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SHEAR WAVE DISSIPATION IN PLANAR MAGNETIC X-POINTS

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ABSTRACT

The resistive dissipation of nonlinear shear wave disturbances is discussed. We consider an incompressible, "open" X-point geometry, in which mass and energy are free to enter and leave the reconnection region. An analytic treatment is possible which unifies many of the dynamic and steady-state X-point solutions obtained previously. We show that while field disturbances in the plane of the X-point have the potential for rapid energy release when suitably driven, perpendicular shear disturbances dissipate slowly, at a rate $\sim\eta^{1/2}$, where η is the plasma resistivity. This behavior can be understood in terms of the absence of flux pileup in nonplanar shear wave disturbances. We conclude that only planar shear waves have the potential for fast magnetic energy release.

Subject headings: MHD — plasmas

1. INTRODUCTION

In a highly conducting plasma, changes in magnetic field topology can occur only by magnetic reconnection, a resistive process involving the cutting and rejoining of field lines at null points in the field. Historically, the simplest models of reconnection involve the advection of magnetic fluid across two-dimensional X-point separatrices, the field lines being reconnected at the neutral point (Forbes & Priest 1987). Analytic models are provided by the linear theory of "closed," arbitrarily compressible X-points (Craig & McClymont 1991, 1993; Hassam 1992). These models are "fast," the reconnection rate being effectively independent of the resistivity, but stalling can occur when the idealization of zero gas pressure is relaxed (see McClymont & Craig 1996a).

Of course, to describe the explosive collapse of large-scale magnetic fields requires a complete nonlinear theory, which probably involves the breakdown of the MHD approximation. Aside from heuristic, semiquantitative approaches, nonlinear work generally requires extensive computer simulation at physically unrealizable levels of plasma resistivity (Biskamp 1994). This requirement holds even for simplified planar MHD systems. Accordingly, most numerical studies are limited either to phenomenology or to assessing the speed of the magnetic merging on the basis of empirical scalings with resistivity.

Quite recently, it has been shown that nonlinear two-dimensional and three-dimensional reconnection solutions can be constructed using incompressible theory in "open" geometries (Craig & Henton 1995; Craig & Fabling 1996). These studies confirm that fast reconnection can occur by the flux pileup of planar field disturbances at the onset of the reconnection region. However, like the compressible linear theory, the solutions imply that the reconnection rate stalls if the hydromagnetic pressure outside the reconnection region is significantly less than the plasma pressure at the neutral point.

One possibility of eluding the pressure problem is to explore shear wave disturbances of magnetic X-points (Bulanov, Shasharina, & Pegararo 1990; Hassam & Lambert 1996). Since shear waves carry magnetic energy towards the neutral point without compressing the plasma, they offer an alternative path towards fast reconnection (as discussed by McClymont & Craig 1996b). This provides the motivation for the present work: can shear waves in "open" planar X-point geometries lead to fast magnetic energy dissipation?

The problem is formulated in § 2. We present a unified treatment in which previous planar incompressible solutions emerge as limiting cases (Clarke 1964; Sonnerup & Priest 1975; Bulanov et al. 1990; Craig & Henton 1995). Section 3 provides a detailed analysis of disturbances with shear perpendicular to the plane of the X-point. An exact eigenfunction analysis is given for the dissipation of "plane wave" solutions; these results are then reinforced by considering the general wave solution for traveling wave packets. Section 4 discusses the propagation of shear disturbances in the plane of the X-point in terms of the Klein-Gordon equation valid in the absence of resistivity. Our conclusions are presented in § 5.

2. PLANAR X-POINT EQUATIONS

2.1. *Introduction*

We assume that the plasma is governed by the incompressible resistive MHD equations. We assume an open geometry, which allows the flow of mass and energy through the bounding surface. Adopting dimensionless variables, in which fluid velocities are expressed in units of the Alfvén speed at the boundary, the momentum and induction equations may be written in the form

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = \boldsymbol{J}\times\boldsymbol{B} - \nabla P\,, \tag{2.1}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) - \eta\nabla\times\boldsymbol{J}\,, \tag{2.2}$$

where $\boldsymbol{J} = \nabla\times\boldsymbol{B}$ and η is the dimensionless plasma resistivity, assumed constant and uniform. We neglect viscous effects for reasons discussed in the Appendix.

The system is conservative apart from the resistive energy losses of the plasma. The small value of the classical resistivity ($\eta \sim 10^{-12}$) means, however, that the ohmic dissipation rate,

$$W_\eta = \int J^2 \, dV \equiv \eta \langle J^2 \rangle , \tag{2.3}$$

is quite negligible unless the plasma contains strong localized currents. The central problem of magnetic reconnection theory is to demonstrate that large current densities can develop over small length scales to enhance the weak coronal energy-loss rate. The aim is to develop two-dimensional and three-dimensional models in which the dissipation rate is "fast," for then W_η scales independently of η and there is a possibility of explaining the explosive energy release of the solar flare.

2.2. *Planar Evolution Equations*

In what follows we assume a planar geometry in which z is the ignorable coordinate. We choose units for which the volume of interest is bounded by the (Gaussian) surfaces $|x| = |y| = 1$. We regard $|x| = 1$ as defining inflow surfaces on which we are free to specify boundary conditions; the form of the solution then determines the outflow conditions on $|y| = 1$.

Taking the flux and stream function representations

$$\boldsymbol{B}(x, y, t) = \nabla\psi \times \hat{z} + Z\hat{z} , \quad \boldsymbol{v}(x, y, t) = \nabla\phi \times \hat{z} + W\hat{z} \tag{2.4}$$

guarantees the conservation equations $\nabla \cdot \boldsymbol{B} = \nabla \cdot \boldsymbol{v} = 0$. In component form, we have $\boldsymbol{B} = (\psi_y, -\psi_x, Z)$ and $\boldsymbol{v} = (\phi_y, -\phi_x, W)$. We refer to the z-components as the perpendicular or normal components of the field.

The planar field components are affected only by planar shear disturbances, which can be isolated by taking the curl of the momentum equation

$$\frac{\partial}{\partial t} (\nabla^2 \phi) + [\nabla^2 \phi, \phi] = [\nabla^2 \psi, \psi] , \tag{2.5}$$

$$\frac{\partial \psi}{\partial t} + [\psi, \phi] = \eta \nabla^2 \psi ; \tag{2.6}$$

the normal components are given by

$$\frac{\partial W}{\partial t} + [W, \phi] = [Z, \psi] \tag{2.7}$$

and

$$\frac{\partial Z}{\partial t} + [Z, \phi] = [W, \psi] + \eta \nabla^2 Z , \tag{2.8}$$

where $[\psi, \phi]$ denotes the Poisson bracket

$$[\psi, \phi] = \psi_x \phi_y - \psi_y \phi_x .$$

As Craig & Henton (1995) have emphasized, it is important to develop solutions in which the Poisson brackets are nonvanishing. Consider, for example, the bracket $[\psi, \phi]$ that describes the advection of the field by the flow. If this bracket vanishes, flux transfer across the separatrices—that is, topological change—can be accomplished only by global resistive diffusion, which is usually very slow. The basic idea of reconnection is to speed up the rate of flux transfer by advecting material across the separatrices; diffusion then occurs only in localized high current regions where the flow vanishes. Thus a necessary condition for reconnection is that $[\psi, \phi] \neq 0$ along the magnetic separatrices.

2.3. *Nonlinear X-Point Disturbances*

It is well known that potential field models offer the simplest equilibrium solutions to the planar reconnection equations. In developing steady-state reconnection solutions, Craig & Henton (1995) consider the superposition $\phi = \alpha p + f(x)$, $\psi = \beta p + g(x)$, where $p(x, y)$ is an arbitrary potential function. In fact, $p = xy$ provides the only allowable construction!

Motivated by these considerations, we consider time-dependent solutions of the form

$$\phi = \alpha xy + f(x, t) , \quad \psi = \beta xy + g(x, t) . \tag{2.9}$$

The field and flow potentials define nonlinear disturbance fields in the plane of the X-point. When $f = g = 0$, we recover an equilibrium X-point in which all flows are constrained to the field lines. Setting $\alpha = 0$ turns off the stagnation point flow. Although this restriction is appropriate to closed magnetic X-points bounded by, say, a rigid superconductor we must allow for the possibility of sustained background flows in open X-points where mass and energy can flow through the boundary surfaces $|x|, |y| = 1$. In fact our solutions will show that, in contrast to the results for closed, *arbitrarily compressible* X-points (Craig & McClymont 1991, 1993; Craig & Watson 1992, Hassam 1992), fast incompressible merging is impossible unless the reconnection is driven by sufficiently strong advective flows.

We model perpendicular disturbances by taking

$$W = W(x, t) , \quad Z = Z(x, t) , \tag{2.10}$$

which represents a plane wave, oriented parallel to the separatrix $x = 0$, propagating disturbances normal to the plane of the X-point. This form describes only one class of shear wave solutions, but does allow a unified mathematical treatment of the dissipation problem. In special cases, however, a more general treatment is possible. We illustrate this in § 3.5, where we consider wave-packet propagation under the assumption $\phi = \eta = 0$.

Figure 1 shows typical magnetic field lines for the disturbed X-point. The planar field disturbance of Figure 1*a* distorts the $y = 0$ separatrix of the equilibrium field. By contrast, the normal field disturbance of Figure 1*b*, being nonreconnective, leaves the equilibrium separatrices intact.

Substituting equations (2.9) and (2.10) into the induction and momentum equations yields the system

$$\frac{df}{dt} = \beta x g_x + 2\alpha f - 2\beta g , \tag{2.11}$$

$$\frac{dg}{dt} = \beta x f_x + \eta g_{xx} , \tag{2.12}$$

$$\frac{dW}{dt} = \beta x Z_x , \tag{2.13}$$

$$\frac{dZ}{dt} = \beta x W_x + \eta Z_{xx} , \tag{2.14}$$

where the Lagrangian operator is given by

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \alpha x \frac{\partial}{\partial x} .$$

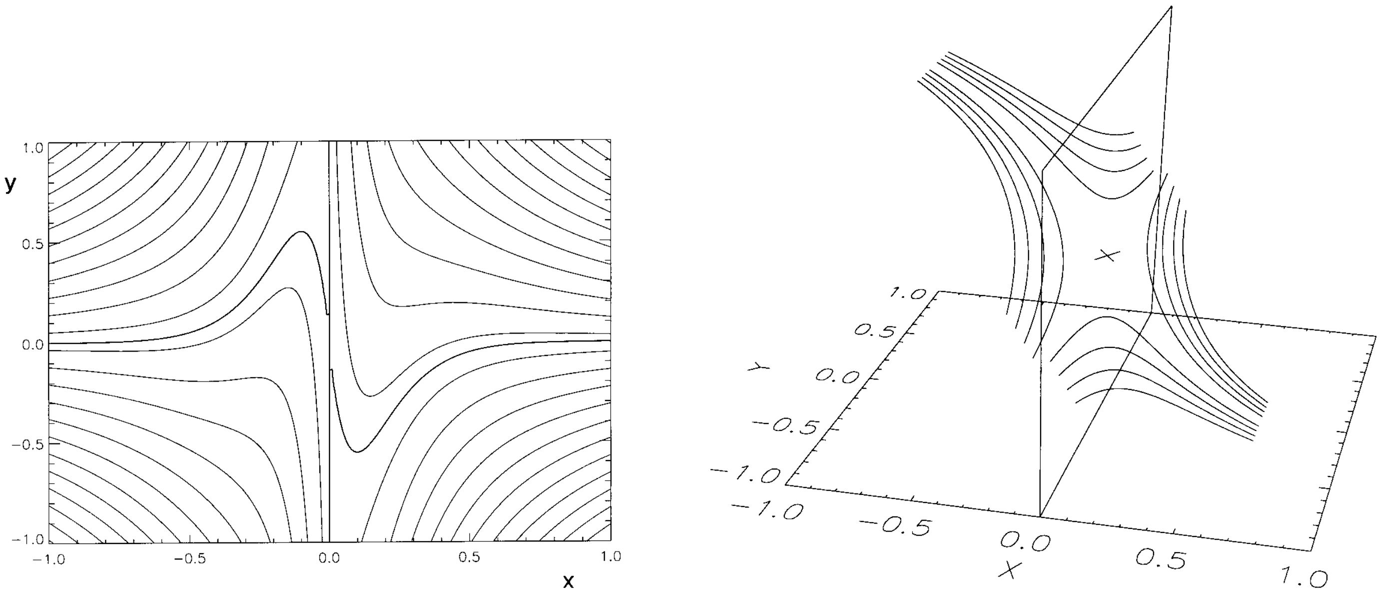

FIG. 1a FIG. 1b

FIG. 1.—(a) Field lines for a typical planar shear disturbance. *Bold lines*: Separatrices $\psi = 0$. (b) Field lines for a normal shear disturbance. This disturbance, in contrast to the planar shear disturbance of (a), does not drive reconnection, since the separatrix lines $x = 0$, $y = 0$ are unchanged by the addition of normal shear.

This system is very convenient to analyze, since the planar and normal disturbances of the field evolve independently. We shall take $\beta \geq 0$ for definiteness. Note that the choice $\alpha < 0$ means that material is being washed into the separatrix surface $x = 0$ by the stagnation point flow, consistent with regarding $x = -1$ and $x = 1$ as inflow surfaces for the fluid. In fact, we shall take these surfaces to be physically equivalent and discuss only the half space $x \geq 0$.

2.4. *Solutions for Special Cases*

Equations (2.11)–(2.14) already incorporates several magnetic merging models discussed in the literature. The steady-state reconnection solution of Craig & Henton (1995) is recovered by setting $g_t \equiv E$ and requiring that all other time derivatives vanish. This solution describes the reconnection of curved field lines washed into the current layer $x \sim \eta^{1/2}$ by the background flow ($\alpha < 0$, $W = Z = 0$). When $\beta = 0$, the annihilation model of Sonnerup & Priest (1975) is obtained: $f = \beta g/\alpha = 0$. These solutions are fast, but they require prohibitive external pressures to sustain the merging.

More specifically, from equation (2.1) we find that the pressure has the form

$$P(x, y, t) = P_0(t) - \tfrac{1}{2}[\alpha^2(x^2 + y^2) + Y^2] - \beta y Y \,, \quad (2.15)$$

where $Y(x, t) \equiv -g_x$ denotes the y-component of the disturbance field. Since the steady-state solution implies that Y scales as $\eta^{-1/2}$ (Craig et al. 1995, see also § 4.2), P_0 must increase as η^{-1} to avoid negative pressures. The implication is that fast merging in the limit of small η is unsustainable (see § 4.3).

The pressure problem is in fact symptomatic of flux pileup in the current layer. Craig et al. (1995) show that flux pileup never develops for steady-state merging of the normal field components ($W = \beta Z/\alpha$). Consequently, the ohmic diffusion rate is slow, $W_\eta \sim \eta^{1/2}$.

Turning now to time-dependent solutions, we note that the simplest model describes the dynamic annihilation of straight field lines ($\beta = 0$) advected by a pure stagnation point flow into the diffusion region. The solution for the planar field $Y(x, t)$ was first given by Clarke (1964). This solution is the dynamical equivalent of the Sonnerup & Priest annihilation model and retains the pressure-scaling problems outlined above.

Finally, we mention the time-dependent shear wave solution of Bulanov et al. (1990). All background flows are now absent and all nonplanar field components vanish ($\alpha = W = Z = 0$). The remarkable property of this solution is that, contrary to the models outlined above, the field decays rapidly without piling up flux at the onset of the sheet, yet the ohmic dissipation remains weak, $W_\eta \sim \eta^{1/2}$. This paradox is explained in § 3.3 below.

3. DISSIPATION OF PERPENDICULAR SHEAR WAVES

3.1. *Solutions for* $\alpha = 0$

We now consider the time-dependent behavior of the normal field components under the restriction of vanishing background flow ($\alpha = 0$). This problem has not been discussed in the literature, although Hassam & Lambert (1996) have provided an interesting analysis of perpendicular field perturbations for X-points in closed geometries. Their analysis indicates—but does not prove—that W_η is slow.

Using equations (2.13) and (2.14) with $\alpha = 0$ implies that

$$W_t = \beta x Z_x \quad (3.1)$$

and

$$Z_t = \beta x W_x + \eta Z_{xx} \,, \quad (3.2)$$

and so nonlinear field disturbances evolve according to

$$Z_{tt} = \beta^2 x (x Z_x)_x + \eta Z_{xxt} \,. \quad (3.3)$$

This expression describes the propagation and dissipation of perpendicular magnetic shear waves.

Consider first the wave solution in the absence of resistivity,

$$Z = W = G(\beta t + \ln x) + H(\beta t - \ln x) \,, \quad \eta = 0 \,, \quad (3.4)$$

where G and H define the initial wave envelopes. As the field G propagates inwards at constant amplitude, the field gradient builds up as x^{-1} and increasingly strong currents are driven close to the origin. The wave can be dissipated resistively only when the disturbance has localized sufficiently. If we compare $\eta\,|Z_{xx}|$ with $|Z_t|$ using equation (3.4), we identify

$$T \simeq \frac{1}{2\beta} \ln \frac{\beta}{\eta} \tag{3.5}$$

as the timescale required to achieve the localization $x \simeq (\eta/\beta)^{1/2}$, which suggests that the wave achieves the current density $J \sim \eta^{-1/2}$ before resistive dissipation sets in.

We now confirm this result using an exact eigenfunction analysis.

3.2. *Normal Mode Solutions*

The field equation (3.3) can be analyzed either as an initial value problem or as an eigenvalue problem. Since we are mainly concerned with the decay rate of the field under arbitrary initial conditions, it is convenient to adopt the eigenfunction approach.

We let $Z(x, t) \rightarrow e^{\lambda t} Z(x)$ and assume that all currents vanish on the boundary surface $x = 1$. The eigenproblem is then

$$\lambda^2 Z = \beta^2 x(xZ')' + \eta\lambda Z'' , \quad Z(0) = Z'(1) = 0 , \tag{3.6}$$

where $Z(x)$ is a complex eigenfunction associated with discrete values of the eigenvalue $\lambda = \nu + i\omega$. Note that we can rescale the equation to eliminate the explicit dependence on resistivity. Specifically, with $Z = Z(s)$, we have that

$$\lambda^2 Z = \beta^2 s(sZ')' + \beta^2 Z'' , \quad s = \beta x/\sqrt{\eta\lambda} , \tag{3.7}$$

which already shows that $x \simeq \eta^{1/2}$ can define the scale of the resistive layer only if λ is effectively independent of the resistivity. A further change of variable, namely

$$z = \sinh^{-1} s = \ln (s + \sqrt{1 + s^2}) , \tag{3.8}$$

yields $\lambda^2 Z = \beta^2 Z_{zz}$, and so

$$Z(s) = \sinh \left(\frac{\lambda}{\beta} z\right) \tag{3.9}$$

is the solution which satisfies the inner condition $Z(0) = 0$.

The dispersion relation is obtained from the condition that the field gradient vanishes on the boundary $x = 1$, that is, $z \approx |\ln [2\beta/(\eta\lambda)^{1/2}]|$. This condition gives

$$\frac{\lambda}{\beta} \ln \frac{2\beta}{\sqrt{\eta\lambda}} = \left(n + \frac{1}{2}\right) i\pi , \tag{3.10}$$

an equation that must generally be solved by iteration. However, for η small enough we anticipate an oscillatory eigenfunction with very little resistive decay. In this case we have $\arg \lambda \simeq \pi/2$ and so

$$\frac{\omega}{\beta} \approx \frac{(n + 1/2)\pi}{\ln (2\beta/\sqrt{\eta})} , \quad \nu\beta = -\frac{1}{2} \frac{\omega^2}{(2n + 1)} . \tag{3.11}$$

These rates are fast since they depend only logarithmically on the resistivity. In fact, they are asymptotically identical to the fast scalings obtained for closed compressible X-points (Craig & McClymont 1991, 1993; Hassam 1992). The key difference here is that the current amplitude is weak, $J \sim \eta^{-1/2}$, and so resistive dissipation cannot account for the rapid field decay.

3.3. *Global Energy Balance*

To understand how weak current structures can be associated with fast decay, it is instructive to consider the global energy of the fluid. We multiply equation (3.1) for W_t by W and integrate over (0, 1). By doing the same for Z_t, we deduce that

$$\frac{\partial}{\partial t} \frac{1}{2} \langle W^2 + Z^2 \rangle = \beta\langle WZ \rangle - \eta\langle J^2 \rangle \tag{3.12}$$

describes the global energy losses of the fluid. We see that the global losses are influenced by a term $\beta\langle WZ \rangle$ unrelated to the ohmic losses of the fluid. This term gives the energy carried out of the volume by shear waves.

To obtain a quantitative result we evaluate $\langle WZ \rangle$ using the test functions

$$Z = \cos (k \ln x) , \quad W = -k \sin (k \ln x) , \quad k \equiv \frac{\pi}{|\ln \eta|} , \tag{3.13}$$

which mimic the fundamental $n = 0$ eigenfunctions. A simple evaluation, assuming that the product WZ is small in the diffusion region $x < \eta^{1/2}$, shows that $\langle WZ \rangle \simeq -\beta^2 k^2$. Clearly, it is the $|\ln \eta|$ dependency of this term—rather than the $\eta^{1/2}$ scaling of W_η—that dominates the decay of the global energy.

The physical interpretation is as follows. Disturbances of the normal field propagate along field lines towards the current sheet $x = 0$. However, most field lines bypass the current layer: only disturbances associated with the field lines for which $\psi < \psi_0 = (\beta\eta)^{1/2}$ can be resistively attenuated. The rest escape via wave propagation out of the volume through the upper and lower surfaces $y = \pm 1$.

3.4. *The Field Decay for $\alpha \neq 0$*

We now investigate whether the resistive decay of the normal field can be enhanced by the inclusion of sustained stagnation point flows. Remembering that inflow corresponds to $\alpha < 0$, we rewrite equations (2.13) and (2.14) in terms of the comoving frame

$$\tau = t , \quad \xi = x \exp (-\alpha t) , \tag{3.14}$$

which gives

$$W_\tau = \beta\xi Z_\xi , \tag{3.15}$$

$$Z_\tau = \beta\xi W_\xi + \eta e^{-2\alpha\tau} Z_{\xi\xi} . \tag{3.16}$$

The general equation for the field now reduces to

$$Z_{\tau\tau} = \beta^2 \xi(\xi Z_\xi)_\xi + \eta(e^{-2\alpha\tau} Z_{\xi\xi})_\tau , \tag{3.17}$$

which is more complicated than equation (3.3). However, in the absence of resistivity, $Z(\xi, \tau)$ is governed by a cylindrical wave equation with wave speed $\beta\xi$. Accordingly, in place of equation (3.4), we have

$$Z = W = G(\beta\tau + \ln \xi) + H(\beta\tau - \ln \xi) , \quad \eta = 0 . \tag{3.18}$$

Let us clarify equation (3.18) for the case if material inflow $\alpha < 0$ through the surface $x = 1$. We have that

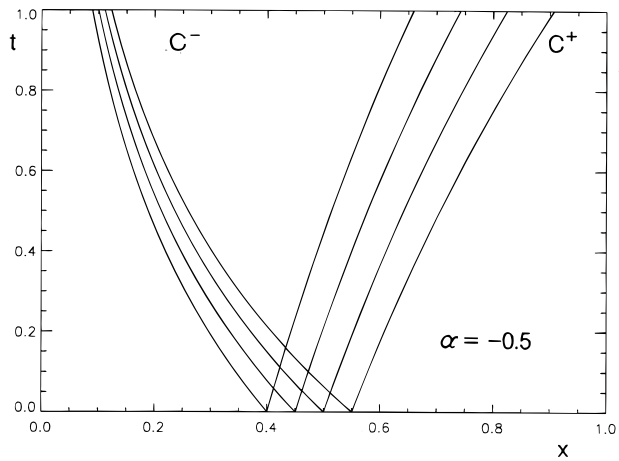

FIG. 2a

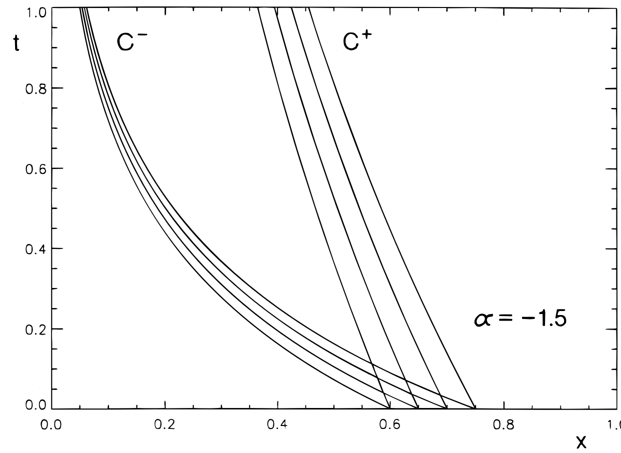

FIG. 2b

FIG. 2.—(a) Inward and outward propagating characteristics for $\alpha = -0.5$, $\beta = 1$. Inward moving waves localize, driving relatively large currents. (b) Inward propagating characteristics for $\alpha = -1.5$, $\beta = 1$. Since $|\alpha| > \beta$, both waves localize as they propagate inwards.

$\xi = xe^{|\alpha|t}$ and so points on the wave profile propagate at the speeds

$$\frac{dx}{dt} = -(\beta + |\alpha|)x\ , \quad \frac{dx}{dt} = +(\beta - |\alpha|)x\ .$$

These define the (real space) characteristics of the wave, namely the lines of constant $C^- = xe^{(\beta+|\alpha|)t}$ and $C^+ = xe^{(|\alpha|-\beta)t}$. Figure 2 shows the $\beta = 1$ characteristics for the cases $\alpha = -0.5$ and $\alpha = -1.5$. It is clear that only inward-moving waves tend to localize. A special case occurs when $|\alpha| = \beta$: there is a standing wave plus a component which propagates inwards at the speed $|2\alpha x|$. For $|\alpha| > \beta$ all waves propagate inward, as shown in Figure 2b.

Returning to equation (3.18), we can use the argument of § 3.2 to show that any wave produces only weak current densities, independent of the advective flow. Thus, comparing the resistive term with $|Z_\tau|$ implies that $x = (\eta/\beta)^{1/2}$ again determines the resistive length scale. It follows that, although the localization time

$$T \simeq \frac{1}{2||\alpha| \pm \beta|} \ln \frac{\beta}{\eta} \tag{3.19}$$

is influenced by the background flow, the strength of the current layer and the ohmic dissipation rate remain unaffected at $J \sim \eta^{-1/2}$, $W_\eta \sim \eta^{1/2}$.

3.5. *Shear Wave Packet Solutions*

Suppose we relax the plane wave constraints $W = W(x, t)$, $Z = Z(x, t)$. Does our conclusion of weak resistive dissipation still hold good?

Returning to equations (2.7) and (2.8) and setting $\phi = 0$, we obtain

$$\frac{\partial W}{\partial t} = [Z, \psi] \tag{3.20}$$

and

$$\frac{\partial Z}{\partial t} = [W, \psi] + \eta\nabla^2 Z\ . \tag{3.21}$$

Since

$$[X, \psi] = \beta(xX_x - yX_y)$$

represents the directional derivative of X along the equilibrium field, we adopt coordinates to simplify this expression. A suitable choice ($\zeta = xy = \psi/\beta$, $\chi = x/y$) leads to $[X, \psi] \to 2\beta\chi X_\chi$, from which it follows that

$$Z = W = \pm G(\psi)H\left(t \pm \frac{1}{2\beta} \ln \chi\right) \tag{3.22}$$

is the general wave solution in the absence of resistivity. This solution confirms that shear waves propagate along each field line according to $\dot{\chi} = \pm 2\beta\chi$.

Let us consider the motion of some point

$$\boldsymbol{r} = (x, y) = (\sqrt{\zeta\chi}, \sqrt{\zeta/\chi})\ , \tag{3.23}$$

on the wave profile. The trajectory is defined by taking $\zeta \propto \psi = \text{const.}$, and so

$$\dot{\boldsymbol{r}} = \pm\beta(x, -y)\ . \tag{3.24}$$

The point follows the path $\boldsymbol{r} = (x_0\, e^{\pm\beta t}, y_0\, e^{\mp\beta t})$ and moves at a rate $\dot{r} = \beta r$ determined by the magnitude of the background field.

Consider a wave packet concentrated around the initial point $x_0 \simeq 1$, $y_0 \ll 1$ and moving inwards, as shown in Figure 3. The packet can localize whenever the rear of the pulse travels faster than the head, that is, whenever $x > y$. It follows that the distance of closest approach to the neutral

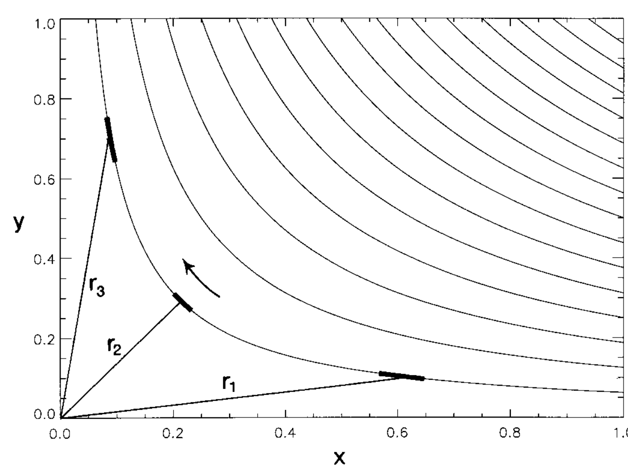

FIG. 3.—Wave packet propagation along a field line. A wave packet moving from right to left localizes as it approaches the neutral point. The extent of the wave is a minimum at the distance of closest approach $r \simeq r_2$.

point (when $x \simeq y$) corresponds to minimum speed and maximum current of the wave.

Under what circumstances can the wave packet be resistively damped? A necessary condition is that the diffusive term $|\eta \nabla^2 Z|$ be comparable to $|Z_t|$ at some point on the trajectory. Since the resistive term is maximum at the distance of closest approach we obtain

$$r < r_g = \left(\frac{\eta}{\beta}\right)^{1/2} \quad (3.25)$$

as the condition for significant dissipation. Waves which propagate along field lines outside the grazing radius r_g can therefore sweep past the dissipation region and exit through the surface $y = 1$ with negligible damping; these waves have initial conditions given by $y_0 > \eta/\beta$, $x_0 \simeq 1$. Even waves that are damped, satisfying $r < r_g$, achieve only a modest current amplitude $J \sim \eta^{-1/2}$.

3.6. *Summary*

Our solutions show that relatively weak current densities are produced by shear wave disturbances perpendicular to the plane of the X-point. The fact that the field can decay rapidly is clearly an artifact of the open geometry: energy loss is dominated by wave motions carrying energy out of the volume rather than ohmic dissipation (see § 3.3). Evidently this result holds good independent of the level of stagnation point flow in the volume. We conclude therefore, that the absence of flux pileup in the perpendicular shear components condemns the ohmic dissipation rate to be slow, $W_\eta \sim \eta^{1/2}$.

4. DISSIPATION OF PLANAR SHEAR WAVES

4.1. *Field Equations*

We now discuss the evolution of the planar field components using equations (2.11) and (2.12). We shall concentrate mainly on the wave propagation characteristics of the solution, since, as we have seen in § 3, wave properties provide an excellent indicator for fast dissipation.

It is convenient to eliminate the potentials and work only with the y-components of the disturbance fields

$$V(x, t) = -f_x , \quad Y(x, t) = -g_x . \quad (4.1)$$

Using the comoving coordinates $\tau = t$, $\xi = x \exp(-\alpha t)$ of equation (3.14), we rewrite equations (2.11) and (2.12) in the form

$$V_\tau = \beta \xi Y_\xi + (\alpha V - \beta Y) , \quad (4.2)$$

$$Y_\tau = \beta \xi V_\xi - (\alpha Y - \beta V) + \eta e^{-2\alpha\tau} Y_{\xi\xi} . \quad (4.3)$$

A detailed discussion of this system is given by Craig & Henton (1997) and we shall present only a summary discussion here. What we wish to emphasize is that fast ohmic dissipation is a possibility only for background flows satisfying $\alpha^2 > \beta^2$.

4.2. *Wave Propagation for* $\eta = 0$

The wave properties of the solution can be examined by setting $\eta = 0$. Eliminating $V(\xi, \tau)$ using equation (4.2), we note that the field evolves according to the Klein-Gordon equation

$$Y_{\tau\tau} = \beta^2 Y_{\chi\chi} + (\alpha^2 - \beta^2) Y , \quad \chi = \ln \xi . \quad (4.4)$$

Equation (4.4) holds for any linear combination of V and Y as a consequence of the intrinsic symmetry in the magnetic and velocity fields of ideal fluids (Elasser 1950).

It is interesting to compare equation (4.4) with the wave equation (3.17) describing the normal shear components. The characteristics are again defined by the local wavespeed $\beta\xi$; in particular, $C^\pm = \beta\tau \mp \ln \xi$. Yet now there is a possibility for growth in the field. This growth is associated with the localization, in real space, of inward-travelling disturbances (see Fig. 2*b*).

More specifically, the solution for an isolated Fourier mode is given by

$$Y_F = e^{\nu\tau \pm ik\chi} , \quad \nu(k) = \pm[\alpha^2 - \beta^2(1 + k^2)]^{1/2} . \quad (4.5)$$

In real space, $Y_F(x, t) = e^{\nu t} e^{\pm ik(\ln x - \alpha t)}$. Either way, the condition for growth is $\alpha^2 > \beta^2(1 + k^2)$. Since long-wavelength modes correspond to $k \simeq 1/|\ln \eta|$ (eq. [3.13]), the localization condition $|\alpha| > \beta$ is sufficient for growth in the limit of small η. The growth of the field is easily interpreted in the case of material inflow through the surface $x = 1$: the inflow effectively forces the shear wave to localize close to the current layer $x \simeq 0$. More surprising is the presence of growth for $\alpha > 0$, but this behavior is unphysical, since it requires an *externally imposed* current sheet on the inflow surface $y = 1$.

More general wave solutions can be constructed using superpositions of the form

$$Y(\xi, \tau) = \int_{-\infty}^{\infty} dk\, A(k) e^{ik \ln \xi} e^{\nu(k)\tau} . \quad (4.6)$$

However, in view of the dispersive nature of the waves, simple wave-packet descriptions can be derived only in special cases.

4.3. *Flux Pileup Solutions* ($\alpha^2 > \beta^2$)

The simplest case to discuss is $\beta = 0$, corresponding to the advection of straight field lines by the background stagnation point flow. In this case there are no shearing wave motions associated with the development of the current layer. Since the growth rate ν is independent of k, we obtain solutions which evolve according to

$$Y(\xi, \tau) = e^{\pm \alpha\tau} Y_0(\xi) , \quad \beta = 0 . \quad (4.7)$$

Legitimate positive growth implies $\alpha < 0$ and so $Y(x, t) = e^{|\alpha| t} Y_0(x e^{|\alpha| t})$ is the required solution. The growth of the field is arrested only when resistive dissipation sets in. The localization time

$$T \simeq \frac{1}{2|\alpha|} \ln \frac{|\alpha|}{\eta} \quad (4.8)$$

corresponds to the diffusive length scale $x \simeq (\eta/\alpha)^{1/2}$. The resultant current amplitude $J \sim \alpha/\eta$ drives the "super-fast" rate $W_\eta \sim \eta^{-1/2}$. Analytic diffusion solutions are given by Clarke (1964).

More generally, for $\beta > 0$ the disturbance comprises inward- and outward-propagating components relative to the comoving coordinate ξ. In real space both components are inward-moving, provided the condition $\alpha^2 > \beta^2$ is satisfied. Although each component is highly dispersive, we can use an approximate treatment to show that strong currents are formed by the slower-moving wave train (Craig & Henton 1996). In this case the current layer $x \sim \eta^{1/2}$ is built up over the timescale

$$T \simeq \frac{1}{2|\mu|} \ln \frac{|\mu|}{\eta} , \quad \mu = \frac{\alpha^2 - \beta^2}{\alpha} , \quad (4.9)$$

which leads to current magnitudes of order η^{-1} and super-fast ohmic dissipation $W_\eta \sim \eta^{-1/2}$.

The price of fast dissipation, however, is not cheap. Since the field at the current layer scales as $Y \sim \eta^{-1/2}$, extremely large pressures $P \sim 1/2\ Y^2 \sim \eta^{-1}$ are present at the neutral point (see eq. [2.15]). This amplitude, which can be interpreted as the level of the external hydromagnetic pressure required to sustain the compressive flow, must eventually saturate, stalling the reconnection. A discussion of this problem is given by Watson & Craig (1997a), (1997b) in the context of steady-state two- and three-dimensional reconnection solutions.

4.4. *Weak Current Solutions* ($\alpha^2 \leq \beta^2$)

When $\alpha^2 < \beta^2$ the stagnation point flow is no longer strong enough to drive flux pileup. In this case the growth rate v is imaginary, so no amplification of the field disturbance is possible. The limiting case $\beta^2 = \alpha^2$, in which $v = \pm i\beta k$, admits wave-packet solutions. In real space the solution reduces to a standing component $G(x)$, say, plus an inward travelling wave $H(xe^{2|\alpha|t})$. The moving pulse can localize in half the time of the $\beta = 0$ wave packet, but the absence of growth implies that only modest currents can be driven, as $J \sim \eta^{-1/2}$.

Finally, we mention planar field solutions when $\alpha = 0$. This case was treated by Bulanov et al. (1990) as an initial value problem, but an eigensolution approach is also possible (Craig 1994). Either way, the conclusion mirrors the analysis of § 3.2. The field decay is fast, but the rate of ohmic dissipation is weak, $W_\eta \sim \eta^{1/2}$. The argument of § 3.3 again holds good, but it is now planar rather than perpendicular shear waves that carry energy out of the volume over the fast timescale $T \sim |\ln \eta|$.

5. CONCLUSIONS

We have considered the development of shear wave disturbances in planar X-point equilibria. Shear waves appear attractive as mechanisms for magnetic energy release because they can produce small length scales in the field without compressing the plasma. Therefore, in contrast to the case of purely compressive "fast mode" mechanisms, there are no strong back pressures to build up and stall the reconnection.

We have considered both perpendicular and planar shear waves in "open" X-point geometries. An exact solution for disturbances normal to the X-point plane shows that although small length scales $x \sim \eta^{1/2}$ can develop, there is no flux pileup at the onset of the current layer. Accordingly, the current sheet is weak, so only slow rates of ohmic dissipation occur, $W_\eta \sim \eta^{1/2}$. The decay of the field is indeed fast, but this result can be attributed to the open geometry—i.e., to shear waves carrying out of the volume—rather than to the resistive losses in the plasma.

Essentially, the same conclusions hold for "undriven" planar shear waves. The ohmic dissipation is again weak, because of the absence of flux pileup at the onset of the sheet: $x \sim \eta^{1/2}$, $J \sim \eta^{-1/2}$, $W_\eta \sim \eta^{1/2}$. It follows that the fast field decay demonstrated by Bulanov et al. (1990) is the result of wave energy escaping the volume, rather than resistive heating of the plasma (see § 3.3).

Can fast reconnection ever occur? The present analysis allows only one mechanism: planar shear waves coupled to strong advective flows driving material towards the neutral point. We have seen that even the simplest stagnation point flows stretch and compress the plasma and dramatically intensify the field. The resulting flux pileup at the current layer ($x \sim \eta^{1/2}$) leads to much stronger sheets ($J \sim \eta^{-1}$) and the super-fast dissipation rate $W_\eta \sim \eta^{-1/2}$.

It cannot be said however, that the problem of fast reconnection has been solved. The inclusion of stagnation point flows presupposes that large external hydromagnetic pressures are available to drive them (of order η^{-1}). In fact, we have simply recovered, in a different guise, the pressure problem associated with fast mode merging. The problem is most severe in the two-dimensional planar models discussed here, but three-dimensional "fan" reconnection solutions still require the scaling $P \sim \eta^{-1/2}$ (Craig & Fabling 1996). Since fast reconnection at arbitrarily small resistivities evidently requires unbounded pressures, the merging rate must eventually saturate. The level of saturation depends, in any concrete case, on the amplitude of the planar disturbances and the value of the coronal resistivity. Recent two-dimensional and three-dimensional steady-state calculations (Watson & Craig 1997a) suggest that the rate of energy dissipation at saturation can be quite large, possibly sufficient even to power modest flares. Clearly, further investigation is required.

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APPENDIX

THE ROLE OF VISCOUS DAMPING

In common with most analytic treatments, the reconnection equations we consider do not account for the finite viscosity of the field. This may seem strange, given that resistive effects are generally very small and viscous dissipation is expected to be significant if strong shearing motions develop in the flow. Although the solutions we discuss do require strong shearing motions in the fluid, the key point to remember is that only reconnection can liberate energy bound up with the global field topology: all nonresistive damping mechanisms are ineffective in this regard.

Consider, for example, some nonlinear disturbance superposed on an equilibrium magnetic X-point. If the connections between the equilibrium field lines are changed, then the disturbance can be thought to contain a "topological" energy component. This component can be isolated by simulating the evolution of a disturbed, highly viscous X-point plasma in which the the resistivity is set to zero (e.g., Rickard & Craig 1993). No matter what form the viscous damping takes, the topological energy eventually accumulates in high current regions overlying the neutral point. Analytic calculations also confirm that current sheet singularities that contain a dominant fraction of the energy in the initial field disturbance develop in two dimensions (Craig 1994). However, current sheets do not, in themselves, guarantee fast reconnection; the exact nature of the singularity is critical.

In the present study we are mainly concerned with determining whether the resistive dissipation of the topological energy is fast. Viscous effects can alter the reconnection rate only through their influence on the global velocity field of the plasma. The viscous influence has been investigated by Fabling & Craig (1996) within the context of the present shear flow solutions. Side-by-side computations confirm that, although viscosity efficiently dissipates the vorticity of the fluid in the high-shear regions close to the neutral point, its influence on the resistive dissipation rate remains negligible, even for extreme levels of viscous damping.

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