# EVENT DISTRIBUTIONS IN
# ONLINE BOOK AUCTIONS

**Sven Bittner & Annika Hinze**

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Sven Bittner and Annika Hinze

Department of Computer Science,
University of Waikato, New Zealand
{s.bittner, a.hinze}@cs.waikato.ac.nz

**Abstract.** Current quantitative evaluations in various research areas for publish/subscribe systems use artificially created event messages to model the system workload. The assumptions made to create these workloads are rather strong and hardly ever described in detail. This does not allow for a repetition of experiments or comparative evaluations of different approaches by different researches.

In this paper, we present an evaluation of the distributions of the values of attributes typically used in online auction scenarios. In particular, we focus on auctions of fiction books. We further show our approach of creating event messages by the help of the gained information. Publishing this information on how to create a typical workload for online auctions should allow for the repetition of experiments and the comparison of different evaluations.

## 1   Introduction

Recent quantitative analyses in various research areas for publish/subscribe systems, for example, filtering algorithms and routing optimizations, use artificial event messages to create the system workload. The assumptions made to create these workloads are rather strong and hardly ever described in detail. This does not allow for a repetition of experiments or comparative evaluations of different approaches by different researches.

To close this gap, in this paper we present the results of an evaluation of the characteristics of online book auctions. Online auctions in particular need active functionalities for an efficient dissemination of process-related information [2]. Thus, they are an application scenario that could benefit from the application of publish/subscribe systems.

The aim of this paper is to derive a typical workload in online auction settings. Event messages in this scenario are the insertion of new auction items, the ending of auctions, and each bid of a user. Additionally, if considering the publish/subscribe functionality as an external extension of current auctioning systems, all items should create status events in certain time intervals. This, for example, allows users to receive notifications not earlier than a day before the end of an auction.

We will show the properties, distributions, and dependencies of various attributes characterizing auction items in Sect. 2. Section 3 describes how to use our results to create a workload of event messages typical for online book auctions. We conclude and summarize this paper in Sect. 4.

To obtain a machine-readable version of the derived distributions, we encourage the reader to contact the authors of this paper.

## 2 Distributions in Online Book Auctions

To obtain realistic data for event messages, we have analyzed the distribution of auction items on eBay[1] on July 8, 2005. We restricted our analysis to book auctions, in particular to fiction books offered in the United States. Our analysis focused on eight attributes describing auction items: `Category`, `Format`, `Special Attribute`, `Condition`, `Ending Within`, `Price`, `Buy It Now`, and `Bids`. We give an overview of these attributes and their associated domains in Table 1.

Additionally, we later on include the attributes `Title` and `Author` in our event messages. That is, each event message specifies ten attribute-value pairs. Our reasoning for choosing these attributes in event messages was depending on the opportunities to analyze auctions on the eBay site, our personal knowledge about and experience with online auctions, and the requirements of typical subscriptions for online book auctions (refer to [1] for such subscriptions).

**Table 1.** Overview of directly analyzed attributes for book auctions

| Attribute | Example | Values |
|-----------|---------|--------|
| Category | Fantasy | 22 |
| Format | Hardcover | 4 |
| Special Attribute | Signed | 3 |
| Condition | New, used | 2 |
| Ending Within | 1 hour | $0 \sec \ldots 10$ days |
| Price | $0.99 | $\$0.00 \ldots \$1000.00$ |
| Buy It Now | Yes, no | 2 |
| Bids | 1 | $0 \ldots 100$ |

### 2.1 Attribute Domains

We now state the exact attribute domains of the attributes involving enumerations of values, that is, `Category`, `Format`, `Special Attribute`, and `Condition`.

The twenty-two possible values for the categories of fiction books in the analyzed online auction system are as follows:

1. Action, Adventure
2. Classics
3. Fantasy

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[1] http://www.ebay.com/

4. Folklore, Mythology
5. Historical
6. Horror
7. Humor
8. Literary Collections
9. Literary Criticism
10. Literature, Ancient
11. Literature, Classic
12. Literature, Modern
13. Military
14. Mystery, Thriller
15. Plays, Screenplays
16. Poetry
17. Pulps
18. Religious, Inspirational
19. Romance
20. Science Fiction
21. Westerns
22. Other

Sellers can specify four different formats for books:

1. Hardcover
2. Softcover
3. Mixed Lot
4. Other

Special attributes might increase the value of books. Sellers can choose among the following options:

1. 1st Edition
2. Signed
3. Unspecified

The condition of a book is one of the following:

1. New
2. Used

In the following subsection, we describe the typical distributions of all attributes given in Table 1.

## 2.2   Distributions of Attribute Values

We have been able to determine all possible combinations of the values of the attributes `Category`, `Format`, `Special Attribute`, and `Condition`. There exist exactly $22 \times 4 \times 3 \times 2 = 528$ combinations. For example, we do know that exactly 11.4 percent of all books are characterized as used "Romance" books without special

attributes and bound as softcover. Books specifying these parameters represent the most offered type of items.

We give an overview of the exact probabilities of all possible combinations of these four attributes in Tables 2 to 7. Each row of the tables specifies a category; the four right columns state a combination of the attributes `Condition`, `Special Attribute`, and `Format`: Table 2 contains the probabilities for used books of the first edition, Table 3 for signed used books, Table 4 for all other used books, Table 5 for new books of the first edition, Table 6 for signed new books, and Table 7 for all other new books. Adding up all probabilities given in these tables results in 1.0. That is, each event message conforms to one of the combinations.

For the attribute `Buy It Now`, we have analyzed the probability that an item of any category is sold as `Buy It Now` item. In all other cases, it is not a `Buy It Now` item. We have given an overview of the results in Table 8. The highest proportion of `Buy It Now` items exists in category "Ancient Literature" and the lowest one in category "Poetry" (in each case compared to the total number of items in the respective category).

We have also analyzed the number of bids depending on the category of items. An overview of our results in given in Tables 9 to 12: Table 9 contains the results for 0 to 3 bids, Table 10 for 4 to 7 bids, Table 11 for 8 to 20 bids, and Table 12 for 21 or more bids. Adding up the probabilities for any category results in 1.0. For example, the largest proportion of zero-bid items (compared to the number of total items of a category) exists for category "Action, Adventure". The highest proportional number of items with 10 or more bids (compared to all items of a category) is found for category "Pulps".

The probability of auction items of all 22 categories depending on the attribute `Price` is shown in Tables 13 to 16: Table 13 gives an overview for prices up to $4.00, Table 14 for prices ranging from $4.01 to $8.00, Table 15 for prices in between $8.01 and $30.00, and Table 16 covers prices over $30.01. Again, the sum of the probabilities for all prices of any category is 1.0. For example, 22.2 percent of all "Fantasy" books are sold for more than $10.00; for "Romance" books this is the case for only 5.9 percent of the items of this category. "Romance" books is also the category having sold the most items for $1.00 or less (38.6 percent).

Table 17 gives an overview of the total number of items for all categories. For example, "Romance" books constitute the overall largest proportion of auction items (27,271 items or 19.2 percent); the smallest proportional number of auction items is found in the category "Ancient Literature" (103 items or 0.07 percent).

The previous results have been obtained using active auctions. We have also analyzed finished auctions on eBay. Their distributions of attribute values have been nearly the same as in active auctions. Thus, the derived distributions approximately hold for both active and finished auctions.

By the help of these distributions of items in online book auctions, we are able to create event messages conforming to the characteristics of book auctions for fiction books. We will describe the creation of event messages in the next section.

# 3   Creation of Online Book Auction Events

We now describe how we create event messages conforming to the derived distributions. Our event messages represent a semi-realistic dataset because we have been unable to determine the exact distributions of all combinations of attribute values. That is, our messages contain the real distributions for certain attributes (e.g., `Category`, `Format`, `Special Attribute`, and `Condition`) but are also built on the assumption of the independence of attributes (e.g., the attribute `Price` does only depend on the attribute `Category`).

In our analysis, we have been unable to derive information about authors (attribute `Author`) and titles (attribute `Title`). This includes knowledge of, for example, the number of unique books per category, the total number of authors, the probability of the same title for different books, and the probability of authors publishing in several categories. So, for the creation of a workload we have to make assumptions about this kind of information, as shown later.

## 3.1   Event Creation For Directly Derived Attributes

For the determination of the values for the four attributes `Category`, `Format`, `Special Attribute`, and `Condition`, we can directly use the information given in Tables 2 to 7. That is, we can derive the values for these four attributes of event messages by picking their given combinations according to the stated probability distribution.

For the attribute `Buy It Now`, the probabilities given in Table 8 allow for the calculation of the attribute value, based on the already known category. Hence, we do know whether items should be sold to users with the highest bid (`Buy It Now` = "No") or for a fixed price (`Buy It Now` = "Yes").

The calculation of the number of bids (attribute `Bids`) works as follows: Based on Tables 9 to 12, we can directly determine the cases of 0 to 9 bids. For a higher number of bids, we can decide on one of the ranges of bids by the help of the given distributions. Within these ranges, we always assume a uniform distribution of bids, for example, for bids in between 11 and 20, all 10 values have the same probability of 0.1. For the last range, more than 50 bids, we assume a maximum of 60 bids.

Our calculation for prices (attribute `Price`) works similarly: Tables 13 to 16 allow for the computation of a range of prices. Within these ranges, we assume a uniform distribution of values. The maximal possible price is defined as $1000.

For the attribute `Ending Within`, we assume a uniform distribution of up to 10 days.

## 3.2   Event Creation For Other Attributes

The attributes `Author` and `Title` require more attention than the eight previous ones because we have been unable to derive their distributions: We do know the total number of items for each category, as given in Table 17. Furthermore, let us assume the following four parameters: $B_{frac}$ describes the number of duplicates for each unique book, $A_{frac}$ specifies the number of books each author has written, $p_{mult}^A$ determines

the probability that an author publishes books listed in several categories, and $p_{mult}^T$ states the probability that different books have the same title.

Parameter $B_{frac}$ lets us determine the number of unique books for each category (dividing the overall number of items per category by $B_{frac}$). And by the help of $A_{frac}$, we can calculate the total number of authors (dividing the number of unique books by $A_{frac}$).

For each author, we then randomly choose a category (assuming a uniform distribution). With the probability of $p_{mult}^A$, each author gets associated a second category. The probability that an author is writing for $n$ categories is $(p_{mult}^A)^{n-1}$.

Finally, for each unique book of a category we choose a random author assigned to this category of the book. With the probability of $p_{mult}^T$, a unique book gets assigned an already used title.

To derive a `Title` for a book of an already determined category, we choose one book that is associated with this category. Since we have previously assigned an author to each book, we have also successfully determined the value of the attribute `Author`.

Using these methods described in the previous and this subsection allows us to determine the values for all ten attributes of event messages in online book auctions. In the next subsection, we describe the meaning of these messages.

## 3.3   Meaning of Created Event Messages

As already stated, event messages created according to our previous description lead to a semi-realistic or, the other way round, a semi-artificial dataset. Even if not obtaining the exact properties of real-world online book auctions, our dataset is by far more accurate than the sole assumption of random distributions within and among all ten attributes.

We have obtained our distributions by analyzing a snapshot of the items offered at eBay. For the attribute `Ending Within`, we have assumed a uniform distribution. This might not necessarily be a realistic assumption if considering a local eBay site, as the one for the USA that has been chosen for our analysis. However, when assuming an international site, this assumption is relatively realistic and sufficient for our purpose of creating a typical event workload.

An analysis of the bidding times in online auctions, for example, shown in [3], reveals the existence of a large proportion of late bidding and, depending on the length of auctions, also early bidding. This typical behavior in bid timing is not explicitly modeled in the created event messages. Although, due to our snapshot analysis, it is incorporated into our results: We did not exactly determine the dependency between `Ending Within` and `Bids`. However, the amount of different numbers of bids per category is known and integrated into the generated event messages. The same holds for prices of items. We do not consider items with more bids as having accumulated a higher price. Instead, our event messages do rather represent the average values for these parameters for the given categories.

An advantage of our snapshot analysis is the possibility to generalize our results: Since event messages represent the average, that is, they are ranging from newly inserted to just finished auctions, they can be used to model real events in auctions. On the one hand, we do not model the history of particular items. But on the other hand, our event messages contain the typical distributions of items at all points in time. That

is, they also represent events other than the insertion of auction items and the ending of particular auctions.

This fact allows us to model the average workload including the real events occurring in online auctions: These real events, next to the insertion and ending of auctions, are bids, that is, users want to pay a higher price than the current highest bidder. Our messages do not exactly model the history of bids for items. Although, they contain the events of certain numbers of bids and associated prices, as existing in the analyzed snapshot. We also do not know about the times of bids. But, if assuming an international site, they should be evenly distributed, as modeled in our messages.

In the next subsection, we present some considerations about the expected frequencies of event messages in an online auction system.

### 3.4   Expected Frequency of Event Messages

As already described, event messages required in online auctions are, at least, insertions of new auctions, endings of existing auctions, and biddings of users. In our analysis, 141,602 items have been listed in the United States. For these items, a total of 38,339 bids has existed. Incorporating this number of bids and two events per item (inserting and ending) leads to 321,543 events in total. Since our analysis included items for the next 10 days, this leads to approximately 0.37 events per second.

This result represents a relatively little system throughput. The reason is obvious: Firstly, our analysis only included fiction books. Secondly, it was restricted to items in the United States. So let us scale the expected amount of event messages to all book auctions hosted at the eBay.com site. On February 14, 2006, there is a total of 9,363,317 book auctions on this site. Thus, the event throughput increases to approximately 24.62 events per second.

And now, let us scale to the total number of items hosted on eBay.com: On February 14, 2006, there are 55,771,229 auctions. So, the throughput increases to 146.58 events per second if assuming the statistics gained from fiction book auctions. When considering the number of auctions internationally, the system throughput increases even more. Obviously, events created by other items than books have a slightly different structure as described in this paper.

As mentioned in Sect. 1, if we want to support subscriptions that can specify to be notified, for example, not earlier than an hour before the end of an auction, and the publish/subscribe system is an external extension to the auctioning site, there should be status information about all items in certain intervals. Assuming intervals of 1 hour and only fiction books (141,602 items), the additional event throughput is 39.33 events per second.

## 4   Summary

In this paper, we have described our analysis of the typical distributions of items in online auctions of fictions books. We have given the typical attributes to describe events in these auctions and presented the probabilities and distributions of attribute values. We

have been able to derive all dependencies among some of the attributes. For other attributes, however, we could only model some of the existing dependencies and assumed their independence of other attributes.

The distributions of attributes given in this paper allow for the creation of event messages describing a typical workload created by online book auctions. We do not claim that our results enable us to exactly model the real events occurring at online book auction sites. However, the semi-realistic dataset one can create by the help of the results of this paper is, firstly, much more accurate and, secondly, makes far less assumptions than pure artificial settings, which have mainly been used in previous analyses.

A goal for further research is to derive more information about the distributions of attribute values and the dependencies among all attributes in online auction settings. Furthermore, the application-related characteristics of other scenarios requiring publish/subscribe systems should be determined. This would allow for the analyses of novel solutions, as well as already introduced approaches, regarding all aspects of publish/subscribe systems using real-world data. The results would show both the applicability of these approaches in real-world settings and the suitability of current assumptions for specific application scenarios.

# References

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**Table 2.** Overview of the probabilities of used books of the first edition for all categories (rows) and formats (columns)

| Category | Other format | Mixed lot | Softcover | Hardcover |
|---|---|---|---|---|
| 1 | $6.469393 \times 10^{-6}$ | $6.469393 \times 10^{-6}$ | $3.234697 \times 10^{-5}$ | $3.752248 \times 10^{-4}$ |
| 2 | 0.000000 | 0.000000 | 0.000000 | $4.528575 \times 10^{-5}$ |
| 3 | 0.000000 | $6.469393 \times 10^{-6}$ | $1.293879 \times 10^{-4}$ | $2.911227 \times 10^{-4}$ |
| 4 | 0.000000 | 0.000000 | $1.293879 \times 10^{-5}$ | $6.469393 \times 10^{-6}$ |
| 5 | $6.469393 \times 10^{-6}$ | 0.000000 | $7.763272 \times 10^{-5}$ | $1.487960 \times 10^{-4}$ |
| 6 | $1.293879 \times 10^{-5}$ | 0.000000 | $5.175515 \times 10^{-5}$ | $3.816942 \times 10^{-4}$ |
| 7 | $1.940818 \times 10^{-5}$ | 0.000000 | $1.552654 \times 10^{-4}$ | $1.940818 \times 10^{-4}$ |
| 8 | $6.469393 \times 10^{-6}$ | 0.000000 | 0.000000 | $6.469393 \times 10^{-5}$ |
| 9 | 0.000000 | 0.000000 | 0.000000 | $6.469393 \times 10^{-6}$ |
| 10 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 11 | 0.000000 | 0.000000 | 0.000000 | $1.940818 \times 10^{-5}$ |
| 12 | $2.587757 \times 10^{-5}$ | 0.000000 | $1.682042 \times 10^{-4}$ | $7.633884 \times 10^{-4}$ |
| 13 | 0.000000 | 0.000000 | $1.293879 \times 10^{-5}$ | $1.811430 \times 10^{-4}$ |
| 14 | $4.528575 \times 10^{-5}$ | 0.000000 | $1.423267 \times 10^{-4}$ | $1.390920 \times 10^{-3}$ |
| 15 | 0.000000 | 0.000000 | $3.234697 \times 10^{-5}$ | $6.469393 \times 10^{-6}$ |
| 16 | $2.587757 \times 10^{-5}$ | $6.469393 \times 10^{-6}$ | $1.293879 \times 10^{-4}$ | $2.587757 \times 10^{-4}$ |
| 17 | 0.000000 | 0.000000 | $6.469393 \times 10^{-6}$ | $6.469393 \times 10^{-6}$ |
| 18 | 0.000000 | 0.000000 | $6.469393 \times 10^{-5}$ | $9.057151 \times 10^{-5}$ |
| 19 | 0.000000 | 0.000000 | $2.070206 \times 10^{-4}$ | $1.811430 \times 10^{-4}$ |
| 20 | $1.293879 \times 10^{-5}$ | 0.000000 | $2.846533 \times 10^{-4}$ | $3.105309 \times 10^{-4}$ |
| 21 | $6.469393 \times 10^{-6}$ | 0.000000 | $3.881636 \times 10^{-5}$ | $6.469393 \times 10^{-5}$ |
| 22 | $1.035103 \times 10^{-4}$ | $6.469393 \times 10^{-6}$ | $1.940818 \times 10^{-4}$ | $6.210618 \times 10^{-4}$ |

**Table 3.** Overview of the probabilities of signed used books for all categories (rows) and formats (columns)

| Category | Other format | Mixed lot | Softcover | Hardcover |
|---|---|---|---|---|
| 1 | $1.811430 \times 10^{-4}$ | $1.293879 \times 10^{-5}$ | $1.759675 \times 10^{-3}$ | $8.584885 \times 10^{-3}$ |
| 2 | $2.587757 \times 10^{-5}$ | $6.469393 \times 10^{-6}$ | $2.781839 \times 10^{-4}$ | $1.261532 \times 10^{-3}$ |
| 3 | $7.116333 \times 10^{-5}$ | $4.528575 \times 10^{-5}$ | $3.836350 \times 10^{-3}$ | $5.304903 \times 10^{-3}$ |
| 4 | $1.293879 \times 10^{-5}$ | $6.469393 \times 10^{-6}$ | $1.552654 \times 10^{-4}$ | $6.340005 \times 10^{-4}$ |
| 5 | $1.293879 \times 10^{-4}$ | $0.000000$ | $5.757760 \times 10^{-4}$ | $3.111778 \times 10^{-3}$ |
| 6 | $1.164491 \times 10^{-4}$ | $2.587757 \times 10^{-5}$ | $1.953757 \times 10^{-3}$ | $5.272556 \times 10^{-3}$ |
| 7 | $9.057151 \times 10^{-5}$ | $0.000000$ | $1.468552 \times 10^{-3}$ | $2.613635 \times 10^{-3}$ |
| 8 | $1.293879 \times 10^{-5}$ | $0.000000$ | $1.487960 \times 10^{-4}$ | $5.951842 \times 10^{-4}$ |
| 9 | $1.293879 \times 10^{-5}$ | $0.000000$ | $4.528575 \times 10^{-5}$ | $1.552654 \times 10^{-4}$ |
| 10 | $0.000000$ | $0.000000$ | $6.469393 \times 10^{-6}$ | $4.528575 \times 10^{-5}$ |
| 11 | $2.587757 \times 10^{-5}$ | $0.000000$ | $1.293879 \times 10^{-5}$ | $4.140412 \times 10^{-4}$ |
| 12 | $4.075718 \times 10^{-4}$ | $6.469393 \times 10^{-6}$ | $1.222715 \times 10^{-3}$ | $6.922251 \times 10^{-3}$ |
| 13 | $1.293879 \times 10^{-5}$ | $1.293879 \times 10^{-5}$ | $8.022048 \times 10^{-4}$ | $1.766144 \times 10^{-3}$ |
| 14 | $2.717145 \times 10^{-4}$ | $9.057151 \times 10^{-5}$ | $2.464839 \times 10^{-3}$ | $1.676220 \times 10^{-2}$ |
| 15 | $3.234697 \times 10^{-5}$ | $0.000000$ | $2.587757 \times 10^{-4}$ | $4.011024 \times 10^{-4}$ |
| 16 | $7.763272 \times 10^{-5}$ | $6.469393 \times 10^{-6}$ | $5.563678 \times 10^{-4}$ | $1.255062 \times 10^{-3}$ |
| 17 | $6.469393 \times 10^{-6}$ | $0.000000$ | $3.364085 \times 10^{-4}$ | $7.116333 \times 10^{-5}$ |
| 18 | $4.528575 \times 10^{-5}$ | $1.293879 \times 10^{-5}$ | $2.652451 \times 10^{-4}$ | $8.798375 \times 10^{-4}$ |
| 19 | $1.293879 \times 10^{-4}$ | $5.822454 \times 10^{-5}$ | $6.165332 \times 10^{-3}$ | $4.535045 \times 10^{-3}$ |
| 20 | $1.811430 \times 10^{-4}$ | $3.234697 \times 10^{-5}$ | $6.197679 \times 10^{-3}$ | $5.686597 \times 10^{-3}$ |
| 21 | $4.528575 \times 10^{-5}$ | $1.293879 \times 10^{-5}$ | $1.300348 \times 10^{-3}$ | $1.352103 \times 10^{-3}$ |
| 22 | $7.181027 \times 10^{-4}$ | $4.528575 \times 10^{-5}$ | $3.791064 \times 10^{-3}$ | $7.925007 \times 10^{-3}$ |

**Table 4.** Overview of the probabilities of used books not specified as signed and first edition for all categories (rows) and formats (columns)

| Category | Other format | Mixed lot | Softcover | Hardcover |
|---|---|---|---|---|
| 1 | $7.569190 \times 10^{-4}$ | $3.881636 \times 10^{-4}$ | $2.761784 \times 10^{-2}$ | $2.664743 \times 10^{-2}$ |
| 2 | $2.587757 \times 10^{-4}$ | $9.704090 \times 10^{-5}$ | $6.275312 \times 10^{-3}$ | $7.879721 \times 10^{-3}$ |
| 3 | $3.881636 \times 10^{-4}$ | $4.593269 \times 10^{-4}$ | $2.851709 \times 10^{-2}$ | $1.143789 \times 10^{-2}$ |
| 4 | $9.704090 \times 10^{-5}$ | $1.293879 \times 10^{-5}$ | $1.889063 \times 10^{-3}$ | $2.160777 \times 10^{-3}$ |
| 5 | $3.493472 \times 10^{-4}$ | $9.704090 \times 10^{-5}$ | $8.759559 \times 10^{-3}$ | $8.112619 \times 10^{-3}$ |
| 6 | $4.140412 \times 10^{-4}$ | $4.269800 \times 10^{-4}$ | $2.172422 \times 10^{-2}$ | $1.339811 \times 10^{-2}$ |
| 7 | $3.687554 \times 10^{-4}$ | $1.617348 \times 10^{-4}$ | $1.653577 \times 10^{-2}$ | $7.892660 \times 10^{-3}$ |
| 8 | $9.704090 \times 10^{-5}$ | $2.587757 \times 10^{-5}$ | $1.345634 \times 10^{-3}$ | $2.775370 \times 10^{-3}$ |
| 9 | $2.587757 \times 10^{-5}$ | $6.469393 \times 10^{-6}$ | $2.975921 \times 10^{-4}$ | $4.011024 \times 10^{-4}$ |
| 10 | $3.881636 \times 10^{-5}$ | $0.000000$ | $1.099797 \times 10^{-4}$ | $2.070206 \times 10^{-4}$ |
| 11 | $1.293879 \times 10^{-4}$ | $5.175515 \times 10^{-5}$ | $1.811430 \times 10^{-3}$ | $2.801247 \times 10^{-3}$ |
| 12 | $7.827966 \times 10^{-4}$ | $2.781839 \times 10^{-4}$ | $1.564299 \times 10^{-2}$ | $1.404505 \times 10^{-2}$ |
| 13 | $8.410211 \times 10^{-5}$ | $1.164491 \times 10^{-4}$ | $6.294720 \times 10^{-3}$ | $4.011024 \times 10^{-3}$ |
| 14 | $1.552654 \times 10^{-3}$ | $1.973165 \times 10^{-3}$ | $5.815985 \times 10^{-2}$ | $4.951674 \times 10^{-2}$ |
| 15 | $4.528575 \times 10^{-5}$ | $3.881636 \times 10^{-5}$ | $1.973165 \times 10^{-3}$ | $1.119205 \times 10^{-3}$ |
| 16 | $2.070206 \times 10^{-4}$ | $3.234697 \times 10^{-5}$ | $2.607166 \times 10^{-3}$ | $4.030432 \times 10^{-3}$ |
| 17 | $6.469393 \times 10^{-6}$ | $0.000000$ | $1.377981 \times 10^{-3}$ | $2.393676 \times 10^{-4}$ |
| 18 | $3.752248 \times 10^{-4}$ | $5.175515 \times 10^{-4}$ | $1.106266 \times 10^{-2}$ | $6.236495 \times 10^{-3}$ |
| 19 | $8.151436 \times 10^{-4}$ | $1.462083 \times 10^{-3}$ | $1.139907 \times 10^{-1}$ | $2.366504 \times 10^{-2}$ |
| 20 | $6.534087 \times 10^{-4}$ | $6.275312 \times 10^{-4}$ | $3.753542 \times 10^{-2}$ | $1.460142 \times 10^{-2}$ |
| 21 | $1.552654 \times 10^{-4}$ | $8.410211 \times 10^{-5}$ | $7.743864 \times 10^{-3}$ | $5.136698 \times 10^{-3}$ |
| 22 | $1.669103 \times 10^{-3}$ | $6.728169 \times 10^{-4}$ | $5.508041 \times 10^{-2}$ | $2.617517 \times 10^{-2}$ |

**Table 5.** Overview of the probabilities of new books of the first edition for all categories (rows) and formats (columns)

| Category | Other format | Mixed lot | Softcover | Hardcover |
|---|---|---|---|---|
| 1 | $1.293879 \times 10^{-5}$ | $0.000000$ | $1.682042 \times 10^{-4}$ | $5.240209 \times 10^{-4}$ |
| 2 | $0.000000$ | $6.469393 \times 10^{-6}$ | $2.587757 \times 10^{-5}$ | $1.423267 \times 10^{-4}$ |
| 3 | $5.175515 \times 10^{-4}$ | $1.293879 \times 10^{-5}$ | $4.075718 \times 10^{-4}$ | $1.125674 \times 10^{-3}$ |
| 4 | $0.000000$ | $0.000000$ | $3.234697 \times 10^{-5}$ | $5.175515 \times 10^{-5}$ |
| 5 | $3.234697 \times 10^{-5}$ | $0.000000$ | $6.469393 \times 10^{-5}$ | $1.358573 \times 10^{-4}$ |
| 6 | $4.528575 \times 10^{-5}$ | $0.000000$ | $2.264288 \times 10^{-4}$ | $7.504496 \times 10^{-4}$ |
| 7 | $6.469393 \times 10^{-6}$ | $0.000000$ | $1.035103 \times 10^{-4}$ | $1.552654 \times 10^{-4}$ |
| 8 | $0.000000$ | $0.000000$ | $1.293879 \times 10^{-5}$ | $9.057151 \times 10^{-5}$ |
| 9 | $0.000000$ | $0.000000$ | $0.000000$ | $0.000000$ |
| 10 | $0.000000$ | $0.000000$ | $6.469393 \times 10^{-6}$ | $0.000000$ |
| 11 | $0.000000$ | $0.000000$ | $0.000000$ | $6.469393 \times 10^{-6}$ |
| 12 | $7.116333 \times 10^{-5}$ | $1.293879 \times 10^{-5}$ | $3.622860 \times 10^{-4}$ | $1.268001 \times 10^{-3}$ |
| 13 | $0.000000$ | $0.000000$ | $3.881636 \times 10^{-5}$ | $1.164491 \times 10^{-4}$ |
| 14 | $1.035103 \times 10^{-4}$ | $0.000000$ | $1.746736 \times 10^{-4}$ | $9.898172 \times 10^{-4}$ |
| 15 | $6.469393 \times 10^{-6}$ | $0.000000$ | $1.293879 \times 10^{-5}$ | $0.000000$ |
| 16 | $2.587757 \times 10^{-5}$ | $0.000000$ | $9.704090 \times 10^{-5}$ | $5.175515 \times 10^{-5}$ |
| 17 | $0.000000$ | $0.000000$ | $1.293879 \times 10^{-5}$ | $0.000000$ |
| 18 | $1.293879 \times 10^{-5}$ | $0.000000$ | $1.940818 \times 10^{-5}$ | $4.528575 \times 10^{-5}$ |
| 19 | $2.587757 \times 10^{-5}$ | $0.000000$ | $1.229185 \times 10^{-4}$ | $7.763272 \times 10^{-5}$ |
| 20 | $3.234697 \times 10^{-4}$ | $0.000000$ | $2.264288 \times 10^{-4}$ | $6.081230 \times 10^{-4}$ |
| 21 | $0.000000$ | $0.000000$ | $0.000000$ | $3.881636 \times 10^{-5}$ |
| 22 | $2.070206 \times 10^{-4}$ | $0.000000$ | $2.587757 \times 10^{-4}$ | $9.121845 \times 10^{-4}$ |

**Table 6.** Overview of the probabilities of signed new books for all categories (rows) and formats (columns)

| Category | Other format | Mixed lot | Softcover | Hardcover |
|---|---|---|---|---|
| 1 | $4.528575 \times 10^{-5}$ | 0.000000 | $6.857557 \times 10^{-4}$ | $3.098839 \times 10^{-3}$ |
| 2 | 0.000000 | $1.293879 \times 10^{-5}$ | $7.116333 \times 10^{-5}$ | $2.587757 \times 10^{-4}$ |
| 3 | $6.275312 \times 10^{-4}$ | $1.293879 \times 10^{-5}$ | $1.863185 \times 10^{-3}$ | $3.706962 \times 10^{-3}$ |
| 4 | $6.469393 \times 10^{-6}$ | $6.469393 \times 10^{-6}$ | $1.035103 \times 10^{-4}$ | $2.717145 \times 10^{-4}$ |
| 5 | $4.528575 \times 10^{-5}$ | 0.000000 | $3.493472 \times 10^{-4}$ | $7.763272 \times 10^{-4}$ |
| 6 | $1.035103 \times 10^{-4}$ | $6.469393 \times 10^{-6}$ | $7.504496 \times 10^{-4}$ | $1.999043 \times 10^{-3}$ |
| 7 | $2.587757 \times 10^{-5}$ | 0.000000 | $4.722657 \times 10^{-4}$ | $9.510008 \times 10^{-4}$ |
| 8 | $1.940818 \times 10^{-5}$ | 0.000000 | $4.528575 \times 10^{-5}$ | $3.493472 \times 10^{-4}$ |
| 9 | 0.000000 | 0.000000 | $6.469393 \times 10^{-6}$ | $2.587757 \times 10^{-5}$ |
| 10 | 0.000000 | 0.000000 | $6.469393 \times 10^{-6}$ | $6.469393 \times 10^{-6}$ |
| 11 | 0.000000 | 0.000000 | $1.293879 \times 10^{-5}$ | $1.940818 \times 10^{-5}$ |
| 12 | $1.487960 \times 10^{-4}$ | $1.293879 \times 10^{-5}$ | $1.442675 \times 10^{-3}$ | $2.969452 \times 10^{-3}$ |
| 13 | $1.293879 \times 10^{-5}$ | 0.000000 | $1.099797 \times 10^{-4}$ | $3.816942 \times 10^{-4}$ |
| 14 | $1.552654 \times 10^{-4}$ | $1.293879 \times 10^{-5}$ | $9.057151 \times 10^{-4}$ | $4.839106 \times 10^{-3}$ |
| 15 | 0.000000 | 0.000000 | $7.116333 \times 10^{-5}$ | $1.293879 \times 10^{-5}$ |
| 16 | $3.881636 \times 10^{-5}$ | 0.000000 | $2.652451 \times 10^{-4}$ | $2.264288 \times 10^{-4}$ |
| 17 | 0.000000 | 0.000000 | $3.234697 \times 10^{-5}$ | $6.469393 \times 10^{-6}$ |
| 18 | $6.469393 \times 10^{-6}$ | 0.000000 | $2.199594 \times 10^{-4}$ | $4.593269 \times 10^{-4}$ |
| 19 | $4.528575 \times 10^{-5}$ | 0.000000 | $1.274470 \times 10^{-3}$ | $1.196838 \times 10^{-3}$ |
| 20 | $3.558166 \times 10^{-4}$ | 0.000000 | $2.561880 \times 10^{-3}$ | $2.814186 \times 10^{-3}$ |
| 21 | 0.000000 | 0.000000 | $2.005512 \times 10^{-4}$ | $2.328982 \times 10^{-4}$ |
| 22 | $2.911227 \times 10^{-4}$ | 0.000000 | $1.953757 \times 10^{-3}$ | $2.600696 \times 10^{-3}$ |

**Table 7.** Overview of the probabilities of new books not specified as signed and first edition for all categories (rows) and formats (columns)

| Category | Other format | Mixed lot | Softcover | Hardcover |
|---|---|---|---|---|
| 1 | $3.364085 \times 10^{-4}$ | $0.000000$ | $4.366840 \times 10^{-3}$ | $9.568233 \times 10^{-3}$ |
| 2 | $5.693066 \times 10^{-4}$ | $1.293879 \times 10^{-5}$ | $9.315926 \times 10^{-4}$ | $1.022164 \times 10^{-3}$ |
| 3 | $9.121845 \times 10^{-4}$ | $7.116333 \times 10^{-5}$ | $5.654250 \times 10^{-3}$ | $6.779924 \times 10^{-3}$ |
| 4 | $7.116333 \times 10^{-5}$ | $1.940818 \times 10^{-5}$ | $5.822454 \times 10^{-4}$ | $8.022048 \times 10^{-4}$ |
| 5 | $1.940818 \times 10^{-4}$ | $6.469393 \times 10^{-6}$ | $1.694981 \times 10^{-3}$ | $1.779083 \times 10^{-3}$ |
| 6 | $2.328982 \times 10^{-4}$ | $3.234697 \times 10^{-5}$ | $3.137656 \times 10^{-3}$ | $3.855758 \times 10^{-3}$ |
| 7 | $2.199594 \times 10^{-4}$ | $6.469393 \times 10^{-6}$ | $3.001798 \times 10^{-3}$ | $3.157064 \times 10^{-3}$ |
| 8 | $9.057151 \times 10^{-5}$ | $0.000000$ | $3.428778 \times 10^{-4}$ | $7.310414 \times 10^{-4}$ |
| 9 | $0.000000$ | $0.000000$ | $1.423267 \times 10^{-4}$ | $6.986945 \times 10^{-4}$ |
| 10 | $0.000000$ | $0.000000$ | $1.293879 \times 10^{-4}$ | $1.229185 \times 10^{-4}$ |
| 11 | $1.293879 \times 10^{-5}$ | $0.000000$ | $4.075718 \times 10^{-4}$ | $3.558166 \times 10^{-4}$ |
| 12 | $2.587757 \times 10^{-4}$ | $2.587757 \times 10^{-5}$ | $4.218044 \times 10^{-3}$ | $4.593269 \times 10^{-3}$ |
| 13 | $5.175515 \times 10^{-5}$ | $0.000000$ | $6.340005 \times 10^{-4}$ | $8.086742 \times 10^{-4}$ |
| 14 | $5.757760 \times 10^{-4}$ | $6.469393 \times 10^{-5}$ | $7.491557 \times 10^{-3}$ | $1.683336 \times 10^{-2}$ |
| 15 | $4.528575 \times 10^{-5}$ | $0.000000$ | $4.011024 \times 10^{-4}$ | $8.410211 \times 10^{-5}$ |
| 16 | $1.099797 \times 10^{-4}$ | $0.000000$ | $7.827966 \times 10^{-4}$ | $8.086742 \times 10^{-4}$ |
| 17 | $2.587757 \times 10^{-5}$ | $0.000000$ | $9.057151 \times 10^{-5}$ | $6.469393 \times 10^{-6}$ |
| 18 | $1.617348 \times 10^{-4}$ | $6.469393 \times 10^{-6}$ | $2.988860 \times 10^{-3}$ | $2.937105 \times 10^{-3}$ |
| 19 | $1.940818 \times 10^{-4}$ | $2.587757 \times 10^{-5}$ | $1.126968 \times 10^{-2}$ | $6.715230 \times 10^{-3}$ |
| 20 | $9.380620 \times 10^{-4}$ | $3.234697 \times 10^{-5}$ | $7.038700 \times 10^{-3}$ | $6.042413 \times 10^{-3}$ |
| 21 | $5.822454 \times 10^{-5}$ | $0.000000$ | $3.946330 \times 10^{-4}$ | $5.563678 \times 10^{-4}$ |
| 22 | $8.086742 \times 10^{-4}$ | $2.523063 \times 10^{-4}$ | $1.195544 \times 10^{-2}$ | $8.947171 \times 10^{-3}$ |

**Table 8.** Overview of the probabilities of books specified as `Buy It Now` items for all categories (rows)

| Category | Buy It Now |
| --- | --- |
| 1 | 0.148564 |
| 2 | 0.137605 |
| 3 | 0.143228 |
| 4 | 0.162313 |
| 5 | 0.116194 |
| 6 | 0.177406 |
| 7 | 0.120268 |
| 8 | 0.136015 |
| 9 | 0.173759 |
| 10 | 0.314286 |
| 11 | 0.186170 |
| 12 | 0.120437 |
| 13 | 0.127722 |
| 14 | 0.169996 |
| 15 | 0.133903 |
| 16 | 0.101506 |
| 17 | 0.043605 |
| 18 | 0.164380 |
| 19 | 0.182857 |
| 20 | 0.135013 |
| 21 | 0.113257 |
| 22 | 0.138597 |

**Table 9.** Overview of the probabilities of book items for all categories (rows) having 0 to 3 bids (columns)

| Category | 0 bids | 1 bid | 2 bids | 3 bids |
|---|---|---|---|---|
| 1 | 0.890891 | 0.071063 | 0.014498 | 0.006369 |
| 2 | 0.856820 | 0.094574 | 0.019970 | 0.007913 |
| 3 | 0.814717 | 0.119045 | 0.024911 | 0.011824 |
| 4 | 0.840345 | 0.117584 | 0.014024 | 0.011866 |
| 5 | 0.873781 | 0.092817 | 0.014189 | 0.007685 |
| 6 | 0.861950 | 0.089958 | 0.017621 | 0.009009 |
| 7 | 0.847547 | 0.100000 | 0.020377 | 0.007170 |
| 8 | 0.871152 | 0.093501 | 0.018244 | 0.004561 |
| 9 | 0.891566 | 0.096386 | 0.008032 | 0.004016 |
| 10 | 0.835294 | 0.129412 | 0.023529 | 0.011765 |
| 11 | 0.866071 | 0.088170 | 0.014509 | 0.008929 |
| 12 | 0.866002 | 0.091617 | 0.016828 | 0.007323 |
| 13 | 0.868811 | 0.081164 | 0.012762 | 0.010720 |
| 14 | 0.884921 | 0.082129 | 0.012797 | 0.006242 |
| 15 | 0.887755 | 0.085034 | 0.017007 | 0.006803 |
| 16 | 0.880086 | 0.092077 | 0.009279 | 0.009279 |
| 17 | 0.815884 | 0.133574 | 0.021661 | 0.007220 |
| 18 | 0.823123 | 0.106225 | 0.023715 | 0.014575 |
| 19 | 0.859675 | 0.096295 | 0.017726 | 0.008275 |
| 20 | 0.850492 | 0.103581 | 0.018532 | 0.008505 |
| 21 | 0.828596 | 0.115872 | 0.024486 | 0.009182 |
| 22 | 0.869191 | 0.090694 | 0.016185 | 0.008324 |

**Table 10.** Overview of the probabilities of book items for all categories (rows) having 4 to 7 bids (columns)

| Category | 4 bids | 5 bids | 6 bids | 7 bids |
|---|---|---|---|---|
| 1 | 0.005950 | 0.003771 | 0.001760 | 0.001844 |
| 2 | 0.004145 | 0.006405 | 0.003391 | 0.001507 |
| 3 | 0.008151 | 0.005510 | 0.004018 | 0.003444 |
| 4 | 0.004315 | 0.004315 | 0.002157 | 0.003236 |
| 5 | 0.003547 | 0.001774 | 0.002069 | 0.001478 |
| 6 | 0.005962 | 0.004240 | 0.002782 | 0.002517 |
| 7 | 0.006226 | 0.006604 | 0.005472 | 0.002075 |
| 8 | 0.003421 | 0.003421 | 0.000000 | 0.003421 |
| 9 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 10 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 11 | 0.006696 | 0.002232 | 0.005580 | 0.001116 |
| 12 | 0.005142 | 0.004363 | 0.004207 | 0.000779 |
| 13 | 0.008167 | 0.005615 | 0.003573 | 0.002552 |
| 14 | 0.003924 | 0.003567 | 0.002051 | 0.001204 |
| 15 | 0.001701 | 0.000000 | 0.000000 | 0.000000 |
| 16 | 0.001428 | 0.002855 | 0.001428 | 0.000714 |
| 17 | 0.003610 | 0.003610 | 0.000000 | 0.003610 |
| 18 | 0.008646 | 0.006423 | 0.005435 | 0.004447 |
| 19 | 0.006263 | 0.004289 | 0.001860 | 0.001632 |
| 20 | 0.005103 | 0.004834 | 0.003133 | 0.002596 |
| 21 | 0.006996 | 0.004373 | 0.004373 | 0.001749 |
| 21 | 0.005318 | 0.003295 | 0.002081 | 0.001561 |

**Table 11.** Overview of the probabilities of book items for all categories (rows) having 8 to 20 bids (columns)

| Category | 8 bids | 9 bids | 10 bids | 11 to 20 bids |
|---|---|---|---|---|
| 1 | 0.001006 | 0.000251 | 0.000587 | 0.001676 |
| 2 | 0.001130 | 0.000377 | 0.000754 | 0.002261 |
| 3 | 0.001492 | 0.000918 | 0.001837 | 0.004018 |
| 4 | 0.000000 | 0.001079 | 0.000000 | 0.001079 |
| 5 | 0.000000 | 0.000591 | 0.000591 | 0.000887 |
| 6 | 0.001722 | 0.001325 | 0.001325 | 0.001590 |
| 7 | 0.001698 | 0.000755 | 0.000943 | 0.001132 |
| 8 | 0.000000 | 0.001140 | 0.001140 | 0.000000 |
| 9 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 10 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 11 | 0.000000 | 0.002232 | 0.001116 | 0.002232 |
| 12 | 0.000779 | 0.000779 | 0.001246 | 0.000779 |
| 13 | 0.001531 | 0.001021 | 0.001531 | 0.002042 |
| 14 | 0.000803 | 0.000713 | 0.000535 | 0.000936 |
| 15 | 0.000000 | 0.000000 | 0.000000 | 0.001701 |
| 16 | 0.000000 | 0.000000 | 0.001428 | 0.000000 |
| 17 | 0.000000 | 0.003610 | 0.003610 | 0.003610 |
| 18 | 0.000988 | 0.000988 | 0.003211 | 0.001976 |
| 19 | 0.000797 | 0.000797 | 0.000683 | 0.001291 |
| 20 | 0.000269 | 0.000806 | 0.001343 | 0.000448 |
| 21 | 0.000000 | 0.000875 | 0.002186 | 0.001312 |
| 22 | 0.000983 | 0.000694 | 0.000751 | 0.000751 |

**Table 12.** Overview of the probabilities of book items for all categories (rows) having 21 or more bids (columns)

| Category | 21 to 30 bids | 31 to 40 bids | 41 to 50 bids | 51 or more bids |
| --- | --- | --- | --- | --- |
| 1 | 0.000000 | 0.000000 | 0.000335 | 0.000000 |
| 2 | 0.000754 | 0.000000 | 0.000000 | 0.000000 |
| 3 | 0.000000 | 0.000000 | 0.000000 | 0.000115 |
| 4 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 5 | 0.000591 | 0.000000 | 0.000000 | 0.000000 |
| 6 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 7 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 8 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 9 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 10 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 11 | 0.001116 | 0.000000 | 0.000000 | 0.000000 |
| 12 | 0.000000 | 0.000156 | 0.000000 | 0.000000 |
| 13 | 0.000000 | 0.000000 | 0.000510 | 0.000000 |
| 14 | 0.000000 | 0.000000 | 0.000178 | 0.000000 |
| 15 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 16 | 0.001428 | 0.000000 | 0.000000 | 0.000000 |
| 17 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 18 | 0.000000 | 0.000000 | 0.000247 | 0.000000 |
| 19 | 0.000000 | 0.000000 | 0.000418 | 0.000000 |
| 20 | 0.000358 | 0.000000 | 0.000000 | 0.000000 |
| 21 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 22 | 0.000173 | 0.000000 | 0.000000 | 0.000000 |

**Table 13.** Overview of the probabilities of book items for all categories (rows) with prices up to $4.00 (columns)

| Category | $0.00 to $1.00 | $1.01 to $2.00 | $2.01 to $3.00 | $3.01 to $4.00 |
|---|---|---|---|---|
| 1 | 0.348310 | 0.166654 | 0.112922 | 0.076866 |
| 2 | 0.234263 | 0.191902 | 0.099524 | 0.109051 |
| 3 | 0.130725 | 0.166924 | 0.103796 | 0.085752 |
| 4 | 0.167331 | 0.121293 | 0.084108 | 0.100930 |
| 5 | 0.212462 | 0.205325 | 0.114741 | 0.108976 |
| 6 | 0.276529 | 0.182667 | 0.105103 | 0.084285 |
| 7 | 0.244309 | 0.201649 | 0.117584 | 0.103155 |
| 8 | 0.221018 | 0.118093 | 0.080715 | 0.098050 |
| 9 | 0.381329 | 0.087025 | 0.050633 | 0.037975 |
| 10 | 0.250000 | 0.096591 | 0.113636 | 0.125000 |
| 11 | 0.206428 | 0.138072 | 0.101856 | 0.081938 |
| 12 | 0.184201 | 0.170094 | 0.110105 | 0.093919 |
| 13 | 0.245265 | 0.241477 | 0.119555 | 0.097301 |
| 14 | 0.323060 | 0.180753 | 0.121580 | 0.089227 |
| 15 | 0.164268 | 0.179509 | 0.101609 | 0.137172 |
| 16 | 0.100659 | 0.135370 | 0.077751 | 0.114196 |
| 17 | 0.098446 | 0.150259 | 0.056995 | 0.111399 |
| 18 | 0.246017 | 0.144734 | 0.148769 | 0.109559 |
| 19 | 0.385819 | 0.191166 | 0.117430 | 0.077845 |
| 20 | 0.210760 | 0.181639 | 0.098466 | 0.092014 |
| 21 | 0.228517 | 0.132477 | 0.103623 | 0.078559 |
| 22 | 0.255159 | 0.254382 | 0.101731 | 0.098515 |

**Table 14.** Overview of the probabilities of book items for all categories (rows) with prices ranging from $4.01 to $8.00 (columns)

| Category | $4.01 to $5.00 | $5.01 to $6.00 | $6.01 to $7.00 | $7.01 to $8.00 |
|---|---|---|---|---|
| 1 | 0.068251 | 0.039918 | 0.026253 | 0.025473 |
| 2 | 0.075536 | 0.044573 | 0.027731 | 0.034536 |
| 3 | 0.080234 | 0.050712 | 0.038020 | 0.042931 |
| 4 | 0.117751 | 0.067729 | 0.056662 | 0.048694 |
| 5 | 0.087977 | 0.059017 | 0.032116 | 0.030744 |
| 6 | 0.074114 | 0.044492 | 0.029562 | 0.026291 |
| 7 | 0.073400 | 0.047679 | 0.028410 | 0.031278 |
| 8 | 0.087216 | 0.060130 | 0.029252 | 0.033044 |
| 9 | 0.083861 | 0.036392 | 0.041139 | 0.050633 |
| 10 | 0.130682 | 0.079545 | 0.034091 | 0.028409 |
| 11 | 0.095971 | 0.035763 | 0.034405 | 0.038026 |
| 12 | 0.076398 | 0.050932 | 0.033484 | 0.034375 |
| 13 | 0.069839 | 0.044744 | 0.021780 | 0.026278 |
| 14 | 0.070070 | 0.037586 | 0.026428 | 0.025606 |
| 15 | 0.080440 | 0.060965 | 0.029636 | 0.049958 |
| 16 | 0.091982 | 0.078445 | 0.031586 | 0.049983 |
| 17 | 0.103627 | 0.077720 | 0.044041 | 0.056995 |
| 18 | 0.085558 | 0.057832 | 0.039727 | 0.028243 |
| 19 | 0.057004 | 0.032857 | 0.023640 | 0.019449 |
| 20 | 0.096204 | 0.049024 | 0.032682 | 0.035993 |
| 21 | 0.088037 | 0.052022 | 0.064027 | 0.061710 |
| 22 | 0.065713 | 0.049767 | 0.021895 | 0.031007 |

**Table 15.** Overview of the probabilities of book items for all categories (rows) with prices ranging from $8.01 to $30.00 (columns)

| Category | $8.01 to $9.00 | $9.01 to $10.00 | $10.01 to $20.00 | $20.01 to $30.00 |
|---|---|---|---|---|
| 1 | 0.017118 | 0.035797 | 0.047234 | 0.013702 |
| 2 | 0.016672 | 0.048826 | 0.064137 | 0.019224 |
| 3 | 0.026708 | 0.052202 | 0.107604 | 0.046904 |
| 4 | 0.036299 | 0.076140 | 0.078353 | 0.019035 |
| 5 | 0.021136 | 0.043234 | 0.053253 | 0.014411 |
| 6 | 0.016893 | 0.039317 | 0.058589 | 0.022841 |
| 7 | 0.021061 | 0.032443 | 0.065782 | 0.015056 |
| 8 | 0.028169 | 0.070423 | 0.086132 | 0.029252 |
| 9 | 0.034810 | 0.104430 | 0.071203 | 0.009494 |
| 10 | 0.022727 | 0.039773 | 0.056818 | 0.011364 |
| 11 | 0.044364 | 0.095971 | 0.071073 | 0.022182 |
| 12 | 0.028955 | 0.058876 | 0.077957 | 0.029475 |
| 13 | 0.018939 | 0.036932 | 0.047585 | 0.012311 |
| 14 | 0.016036 | 0.035979 | 0.049099 | 0.012485 |
| 15 | 0.033870 | 0.055885 | 0.055038 | 0.024555 |
| 16 | 0.045817 | 0.091982 | 0.108990 | 0.032280 |
| 17 | 0.082902 | 0.054404 | 0.090674 | 0.044041 |
| 18 | 0.017277 | 0.032899 | 0.057107 | 0.014277 |
| 19 | 0.011525 | 0.024688 | 0.041828 | 0.009266 |
| 20 | 0.020112 | 0.052837 | 0.070980 | 0.025895 |
| 21 | 0.021904 | 0.046967 | 0.064869 | 0.029697 |
| 22 | 0.018224 | 0.029587 | 0.045881 | 0.011604 |

**Table 16.** Overview of the probabilities of book items for all categories (rows) with prices from $30.01

| Category | $30.01 to $40.00 | $40.01 to $50.00 | $50.01 to $100.00 | $100.00 or more |
|---|---|---|---|---|
| 1 | 0.005830 | 0.005199 | 0.006276 | 0.004196 |
| 2 | 0.006975 | 0.006125 | 0.011398 | 0.009527 |
| 3 | 0.024004 | 0.013575 | 0.019589 | 0.010319 |
| 4 | 0.006197 | 0.005755 | 0.007525 | 0.006197 |
| 5 | 0.005078 | 0.003843 | 0.005764 | 0.001921 |
| 6 | 0.012788 | 0.008506 | 0.011301 | 0.006721 |
| 7 | 0.005736 | 0.004571 | 0.006722 | 0.001165 |
| 8 | 0.012459 | 0.014085 | 0.021668 | 0.010293 |
| 9 | 0.004747 | 0.001582 | 0.003165 | 0.001582 |
| 10 | 0.005682 | 0.000000 | 0.000000 | 0.005682 |
| 11 | 0.012675 | 0.009507 | 0.007696 | 0.004074 |
| 12 | 0.015591 | 0.010840 | 0.015963 | 0.008835 |
| 13 | 0.005208 | 0.003314 | 0.006155 | 0.003314 |
| 14 | 0.004766 | 0.002523 | 0.003420 | 0.001383 |
| 15 | 0.008467 | 0.009314 | 0.003387 | 0.005927 |
| 16 | 0.009719 | 0.011107 | 0.013190 | 0.006942 |
| 17 | 0.018135 | 0.005181 | 0.005181 | 0.000000 |
| 18 | 0.009311 | 0.003621 | 0.004138 | 0.000931 |
| 19 | 0.003291 | 0.001965 | 0.001948 | 0.000278 |
| 20 | 0.011565 | 0.007039 | 0.009134 | 0.005657 |
| 21 | 0.013269 | 0.005265 | 0.005687 | 0.003370 |
| 22 | 0.005976 | 0.003082 | 0.004690 | 0.002787 |

**Table 17.** Overview of the number of items for all categories (rows)

| Category | Number of items |
|----------|-----------------|
| 1        | $12,244$        |
| 2        | $2,833$         |
| 3        | $8,951$         |
| 4        | $9,84$          |
| 5        | $3,514$         |
| 6        | $7,799$         |
| 7        | $5,456$         |
| 8        | 911             |
| 9        | 262             |
| 10       | 103             |
| 11       | 911             |
| 12       | $6,673$         |
| 13       | $2,051$         |
| 14       | $23,292$        |
| 15       | 627             |
| 16       | $1,450$         |
| 17       | 271             |
| 18       | $4,142$         |
| 19       | $27,172$        |
| 20       | $11,407$        |
| 21       | 2343            |
| 22       | $18,206$        |