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# A Model for Large $\theta_{13}$ Constructed using the Eigenvectors of the $S_4$ Rotation Matrices

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**Abstract.** A procedure for using the eigenvectors of the elements of the representations of a discrete group in model building is introduced and is used to construct a model that produces a large reactor mixing angle,  $\sin^2 \theta_{13} = \frac{2}{3} \sin^2 \frac{\pi}{16}$ , in agreement with recent neutrino oscillation observations. The model fully constrains the neutrino mass ratios and predicts normal hierarchy with the light neutrino mass,  $m_1 \approx 25$  meV. Motivated by the model, a new mixing ansatz is postulated which predicts all the mixing angles within  $1\sigma$  errors.

## 1. Introduction

We use the group  $SU(3)$  and its discrete subgroup  $S_4$  for model building. Both of these groups had been studied extensively as flavour symmetry groups. The  $S_4$  group has the presentation

$$\langle a, b | a^2 = b^3 = (ab)^4 = e \rangle. \quad (1)$$

$S_4$  is the symmetry group of the cube (Fig. 1) and the elements of the group can be represented as the orientation preserving rotations of the cube. The matrices representing the generators can be written as

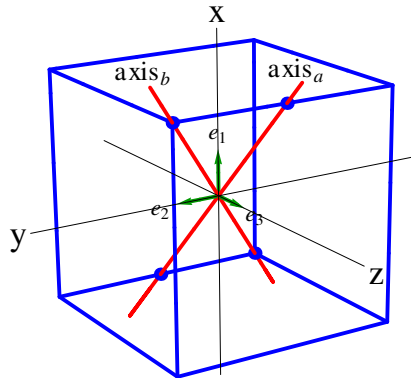
$$a = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \quad (2)$$

Here the basis vectors  $e_1 = (1, 0, 0)^T$ ,  $e_2 = (0, 1, 0)^T$  and  $e_3 = (0, 0, 1)^T$  form the symmetry axes of the cube passing through face centres. If we define the left-handed leptons,  $L = (L_e, L_\mu, L_\tau)^T$  where  $L_e = (\nu_{eL}, e_L)^T$  etc., as a triplet in this basis, the flavour states  $L_e, L_\mu, L_\tau$  correspond to  $e_1, e_2, e_3$  respectively. Usually in models a set of flavons are introduced whose vacuum expectation values (vevs) produce the desired texture for the fermion mass matrices. In other words, the orientation of fermion flavour states as well as the flavon vevs in the flavour space determines the form of the mass matrices.

Axes of the orientation preserving rotations of the cube are nothing but eigenvectors of the corresponding rotation matrices with eigenvalue equal to +1. The basis vectors  $e_1, e_2$  and  $e_3$  are examples. There are also other vectors like the ones passing through the opposite edge centres (e.g. axis<sub>a</sub> in Fig. 1) and the ones passing through the opposite vertices (e.g. axis<sub>b</sub> in Fig. 1). Compared to vectors pointing in random directions, these vectors are “special” in the context of the  $S_4$  symmetry. The rotation matrices are unitary and so their eigenvalues in general are complex numbers with unit modulus. If non-degenerate, these eigenvalues also correspond to



unique eigenvectors and the author argues that these eigenvectors are also “special” like the rotation axes.



**Figure 1.** The generators  $a$  and  $b$  represent  $\pi$ -rotation about axis $_a$  and  $\frac{2\pi}{3}$ -rotation about axis $_b$  respectively.

As an example consider  $v = \frac{1}{\sqrt{3}}(1, \bar{\omega}, \omega)^T$ , the normalised ( $v^\dagger v = 1$ ) eigenvector of the matrix  $b$  in Eq. (5), corresponding to the eigenvalue  $\omega$ , where  $\omega = e^{i\frac{2\pi}{3}}$  and  $\bar{\omega} = e^{-i\frac{2\pi}{3}}$ . Since  $e^{i\theta}v$ , where  $e^{i\theta}$  is an arbitrary phase, is also a normalised eigenvector, we impose the following condition to uniquely fix the phase: The component of the eigenvector in the direction of one of the basis vectors should have zero phase ie.  $\arg(v^\dagger e_i) = 0$ , where  $i = 1, 2$  or  $3$ . This is intuitive since the basis vectors are used to define the fermion flavour states. Thus the allowed choices for  $v$  are  $\frac{1}{\sqrt{3}}(1, \bar{\omega}, \omega)^T$ ,  $\frac{1}{\sqrt{3}}(\omega, 1, \bar{\omega})^T$  and  $\frac{1}{\sqrt{3}}(\bar{\omega}, \omega, 1)^T$ .

Let  $g$  represent an element of the group and  $l$  be one of its non-degenerate eigenvalues. The corresponding “special” eigenvector,  $\text{eig}(g, l)_i$ , is defined using the following normalisation condition and the phase condition:

$$\text{eig}(g, l)_i^\dagger \text{eig}(g, l)_i = 1, \quad \arg\left(\text{eig}(g, l)_i^\dagger e_i\right) = 0. \quad (3)$$

For example, the basis vector  $e_3$  is  $\text{eig}(ab, 1)_3$ . Such eigenvectors which will be used later in the model are listed below:

$$\begin{aligned} \text{eig}(b, 1)_1 &= \frac{1}{\sqrt{3}}(1, 1, 1)^T, & \text{eig}(a, 1)_1 &= \frac{1}{\sqrt{2}}(1, 0, 1)^T, \\ \text{eig}(b, \omega)_1 &= \frac{1}{\sqrt{3}}(1, \bar{\omega}, \omega)^T, & \text{eig}(c, i)_1 &= \frac{1}{\sqrt{2}}(1, 0, i)^T, \\ \text{eig}(b, \bar{\omega})_1 &= \frac{1}{\sqrt{3}}(1, \omega, \bar{\omega})^T, & \text{eig}(c, i)_3 &= \frac{1}{\sqrt{2}}(-i, 0, 1)^T, \\ \text{eig}(d, 1)_1 &= \frac{1}{\sqrt{2}}(1, 0, -1)^T, & \text{eig}(c, -i)_1 &= \frac{1}{\sqrt{2}}(1, 0, -i)^T, \\ \text{eig}(d, 1)_3 &= \frac{1}{\sqrt{2}}(-1, 0, 1)^T, & \text{eig}(c, -i)_3 &= \frac{1}{\sqrt{2}}(i, 0, 1)^T \end{aligned} \quad (4)$$

where

$$c = bab = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad d = a(bab)^2 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}. \quad (5)$$

## 2. The Model

Daya Bay reactor neutrino experiment [1] showed that the reactor mixing angle,  $\theta_{13}$ , is non-zero. In this paper we propose a flavour model to accommodate the observed non-zero  $\theta_{13}$ . We use the Standard Model (SM) framework with the addition of the right-handed neutrino triplet,  $\nu_R = (\nu_{1R}, \nu_{2R}, \nu_{3R})^T$  in the context of the type-1 seesaw mechanism. We postulate a global flavour symmetry group,

$$G_f = SU(3)_1 \times SU(3)_2 \times U(1)_f. \quad (6)$$

The fermion fields and a set of postulated flavons belong to specific representations of  $G_f$  as shown in Table 1. The  $U(1)$  group is introduced to ensure that the flavons couple to only the desired fermions. We write the mass terms at the lowest order, containing the fermions and the minimum number of flavons, invariant under  $G_f$  and the SM gauge group. The eigenvectors of the elements of the  $S_4$  subgroup of the  $SU(3)$  group are used to construct the vevs of the flavons. The vevs break the flavour symmetry and the required mass matrices are obtained.

	$L$	$e_R$	$\mu_R$	$\tau_R$	$\nu_R$	$\phi_e$	$\phi_\mu$	$\phi_\tau$	$\phi$	$\xi_1$	$\xi_2$
$SU(3)_1$	<b>3</b>	<b>1</b>	<b>1</b>	<b>1</b>	$\bar{\mathbf{3}}$	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>6</b>	<b>1</b>
$SU(3)_2$	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>3</b>	<b>1</b>	$\bar{\mathbf{6}}$
$U(1)_f$	$f_l$	$f_l + f_e$	$f_l + f_\mu$	$f_l + f_\tau$	0	$-f_e$	$-f_\mu$	$-f_\tau$	0	$f_l$	0

**Table 1.** The fields  $\phi_e, \phi_\mu, \phi_\tau, \phi, \xi_1, \xi_2$  are the flavons. For the  $U(1)$  group the tabulated values are the generators, e.g.  $f_l \equiv e^{if_l\theta}$ . The SM Higgs is a flavour singlet.

The tensor product expansion of the fundamental representations of  $SU(3)$  are

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}, \quad \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \bar{\mathbf{6}}, \quad \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}. \quad (7)$$

For the charged leptons, the lowest order mass term is

$$\bar{L}(y_e \phi_e e_R + y_\mu \phi_\mu \mu_R + y_\tau \phi_\tau \tau_R) H \frac{1}{\Lambda_{\phi_l}} + H.C. \quad (8)$$

where  $y_\alpha$  are the coupling constants,  $H$  is the SM Higgs,  $\Lambda_{\phi_l}$  is the cut-off scale for the flavons  $\phi_e, \phi_\mu$  and  $\phi_\tau$ . The  $S_4$  group has four irreducible representations: **1**, **2**, **3** and **3'** [2]. The orientation preserving rotations of the cube discussed earlier belong to **3'**. The flavon triplets  $\phi_e, \phi_\mu$  and  $\phi_\tau$  belong to the representation **3** of  $SU(3)_1$ . The restriction of the representation **3** of  $SU(3)$  to its subgroup  $S_4$  is the representation **3'** of  $S_4$ . Therefore the ‘‘special’’ eigenvectors of the representation matrices of **3'** of  $S_4$  are used to construct the vevs of the flavons. We assign:

$$\langle \phi_e \rangle = \text{eig}(b, 1)_1, \quad \langle \phi_\mu \rangle = \text{eig}(b, \omega)_1, \quad \langle \phi_\tau \rangle = \text{eig}(b, \bar{\omega})_1 \quad (9)$$

where the angular brackets are used to denote vevs. We do not discuss the mechanism of flavour symmetry breaking in this paper. To avoid Goldstone bosons, it is necessary to add explicit symmetry breaking terms for the flavon potentials, which break the continuous flavour group  $G_f$ , Eq. (6), into an unknown discrete group. The flavon vevs spontaneously break this discrete flavour symmetry. Also the Higgs vev,  $(0, h_o)^T$ , breaks the weak gauge symmetry. After the symmetry breaking, the charged-lepton mass term, Eq. (8), takes the form

$$\bar{l}_L \mathcal{T}^\dagger M_d l_R + H.C. \quad (10)$$

where

$$\mathcal{T}^\dagger = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \bar{\omega} & \omega \\ 1 & \omega & \bar{\omega} \end{pmatrix}, \quad (11)$$

$l_L = (e_L, \mu_L, \tau_L)^T$ ,  $l_R = (e_R, \mu_R, \tau_R)^T$  and  $M_d = \text{diag}(m_e, m_\mu, m_\tau)$  with  $m_e = \frac{y_e h_o}{\Lambda_{\phi_e}}$  etc. The charged-lepton mass matrix,  $\mathcal{T}^\dagger M_d$ , when left-multiplied with  $\mathcal{T}$ , is diagonalised giving the charged-lepton masses  $m_e, m_\mu, m_\tau$ .  $\mathcal{T}$  is the Trimaximal mixing matrix [3]. For later use we define  $2 \times 2$  maximal matrices,  $\mathcal{B}_2$  and  $\mathcal{B}_3$ :

$$\mathcal{B}_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \mathcal{B}_3 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (12)$$

For the neutrinos, the lowest order Dirac mass term is

$$2y_w \bar{L} \xi_1 \nu_R \tilde{H} \frac{1}{\Lambda_{\xi_1}} + H.C. \quad (13)$$

where  $\tilde{H}$  is the conjugate Higgs and  $y_w$  is the coupling. The flavon  $\xi_1$  belongs to the representation  $\mathbf{6}$  of  $SU(3)_1$  and can be written as a  $3 \times 3$  complex-symmetric matrix. The restriction of  $\mathbf{6}$  of  $SU(3)$  to the  $S_4$  subgroup is the direct sum of  $\mathbf{1}$ ,  $\mathbf{2}$  and  $\mathbf{3}$  of  $S_4$ . We assign a very simple choice of vev for  $\xi_1$  where  $\mathbf{2}$  and  $\mathbf{3}$  parts vanish, ie.  $\langle \xi_1 \rangle$  becomes the identity when written in the matrix form. After the symmetry breaking, the Dirac mass term, Eq. (13), takes the form

$$m_w \left( \bar{\nu}_L \nu_R + \overline{(\nu_R)^c} (\nu_L)^c \right) + H.C. \quad (14)$$

where  $\nu_L = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})^T$ ,  $m_w = \frac{y_w h_o}{\Lambda_{\xi_1}}$ .

The lowest order Majorana mass term for the neutrinos is

$$y_G \overline{(\nu_R)^c} \phi \xi_2 \phi^T \nu_R \frac{1}{\Lambda_{\xi_2} \Lambda_\phi^2} + H.C. \quad (15)$$

Note that  $\phi$  transforms as a  $\mathbf{3}$  under both  $SU(3)_1$  and  $SU(3)_2$ . Therefore  $\phi$  can be written as a  $3 \times 3$  matrix,  $\phi_{ij}$ , the row index  $i$  representing  $SU(3)_1$  and the column index  $j$  representing  $SU(3)_2$ . The flavon  $\xi_2$  belongs to the representation  $\bar{\mathbf{6}}$  of  $SU(3)_2$ . A  $\bar{\mathbf{6}}$  of  $SU(3)$ , just like a  $\mathbf{6}$ , contains a  $\mathbf{1}$ , a  $\mathbf{2}$  and a  $\mathbf{3}$  of the  $S_4$  subgroup. As was done earlier for the case of  $\langle \xi_1 \rangle$ , here we assign  $\langle \xi_2 \rangle$  also to be equal to the identity. After the symmetry breaking, the Majorana mass term, Eq. (15), takes the form

$$m_G \overline{(\nu_R)^c} \langle \phi \rangle \langle \phi \rangle^T \nu_R + H.C. \quad (16)$$

where  $m_G = \frac{y_G}{\Lambda_{\xi_2} \Lambda_\phi^2}$ . The matrix  $\langle \phi \rangle \langle \phi \rangle^T$  is complex-symmetric and  $\langle \phi \rangle \langle \phi \rangle^T$  contains all the interesting physics in our model. The reason why we use two  $SU(3)$ s in the flavour group  $G_f$ , Eq. (6), is to ensure that the mass matrix, Eq. (16), contains the symmetric product of two  $\langle \phi \rangle$ s.

To assign vev for the flavon  $\phi_{ij}$ , we use  $(S_4)_1 \times (S_4)_2$ , the subgroup of  $SU(3)_1 \times SU(3)_2$ . The group  $(S_4)_1 \times (S_4)_2$  has  $24 \times 24 = 576$  elements. Let  $g_1$  and  $g_2$  be the elements of  $(S_4)_1$  and  $(S_4)_2$  respectively. If  $v_1$  and  $v_2$  are the eigenvectors of  $g_1$  and  $g_2$  corresponding to the eigenvalues  $a_1$  and  $a_2$ , then the direct product  $v_1 \times v_2$  will be an eigenvector of  $g_1 \times g_2$  with an eigenvalue  $a_1 a_2$ . Now we make the following assumption:

$$\langle \phi \rangle = v_1^1 \times v_2^1 + v_1^2 \times v_2^2 + v_1^3 \times v_2^3 + v_1^4 \times v_2^4 \quad (17)$$

where the RHS of Eq. (17) is the sum of four eigenvectors. Based on the choices for  $v_1^i \times v_2^i$ s we get a set of similar cases of solutions described in the following sections. The assumed form of  $\langle \phi \rangle$  given in Eq. (17) and the choices for  $v_1^i \times v_2^i$ s were obtained through educated guesses and also through trial and error to fit the experimental data.

### 2.1. Case 1

Here we assign

$$\begin{aligned} v_1^1 &= e_2, & v_1^2 &= \text{eig}(a, 1)_1, & v_1^3 &= \text{eig}(d, 1)_1, & v_1^4 &= \text{eig}(d, 1)_3, \\ v_2^1 &= e_2, & v_2^2 &= e_1, & v_2^3 &= e_1, & v_2^4 &= \text{eig}(c, -i)_1. \end{aligned} \quad (18)$$

Using Eq. (17), Eqs. (18) and Eqs. (4), we get

$$\langle \phi \rangle = \begin{pmatrix} -\frac{1}{2} + \sqrt{2} & 0 & \frac{i}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & -\frac{i}{2} \end{pmatrix} \quad (19)$$

in matrix form, where the row and the column indices of the matrix correspond to the  $(S_4)_1$  and the  $(S_4)_2$  indices respectively. If  $m_G \gg m_w$ , the Majorana mass matrix,  $m_G \langle \phi \rangle \langle \phi \rangle^T$  from Eq. (16), becomes much larger than the Dirac mass matrix,  $m_w \times \text{Identity}$  from Eq. (14), resulting in the type-1 see-saw mechanism. We get an effective see-saw mass matrix [4],  $M_{ss}$ , proportional to the inverse of  $\langle \phi \rangle \langle \phi \rangle^T$ :

$$M_{ss} = -k (\langle \phi \rangle \langle \phi \rangle^T)^{-1} = -k \begin{pmatrix} 2 - \sqrt{2} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}^{-1} \quad \text{where } k = \frac{m_w^2}{m_G}. \quad (20)$$

$M_{ss}$  is diagonalised using the unitary matrix  $U_\nu$ :

$$U_\nu^\dagger M_{ss} U_\nu^* = -\text{diag}(m_1, m_2, m_3) \quad (21)$$

where the neutrino masses  $m_1, m_2, m_3$  are given by

$$m_1 = \frac{k(2 + \sqrt{2})}{1 + \sqrt{2(2 + \sqrt{2})}}, \quad m_2 = k, \quad m_3 = \frac{k(2 + \sqrt{2})}{-1 + \sqrt{2(2 + \sqrt{2})}} \quad (22)$$

and

$$U_\nu = \mathcal{B}_2 \mathcal{I} \mathcal{B}_2 \mathcal{E} \mathcal{B}_2^T \mathcal{P} \quad (23)$$

with

$$\mathcal{I} = \text{diag}(1, 1, i), \quad \mathcal{E} = \text{diag}(e^{i\frac{\pi}{8}}, 1, 1), \quad \mathcal{P} = \text{diag}(e^{-i\frac{\pi}{16}}, 1, e^{-i\frac{\pi}{16}}). \quad (24)$$

The PMNS matrix becomes

$$U = \mathcal{T} U_\nu = \mathcal{T} \mathcal{B}_2 \mathcal{I} \mathcal{B}_2 \mathcal{E} \mathcal{B}_2^T \mathcal{P}. \quad (25)$$

The mixing obtained is a constrained form of the Trichimaximal ( $\text{T}\chi\text{M}$ ) mixing [5] with  $\chi = \frac{\pi}{16}$ :

$$|U| = |\text{T}\chi\text{M}_{(\chi=\frac{\pi}{16})}| \quad \text{where} \quad \text{T}\chi\text{M} = \begin{pmatrix} \frac{\sqrt{2}}{3} \cos \chi & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{3} \sin \chi \\ -\frac{\cos \chi}{\sqrt{6}} - i \frac{\sin \chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & i \frac{\cos \chi}{\sqrt{2}} - \frac{\sin \chi}{\sqrt{6}} \\ -\frac{\cos \chi}{\sqrt{6}} + i \frac{\sin \chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -i \frac{\cos \chi}{\sqrt{2}} - \frac{\sin \chi}{\sqrt{6}} \end{pmatrix}. \quad (26)$$

The modulus sign is used throughout this paper to indicate that the expression for the mixing matrix is valid only upto right and left multiplication with diagonal phase matrices (which do not affect the phenomenon of neutrino oscillation). The right multiplying diagonal phase matrices,

like  $\mathcal{P}$  in Eq. (25), do contribute to Majorana phases, the study of which is beyond the scope of this paper. From Eq. (26) and using Eq. (12) in [5] we get

$$|U_{e3}|^2 = \frac{2}{3} \sin^2 \frac{\pi}{16} \Rightarrow \sin^2 \theta_{13} = 0.025, \quad (27)$$

$$|U_{e2}|^2 = \frac{1}{3} \Rightarrow \sin^2 \theta_{12} = 0.342, \quad (28)$$

$$\sin^2 \theta_{23} = \frac{1}{2}, \quad (29)$$

$$\delta_{CP} = \frac{\pi}{2}. \quad (30)$$

From Eq. (22) we get the ratios of the neutrino masses,  $m_1 : m_2 : m_3 = 0.945 : 1 : 2.117$ . These ratios are compatible with the mass-squared differences measured experimentally [6, 7] within  $1\sigma$  errors and thus we can predict the light neutrino mass:

$$24.7 \text{ meV} \lesssim m_1 \lesssim 25.5 \text{ meV}. \quad (31)$$

Even though the set of eigenvectors given in Eqs. (18) results in the matrix  $\langle\phi\rangle$  given in Eq. (19), other choices of eigenvectors also exist which produce the same  $\langle\phi\rangle$ . The matrix  $\mathcal{T}^\dagger \langle\phi\rangle \langle\phi\rangle^T \mathcal{T}^*$  is a highly constrained form of the complex-symmetric ‘‘Simplest’’ texture [8, 9].

Let  $x = i^n$  where  $n$  is an integer and let

$$\Phi_x = \begin{pmatrix} i\frac{x}{2} + \frac{1-ix}{\sqrt{2}} & 0 & -\frac{x}{2} \\ 0 & 1 & 0 \\ -i\frac{x}{2} + \frac{1+ix}{\sqrt{2}} & 0 & \frac{x}{2} \end{pmatrix}. \quad (32)$$

Using  $\Phi_x$ , Eq. (19) can be rewritten as

$$\langle\phi\rangle = \Phi_i^*. \quad (33)$$

## 2.2. Case 2

Assigning

$$\begin{aligned} v_1^1 &= e_2, & v_1^2 &= \text{eig}(c, i)_1, & v_1^3 &= \text{eig}(c, i)_3, & v_1^4 &= \text{eig}(d, 1)_1, \\ v_2^1 &= e_2, & v_2^2 &= e_1, & v_2^3 &= e_1, & v_2^4 &= \text{eig}(c, -i)_3. \end{aligned} \quad (34)$$

we get

$$\langle\phi\rangle = \Phi_{i^2}^*. \quad (35)$$

In this case, the resulting PMNS matrix is

$$U = \mathcal{T}U_\nu = \mathcal{T}\mathcal{B}_2\mathcal{I}^2\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T\mathcal{P} \quad (36)$$

and it is a constrained form of the Triphimaximal (T $\phi$ M) mixing [5] with  $\phi = -\frac{\pi}{16}$ :

$$|U| = |\text{T}\phi\text{M}_{(\phi=-\frac{\pi}{16})}| \quad \text{where} \quad \text{T}\phi\text{M} = \begin{pmatrix} \sqrt{\frac{2}{3}} \cos \phi & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \sin \phi \\ -\frac{\cos \phi}{\sqrt{6}} - \frac{\sin \phi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos \phi}{\sqrt{2}} - \frac{\sin \phi}{\sqrt{6}} \\ -\frac{\cos \phi}{\sqrt{6}} + \frac{\sin \phi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos \phi}{\sqrt{2}} - \frac{\sin \phi}{\sqrt{6}} \end{pmatrix}. \quad (37)$$

From Eq. (37) and using Eq. (9) in [5] we get

$$|U_{\mu 3}|^2 = \frac{2}{3} \sin^2 \left( \frac{2\pi}{3} - \frac{\pi}{16} \right) \Rightarrow \sin^2 \theta_{23} = 0.613, \quad (38)$$

$$\delta_{CP} = \pi. \quad (39)$$

Here, as well as in the next two cases, Eq. (22) which gives the neutrino masses and Eqs. (27, 28) which give  $\sin^2 \theta_{13}$  and  $\sin^2 \theta_{12}$  remain valid.



### 2.3. Case 3

Assigning  $v_1^1 = e_2$ ,  $v_2^1 = e_2$ ,  $v_1^2 = \text{eig}(a, 1)_1$ ,  $v_2^2 = e_1$ ,  $v_1^3 = \text{eig}(d, 1)_3$ ,  $v_2^3 = e_1$ ,  $v_1^4 = \text{eig}(d, 1)_1$  and  $v_2^4 = \text{eig}(c, -i)_1$  we get

$$\langle \phi \rangle = \Phi_{i3}^* \quad (40)$$

$$U = \mathcal{T}\mathcal{B}_2\mathcal{I}^3\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T\mathcal{P} \Rightarrow |U| = |\text{T}\chi\text{M}_{(\chi=-\frac{\pi}{16})}|, \quad (41)$$

$$\sin^2 \theta_{23} = \frac{1}{2}, \quad (42)$$

$$\delta_{CP} = \frac{3\pi}{2}. \quad (43)$$

### 2.4. Case 4

Assigning  $v_1^1 = e_2$ ,  $v_2^1 = e_2$ ,  $v_1^2 = \text{eig}(c, -i)_1$ ,  $v_2^2 = e_1$ ,  $v_1^3 = \text{eig}(c, -i)_3$ ,  $v_2^3 = e_1$ ,  $v_1^4 = \text{eig}(d, 1)_3$  and  $v_2^4 = \text{eig}(c, -i)_3$  we get

$$\langle \phi \rangle = \Phi_{i4}^* \quad (44)$$

$$U = \mathcal{T}\mathcal{B}_2\mathcal{I}^4\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T\mathcal{P} \Rightarrow |U| = |\text{T}\phi\text{M}_{(\phi=\frac{\pi}{16})}|, \quad (45)$$

$$|U_{\mu 3}|^2 = \frac{2}{3} \sin^2 \left( \frac{2\pi}{3} + \frac{\pi}{16} \right) \Rightarrow \sin^2 \theta_{23} = 0.387, \quad (46)$$

$$\delta_{CP} = 2\pi. \quad (47)$$

The values predicted by all the four cases of the model are within  $3\sigma$  errors of the experimental best fits [6, 7, 10]. In fact the generic prediction  $\sin^2 \theta_{13} = 0.025$ , Eq. (27), is within  $1\sigma$  errors. However the global analysis [6] shows more than  $2\sigma$  tension with  $\sin^2 \theta_{23} = \frac{1}{2}$ , the  $\text{T}\chi\text{M}$  value (Cases 1 and 3, Eqs. (29, 42)). On the other hand the  $\text{T}\phi\text{M}$  values,  $\sin^2 \theta_{23} = 0.613$  from Eq. (38) in Case 2 and  $\sin^2 \theta_{23} = 0.387$  from Eq. (46) in Case 4, are well within  $1\sigma$  errors calculated in [7] and [6] respectively. All the cases predict  $\sin^2 \theta_{12} = 0.342$ , Eq. (28), which is at the edge of the  $2\sigma$  error range in [6]. A new mixing ansatz called the VS mixing<sup>1</sup> is proposed in the following section which modifies  $\theta_{12}$  as well as  $\delta_{CP}$ .

## 3. The VS Mixing Ansatz

The mixing obtained using the model, Eqs. (25, 36, 41, 45), is of the form

$$|U| = |\mathcal{T}\mathcal{B}_2\mathcal{I}^n\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T|. \quad (48)$$

The matrix  $|\mathcal{T}\mathcal{B}_2\mathcal{I}^n|$  gives the Tribimaximal (TBM) mixing [11]. Multiplying  $\mathcal{T}\mathcal{B}_2\mathcal{I}^n$  with  $\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T$  mixes the first and the third columns of the TBM matrix. Similarly we may also mix the first and the second columns resulting in the new ansatz defined by

$$|\text{VS}_{in}(\alpha)| = |\mathcal{T}\mathcal{B}_2\mathcal{I}^n\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T\mathcal{B}_3\mathcal{E}'\mathcal{B}_3^T| \quad (49)$$

where

$$\mathcal{E}' = \text{diag}(e^{i\alpha}, 1, 1). \quad (50)$$

Note that the Cases 1 to 4 are simply  $\text{VS}_{in}(0)$  with  $n = 1$  to 4 respectively. Eq. (49) on simplification gives

$$|\text{VS}_{in}(\alpha)| = |\mathcal{T}\mathcal{B}_2\mathcal{I}^n\mathcal{H}_2\mathcal{S}\mathcal{H}'_3| \quad (51)$$

<sup>1</sup> Dedicated to my father K Venugopal and mother J Saraswathi Amma

where

$$\mathcal{S} = \text{diag}(e^{i\frac{\pi}{16}}, 1, 1), \quad \mathcal{H}_2 = \begin{pmatrix} \mathbf{c} & 0 & i\mathbf{s} \\ 0 & 1 & 0 \\ i\mathbf{s} & 0 & \mathbf{c} \end{pmatrix}, \quad \mathcal{H}'_3 = \begin{pmatrix} \mathbf{c}' & i\mathbf{s}' & 0 \\ i\mathbf{s}' & \mathbf{c}' & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (52)$$

with

$$\mathbf{c} = \cos \frac{\pi}{16}, \quad \mathbf{s} = \sin \frac{\pi}{16}, \quad \mathbf{c}' = \cos \frac{\alpha}{2}, \quad \mathbf{s}' = \sin \frac{\alpha}{2}. \quad (53)$$

We get  $\sin^2 \theta_{12}$  within  $1\sigma$  errors for  $0.08\pi \lesssim \alpha \lesssim 0.26\pi$ . Table 2 lists a few cases of the VS mixing along with the predicted values of the mixing angles. The author finds the choice  $\alpha = \frac{\pi}{8}$  to be aesthetically pleasing. When  $\alpha = \frac{\pi}{8}$  we get  $\mathcal{E}' = \mathcal{E}$  and also  $\mathbf{c}' = \mathbf{c}$ ,  $\mathbf{s}' = \mathbf{s}$ .

	VS <sub>-1</sub> ( $\frac{\pi}{4}$ )	VS <sub>1</sub> ( $\frac{\pi}{4}$ )	VS <sub>-1</sub> ( $\frac{\pi}{6}$ )	VS <sub>1</sub> ( $\frac{\pi}{6}$ )	VS <sub>-1</sub> ( $\frac{\pi}{8}$ )	VS <sub>1</sub> ( $\frac{\pi}{8}$ )
$\sin^2 \theta_{23}$	0.613	0.387	0.613	0.387	0.613	0.387
$\sin^2 \theta_{12}$	0.323	0.323	0.317	0.317	0.319	0.319
$\delta_{CP}$	$1.27\pi$	$0.27\pi$	$1.18\pi$	$0.18\pi$	$1.13\pi$	$0.13\pi$

**Table 2.** Note that  $\sin^2 \theta_{13} = 0.025$  is a generic feature of the VS mixing. Conjugation,  $\text{VS}_{in}^*(\alpha)$ , changes the sign of  $\delta_{CP}$  without affecting the mixing angles  $\theta_{12}$ ,  $\theta_{23}$  and  $\theta_{13}$ .

#### 4. Summary

The symmetries represented by a discrete group are related to the eigenvectors of the group elements. We develop a notation to uniquely identify the eigenvectors and use it to assign vevs for the flavons. An orthonormal set of eigenvectors define the fermions' flavour states. The model thus constructed predicts the reactor mixing angle,  $\sin^2 \theta_{13} = 0.025$ , as well as the ratios of the neutrino masses,  $m_1 : m_2 : m_3 = 0.945 : 1 : 2.117$ . The T $\phi$ M versions of the model provide solutions for  $\theta_{23}$  in the first octant,  $\sin^2 \theta_{23} = 0.387$ , as well as in the second octant,  $\sin^2 \theta_{23} = 0.613$ . The T $\phi$ M as well as the T $\chi$ M versions give  $\sin^2 \theta_{12} = 0.342$ . A new mixing ansatz,  $\text{VS}_{in}(\alpha)$ , is introduced which gives reduced values for  $\theta_{12}$ . The ansatz also predicts various values for  $\delta_{CP}$ .

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