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# Varieties of concretisation: an illustrative case study

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## ABSTRACT

This paper illustrates, compares and contrasts a variety of different constructions that can be viewed as serving a role in *concretisation* – the process of developing concrete artefacts to support a learner in understanding abstract concepts. Our illustrative examples are based around the theme of introducing simple number-theoretic notions by representing operations on integers by actual or simulated operations carried out on two liquid containers (so-called “jugs”). Different varieties of concretisation are discussed with reference to phenomenalisation, visualisation and physical embodiment, and in their broader relation to constructionism and learning in cultural context. The importance of concretisations that are both accessible in everyday situations and offer rich experiential potential is highlighted.

## Keywords

Empirical Modelling, concretisation, robotics, JUGS, Lego.

## 1. INTRODUCTION

The collaboration described in this paper brings together two research groups interested in the role that interactive artefacts can play in experiential learning. It specifically targets concretisation as a concept with critical relevance to how abstract and symbolic knowledge is related to what is encountered in experience.

The educational principles that motivate concretisations are rooted in Jean Piaget’s theories of cognitive development [8]. Seymour Papert built on these theories in his notion of ‘constructionist learning’. According to constructionist principles, the active learner is the centre of the learning process. Learners enlarge their knowledge by manipulating and constructing objects [8]. A student can create concrete new knowledge and learn in a constructionist way by interacting with real world objects [1].

The most direct way in which to approach concretisation is to transform standard algorithmic procedures into explicit ‘real-world’ routines to be executed by tangible physical devices. Robotics, for example, has been used widely in this spirit to teach traditional computer science concepts. Programming, networking, artificial intelligence and many other topics have all been taught with robots – especially to novices. A key idea is that robotics can help to motivate students to learn an abstract topic.

Besides fostering motivation, concretisations can also be used to contextualise abstract concepts, such as programming. For example, concrete learning artefacts have been developed to teach basic programming in developing countries. Most of the present ICT tools are based on a cultural norm that has put its primary educational emphasis on symbolism and logic, but this does not necessarily correspond to the way of thinking in all

cultures, nor can it be readily supported using the available resources. This has been the motivation, for example, for teaching programming in the context of Tanzania by using intelligent building blocks, or *I-Blocks*. These blocks support learning by construction or, more specifically, “programming by building” [9].

An alternative approach to the goal of developing interactive artefacts to support experiential learning is offered by Empirical Modelling (EM), as developed at the University of Warwick (see <http://www.dcs.warwick.ac.uk/modelling>). EM has so far made limited use of the new technologies that are prominent in research on concretisation. It is however more radical in another respect, in that it favours a stance on model-building that gives experiential aspects priority over logic. Our collaboration targets concretisations with the contextual qualities of everyday artefacts that also afford rich learning experiences.

## 2. VARIETIES OF CONCRETISATION

Our study of concretisation has involved elaborating on a simple educational program called JUGS that was initially developed for the BBC Microcomputer in the 1980s. This section introduces four concretisations linked with the JUGS theme, briefly describing their construction and the functions that they serve.

The underlying educational objective of constructing JUGS was to familiarise children with elementary concepts of number theory. The idea was to create an environment for exploration in which pupils can come to appreciate that what can be derived by repeatedly applying a restricted set of simple operations of addition and subtraction to two given positive integers is determined by their highest common factor (hcf). The abstract mathematical observation that underlies the use of JUGS is that, given two integers  $m$  and  $n$ , the set  $AS(m,n)$  of numbers that can be generated from  $m$  and  $n$  by additions and subtractions alone is the set of multiples of  $hcf(m,n)$ . In applying fill/empty/pour operations to two jugs that have integer capacities, it follows that every operation generates a quantity of liquid that is in  $AS(m,n)$  and that this quantity is restricted to be positive and cannot exceed  $\max(m,n)$ . It is also true – but not quite as easy to prove – that all quantities satisfying these constraints can be derived in this way.

### 2.1 A real physical concretisation

The simplest concretisation makes use of real jugs and liquids (Figure 1). This concretisation does not need any computer or special technology; everyday items could be used. The only requirements would be a large range of jugs with different integral capacities and a suitable source of liquid. With this concretisation, pupils are able to physically interact with the jugs and perform the filling, pouring, and emptying operations themselves.

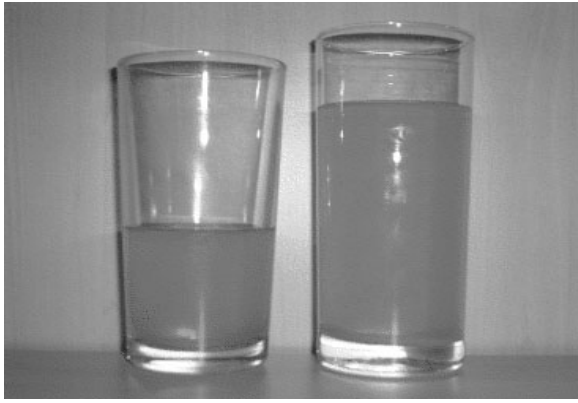


Figure 1: A real physical concretisation

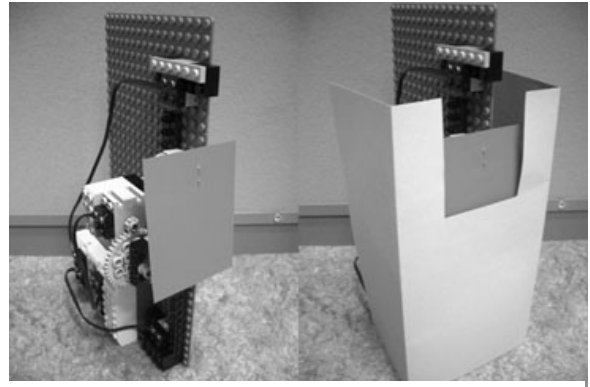


Figure 3: A robotics concretisation using Lego

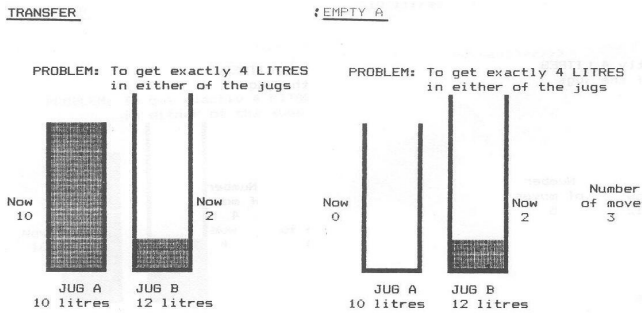


Figure 2: A software concretisation (BBC JUGS)

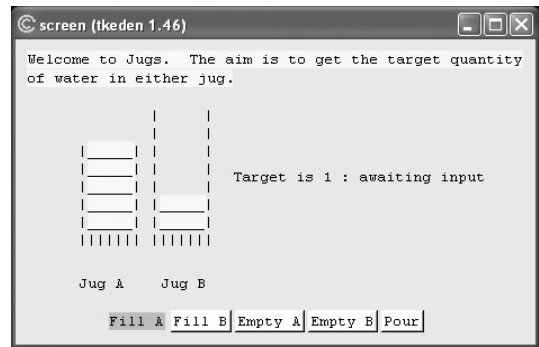


Figure 4: An EM concretization

## 2.2 A software concretisation

The BBC Microcomputer implementation of JUGS [2] was designed to simulate interaction with real jugs and liquid. The interface displays two jugs with integer capacities (Figure 2) and provides the student with a button interface through which three kinds of operation can be simulated. The operations are defined as: *fill* (to fill a jug with liquid), *empty* (to remove all the liquid from a jug), and *pour* (to pour the liquid from one jug into the other). The JUGS program is designed to be used in an exploratory manner, where the principal objective for a pupil is to use the operations to produce a specified target quantity of liquid.

## 2.3 A robotics concretisation

In keeping with previous work on concretisation using robotics [7], a physical model of jugs can be constructed that offers the advantages of the software program as well as the benefits of real-world interaction. To explore this possibility, Harfield and Jormanainen constructed an experimental concretisation of the JUGS program using Lego, as will now be briefly described.

The first phase of the construction involved building a physical jug with Lego building blocks. This model incorporated a motor and two touch sensors. With the motor and sensors it was possible to control the filling, pouring and emptying operations of the model. The motor and sensors were connected to a RCX unit – an autonomous small-scale computer with connectors for motors and sensors. With this hardware, it is possible to produce autonomous robots, which can also communicate, during execution, with a computer or with each other via an infrared device.

The next phase of the construction was to develop the software for our model. Visual Basic was used for the development work together with the *Phantom* software component which allows the programmer to access the RCX unit easily. The *immediate mode* of the component was used to control the motor and read

the sensors without downloading any software to the robot. This meant that the application served as a remote controller.

The interface for interacting with the jugs followed a similar design to the JUGS program, with operations for filling, pouring and emptying. However, no on-screen visualisation was necessary because the physical representation of the jugs enables the student to observe the content level (Figure 3).

## 2.4 An Empirical Modelling concretisation

The motivation for using EM to build a jugs model (Figure 4, taken from [9: jugsPavelin2002]) is that it represents a quite different way of using the computer to express the experiential aspects that are an essential ingredient of concretisation. Whereas the JUGS program captures only a narrow and specific range of operations that can be performed on actual jugs, EM grapples with the much wider – and no longer circumscribed – repertoire of interactions that is associated with the concept of a 'jug' as it is negotiated in experience. This has the effect of providing a more open environment to engage the student in exploring jugs than the JUGS program. Whilst the JUGS program allows specific actions to be performed and specific parameters to be changed in ways that are limited by the specification of the programmer who created the concretisation, such limitations do not apply to the EM jugs model. The learner acts in a role more like that of a modeller, and the distinction between model-builder and model-user is blurred. A modeller is able to guide their learning by exploring the model and investigating the effects of making changes to the model. This modelling activity is supported by the key concepts of observation, dependency and agency, as described in [3, 4, 5].

By way of illustration, modelling jugs in the EM environment proceeds by identifying the relevant observables, dependency and agency. Suitable observables might be the capacity of the jug, the quantity of liquid in the jug, and the colour of the liquid. A dependency between these observables might then be that the jug is full when the quantity of liquid in the jug equals its capacity. Agency in the environment might be associated

with the state-changing actions involved in filling or emptying a jug. It is important to notice that this model building does not necessarily involve any traditional procedural programming. Instead, it more closely resembles the activity of developing and using a spreadsheet by interactively creating observables and dependencies and acting out the roles of agents.

The original EM jugs model was devised with specific reference to novel implementation of the JUGS program, and several variants have since been developed. Some of these variants place more emphasis on realistic characteristics of actual jugs. For instance, one variant makes use of floating point numbers to represent the capacity of a jug [9: jugsextensionsRunbol2002], and takes account of evaporation (or spillage) of liquid. It is characteristic of EM that there is no objective criterion by which such variants of the original EM jugs model can be deemed to be distinct models – it is more appropriate to regard them as associated with different loci for interaction in an amorphous space of jugs-related models to which the human interpreter can migrate at their discretion (cf. *The JUGS model: theme and variations* poster in [9:kaleidoscopeBeynon2005]).

### 3. COMPARING CONCRETISATIONS

The four concretisations described in the previous section have very diverse characteristics. These are discussed with reference to: the intended function of the concretisations; the nature of their construction; their qualities in use; their limitations.

#### 3.1 Concretisation by physical artefacts

The use of actual jugs and liquid most directly captures the core concept of concretisation: the design of a physical artefact that is intended to be used interactively to promote the learner's understanding of an abstract mathematical concept. Whilst there may seem to be little involved in the construction of such a concretisation, there are significant issues associated with the use of liquid to represent an integral quantity. If the quantity of liquid is to be used as an intrinsic measure, then it is possible to imagine how blemishes in making a jug, or careless execution of pouring operations, might result in discrepancies – for instance, such as correspond to replacing integer parameters 2 and 3 by 199 and 301 respectively – with very significant consequences for what quantities can be generated by sequences of operations. If such discrepancies could be detected and monitored, they might indeed add something instructive by way of insight into the complex relationship between additive operations and multiplicative properties. Otherwise, in order to avoid such pathological number-theoretic diversions, it might be appropriate to mark off the quantity of liquid by providing a calibration for each jug. Whilst the explicit physical nature of the concretisation and the familiar commonplace nature of the operations accords well with the idea of placing learning activity in an everyday context, it also renders adaptation difficult: it would not be an easy matter to change the capacity of a jug, for instance, to change the colour of the liquid arbitrarily, or to apply scaling to the size of jugs.

#### 3.2 Concretisation by the JUGS program

It is clear that the JUGS program in some sense serves a similar function to that of the physical concretisation using actual jugs. By comparison with the actual jugs, the quantities of liquid displayed are robustly integral, and the potential messiness and imprecision of the physical operations is eliminated. This idealisation of the physical experience that the program offers is in some respects out of keeping with the educational motivation for concretisation. Perhaps on this account, a common reaction

of teachers, especially when computer technology in schools was still scarce, was to question the value of replacing such simple artefacts as actual jugs by a computer simulation. The supplementary notes on use initially issued with the JUGS program implicitly acknowledge this kind of objection, proposing that pupils are first exposed to the physical concretisation by jugs and liquid, and only subsequently to the "abstract experience" that the program affords. It is nevertheless clear that the JUGS program more readily meets the declared objective of "[allowing] pupils to investigate open-ended problems", and whilst there is no explicit reference to the underlying abstract number-theoretic concepts, comprehensive answers to the questions for investigation demand some engagement with these. For instance, a pupil has to appeal to the notion of 'highest common factor' in some way in order to explain why it is impossible to achieve a certain target. Despite the purely visual nature of its realisation of state, the JUGS program can still be seen as a concretisation in that it serves to 'phenomenalise' abstract mathematical concepts in the sense of [6]. Because of its less tangible embodiment, the JUGS program is also in principle much easier to adapt than a pair of actual jugs, subject to the program being developed so that parameters can be easily modified and the well-known problems of adapting to a modified specification are overcome.

#### 3.3. The LEGO JUGS concretisation

The purpose of the LEGO concretisation is to exploit the more sophisticated computer-related technologies for devising physical artefacts that have been developed since 1982. The objective for this concretisation is to move beyond mere visualisation to a physical realisation of state that allows the kind of direct interaction in the world that actual jugs afford. As the account of the construction of the LEGO model in section 2.3 indicates, the electronic counterpart of the engineering problem of manufacturing jugs with precise integer capacities presents software and hardware challenges that subvert the idea of marrying the physicality of 'real-world' artefacts with the programmability of computer technology. This means that it is technically hard to build an entirely satisfactory concretisation of this nature in practice. When considering the role of such a concretisation, it is of interest to note that virtual reality technology now potentially offers yet another approach to concretisation that might be seen as overcoming some of the limitations of the original JUGS program. The significance of supporting stand-alone interactions that can be carried out in a commonplace environment without the need for special apparatus must also be acknowledged.

#### 3.4. The EM Jugs model as a concretisation

Superficially, EM does not seem to address the goals of concretisation where physical embodiment and realism are concerned. Like the JUGS program, the EM Jugs model makes use of a standard computer display interface, and to this extent offers a similar kind of 'abstract experience'. Taking the full range of possible interactions with the EM Jugs model into account, there is exceedingly wide scope for state-changing activities beyond the menu selection and direct manipulation in a conventional user-interface, since there is always scope to augment and revise the model through modifying the observables and dependencies on-the-fly. This interaction involves a relatively sophisticated activity similar to introducing and manipulating spreadsheet definitions however, and cannot generally be carried out by a naïve user. The experiences invoked through such interaction, far from being 'realistic' in the narrow sense, can have a surreal quality. For instance, by

the simplest most direct forms of redefinition, the jugs can be configured so that the content of a jug exceeds its capacity, the colour of the liquid depends on the content, or the capacity of one jug is determined by the content of the other.

Viewed as an artefact that can be the subject of experiment, an EM model may nonetheless be seen as having some of the characteristic qualities of a concrete object. Interaction with it has a degree of openness, both in respect of scope, and interpretation, that is associated with 'real-world' artefacts. In the other examples of concretisation discussed, the attention of the learner is directed in a highly constrained and specific way to particular observables and operations that are deemed meaningful. Indeed, as the above discussion has illustrated, this degree of precise framing of function and interpretation is vital in constructing the JUGS program and its associated LEGO concretisation: it is an essential prerequisite to the specification of the procedural mechanisms that support their implementation. In EM, in contrast, the significance of one's experience of a model is not in general considered to be objectively determined, but is shaped primarily by interaction and interpretation with the model and its broader environment. As has been discussed at length elsewhere [3,4], the peculiar qualities of an EM model derive from the way in which it serves as a 'construal', embodying patterns of observation, dependency and agency attributed to a situation. This makes it possible to integrate model-building with the acquisition of domain understanding in a manner that is conspicuously absent in the development of other concretisations – a quality that leads us to identify EM models as much better suited to the goals of constructionist learning than traditional programs [3,4].

#### 4. CONCLUSIONS

Our study of varieties of concretisation highlights two particularly significant and complementary concerns:

- the importance of developing learning artefacts that are matched to their cultural and technological context, so that they can participate in everyday situations alongside ordinary objects and activities;
- the vital role in learning for artefacts that support the same quality of exploratory interaction and experiment that is characteristic of lived experience, especially in our first vivid encounters with new phenomena and situations.

In creating artefacts that blend into an existing cultural context, the challenge is to avoid needless circumscription of interaction and meanings. In developing artefacts that can support the highly imaginative and speculative processes of tentative construction and reflection that underlie the constructivist ideal, the challenge is to make such development as accessible, natural and inviting as possible. In deepening the international collaboration between the two research groups contributing to this paper, these two challenges help to frame the agenda for our future work. Possible topics for further exploration and research suggested by the above account include: using EM principles to support the construction of concretisation using robotics (cf. [3]); developing more concrete and readily accessible interfaces to support EM; integrating EM with virtual or blended realities.

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