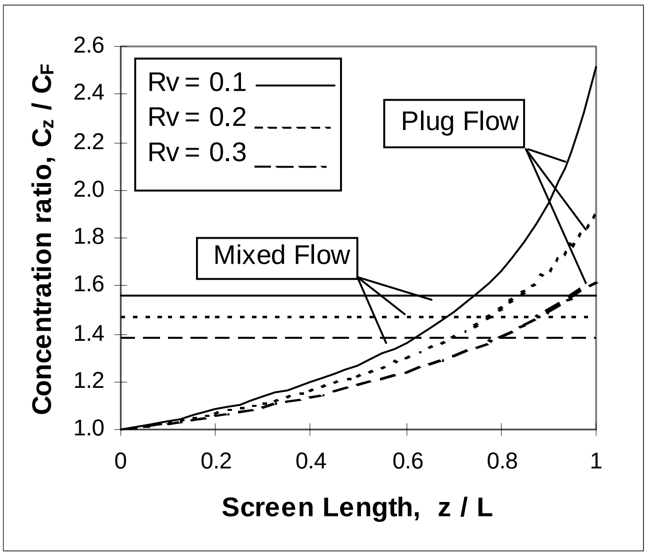


Figure 4. Pulp concentration profile along the length of a screen as predicted by the mixed flow and plug flow models for three rejects rates and *P = 0.6*.

PLUG FLOW MODELLING

In plug flow modelling the pulp is assumed to flow as a small well-mixed plug through the annular gap between the rotor and the screen plate. The length to annular gap ratio of the screen is important for this flow condition.

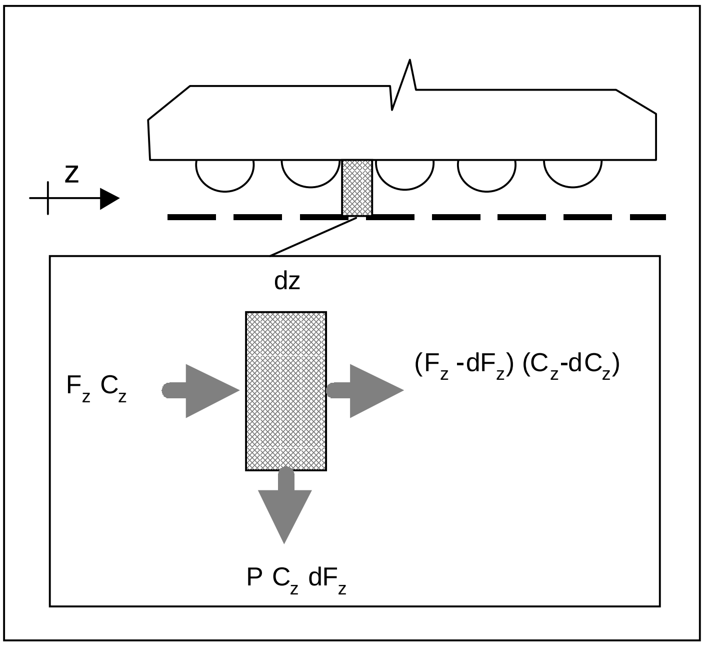


Figure 5. Plug flow screening modelled by considering a series of annual elements in the narrow screening zone between the rotor and screen plate.

As the pulp progresses along the screen, the pulp on the screening zone side of the screen progressively increases in concentration (see Figure 4) and the screening condition changes within the screen. This situation is quite different to the mixed flow case where the screening concentration is uniform throughout the entire screen. Plug flow screening is also called *series screening*. Screens with closed rotors, like the bump, S-rotor and closed foil rotor are more likely to exhibit this behaviour.

For the case of *plug flow screening* no axial mixing is assumed to occur in the screening zone (i.e. no mixing along the screen), and perfect radial mixing is assumed between the rotor and the screen plate (see Figure 5).

A differential mass balance over an element of thickness *dz* yields Equation 4.

$$F_zC_z = [(F_z\text{-}dF_z)(C_z\text{-}dC_z)] + PC_z\,dF_z \qquad (4)$$

where
F_z = axial flow rate
C_z = average concentration of pulp at a distance z from the entry to the screening zone
P = the fibre passage ratio

Equation 4 can be rewritten as

$$dC_z / C_z = (P\text{-}1)\; dF_z / F_z \qquad (5)$$

Equation 5 can be solved by integration if P is assumed to be constant and independent of C_z and F_z, which is reasonable as a first approximation. The rotor induces a high tangential velocity which dominants the flow field making changes in F_z have little affect on P. The assumption that P is independent of C_Z holds true only if the fibres act independently of each other for the range of concentrations present in the screening zone. This is only likely to be the case at very high levels of turbulence and very low fibre concentrations.

Plug flow model - assuming *P* constant

Assuming P is constant and independent of C_z and F_z is a useful assumption for a first approximation solution of Equation 5. Setting boundary conditions of :

For $z = 0$, $C_Z = C_F$ and $F_Z = F_F$

For $z = L$ $C_Z = C_R$ and $F_Z = F_R$

where L is the axial length of the screen, Equation 5 can be integrated to give:

$$C_R / C_F = (F_R / F_F)^{(P\text{-}1)} \qquad (6)$$

In terms of pulp thickening $T = C_R / C_F$ and volumetric rejects rate $Rv = F_R / F_F$:

$$T = Rv^{(P\text{-}1)} \qquad (7)$$

Equation 7 is presented graphically in Figure 5.

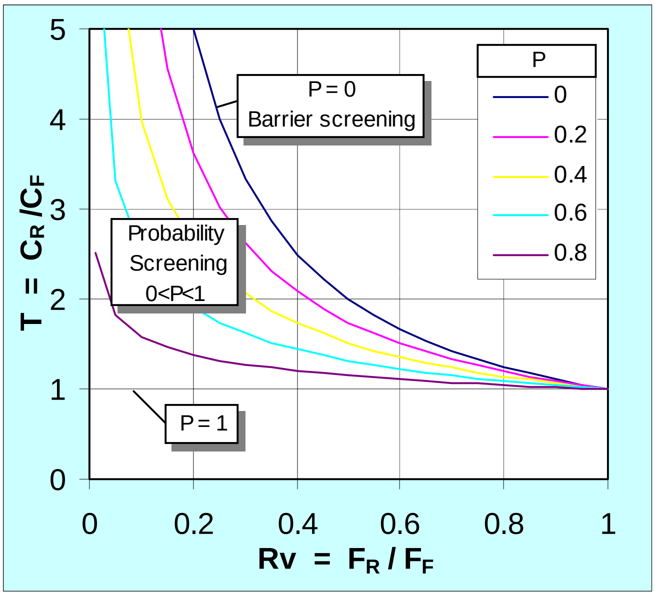


Figure 5. Pulp thickening behaviour in a screen predicted by the plug flow model.

The thickening behaviour follows similar trends to the mixed flow model (compare with Figure 3). Thickening increases with decreasing R_v and decreasing P. However, the degree of thickening is much greater than that predicted by the mixed flow model. This is illustrated in Figure 6 where a comparison is made between the thickening curves of the mixed and plug flow models at the same P value.

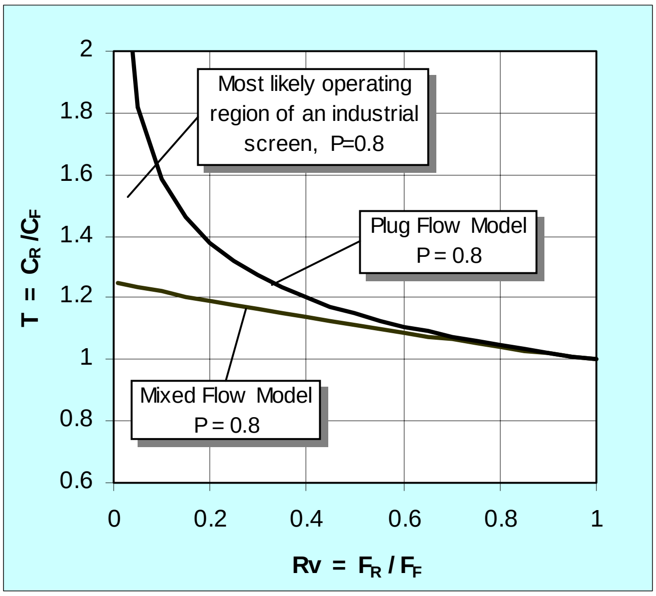


Figure 6. Comparison of predicted thickening behaviour of a screen according to the mixed and plug flow models at P= 0.8.

With decreasing rejects rate, thickening increases for both cases, but the rate of increase is much less for mixed flow compared to the plug flow. This result is significant and illustrates why experimental determination of P is prone to error. The thickening and rejects rate values alone do not isolate P, as P can vary significantly depending on the type of mixing occurring in the screen.

Effect of screen length on pulp concentration

Equation 7 can be re-written in terms of the pulp concentration C_Z and flow rate F_Z at distance z along the screen axis.

$$C_Z / C_F = (F_Z / F_F)^{(P-1)} \qquad (8)$$

The flow rate of F_Z varies linearly with distance z along the screen (see Figure 9) since the aperture flow rate is assumed to be constant.

Hence:

$$z/L = (F_Z - F_R)/(F_F - F_R) \qquad (9)$$

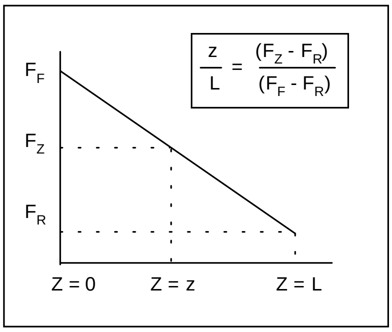


Figure 9. Variation of internal screen flow rate F_Z with distance z along the screen.

Equation 9 rearranges to:

$$F_Z / F_F = (1 - z/L)\,(1- R_v) + R_v \qquad (10)$$

Equations 8 and 10 can then be combined to form an equation for the concentration profile along the screen.

$$C_Z = C_F\,[\,(1 - z/L)\,(1- R_v) + R_v\,]^{(P-1)} \qquad (11)$$

Equation 11 is presented graphically in Figure 3 for three values of R_v at a $P=0.6$. In comparison to the mixed flow concentration profile the plug flow concentration begins at the feed concentration and gradually increases until towards the end of the screen the concentration increases above the mixed flow concentrations. For the same R_v and P values plug flow behaviour results in higher levels of thickening and higher possibility of matting near the reject end of the screen.

PASSAGE RATIOS IN SCREENS

To accurately determine P values in an industrial pressure screen it is necessary to know the flow characteristics of the screen. Industrial screens are likely to operate with flow behaviour that is a combination of mixed and plug flow. Hence thickening data will lie somewhere between the mixed and plug flow curves for a given P value. A method of determining P values in a screen is to shorten the screen temporarily to give flow conditions similar to mixed flow.

To investigate the effect of fibre concentration, rotor speed and other important screen variables on passage ratio *P* a laboratory scale 200mm diameter pressure screen was modified to have a screening length *L* of 25mm. Pulp thickening data were collected using previously developed methods (9) and data were plotted as *1/T* versus R_v to determine *P* values.

Equation 1 for mixed flow can be rearranged to give

$$1/T = R_v(1-P) + P \qquad (12)$$

Hence a plot of *1/T* versus R_v should give a straight line of slope *(1-P)* and intercept *P*. If mixed flow behaviour is present then the *P* value derived from the slope should equal the intercept *P* value.

Typical thickening data are presented in this form in Figure 10 for two different *L/D* ratios. The small *L/D* ratio gave a straight line and near perfect match between the *P* values for the intercept and from the slope. In contrast the full-length screen gave *P* values that were up to 60% different and therefore the mixed flow method is not valid.

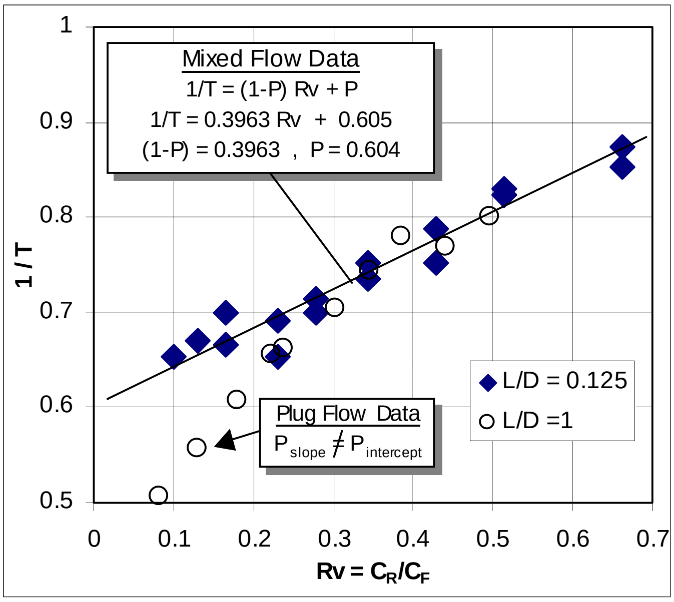


Figure 10. Thickening data showing determination of passage ratio *P* in a pressure screen for a narrow length screen.

For the full length screen the plug flow model (Equation 7) gives a better fit to the data and an average *P* value can be estimated using curve fitting methods. However, the *P* value is for an average range of pulp concentrations and does not isolate the true effect of pulp concentration on *P* within the screen.

Effect of pulp concentration on *P*

The mixed flow method across a short length of screen was used to study the effect of pulp concentration on *P*. The *P* values were derived from pulp thickening data obtained across a 25mm section of screen. The very short section of screen gave good mixed flow characteristics as discussed, which made for easy calculation of *P* directly from rearrangement of the mixed flow Equation 1 to:

$$P = (1/T - R_v)/(1 - R_v) \qquad (13)$$

Figure 11 is a plot of *P* versus pulp feed concentration C_F for pine and eucalyptus fibres. As shown the assumption that fibre concentration does not affect *P* is only valid at very low concentrations. For eucalyptus fibres this is below 0.1% for this rotor speed, and for pine fibres this is below 0.03%. Above these critical concentrations integration of Equation 5 by assuming *P* is independent of C_Z will result in under estimation of the thickening effect, especially at low rejects rates.

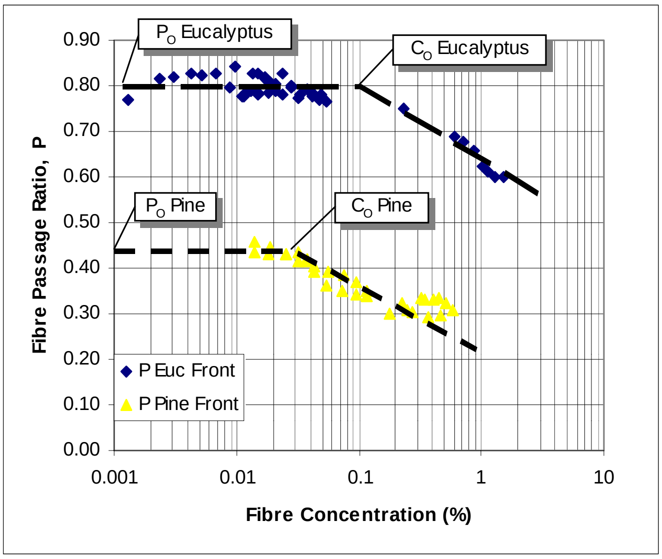


Figure 11. Affect of screen fibre concentration on fibre passage ratio for mixed flow screening across a 25mm length of screen, R_v = 0.2.

Plug Flow Modelling – changing *P*

From short screen studies it was established that *P* varies with pup concentration beyond a critical concentration C_O. The variations of *P* forms an approximate straight line on a linear-log plot (see Figure 11). A simplification of this trend is to linearise the decrease in *P* beyond C_O. such that:

$$P = P_O \qquad C_Z < C_O \qquad (14a)$$

$$P = P_O - \alpha(C_Z - C_O) \qquad C_Z > C_O \qquad (14b)$$

where α = slope of *P* decreasing with C_Z.

Substituting Equation 14b into Equation 5 and integrating across the screen with the same boundary conditions as previously gives:

$$T = [(P_R - 1)/(P_F - 1)]\ R_v^{\ (Po - 1 + \alpha Co)} \qquad (15)$$

where P_R and P_F represent the passage ratios of the pulp at the rejects end and feed end of the screen.

For screening below C_O the passage ratio P is constant at P_O, $P_R = P_F$ and $\alpha = 0$, hence Equation 15 reduces to:

$$T = R_v^{\ (Po - 1)} \qquad (16)$$

To illustrate the effect of P varying with pulp concentration Equation 15 and Equation 16 have been graphed for a set of typical P_O, C_O and α values (see Figure 12). Equation 15, which accounts for P decreasing with increasing pulp concentration, thickens to a higher level than Equation 16, the constant P case. The difference in thickening is significant and the extent of the difference shows that curve fitting the plug flow model to screen thickening data can result in P values that are not correct even though the data fits the plug flow model curve.

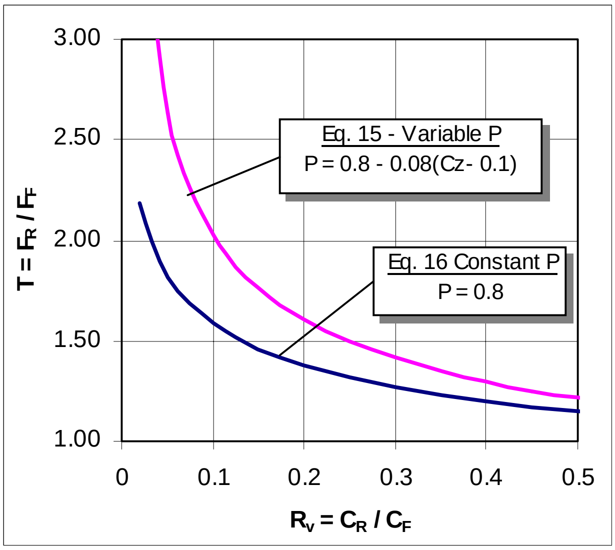


Figure 12. Comparison of the plug flow model assuming constant *P* (Eq. 16) and variable *P* (Eq.15).

MODELLING SCREEN PERFORMANCE

Screen performance can be modelled by using the mixed flow and plug flow thickening equations previously derived (2). In this discussion the simplified models which yielded Equations 1 and 7 will be used

Measuring screen performance

Like all separation processes the performance of the screen is best characterised by at least two measures. The measures used depend on the emphasis of the screening process. Where the screen is used to clean pulp, i.e. to remove low value material (contaminants) from high value material (fibre), one measure is the mass fraction of contaminants removed to the reject stream. The other measure is the mass fraction of fibres lost to the reject stream. The process could equally be characterised by the mass fraction of fibre recovered to the accept stream and the cleanliness of the fibre recovered to that stream. If the screen is used to separate two equally important materials, for example in separating long and short fibre in a fractionation process, the measure of screen performance will be different again. It may be based on the relative purity of the two exit streams for a desired mass split ratio. On the other hand, it may be based on the pulp properties of each stream, or the mass fraction of a certain size fraction of long fibre.

Predicting screen performance

For the case of pulp cleaning, screen performance equations have been proposed which relate the efficiency of capturing contaminants to the reject stream E_R to the mass rejects rate R_m and a performance number, either the screen quotient Q or the screening index α.

Nelson (10) proposed:

$$E_R = \frac{R_m}{1 - Q + Q\,R_m} \qquad (17)$$

Kubat and Steenberg (3) proposed:

$$E_R = R_m^{\ \alpha} \qquad (18)$$

For both equations, the screen efficiency E_R is defined as the mass of contaminants in the rejects stream as a percentage of shives in the feed flow. The mass rejects rate R_m is defined as the overall mass fraction of pulp and contaminants (dry solids) that pass through to the reject stream. Both equations are presented graphically in Figure 13, for a screen quotient of $Q = 0.9$ and screen index $\alpha = 0.1$.

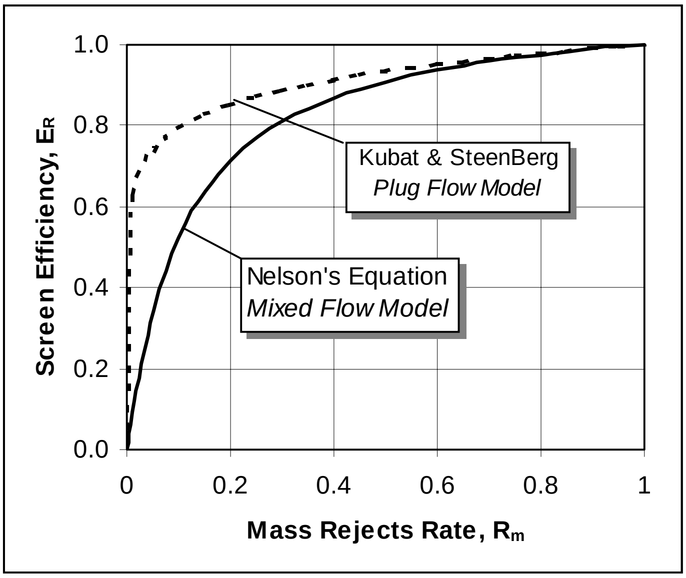


Figure 13. Comparison of screen efficiency equations at equivalent performance values of *Q=0.9, α = 0.1.*

Nelson's screen performance equation is not based on any mathematical proof but was derived empirically from extrapolating experimental data. The variable Q is the screen quotient, a characteristic of the screen hardware and operational variables such as rotor speed and average aperture passing velocity . It is approximately independent of rejects rate. A high Q denotes good screening performance and a low Q denotes poor screening. When $Q = 0$, the screen is acting like a flow splitter in a pipe and $E_r = R_m$.

Kubat and Steenberg's equation was not proposed in the form presented in Equation 18, but similar to it and was based on modelling a screen as a row of apertures acting in series. The screening index is similar in concept to the screen quotient. However, $\alpha = (1-Q)$ and decreasing values of α represent better screening.

The screen efficiency equations have also been derived by Gooding and Kerekes (G2) from the screen thickening equations they developed for predicting the thickening of fibre and contaminants across a screen. They showed that Equation 17 represented the case of perfect mixing adjacent to the screen plate *(mixed flow model)* and is equivalent to:

$$E_R = \frac{R_m}{R_m + (P_2/P_1) - (P_2/P_1)\,R_m} \qquad (19)$$

where P_1 and P_2 are the "passage ratios" of solid 1 (pulp) and solid 2 (contaminant) being separated and

$$Q = 1 - (P_2/P_1) \qquad (20)$$

They also showed that Kubat and Steenberg's Equation 18 could be derived from the thickening equations of the *plug flow model* and represents the case of perfect radial mixing with no axial mixing. The equation is :

$$E_R = R_m^{\,(P_2/P_1)} \qquad (21)$$

Equation 21 is similar to Equation 18 where :

$$\alpha = (P_2/P_1) \qquad (22)$$

Modelling screen performance

Equations 19 and 21 can be used to model screen performance. They represent the two types of mixing behaviour that arise within the screen. To use these equations however, the mass reject rate R_m , which is a dependent variable needs to be related to the volumetric rejects rate R_v which is an independent variable and can be manipulated in an operating situation.

By definition of mass rejects rate:

$$R_m = M_R / M_F = (F_R\ C_R) / (F_F\ C_F) = (F_R / F_F)\,(C_R / C_F)$$

Hence

$$R_m = R_v\, T \qquad (23)$$

Equation 23 can be combined with either Equation 19 or 21 to derive an overall screen performance equation. This equation is based on the passage ratios of the two solids being separated, the volumetric rejects rate, and the mass fraction of the two solids.

For the plug flow case:

$$E_R = T^{\,(P_2/P_1)}\, R_v^{\,(P_2/P_1)} \qquad (24)$$

where

$$T = \frac{T_1}{[1 + C_{F2}/C_{F1}]} + \frac{T_2}{[C_{F1}/C_{F2} + 1]} \qquad (25)$$

and

$$T_1 = C_{R1}/C_{F1} = Rv^{\,(P1-1)} \qquad (26)$$

$$T_2 = C_{R2}/C_{F2} = Rv^{\,(P2-1)} \qquad (27)$$

The notation used for the analysis also gives

$C_{F1} = M_{F1}/F_F$
$C_{F2} = M_{F2}/F_F$
$C_{R1} = M_{R1}/F_R$
$C_{R2} = M_{R2}/F_R$

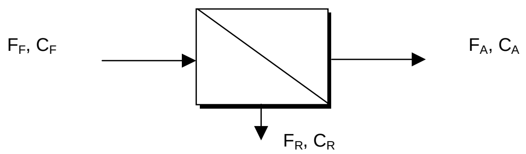
where M is the mass of solid in a stream and C is the solids concentration in a stream. For example, M_{F1} is the mass of solid component 1 in the feed stream and C_{F1} is the concentration of solid component 1 in the feed stream.

Table 1. Spreadsheet for modelling screen performance

Modelling Screen Performance - Plug Flow

Assume : P constant with fibre concentration

P_1 =	0.6	M_{F1} =	99
P_2 =	0.1	M_{F2} =	1
P_2/P_1 =	0.166667	C_{F1}/C_{F2} =	99
Q =	0.833333		



Rv	T1	T2	T	R=RvT	E_R
0				0.00	0.00
0.001	15.85	501.19	20.70	0.02	0.52
0.01	6.31	63.10	6.88	0.07	0.64
0.05	3.31	14.82	3.43	0.17	0.75
0.1	2.51	7.94	2.57	0.26	0.80
0.15	2.14	5.51	2.17	0.33	0.83
0.2	1.90	4.26	1.93	0.39	0.85

A spreadsheet has been developed using these equations to calculating the screen performance for specific sets of variables. Table 1 shows a typical view of the spreadsheet. An example performance curve is presented in Figure 14.

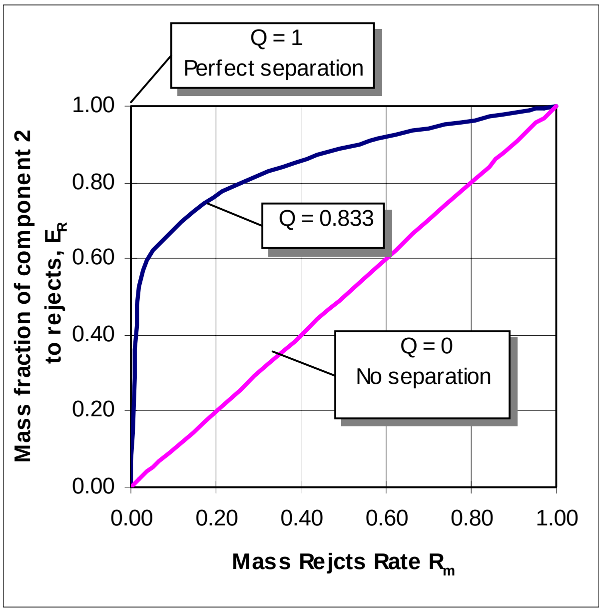


Figure 14. Screen performance curve for Table 1 conditions, $P_1 = 0.6$, $P_2 = 0.1$, $M_{F1}/M_{F2} = 99/1$.

CONCLUSIONS

The plug flow and the mixed flow methods of modelling a screen have been used to predict thickening across a screen, the concentration profile along the screen, and the screen performance in terms of efficiency of separation. Plug flow behaviour leads to greater thickening than mixed flow behaviour especially at low reject rates, and at the rejects end of the screen. Passage ratio decreases with increasing pulp concentration above a critical concentration, hence the assumption that passage ratio is independent of pulp concentration is not completely valid. Determining passage ratio values from short length screens using the mixed flow model is more accurate than from full-length screens using the plug flow model. Screen performance can potentially be predicted from passage ratio measurements from narrow length screen sections.

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