SUPER (a,d)-EDGE ANTIMAGIC TOTAL LABELING OF CONNECTED LAMPION GRAPH

Robiatul Adawiyah[13], Dafik[14], Slamin[15]

Abstract. *A G graph of order p and size q is called an (a,d)-edge antimagic total if there exist a bijection f: V(G)∪E(G) →{1,2,…,p+q} such that the edge-weights, w(uv)=f(u)+f(v)+f(uv), uv ∈ E(G), form an arithmetic sequence with first term a and common difference d. Such a graph G is called super if the smallest possible labels appear on the vertices. In this paper we study super (a,d)-edge-antimagic total properties of connected* $£_{n,m}$ *by using deductive axiomatic and the pattern recognition method. The result shows that a connected Lampion graphs admit a super (a,d)-edge antimagic total labeling for d = 0,1,2 for* $n \geq 1$. *It can be concluded that the result of this research has covered all the feasible d.*

Key Words*: (a,d)-edge antimagic vertex labeling, super (a,d)-edge antimagic total labeling, Lampion Graph.*

INTRODUCTION

Mathematics as a basic of science hold an important role in technology development. One of an interesting topic in mathematics is graph theory as one of the prime objectsstudy in discrete mathematics. There are many topic in graph theory. In this paper, we will learn about super (a,d)-edge antimagic total labeling of lampion graph ($£_{n,m}$).

Lampion graph is the family of triangular book graph denoted by $£_{n,m}$ with $m \geq 1$ and $n \geq 1$. This graph is developed from combining single triangular book graph and adding an edge in one of the top vertices in triangular book graph so that it become a connected graph. The shape is also being modified become a circle shape therefore the shape seems like a connected lampion.

A labeling of a graph is any mapping that sends some set of graph elements to a set of positive integers. If the domain is the vertex-set or the edge-set, the labeling are called, respectively, vertex labelings or edge labelings. Moreover, if the domain is V (G) U E(G) then the labelings are called total labelings. We define the edge-weight of an edge uv ϵ E(G) under a total labeling to be the sum of the vertex labels corresponding to vertices u, v and edge label corresponding to edge uv. If such a labeling exists then G is said to be an (a; d)-edge-antimagic total graph. Such a graph G

[13] Student of Mathematics Education Department Jember University
[14] Lecturer of Mathematics Education Department Jember University
[15] Lecturer of Information System Department Jember University

is called super if the smallest possible labels appear on the vertices. Thus, a super (a; d)-edge-antimagic total graph is a graph that admits a super (a; d)-edge-antimagic total labeling.

In this paper will be discussed about super (a; d)-edge-antimagic total labeling because it has not been found before. Such that in this paper we investigate the existence of super (a; d)-edge-antimagic total labelings of lampion graph and it will be concentrated on the connected Lampion graph $£_{n,m}$.

RESEARCH METHODS

Research methods a super (a; d)-edge-antimagic total labeling of Lampion graph are deductive axiomatic and the pattern recognition. The research techniques are as follows: (1) calculate the number of vertex p and size q of graph $£_{n,m}$;(2) determine the upper bound for values of d; (3) determine the label of *EAVL* (edge-antimagic vertex labeling) of $£_{n,m}$; (4) if the label of *EAVL* is expandable, then we continue to determine the bijective function of *EAVL*; (5) label the graph $£_{n,m}$ with *SEATL* (super-edge antimagic total labeling) with feasible values of d by using Lemma 1 and (6) determine the bijective function of super-edge antimagic total labeling of graph $£_{n,m}$.

Lemmas

We start this section by a necessary condition for a graph to be super (a; d)-edge antimagic total, providing a least upper bound for feasible values of d. This lemma can be found in [18].

Lemma 1 *If a (p,q)-graph is super (a,d)-edge-antimagic total then* $d \leq \frac{2p+q-5}{q-1}$.

Proof. Assume that a *(p,q)*-graph has a super *(a,d)*-edge-antimagic total labeling $f: V(G) \cup E(G) \rightarrow \{1,2,..., p+q\}$. The minimum possible edge-weight in the labeling *f* is at least 1+ 2 + p+1 = p+4. Thus, a ≥ p+4. On the other hand, the maximum possible edge-weight is at most (p-1) + p + (p+q) = 3p +q - 1. So we obtain a + (q-1)d ≤ 3p + q - 1 which gives the desired upper bound for the difference *d*

Another important lemma obtainded by Figueroa-Centeno et al [6], gives an easy way to find a total labeling for super edge-magicness of graph.

Lemma 2 A *(p,q)*-graph G is super edge-magic if and only if there exists a bijective function $f: V(G) \rightarrow \{1,2,\dots, p\}$ such that the set $S=\{f(u)+f(v): uv \in E(G)\}$ consists of q consecutive integers. In such a case, *f* extends to a super edge-magic labeling of G with magic constant a = p + q + s, where s = min(S) and S= { a- (p+1), a- (p+2), ….,a-(p+q)} In our terminology, the previous lemma states that a *(p,q)*-graph G is super *(a,0)*-edge-antimagic total if and only if there exists an (a – p - q, 1)-edge-antimagic vertex labeling.

RESULT AND DISCUSSIONS

If Lampion graph, has a super (a,d)-edge-antimagic total labeling then, for p = 4n + 1 and q = 8n - 1, it follows from Lemma 1 that the upper bound of d is d ≤ 2 or d ∈ {0,1,2}. The following lemma describes an (a,1)-edge-antimagic vertex labeling for Lampion.

Definition of Lampion Graph

Lampion graph denoted by $\pounds_{n,m}$ is a connected graph with vertex set $V(\pounds_{n,m}) = \{x_i, x_{i,1,j}, x_{i,2,j}; 1 \le i \le n+1, 1 \le j \le m\}$ and $E(\pounds_{n,m}) = \{x_i x_{i,1,j},; 1 \le i \le n+1, 1 \le j \le m\} \cup \{x_i x_{i,2,j}; 1 \le i \le n+1, 1 \le j \le m\} \cup \{x_{i,1,j} x_{i+1}; 1 \le i \le n+1, 1 \le j \le m\} \cup \{x_{i,2,j} x_{i+1}; 1 \le i \le n+1, 1 \le j \le m\} \cup \{x_{i,1,1} x_{i,2,1}; 1 \le i \le n+1\}$. Thus $|V(\pounds_{n,m})| = p = 2nm + n + 1$ and $|E(\pounds_{n,m})| = q = 4nm + 2n - 1$. We can see the example of Lampion graph in the figure 1

THE RESULTS OF RESEARCH

If Lampion graph, has a super (a, d)-edge-antimagic total labeling then, for $p = 2nm + n + 1$ and $q = 4nm + 2n - 1$, it follows from Lemma 1 that the upper bound of d is $d \le 2$ or $d \in \{0, 1, 2\}$. The following lemma describes an $(a, 1)$-edge-antimagic vertex labeling for Lampion graph.

Lemma 3 *If* $m \ge 1$ and $n \ge 1$ *then the Lampion graph* $\pounds_{n,m}$ *has an (3, 1)-edge-antimagic vertex labeling.*

Proof. define the vertex labeling $\propto_1 : V(\pounds_{n,m}) \to \{1, 2, 3, \dots, 3n + 2)$ in the following way:

$$\alpha_1(x_i) = 2mi + i - 2m, \text{for } 1 \le i \le n$$
$$\alpha_1(x_{i,l,j}) = 2mi + i - j(-1)^{i+l} - m + (\frac{1 + (-1)^{i+l}}{2}) \quad \text{for any i and } l = 1, 2$$

The vertex labeling $\propto_1$ is a bijective function. The edge-weights of $\pounds_{n,m}$ for any i, j, and l = 1; 2, under the labeling $\propto_1$, constitute the following sets:

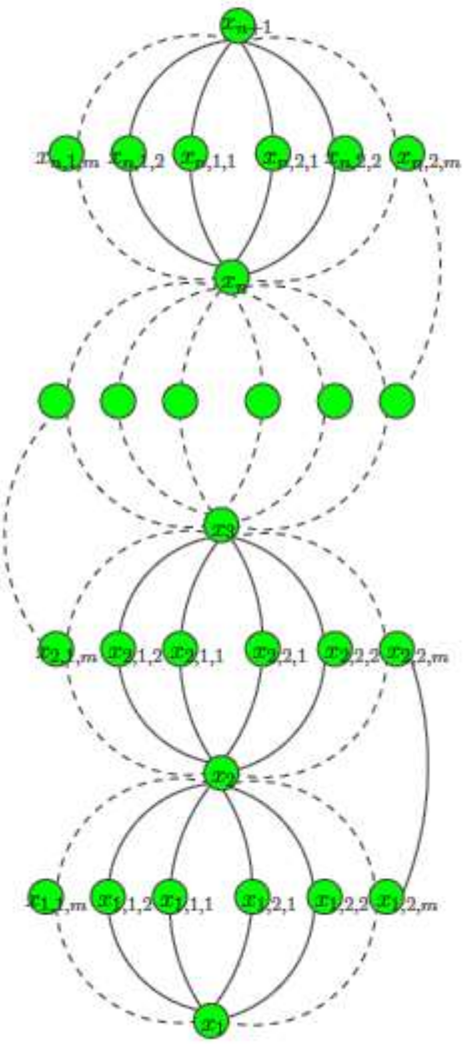

Figure 1: Lampion Graph $£_{n,m}$

$$\begin{aligned}
w_{\alpha_1}(x_{i,1,1}x_{i,2,1}) &= 4mi - 2m + 2i + 1\\
w_{\alpha_1}(x_i x_{i,l,j}) &= 4mi - 3m + 2i - j(-1)^{i+l} + (\frac{1+(-1)^{i+l}}{2})\\
w_{\alpha_1}(x_{i,l,j}x_{i+1}) &= 4mi - m + 2i - j(-1)^{i+l} + 1 + (\frac{1+(-1)^{i+l}}{2})\\
w_{\alpha_1}(x_{i,l,j}x_{i+1,l,j}) &= 4mi + i + 2
\end{aligned}$$

From the formula of edge-weights above, we can see that the set $w_{\propto 1} = \{3, 4, 5, \ldots, 4nm + 2n + 1\}$ consists of consecutive integers. Thus $\propto_1$ is a (3, 1)-edge antimagic vertex labeling. Figure 2 is an example of (3,1)-edge antimagic vertex labeling and edge-weights EAV L of lampion graph $£_{n,m}$.

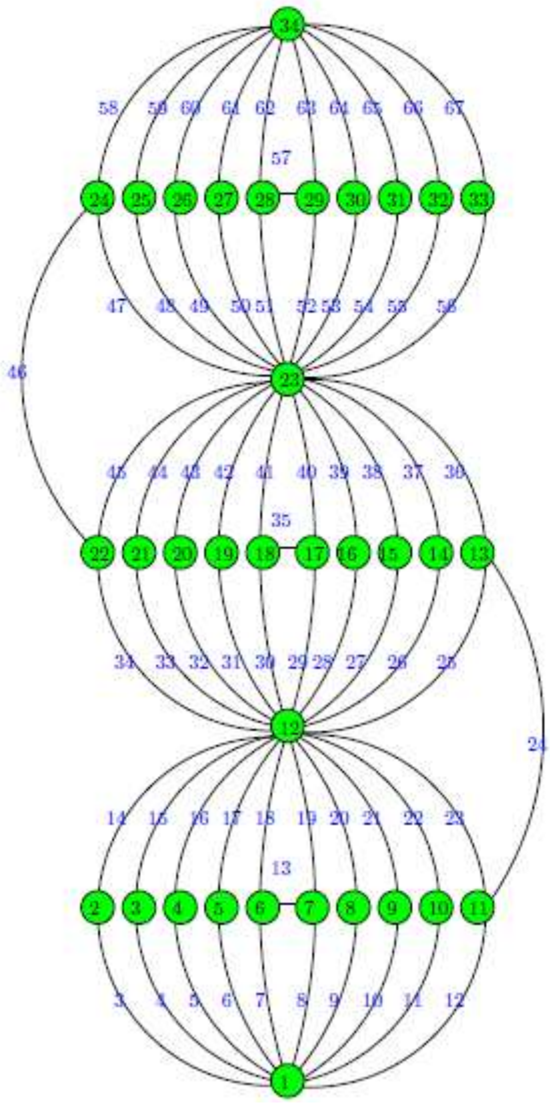

Figure 2: super(3,1)-edge total labeling of $\pounds_{3,5}$

Theorem 1 *If $m \geq 1$ and $n \geq 1$ then the graph* $\pounds_{n,m}$ *has a super (6mn + 3n + 3,0)-edge-antimagic total labeling*

Proof.

we use the formula of vertex labeling to define the label of vertex in lampion graph $\pounds_{n,m}$, then definene the edges labeling as $\propto_2 : E\left(\pounds_{n,m}\right) \rightarrow \{2nm + n + 2, 2nm + n + 3, 2nm + n + 4, \ldots, 4nm + 2n - 1)$, such that the formula of super (a,0) edge-antimagic total labeling for any i, j, and l = 1,2 can be defined as follow:

$$\begin{aligned}
\alpha_2(x_{i,1,1}x_{i,2,1}) &= 6mn - 4mi + 2m + 3n - 2i + 2 \\
\alpha_2(x_i x_{i,l,j}) &= 6mn - 4mi + 3n + 3m - 2i + j(-1)^{i+j} + 2 + (\frac{1-(-1)^{i+l}}{2}) \\
\alpha_2(x_{i,l,j}x_{i+1}) &= 6mn - 4mi + 3n + m - 2i + j(-1)^{i+j} + 1 + (\frac{1-(-1)^{i+l}}{2}) \\
\alpha_2(x_{i,l,j}x_{i+1,l,j}) &= 6mn - 4mi + 3n - 2i + 1
\end{aligned}$$

We can find the total labeling W_{α_2} with summing $w_{\alpha_1} = w_{\alpha_2}$ with edge label α_2. It is not difficult to see that the set W_{α_2}= {6mn + 3n +3 , 6mn + 3n +3,…, 6mn + 3n +3 } contains an arithmetic sequence with the first term 6mn + 3n +3 and common difference 0. Thus W_{α_2} is a super (6mn + 3n +3,0)-edge-antimagic total labeling. This concludes the proof.

Theorem 2 *If $m \geq 1$ and $n \geq 1$ then the graph* $₤_{n,m}$ *has a super (2nm + n + 5,2)-edge-antimagic total labeling*

Proof. Let we use the formula of vertex labeling to define the label of vertex in lampion graph $₤_{n,m}$, then definene the edges labeling as $\propto_3 : E\left(₤_{n,m}\right) \to \{2nm + n + 2, 2nm + n + 3,\ 2nm + n + 4, \dots, 4nm + 2n - 1)$, such that the formula of super (a,2) edge-antimagic total labeling for any i, j, and l = 1,2 can be defined as follow:

$$\begin{aligned}
\alpha_3(x_{i,1,1}x_{i,2,1}) &= 2nm + 4mi + n + 2i - 2m \\
\alpha_3(x_i x_{i,l,j}) &= 2mn + 4mi + n - 3m + 2i - j(-1)^{i+l} - (\frac{1-(-1)^{i+l}}{2}) \\
\alpha_3(x_{i,l,j}x_{i+1}) &= 2mn + 4mi + n - m + 2i - j(-1)^{i+j} + (\frac{1+(-1)^{i+l}}{2}) \\
\alpha_3(x_{i,l,j}x_{i+1,l,j}) &= 2nm + 4mi + n + 2i + 1
\end{aligned}$$

The total labeling $\propto_3$ is a bijective function from V ($₤_{n,m}$) U E($₤_{n,m}$) . The edge-weights of $₤_{n,m}$ for any i, j, and l = 1,2 can be defined as follow:

$$\begin{aligned}
w_{\alpha_3}(x_{i,1,1}x_{i,2,1}) &= 2nm + 8mi + n + 4i - 4m + 1 \\
w_{\alpha_3}(x_i x_{i,l,j}) &= 2nm + 8mi + n - 6m + 4i - 2j(-1)^{i+l} + (-1)^{i+l} \\
w_{\alpha_3}(x_{i,l,j}x_{i+1}) &= 2nm + 8mi + n - 2m + 4i - 2j(-1)^{i+l} + 2 + (-1)^{i+l} \\
w_{\alpha_3}(x_{i,l,j}x_{i+1,l,j}) &= 2nm + 8mi + n + 4i + 3
\end{aligned}$$

It is not difficult to see that the set W_{α_3}= {2nm + n + 5, 2nm + n + 7, 2nm + n + 9, …, 10mn + 5n + 1 } contains an arithmetic sequence with the first term 2nm + n + 5and common difference 2. Thus W_{α_3} is a super (2nm + n + 5,2)-edge antimagic total labeling. This concludes the proof. □

Theorem 3 *If $m \geq 1$ and $n \geq 1$ then the graph $£_{n,m}$ has a super $(4nm + 2n + 4,1)$-edge-antimagic total labeling.*

Proof. Let we use the formula of vertex labeling to define the label of vertex in lampion graph $£_{n,m}$, then definene the edges labeling as $\propto_4 : E\left(£_{n,m}\right) \rightarrow \{2nm + n + 2, 2nm + n + 3, 2nm + n + 4, \ldots, 4nm + 2n - 1)$, such that the formula of super (a,1) edge-antimagic total labeling for any i, j, and l = 1,2 can be defined as follow:

$$
\begin{aligned}
\alpha_4(x_{i,1,1}x_{i,2,1}) &= 4nm - 2mi + 2n + m - i + 2\\
\alpha_4(x_i x_{i,l,j}) &= \frac{16mn + 4mn(1-(-1)^{i+j+l}) - 8mi + 8n + 2n(1-(-1)^{i+j+l})}{4} +\\
&\quad \frac{12m + 2j(-1)^{i+l} + (-1)^{i+l}(1-(-1)^j) - 4i + 8 - 2(-1)^{i+l}}{4} +\\
&\quad \frac{2(-1)^{i+j+l}}{4}\\
\alpha_4(x_{i,l,j}x_{i+1}) &= \frac{16mn + 4mn(1+(-1)^{i+j+l}) - 8mi + 8n + 2n(1+(-1)^{i+j+l})}{4} +\\
&\quad \frac{2m + 2j(-1)^{i+l} + (-1)^{i+l}(1-(-1)^j) - 4i + 6 - 2(-1)^{i+l}}{4}\\
\alpha_4(x_{i,l,j}x_{i+1,l,j}) &= 6nm - 2mi + 3n - i + 1
\end{aligned}
$$

If W_{α_4} is defined as edge-weight total labeling based on $\propto_4$ labeling formula, so The edge-weights W_{α_4} of $£_{n,m}$ for any i, j, and l = 1,2 can be defined as follow:

$$
\begin{aligned}
w_{\alpha_4}(x_{i,1,1}x_{i,2,1}) &= 4nm - 2mi + 2n + i + 3\\
w_{\alpha_4}(x_i x_{i,l,j}) &= 4mn + 2mn(\frac{1-(-1)^{i+j+l}}{2}) + 2mi + 2n + n(\frac{1-(-1)^{i+j+l}}{2}) -\\
&\quad \frac{3m}{2} + i - \frac{j(-1)^i + l}{2} + \frac{(-1)^{i+l}(1-(-1)^j)}{4} + 2 + \frac{(1+(-1)^{i+j+l}}{2}\\
w_{\alpha_4}(x_{i,l,j}x_{i+1}) &= 4mn + 2mn(\frac{1+(-1)^{i+j+l}}{2}) + 2mi + 2n + n(\frac{1+(-1)^{i+j+l}}{2}) -\\
&\quad \frac{m}{2} + i - \frac{j(-1)^i + l}{2} + \frac{(-1)^{i+l}(1-(-1)^j)}{4} + 3\\
w_{\alpha_4}(x_{i,l,j}x_{i+1,l,j}) &= 6nm + 2mi + 3n + i + 3
\end{aligned}
$$

It is not difficult to see that the set W_{α_4}= {4mn + 2n+4, 4mn + 2n+5, 4mn + 2n+6, …, 8mn +4n +2 } contains an arithmetic sequence with the first term 4mn + 2n+4 and common difference 1. It can be concluded that lampion graph has super (4mn + 2n+4, 1) – edge antimagic total labeling. □

CONCLUSION

Finally, we can conclude that the graph $£_{n,m}$ admit a super (a,d)-edge antimagic total labeling for all feasible d and m, n ≥ 1

REFERENCES

A. Kotzig and A. Rosa, Magic valuations of finite graphs, *Canad. Math. Bull.*13 (1970), 451--461.

Chartrand, G. 2012. *Introductory Graph Theory*. United Stated of America: dover publication inc.

Dafik, M. Miller, J. Ryan and M. Ba·ca, *Super edge-antimagic total labelings of* $mK_{n,n,n}$, *Ars Combinatoria* (2006), in press.

Dafik, Slamin, Fuad and Riris. 2009. *Super Edge-antimagic Total Labeling of Disjoint Union of Triangular Ladder and Lobster Graphs*. Yogyakarta: Proceeding of IndoMS International Conference of Mathematics and Applications (IICMA) 2009.

R.M. Figueroa-Centeno, R. Ichishima, F.A. Muntaner-Batle. *The place of super- edge-magic labelings among ather classes of labelings*, Discrete Mathematics, 231 (2001), 153-168.

Gallian, J.A. 2009. *A Dynamic Survey of Graph Labelling*. [serial on line]. http://www.combinatorics.org/Surveys/ds6.pdf. [17 Agustus 2010].

Grifin, C. 2012. *Graph Theory*. United Stated: Creative Commons Attribution-Noncommercial-Share.

Hartsfield, N., and Ringel, G. 1994. *Pearls in Graph Theory*. London: Accademic Press Limited.

Johnsonbaugh, Richard. 2009. Discrete Mathematics, seventh edition. New Jersey: Pearson Education, Inc.

Joseph A Gallian. 2013. A *Dinamic Survey Of Graph Labeling*. Jember: Gallian Survey.124-128.

Lee, Ming-ju.2013. *On Super (a,1)-edge Antimagic Total Labelings Of Subdifition Of Stars* .Miaoli: Jen-Teh Junior Collage Of Madicine.1-10.

M. Ba·ca, Y. Lin, M. Miller and R. Simanjuntak. *New constructions of magic and antimagic graph labeling.* Utilitas Math. 60 (2001), 229{239.

R. Simanjuntak, F. Bertault and M. Miller. *Two new (a, d)-antimagic graph labelings, Proc. of Eleventh Australasian Workshop on Combinatorial Algorithms* (2000), 179{189.