International Electronic Journal of
Mathematics Education
Volume 1, Number 1, October 2006 www.iejme.com

PERSONAL EXPERIENCES AND BELIEFS IN PROBABILISTIC REASONING: IMPLICATIONS FOR RESEARCH

Sashi Sharma

ABSTRACT. Concerns about students' difficulties in statistical thinking led to a study which explored form five (14 to 16 year olds) students' ideas in this area. The study focussed on probability, descriptive statistics and graphical representations. This paper presents and discusses the ways in which students made sense of probability concepts used in individual interviews. The findings revealed that many of the students used strategies based on beliefs, prior experiences (everyday and school) and intuitive strategies. From the analysis, I identified a four category rubric that could be considered for describing how students construct meanings for probability questions. While students showed competence with theoretical interpretation, they were less competent on tasks involving frequentist definition of probability. This could be due to instructional neglect of this viewpoint or linguistic problems. The paper concludes by suggesting some implications for further research.

KEYWORDS. Probabilistic Reasoning, High School Students, Interviews, Intuitive Strategies, Beliefs, Experiences, Culture, Contexts.

INTRODUCTION

Over the past years, there has been a movement in many countries to include statistics and probability at every level in the mathematics curricula. In western countries such as Australia (Australian Education Council, 1991), New Zealand (Ministry of Education, 1992) the United Kingdom (Holmes, 1994) and the United States (Shaughnessy and Zawojewski, 1999) these developments are reflected in official documents and in resources produced for teachers. In line with these moves, Fiji has also produced a new mathematics prescription at the primary level that places a stronger emphasis on statistics at this level (Fijian Ministry of Education, 1994).

Lajoie and Romberg (1998) note that in spite of its decade-long presence in curricular reform in elementary mathematics education, statistics is an area still in its infancy. Research shows that many students find probability difficult to learn and understand in both formal and everyday contexts and that we need to better understand how learning and understanding may be influenced by ideas and intuitions developed in early years (Amir and Williams. 1999; Barnes,

ISSN: 1306-3030

1998; Fischbein and Schnarch, 1997; Garfield, 1995;). Shaughnessy and Zawojewski (1999) report that even when secondary students were successful on such items as identifying the probability of a simple event on the National Assessment of Educational Progress (NAEP) study, they experienced difficulty applying this information in problem-solving situations.

Recently there has been evidence pointing to an influence of social settings and culture on mathematics thinking in general. Literature on ethno-mathematics (Barton, 1996) show strong influences of culture on mathematical thinking of adults. Carraher and Schliemann (2002) were intrigued by the ways Brazilian street vendors who seemed to be unskilled in mathematics in school settings could solve problems with no difficulty in non-school settings. The vendors often did not see school mathematics as connected to the real world, and as a result they often did not employ school taught procedures but used alternative flexible strategies to solve the problems. However, research illuminating the influence of culture on probabilistic thinking is sparse. Most of the research in probability has been done in just a very few western countries. It needs to be determined how culture influences conceptions of probability, whether biases and misconceptions are artifacts of western culture, or whether they vary across cultures. For instance Watson and Callingham (2003) argue that students in 'other cultural settings' may respond differently to their Australian counterparts, particularly to context-based items used in their studies.

Concerns about the importance of statistics in everyday life and in schools, a lack of research in this area and students' difficulties in statistical reasoning, determined the focus of my study. Overall, the study was designed to investigate the ideas form five students have about statistics (probability, descriptive statistics, graphical representations), and how they construct them. This paper presents and discusses data obtained from the probability tasks. Prior to discussing the details of my own research, I will briefly mention the theoretical framework and some examples of work on probability by students.

THEORETICAL FRAMEWORK

Much recent research suggests that constructivist models of knowledge provide a useful model of how students learn mathematics. Constructivism in its various forms, is based on the principle that learners actively construct ways of knowing as they strive to reconcile present experiences with already existing knowledge (von Glasersfeld, 1993). Students are no longer viewed as passive absorbers of mathematical knowledge conveyed by adults; rather they are considered to construct their own meanings internally by transforming, organising and reorganising previous knowledge (Cobb, 1994) as well as externally, through environmental and social factors that are influenced by culture, language and interactions with others (Mills, 2003). The constructions, based on beliefs and past experiences, appear to be the foundation for most learning. This active construction process results in misconceptions and alternative views as well as the student learning the concepts intended by the teacher. The research on student's misconceptions in probability (Amir and Williams, 1999, 1994; Fischbein and Schnarch, 1997; Nicolson, 2005; Shaughnessy,1997) provides evidence of the constructive process. Fischbein and Schnarch report that students use a variety of intuitive strategies to solve probabilistic problems. The researchers speculated that students are likely to have intuitions which are at variance with the commonly accepted reasoning.

Constructivism provides the theoretical basis for understanding how students learn in this study. This research was therefore designed to identify students' ideas, and to examine how they construct them. Although I conducted the study within a constructivist framework, in that the students learning was seen as building their own constructions, the teaching which occurred during the study was, not unexpectedly, based on a transmissive view of teaching. There was mostly teacher telling, followed by text-book based students' practice. Group work, problem solving and other desirable teaching/learning activities such as use of relevant contexts were not picked up by the teacher even though he appeared to be familiar with these ideas.

PREVIOUS RESEARCH

A number of research studies from different theoretical perspectives seem to show that students tend to have intuitions which impede their learning of probability concepts. Some prevalent ways of thinking which inhibit the learning of probability are:

- Representativeness: According to this strategy students make decisions about the likelihood of an event based upon how similar the event is to the population from which it is drawn or how similar the event is to the process by which the outcome is generated (Tversky and Kahneman, 1974). For instance, a long string of heads does not appear to be representative of the random process of flipping a coin, and so those who are employing representativeness would expect tails to be more likely on subsequent tosses until things evened out. Of course, the belief violates independence construct which is a fundamental property of true random sampling.
- Equiprobability bias: Students who use this bias tend to assume that random events are equiprobable by nature. Hence, the chances of getting different outcomes, for instance, three fives or one five on three rolls of a die are viewed as equally likely events (Lecoutre, 1992).
- Beliefs: Research shows that a number of children think that their results depend on a force, beyond their control, which determines the eventual outcome of an event. Sometimes this force is God or some other force such as wind, other times wishing or pleasing (Truran, 1994).

• Human Control: Research designed to explore children's ability to generalise the behaviour of random generators such as dice and spinners show that a number of children think that their results depend on how one throws or handles these different devices (Nicolson, 2005).

Representativeness; To illustrate the undue confidence that people put in the reliability of small samples, Tversky and Kahneman (1974) presented the following problem to tertiary students:

> Assume that the chance of having a boy or girl baby is the same. Over the course of a year, in which type of hospital would you expect there to be more days on which at least 60% of the babies born were boys?
>
> (a) In a large hospital
>
> (b) In a small hospital
>
> (c) It makes no difference

Most subjects in Tversky and Kahneman's study (1974) judged the probability of obtaining more than 60% boys to be the same in the small and in the large hospital. However, the sampling theory entails that the expected number of days on which more than 60% of the babies are boys is much more likely to occur in a small hospital because a large sample is less likely to stray from 50%. According to Tversky and Kahneman (1974) the representativeness heuristic underlies this misconception.

Shaughnessy (1997) provides evidence that students may actually superimpose a sampling setting on a question where none is there to begin with, in order to establish a centre from which to predict. For instance, consider the following task given to a sample of tertiary students at the beginning of a class in statistics:

> A fair coin is flipped 5 times in succession. Which do you feel is more likely to occur for the five flips?
>
> (a) HTTHT (b) HHHHH (c) they have the same chance of happening.

Below are some typical responses. The responses indicate a great variety of conceptions, and interpretations of the problem.

> I would go with (a) only because it more closely approximates the ratio 50-50, but in such a small sample anything is possible.
>
> I would say both are equally likely on any particular instance, although the long term results would gravitate to a result more like (a).
>
> More likely to have a series of two of the same, than to have five of the same.

The notion of a representative sample that is so helpful in the Tversky and Kahneman (1974) survey can cause problems when applied in the above context. There is no sample in the above question, there is just the sample space and yet some of the students appeared to superimpose a sampling context on the original question inorder to employ the representativeness strategy in their responses.

Equiprobability: Lecoutre (1992) used the following question in an experimental study of 1000 students with various backgrounds in probability:

> Two dice are simultaneously thrown, and the following two results are obtained:
>
> R1: 5 and 6 are obtained
>
> R2: a 6 is obtained twice
>
> Do you think the chance of obtaining each of these results is equal?
>
> Or is there more chance of obtaining one of them, and if so, which, R1 or R2?
>
> Or is it impossible for you to give an answer, and if so, why?

Lecoutre reports that most of the subjects answered incorrectly, that the two events have the same probability. From a systematic analysis of the justifications provided by students, it appeared that the most frequent cognitive model (65%) was based on the following type of argument: *The two results are equiprobable because it is a matter of chance.* Lecoutre also found that the equiprobability bias was highly resistant to change. For instance, it was found that increasing age, greater experience and experimental context (combinatorial information, frequency information) had little or no impact on this bias.

Beliefs: Amir and Williams (1994) propose that beliefs appear to be the elements of culture with the most influence on probabilistic thinking. They interviewed thirty-eight 11 to 12-year- old children about their concepts of chance and luck, their beliefs and attributions, their relevant experiences and their probabilistic thinking. Some pupils thought God controls everything that happens in the world while others thought God chooses to control, or does not control anything in the world. Several pupils believed in superstitions, such as walking under a ladder, breaking a mirror and lucky and unlucky numbers. There were also beliefs directly related to coins and dice; for instance, when throwing a coin tails is luckier. A majority of children in the Amir and William study concluded that it is harder to get a 6 than other numbers (17 out of 21 interviewees). The children remembered from their experience of beginning board games waiting a long time for a 6 on the die.

Human control: Fischbein and Schnarch (1997) asked 139 junior high school students (prior to instruction) to compare the probability of obtaining three fives by rolling one die three times versus rolling three dice simultaneously. Two main types of unequal probabilities were mentioned by about forty percent of the students. Of these students, about three-fifths considered that, by successively throwing the die, they had a higher chance of obtaining the expected result, and about two-fifths considered that by throwing three dice simultaneously they would have a higher chance of obtaining the expected results. Some of the justifications provided by the students for successive trials were:

> By rolling one die at a time, one may use the same type of rolling because the coins do not knock against each other and therefore do not follow diverse paths

The opposite solution, that is, there was a greater chance of getting three heads by tossing the coins simultaneously also had its proponents. Reasons given included:

Because the same force is imparted.

One can launch in the same way.

The explanations indicate a belief that the outcomes can be controlled by the individual.

In the Truran study (1994), many different methods of tossing coins and dice were described by children in order to get the result they wanted. For example, for tossing three dice together, some children thought that it is better to throw the dice one at a time because (when tossed together) dice can bump into each other and change the numbers which would otherwise have come up. Even if their carefully explained and demonstrated method did not work, children were still convinced that if they did everything right, it would work the next time.

Shaughnessy and Zawojewski (1999) documented similar statements among twelfth graders on probability tasks. Rather than using the sample space or multiplication principle, the students attributed to some sort of physical property of the spinners such as the rate at which the spinners were spun, the initial position of the arrows, or even some external influence, for instance, the wind.

Whether one explains the misconceptions in probabilistic thinking by using naive strategies such as representativeness and equiprobability or by deterministic belief systems such outcomes can be controlled, the fact remains that students seem very susceptible to using these types of judgements. In some sense, all of these general claims seem to be valid. That is, a student has knowledge about a variety of uncertain situations. Different problems address different pieces of this knowledge. Hence, in one problem, a student may reason according to the representativeness model, whereas in another, the same person may reason according to the control strategy.

The belief that outcomes can be controlled by the individual is not generally false. We can imagine throwing the die in a way that we can predict the outcome (pushing them smoothly from a height of 1 cm). It depends on the situation and the context. Even with a fair die, the side that it lands on is virtually completely determined by a number of factors such as which way up it started and the degree of spin. If we knew all this then with sufficient expertise in physics we could write down some equations which are thought to govern the motion of the die and use these to work out which way up the die should land.

Moreover, some of the misconceptions addressed in the research literature may actually be due to misinterpretation of the questions. Given the subtleties of interpretations that have been reported, it is unlikely that the research items used in the research described in this section would have discriminated finely enough. Some of these misconceptions may actually be due to misinterpretation of the questions. The study by Amir and Williams (1994) is one of the few that has used individual interviews to explore probabilistic reasoning in any depth. Interviews have provided the researchers with much detailed information as to how pupils' knowledge and misconceptions are constructed. Additionally, mathematics educators should realise that the students they teach are not *new slates* waiting to have the formal theories of probability written upon them. The students already have their own built-in heuristics, biases and beliefs about probability and these cannot, as it were, be simply wiped away. If student conceptions are to be addressed in the process of instruction, then it is important for teachers of probability to become familiar with the alternative conceptions that students bring to classes. Moreover, the research discussed in this section has been done in a very few western countries, it is important to investigate how culture influences conceptions of probability, whether intuitive strategies and preconceptions such as representativeness, equiprobability are artifacts of western culture or whether they vary across cultures. Fischbein and Schnarch (1997) and Metz (1997) argue that to adequately understand students' cognitive constructions and beliefs, we need to consider the culture in which students participate. In order to help inform teachers and curriculum designers, it appears to be crucial to carry out investigations at the secondary level. Such information may help teachers plan learning activities and students overcome their difficulties. In the current interview-based study, open-ended tasks were used to determine specific student conceptions and the factors that contribute to these constructs. An overview of the research design follows, after which I will discuss the results of my study.

OVERVIEW OF THE STUDY

In Fiji, selected topics in statistics are taught in schools from class 8 to form 7 (grades 8 to 13) as part of the mathematics curriculum (see Appendix 1). Primary schools in Fiji are classes 1 to 8 (in some cases 6) and cater for children aged 6 to 13 (or 11). Secondary schools start at Form 1 (class 7) or Form 3 and go to Form 4 or Form 7. The new mathematics prescription for classes 1 to 6 (Fijian Ministry of Education, Women, Culture, Science and Technology, 1994) gives greater emphasis to statistics at these age levels (6 to 11-year-olds). Statistics has been added into mathematics at these lower levels probably because it appears in the New Zealand Mathematics Curriculum document and Fiji had followed the New Zealand curriculum up to 1988. However, probability is not taught until the senior years of high school.

Sample

The study took place in a co-educational private secondary school in Fiji. The school roll was about 400, the majority being Indians. The school draws students mainly from rural contributing schools; the students' families came from a farming community. It was an average high school, and the staff were interested in being involved in the research project. The study focused on form 5 students, for two main reasons. Firstly, this was the age group that I had taught frequently during my teaching career and therefore felt very comfortable with. Secondly, since form five students in Fiji do not sit any external examinations, it was felt that the class teacher would be free of external constraints and more willing to be involved in the project.

The class consisted of 29 students aged 14 to 16 years. Fourteen students were chosen from the class and this constituted the research sample. The students were from different religious backgrounds (Sanatani, Samaji, Muslims and Christians). According to the teacher, none of the students in the sample had previously received any in-depth instruction on statistics. The sample was selected by the researcher in consultation with the class teacher. The criteria for selection included gender and achievement.

Tasks

The investigation dealt with a number of probability concepts (equally likely, independence, compound events, proportional reasoning) in different contexts. The appropriateness of these interview tasks for Fijian students was established by checking them with the class teacher and the HOD mathematics at the school. In the present paper, findings relating to equally likely items are discussed.

The single die question (Item 1A) was used to elicit students' ideas about theoretical probabilities embedded in chance generating devices. In explaining their answers, students could consider the sample space or symmetrical properties of the die. The advertisement regarding the sex of a baby (Item 1B) and the Fiji Sixes item (Item 1C) in Figure 1 attempted to explore students' understanding of theoretical and experimental probabilities in social contexts. In contrast to Item 1A, these questions require engagement with probabilistic and contextual knowledge. Responses demanded both numerical and qualitative descriptions. In all these questions, the students had to consider equally likely assumption related to the events, hence this is the central notion to which I refer in these questions.

Item 1A: Single die problem

Manoj feels that a six is harder to throw than any other score on a die.

What do you think about this belief? Why do you think so?

Item 1B: Advertisement involving sex of a baby

Expecting a baby? Wondering whether to buy pink or blue?

I can GUARANTEE to predict the sex of your baby correctly.

Just send $20 and a sample of your recent handwriting.

Money-back guarantee if wrong!

Write to…..

What is your opinion about this advertisement?

Figure 1: Item 1C, Fiji Sixes task (Fiji Sixes is a lotto game. At a cost of 50 cents, a player can purchase a "board" from The Fiji Times which allows him or her to choose 6 different numbers between 1 and 45. Players then post their entries to the Fiji Sixes office. A sampling machine draws six balls at random without replacement. Prizes are awarded according to how many of the winning numbers the player has picked. Winners are announced in the Fiji Times.)

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45

Interviews

Each student was interviewed individually by myself in a room away from the rest of the class. The interviews were audio taped for analysis, and each interview lasted about 40 to 50 minutes. Paper, pencil and a calculator were provided for the student if he or she needed it. I believed that this qualitative approach would allow students to demonstrate statistical understanding and questioning which would not have been possible in a survey format.

ANALYSIS OF DATA

The data analysis was conducted using the transcripts which were read and re-read by myself and common themes identified. The data revealed that many of the students held beliefs and used strategies based on prior knowledge. I created a simple four category rubric that could be helpful for describing research results relating to students' statistical conceptions, planning instruction and dissemination of findings to mathematics educators. The four categories in the rubric are: non response, non-statistical, partial-statistical and statistical. These are described in Table I. The non-statistical responses were based on beliefs and experiences while the students using the partial-statistical responses applied rules and procedures inappropriately or refered to intuitive strategies. The term statistical is used in this paper for the appropriate responses. However, I am aware that such a term is not an absolute one. Students possess interpretations and representations which may be situation specific and hence these ideas have to be considered in their own right. This category has been used mainly to present and discuss the results. Statistical simply means what is usually accepted in standard mathematics text-books in Fiji.

Table 1. Characteristics of the Four Categories of Responses

Response Type	Descriptor	Item
Non-response	Complete silence, I don't know, I have forgotten the rule, I just guessed	Items 1B, 1C
Non-statistical responses	Refereed to everyday experiences	Item 1A, 1B, 1C
	Made inappropriate connections with learning in other areas	
	Language difficulties	Items 1B, 1C
	Refer to beliefs-luck, lucky numbers	Items1A, 1C
	Refer to religious beliefs	Item 1B
	Hold the pervasive belief that they can control outcomes of random generators	Items 1A
Partial-statistical responses	Adapted the rules or applied them inappropriately.	Items 1A
	Refer to representativeness,	Items 1C
	Refer to unpredictability bias,	Items 1A, 1B
	Could not explain reasoning	Item 1B
Statistical responses	Able to justify reasoning by using classical or frequentist interpretation	All items
	Refer to symmetries of dice and coins	Items 1A
	Used correct numerical probabilities to justify reasoning.	Item 1A, 1B
	Refereed to complete sample space	Item 1A

RESULTS

This section describes the patterns of thinking identified in response to the equal chance Item. The types of responses are summarised and the ways that the students have constructed these ideas is described. Extracts from typical individual interviews are used for illustrative purposes. Throughout the discussion, I is used for the interviewer and Sn for the nth student. The responses to the three questions (Items 1A, 1B and 1C) are summarised in Table II.

Table II. Response types Item 1 (n = 14)

Response type	Single die ask	Ad for baby task	Fiji Sixes task	[All tasks]
Non-response	–	2 (S6, S9)	1 (S9)	–
Non-statistical	6 (S3, S9, S17, S20, S21, S26)	7 (S2, S3, S5, S14, S17, S21, S25)	6 (S3, S14,S20, S25, S26)	[2]
Partial-statistical	2 (S6, S14)	5 (S12, S20, S22, S26, S29)	5 (S2, S5, S6, S17, S29)	–
Statistical	6 (S2, S5, S12, S22, S25, S29)	–	2 (S12, S22)	–

Statistical responses: Table II data reveal that six students showed some grasp of probability theory underlying equal chance on the single die task (Item 1A) and two did so on the Fiji Sixes card (Item 1C). The students not only reasoned that all the outcomes had the same theoretical probability but provided correct explanations. For example, student 2 said that she did not believe that a six is hardest to get *because there is only one face with six dots and there are six faces and so the probability will be only one upon six*. In addition, student 25 not only believed that the chance of getting a six on one roll of a die was one-sixth, but was also able to provide a reason why some people believe that a six is hardest to throw. He explained that it is often the side which one needs and so one's attention is focussed on the chance of a six coming up, rather than with seeing how often the six comes up compared with each of the other outcomes. Two students (S12, S22) listed the sample space and explained that all numbers had the same chance of occurring. For instance, student 12 said that she did not agree with Manoj because there are six outcomes and only one of them is a 6 and so the chance of getting a six is one-sixth. Two students (S5, S29) calculated the correct probability by using the n(E)/n(S) rule.

We can’t increase our chance of winning a prize in the Fiji Sixes card (Item 1C) because any set of six numbers is just as likely to be chosen as another, although we can increase the amount we will win if we win the jackpot by avoiding numbers that others are likely to choose. Two students (S12, S22) based their responses on the equally likely concept. For instance, student 12 said: *All combinations of numbers are equally likely and it doesn’t really matter which numbers she chooses*.

Anyone can expect to be right in half the number of cases just by guessing (Item 1B). Even if predictions are made incorrectly, some will not bother to complain anyway. Even if they did, a clear profit can be made on 50% of all the $20 payments sent in. None of the students were considered statistical on this item. One possible explanation for this could be a lack of emphasis in Fiji classrooms on experimental probability. Since the students lacked experiences in interpreting chance from this perspective, they were more likely to use the other categories. The other reason could be that students experienced difficulties in communicating their ideas or they did not interpret the question in the way intended by myself.

Non-statistical responses: The non-statistical category consisted of students' responses which related the data to their beliefs and everyday experiences. The results of this study provide information that beliefs about random generators sometimes cause students to see these common random devices in a non-statistical way. Two students whose responses were classified as non-statistical on the single die task (Item 1A), believed that outcomes can be controlled by individuals. This is revealed in the following interview:

S26: It is not harder. It depends on how you throw it.

I: How do you throw so that you get a six?

S26: If you put six down and then throw you can get a six.

I: Can you try that?

S26: [Throws and gets a five].

When asked if he still could get a six, the response was ‘Yes.’ Clearly Student 26’s responses show that he is used to finding rational explanations for random experiments. Hence, his thoughts about rolling dice are based on deterministic rather than probabilistic reasoning. Student 17 did not believe in Manoj's claims because the same force is applied when throwing a die.

Luck formed an important component of some of these students' explanations. Two of the pupils (S9, S20) responded on the basis of good luck on Item 1A. For example, student 20 responded:

S20: Eh ... because six is a number that starts a game eh, so if you put a six the game starts and this is luck eh. You would be lucky and you will be able to throw a six.

I: What do you mean by lucky?

S20: Like when you playing cards eh so if your luck is not there, you can lose. So if you get a six you are lucky.

Three of the pupils (S14, S20, S21) responded on the basis of lucky and unlucky numbers on the Fiji Sixes task (Item 1C). The students thought that a person can increase his/her chance of winning in the Fiji Sixes by selecting lucky or birthday numbers. Manifestations of the lucky number aspect are reflected in the following interview:

S14: She should have followed some other methods like I have followed, the members of the family or her lucky number.

I: What do you mean by lucky numbers?

S14: Like for me the lucky number is 18.

I: Why is 18 your lucky number?

S14: Because it is my birthday.

In addition to basing their thinking on beliefs such as luck and lucky numbers, three students based their reasoning on their religious beliefs and everyday experiences. Strong influences of religious beliefs were apparent when students were asked to comment on the advertisement regarding the sex of a baby (Item 1B). Even when challenged about how the people placing the advertisement could make money, the students could not see that roughly half the babies born would be girls and half would be boys. The powerful nature of their religious beliefs is reflected in the response of student 17:

> As I have told you before that God creates all human beings. He is the one who decides whether a boy is born or a girl is born. Unless and until like now they have made a machine if one is pregnant and they can go there and they tell you whether the baby is a girl or a boy. But they can't tell until the baby is 8 months old. So that it means that the God created like that before we can't tell that the baby is a male or a female.

Two students referred to previous experience on Item 1A; they tended to think that it is harder to throw a six with a single die than any other score. The explanations provided by student 3 seemed to indicate that she remembered from her experience with chance games waiting a long time for a 6 on the die, often needed to begin a game. Student 21 thought that since six is a bigger number, it is difficult to get and one would be lucky to get a six.

Three students referred to previous experience when commenting on the advertisement regarding the sex of the baby. Two said that the advertisement was placed just to earn money, while student 25 offered the following explanations:

> Sometimes ... my sister is a nurse, she tells this is the time only for boys to be born. She came at our place. She said that this time only boys are being born. My aunt, she was expecting a baby; she said it will be a boy because it is the season. My aunt had a boy.

Three students referred to previous experience when picking numbers from a Fiji Sixes card. For every Fiji Sixes game in the Fiji Times, there is always a sample which shows people how to play the game. In the sample, a number is crossed from each row. It seems that student 26 thought this is how numbers should be crossed, one number from each row, and experience seemed to confirm this.

> S26: You have to select one number from each row.
>
> I: Why do you say that?
>
> S26: Because it is written in the Fiji Times example; they cross the numbers from each row. I have also seen people who play Fiji Sixes; they put numbers from each line.

Student 25 had considerable faith in the numbers 3 and 9.

> I: Why is 3 your favourite number?
>
> S25: When I was in class three, there was a Women's Club eh. They raffle tickets. Every time I bought tickets 9, 3 and 13. Once I bought the tickets, I won both the prizes. So every time I buy tickets, I check those numbers.

Student 3 referred to previous experience of how numbers have appeared in the past draws. For Item IB, student 3 thought that this problem was really to do with a doctor charging a $20 consulting fee to inform the parents of the sex of their unborn baby. Even when challenged about how the people placing the advertisement could make money, the student could not see that roughly half the babies born would be girls and half would be boys. The powerful nature of their everyday reading strategies of skimming and using the context or knowledge of the world to support comprehension are reflected in the following interview:

S3: The doctor may be charging $20……No.

I: What do you mean by "No"?

S3: Because it's a bad thing. They change the sex.

I: They are not changing the sex, they are just predicting the sex of the baby by looking at the hand writing. So what's your opinion about it now?

S3: That when anything goes wrong then they must know the hand writing of the person.

Student 3 had an idea that the problem was to do with determining the sex of the baby but then seemed to associate it with notions of childbirth difficulties.

Partial-statistical responses: Students who displayed partially statistical responses on the three items showed some understanding of chance. The procedures and intuitive strategies (representativeness) they used do not appear to be really statistical because they work only in some settings. I believe that an unpredictability bias needs to be added to the list of intuitive strategies suggested in literature. Students who base their explanations on the unpredictability bias tend to assume random events to be unpredictable by nature and over-respond to it on various contexts. More details of this unpredictability bias is given in part 2.

Four students (S2, S5, S6, S17) who used the representativeness strategy for the Fiji Sixes item thought that the chance of getting six consecutive numbers out of 45 in a Fiji Sixes card was less than getting other numbers because it did not represent a random process of generating these numbers. The students thought that they should choose numbers that are distributed throughout the range of choices. They did not realise that any set of six numbers is just as likely to be chosen as another. The following justification is indicative of this argument:

> Yeah … Like she doesn't have much chance. It is really very unlikely to happen. More likely to be from all over the place. She should choose one from each line, one from 1-10.

Two students displayed the unpredictability bias with the single die task, three with the prediction task and one with the Fiji Sixes problem. For example, student 29 explained that he did not believe in the advertisement because one can not predict the sex of a baby. For the Fiji Sixes item, the student offered the following justification:

> We don't know what will happen in future. The numbers that people will pick, they can pick any number.

Two students said the two outcomes on Item 1B were equiprobable. However, they did not realise that probability could be constructed in different ways. For example, student 12 said that the two outcomes were equally likely but she could not explain how the advertisers could make money.

DISCUSSION

This section first discusses the results in a broader context. Then limitations of the study are discussed and suggestions made for directions for further research.

Probability: A broader Context

Human Control: The results show that quite a number of students think that outcomes on random generators such as dice (Item 1A) can be controlled by individuals. The general belief is that results depend on how one throws or handles these different devices. Even if their carefully explained and demonstrated method did not work, students were still convinced that if they did everything right, or if luck was with them, it would work the next time. The finding concurs with the results of studies by Amir and Williams (1994), Fischbein et al. (1991), Shaughnessy and Zawojewski (1999) and Truran (1994). For instance, Shaughnessy and Zawojewski (1999) reported that rather than using the sample space or multiplication principle, the students in the NAEP attributed to some sort of physical property of the spinners such as the rate at which the spinners were spun. It must be noted that the students using the control strategy in my study were boys. One explanation for this could be that boys are more likely to play sports and chance games that involve flipping coins and rolling dice to start these games.

Although this study provides evidence that reliance upon control assumption can result in biased, non-statistical responses, in some cases this strategy may provide useful information for other purposes. For example, student 20's knowledge of physics may have been reasonable. The students using this approach have drawn on relevant common sense information. The responses raise further questions. Is there a weakness in the wording of this task in that it is completely open-ended and does not focus the students to draw on other relevant knowledge? Perhaps, including cues such as "fair" in the item would have aided in the interpretation of this question. Are the students aware of the differences in probabilistic reasoning compared with reasoning in other curriculum areas? Although we consider the flip of a coin and the throw of a die as random, deterministic physical laws govern what happens during these trials. It does not make sense to say that the die has a probability of one-sixth to be sixes because the outcome can be completely determined by the manner in which it is thrown. Additionally, a good Bayesian statistician might not give 50/50 heads/tails as the likely outcome after a run of heads with a particular coin. Such a person might start looking at prior experience to inform a particular situation.

Beliefs and Experiences: With respect to students' beliefs, experiences and learning, it is evident that other researchers have encountered similar factors. Amir and Williams (1994) note that children's reasoning appeared to be related to their religious, superstitious and causal beliefs. In some respects, the findings of the present investigation go beyond those discussed above. The findings demonstrate how students' other school experiences also influence their construction of statistical ideas. At times the in-school experiences appear to have had a negative effect on the students. An example of negative effect that arose from other school experiences was the student who was deeply convinced that he could control the outcomes on a die.

Intuitive Strategies: In the present study, the use of representativeness and equiprobability bias was not as common as that discussed in the literature. One reason for this could be that Shaughnessy (1997) and Tversky and Kahneman's (1974) items are significantly different and students may have responded differently to these situations. The research studies discussed in the literature review have mostly been carried out with tertiary students who would have had more cognitive experiences than the students in my study. Consequently, they would be more likely to use intuitive strategies such as representativeness and equiprobability bias. The students in my study had some rules-based teaching in probability. Their ideas of probability seemed to have been distorted and influenced by rote learning, so they tended to resort to incorrect rules and previous experiences. Another explanation for this discrepancy could be that these researchers only surveyed students' understanding in a few contexts using multiple-choice format which are too narrow to provide adequate information about student thinking. In the present study, students' understanding in both school and out-of-school contexts were explored. Hence students were more likely to use approaches other than naive strategies.

There are two major interpretations of probabilities, theoretical assumption and empirical data. One of the keys to understanding a probability model is balancing the ideas of theoretical and empirical probability. It is true that individual events are unpredictable, however long run frequencies are predictable. However, because they are contradictory when considered separately, students appear to over-respond to one or the other on various contexts. Over-reliance on theoretical probability is likely to lead to the belief that all outcomes are unpredictable. Students who base their explanations on the unpredictability bias tend to assume random events to be unpredictable by nature. The results concur with the findings of Nicolson (2005) who claims that predicting the unpredictable is a major challenge for anyone studying probability.

Relevant Contexts: It appears that background knowledge can be used helpfully and unhelpfully in tackling a problem in probability. The background knowledge as it were obscured the mathematical core of the problem. In the study described here, background knowledge, that is often invoked to support a student's mathematical understanding, is getting in the way of efficient problem solving. Given how statistics is often taught through examples drawn from "real life" teachers need to exercise care in ensuring that this intended support apparatus is not counterproductive. This is particularly important in light of current curricula calls for pervasive use of contexts (Meyer, Dekker, and Querelle, 2001; Ministry of Education, 1992) and research showing the effects of contexts on student' ability to solve open ended tasks (Cooper and Dunne, 1997; Sullivan, Zevenbergen, and Mousley, 2002).

Cultural Context: The students' beliefs and experiences can be understood in the context of the Indian culture. Some of these are discussed in this section and are based mainly on my own knowledge, as a Fiji Indian, of schooling and the way of life in Fiji. In many respects, students' attitudes and beliefs about luck and lucky numbers are consistent with the way Fijian Indian people view cultural conviction, such as one's fate being pre-determined and there always being a guiding force influencing the outcome of events. People often use phrases such as takdeer ki baat hai [it depends on fate] suggesting they feel these people are particularly helpless, and the events outside their control. Indeed, almost all events in real life can be explained in terms of the causality perspective. Perhaps believing that some outside factor influences one's behaviour means one does not feel so responsible when things go wrong.

The prominence of the view that God decides the sex of the baby could be attributed to certain aspects of the Indian culture. For instance, the sex of a baby is considered to be determined by God. Thus, in everyday life, children hear phrases like Bhagwan ki upaar hai, chai ladki dei yeh ladka dei, which literally means God decides the sex of the baby. Thus in everyday life, parents might pray at home or even go to a mandir [temple] in order to have a boy or a girl. It appears that there are certain aspects of the culture that are at odds with the probabilistic thinking that is aimed at in the statistics curriculum.

Limitations

The findings reveal that many of the students used beliefs and personal and social experiences to explain their thinking. It must be acknowledged that the open-ended nature of the tasks and the lack of guidance given to students regarding what was required of them certainly influenced how students explained their understanding. The students may not have been particularly interested in these types of questions as they are not used to having to describe their reasoning in the classroom. Some students in this sample clearly had difficulty explaining explicitly about their thinking. Students who realised that it costs money to put an advertisement in the newspaper (Item 1B) had a difficult time articulating exactly how people would make money in such situations. Although the study provides some valuable insights into the kind of thinking that high school students use, the conclusions cannot claim generality because of a small sample. Additionally, the study was qualitative in emphasis and the results rely heavily on my skills to collect information from students. Some directions for future research are implied by the limitations of this study.

Implications for Further Research

One direction for further research could be to replicate the present study and include a larger sample of students from different backgrounds (Fijian Indian and Indigenous Fijian).

Secondly, this small scale investigation into identifying and describing students' reasoning from constructivism has opened up possibilities to do further research at a macro-level on students' thinking and to develop more explicit categories for each level of the framework. Such research would validate the framework of response levels described in the current study and raise more awareness of the levels of thinking that need to be considered when planning instruction and developing students' statistical thinking. The place of statistics has changed in the revised mathematics prescription. Statistics appears for the first time at all grade levels (Fijian Ministry of Education, Women, Culture, Science and Technology, 1994). Like the secondary school students, primary school students are likely to resort to non-statistical or deterministic explanations. Research efforts at this level are crucial in order to inform teachers, teacher educators and curriculum writers.

CONCLUDING THOUGHTS

Although subjective types of thinking permeate our students' lives, statistical curricula in Fiji presumes the "correctness" of the theoretical model and ignores beliefs and experiences that do exist in real life situations. This presents a real dilemma which needs to be resolved if statistics education is to flourish in Fiji. For example, if students come to the class with the religious view that God decides the sex of a baby and the teacher is trying to teach the mathematical view that chance is blind and not controlled by prior knowledge, then how this can be done in a way that does not denigrate the first two views needs to be investigated. It is not adequate to consider this just as a "misconception", the students, after all, require the mathematical view to pass examinations. We as teachers can convince them by data and by theoretical argument that the birth can be modelled by a random experiment. Perhaps, it is also important to point out to students that there are alternative points of view. It is hoped that the findings of this study will generate more interest in research with respect to subjective ideas that students possess and teaching approaches. Teachers, curriculum developers and researchers need to work together to find better ways to help students learn this topic.

REFERENCES

Amir, G. & Williams, J. (1999). Cultural Influences on Children's Probabilistic Thinking. *Journal of Mathematical Behavior*, 18(10), 85-107.

Amir, G. & Williams, J. (1994). The influence of children's culture on their probabilistic Thinking. In J. da Ponte & J. Matos (eds.), *Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education*, 29 July-3 August, 1994, Vol. II (pp. 24-31). Lisbon, Portugal, The Program Committee of the 18th PME Conference.

Australian Education Council. (1991). *A National Statement on Mathematics for Australian Schools*, Canberra, Curriculum Corporation.

Barnes, M. (1998). Dealing with misconceptions about probability. *Australian Mathematics Teacher*, 54(10), 17-20.

Barton, B. (1996). Making sense of ethno-mathematics: ethno-mathematics is making sense. *Educational Studies in Mathematics*, 31, 201-133.

Carraher, D. & Schliemann A. (2002). Modeling Reasoning. In K. Gravemeijer, R. Lehrer, B. V. Oers & L. Verschaffel (eds.), *Symbolizing, Modeling and Tool Use in Mathematics Education* (pp. 295-304). Dordrecht: Kluwer Academic Publishers.

Cobb, P. (1994).'Where is the mind? Constructivist and sociocultural perspectives on mathematical development' *Educational Research*, 23(7), 13-20.

Cooper, B. and Dunne, M. (1999). *Assessing Children's Mathematical Ability. Buckingham*, Open University Press.

Fijian Ministry of Education, Women, Culture, Science and Technology (1994). *Primary Mathematics Prescription*s. Suva: Curriculum Development Unit.

Fischbein, E., Nello, M. S. & Marino, M. S. (1991). Factors affecting probabilistic judgements in children and adolescents'. *Educational Studies in Mathematics*, 22, 523-549.

Fischbein, E. & Schnarch, D. (1997). The evolution with age of probabilistic, intuitively based misconceptions. *Journal for Research in Mathematics Education*, 28, 96-105.

Garfield, J. B. (1995). How students learn statistics, International Statistical Review, 63(1), 25–34.

Holmes, P. (1994). Teaching statistics at school level in some European countries, in L. Brunelli and G. Cicchutelli (eds.), *Proceedings of the First Scientific Meeting* (pp. 3-11), University of Perugia,

Lajioe, S. P. and Romberg, T. A.: 1998: ' Identifying an agenda for statistics instruction and assessment in K-12'. In S.P. Lajoie (ed.), *Reflections on Statistics: Learning, Teaching, and Assessment in Grades K-12.*.Mahwah NJ, Lawrence Erlbaum Associates.

Lave, J. (1991). *Cognition in practice; Mind, mathematics and culture in everyday life*. New York: Cambridge University Press.

Lecoutre, M. (1992). Cognitive models and problem spaces in purely random situations. *Educational Studies in Mathematics*, 23, 557-568.

Metz, K. E. (1997). Dimensions in the assessment of students' understanding and application of Chance. In I. Gal and J. B. Garfield (eds.), *The Assessment Challenge in Statistics Education* (pp. 223-238). Amsterdam: The Netherlands, IOS Press,

Meyer, M. R, Dekker, T. & Querelle, N. (2001). Context in mathematics curricula. *Mathematics Teaching in the Middle School*, 6(9), 522-527.

Mills, J. D. (2003). A theoretical framework for teaching statistics. *Teaching Statistics,* 25(2), 56-58.

Ministry of Education. (1992). *Mathematics in the New Zealand Curriculum*. Wellington: Ministry of Education.

Nicolson, C. P. (2005). Is chance fair? One student's thoughts on probability. *Teaching Children Mathematics*. 12(2), 83-89.

Shaughnessy, J. M. (1997). Missed opportunities in research on the teaching and learning of data and chance. In F. Biddulph & K. Carr (eds.), People in mathematics education. Mathematics Education Beyond 2000 (pp. 6-22). *Proceedings of the 20d Annual Conference of the Mathematics Education Research Group of Australasia*. Hamilton, MERGA INC,

Shaughnessy, J.M. & Zawojewski, J. S. (1999). Secondary students' performance on data and chance in the 1996 NAEP. *The Mathematics Teacher*, 92(8), 713-718.

Sullivan, P., Zevenbergen, R., & Mousley, J. (2002). Contexts in Mathematics teaching: snakes or ladders? In B. Barton, K. C. Irwin, M. Pfannncuch and M. Thomas (eds.), Mathematics Education in the South Pacific (pp 649-656). *Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia*. Auckland.

Truran, K. M. (1994). Children's understandings of random generators. In C. Beesey &D. Rasmussen (eds.), *Mathematics Without Limits, Proceedings of the 31st Annual Conference of the Mathematical Association of Victoria* (pp. 356-362). Melbourne: Mathematics Association of Victoria.

Tversky, A. & Kahneman, D. (1974). Judgement under uncertainty: Heuristics and biases. *Science*, 185, 1124-1131.

Von Glasersfeld, E. (1993). Questions and answers about radical constructivism. In K. Tobin (ed.), *The practice of constructivism in science education* (pp. 24-38). Washington: American Association for the Advancement of Science.

Watson, J. M. & Callingham R. (2003). Statistical literacy: A complex hierarchical construct. *Statistics Education Research Journal*, 2(2), 3-46.

APPENDIX 1

MATHEMATICS EDUCATION IN FIJI SCHOOLS

BACKGROUND

Primary schools teach classes 1 to 8 (in some cases classes 1 to 6) whereas secondary schools teach Forms III to VII, and Junior secondary schools teach Forms I to IV. The following external examinations involving maths are taken by pupils while at school:

Class 6	Fiji Intermediate Examination in which mathematics is compulsory.
Form II	Fiji Secondary Schools' Entrance Examination in which mathematics is compulsory
Form IV	Fiji Junior Certificate Examination in which mathematics is compulsory
Form VI	Fiji School Leaving Certificate Examination in which mathematics is not compulsory
Form VII	Fiji Seventh Form Certificate Examination in which mathematics is not compulsory

STATISTICS

Statistics is taught as one of the topics in the mathematics syllabus and is first introduced in Form 11 (Class 8) where the average age of a pupil is 13 years. Statistics is then taught in bits and pieces right up to Form VII. A brief summary of statistics taught at different levels is given below:

Form II (Class 8)
Collecting information, representing information, interpreting representations, average (mean), pictograms, frequency tables, bar graphs, pie charts.

Form III
Representing statistical data in graphical or chart form, computing sample mean, mode, median and range, organising data from a sample into a frequency distribution and representing this by a frequency line graph or a histogram.

Form IV
Computing statistics: mean, median, upper and lower quartiles, interquartile range. Representing data: frequency polygon, cumulative frequency table, cumulative frequency graph. Elementary ideas in probability.

Form V
Classification of data, statistical graphs, including frequency and cumulative frequency curves, median and mean as measures of central tendency, ideas of spread, including standard deviation, simple probability and relative frequency.

Form VI
Probability: sample space, mutually exclusive events, independent events. Populations and samples: mean, median, standard deviation and range as examples of population parameters, samples, random samples, frequency distributions. sample statistics. Distributions: the binomial; distribution taken as an example of a discrete distribution, mean of binomial distribution. The normal distribution as an example of a continuous distribution, z score.

Form VII
Probability, Statistics and Computing
Choose only one of the options
Option A: Probability and Statistics
Option B: Probability and Computing
Option C: Statistics and Computing

<u>Sashi Sharma</u>

Department of Mathematics, Science and Technology Education

The University of Waikato

Hamilton

New Zealand

Phone: +61 7 838 4500 ext 6298

Email: sashi@waikato.ac.nz