SUPER (a,d)-EDGE-ANTIMAGIC TOTAL LABELING OF SILKWORM GRAPH

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Abstract. *An (a, d)-edge-antimagic total labeling of G is a one-to-one mapping f taking the vertices and edges onto* $\{1, 2, 3, \ldots, p + q\}$ *Such that the edge-weights* $w(uv) = f(u) + f(v) + f(uv)$, $uv \in E(G)$ *form an arithmetic sequence* $\{a, a+d, a+2d, \ldots, a+(q-1)d\}$, *where first term* $a > 0$ *and common difference* $d \geq 0$. *Such a graph G is called super if the smallest possible labels appear on the vertices. In this paper we will study a super edge-antimagic total labelings properties of connective* Sw_n *graph. The result shows that a connected Silkworm graph admit a super (a, d)-edge antimagic total labeling for* $d = 0, 1, 2$. *It can be concluded that the result of this research has covered all the feasible n, d.*

Key Words: *(a, d)-edge-antimagic total labeling, super (a, d)-edge-antimagic total labeling, Silkworm graph.*

INTRODUCTION

In mathematics and computer science, graph theory is used to model pairwise relations between objects from a certain collection. A ”graph” in this context refers to a collection of vertices or ’nodes’ and a collection of edges that connect pairs of vertices. A graph may be undirected, it means for two vertices u, v the edge uv = edge vu, or may be directed from one vertex to another. In this study we focus for undirected graph, and how to assign label on either vertex and edge.

A labeling of a graph is any mapping graph that sends some set of graph elements to a set of positive integers. If the domain is the vertex-set or the edge-set, the labelings are called, respectively, vertex labelings or edge labelings. Moreover, if the domain is $V(G) \cup E(G)$ then the labelings are called *total* labelings. We define the *edge-weight* of an edge $uv \in E(G)$ under a total labeling to be the sum of the vertex labels corresponding to vertices u, v and edge label corresponding to edge uv. If such a labeling exists then G is said to be an (a, d)-*edge-antimagic total graph*. Such a graph G is called *super* if the smallest possible labels appear on the vertices. Thus, a *super* (a, d)-*edge-antimagic total graph* is a graph that admits a super (a, d)-edge-antimagic total labeling.

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This paper we investigate the existence of super (a, d)-edge-antimagic total labelings of Silkworm graph, and concentrate on the connected Silkworm graph denoted by Sw_n with vertex set $V(Sw_n) = \{x_i, y_i, z_j\ ;\ 1 \le i \le n, 1 \le j \le n+1\}$ and $E(Sw_n) = \{x_i z_i, x_i z_{i+1}, z_i z_{i+1}, y_i z_i, y_i z_{i+1}; 1 \le i \le n\} \cup \{y_i x_{i+1}; 1 \le i \le n-1\}$. Thus $|V(Sw_n)| = p = 3n + 1$ and $|E(Sw_n)| = q = 6n - 1$.

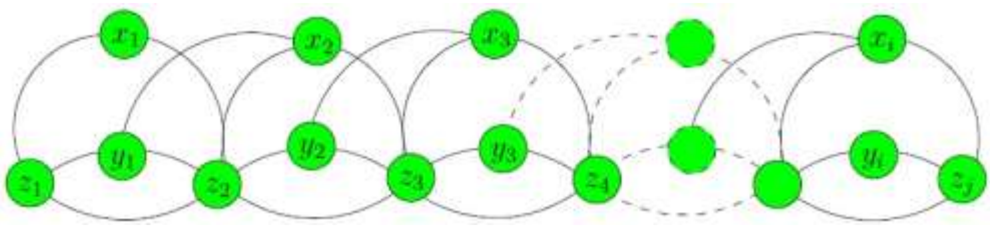

Figure 1: Silkworm graph Sw_n

RESEARCH METHODS

Research methods a super (a, d)-edge-antimagic total labeling of Silkworm graph are deductive axiomatic and the pattern recognition. The research techniques are as follows: (1) calculate the number of vertex p and size q on the graph Sw_n; (2) determine the upper bound for values of d; (3) determine the *EAVL* (edge-antimagic vertex labeling) of Sw_n; (4) if the label of *EAVL* is expandable, then we continue to determine the bijective function of *EAVL*; (5) label the graph Sw_n with *SEATL* (super-edge antimagic total labeling) with feasible values of d and (6) determine the bijective function of super-edge. □

Lemmas

We start this section by a necessary condition for a graph to be super (a, d)-edge antimagic total, providing a least upper bound for feasible values of d. This lemma can be found in [12]

Lemma 1. *If a* (p, q)*-graph is super* (a, d)*-edge antimagic total then* $d \le \frac{2p+q-5}{q-1}$

Proof. Assume that a (p, q)-graph has a super (a, d)-edge antimagic total labeling $f: V(G) \cup E(G) \to \{1, 2, \dots, p+q\}$ and the edge-weights $\{a, a+d, a+2d, \dots, a+(q-1)d\}$. The minimum possible edge-weight in the labeling f is at least $1 + 2 + p + 1 = p + 4$. Thus, $a \ge p + 4$. On the other hand, the maximum possible edge-weight is at most $(p-1)+p+(p+q)=3p+q-1$. So we obtain $a+(q-1)\,d \leq 3p+q-1$ which gives the desired upper bound for the difference d. □

Another important lemma obtainded by Figueroa-Centeno et al [6], gives an easy way to find a total labeling for super edge-magicness of graph.

Lemma 2. A (p, q)-graph G is super edge-magic if and only if there exists a bijective function $f: V(G) \rightarrow \{1,2,\dots,p\}$ such that the set $S=\{f(u)+f(v): uv \in E(G)\}$ consists of q consecutive integers. In such a case, f extends to a super edge-magic labeling of G with magic constant $a=p+q+s$, where $s=min(S)$ and $S=\{a-(p+1), a-(p+2), \dots, a-(p+q)\}$.

The two above lemma will be used for develop theorem 1.

RESULT AND DISCUSSIONS

If Silkworm graph has a super (a, d)-edge-antimagic total labeling then, for $p=3n+1$ and $q=6n-1$, it follows from Lemma 1 that the upper bound of d is $d \leq 2$ or $d \in \{0,1,2\}$. The following lemma describes an $(a, 1)$-edge-antimagic vertex labeling for Silkworm graph.

Lemma 3 *If* $n \geq 2$ *then the Silkworm graph* Sw_n *has an* (3, 1)-*edge-antimagic vertex labeling.*

Proof. Define the vertex labeling $f_1 : Sw_n \rightarrow \{1,2,\dots,3n+1\}$ in the following way:

$f_1(x_i) = 3_i - 1, for\ 1 \leq i \leq n$

$f_1(y_i) = 3_i, for\ 1 \leq i \leq n$

$f_1(z_j) = 3_j - 2, for\ 1 \leq j \leq n+1$

The vertex labeling is a bijective function. The edge-weights of Sw_n, under the labeling f_1, constitute the following sets

$wf_1(x_i z_i) = 6i - 3, for\ 1 \leq i \leq n$

$wf_1(y_i z_i) = 6i - 2, for\ 1 \leq i \leq n$

$wf_1(z_i z_{i+1}) = 6i - 1, for\ 1 \leq i \leq n$

$wf_1(x_i z_{i+1}) = 6i\ , for\ 1 \leq i \leq n$

$wf_1(y_i z_{i+1}) = 6i + 1, for\ 1 \leq i \leq n$

$wf_1(y_i x_{i+1}) = 6i + 2, for\ 1 \leq i \leq n - 1$

It is not difficult to see that the set $wf_1 = \{3, 4, 5, \ldots, 6n + 1\}$ consists of consecutive integers. Thus f_1 is a (3, 1)-edge antimagic vertex labeling. □

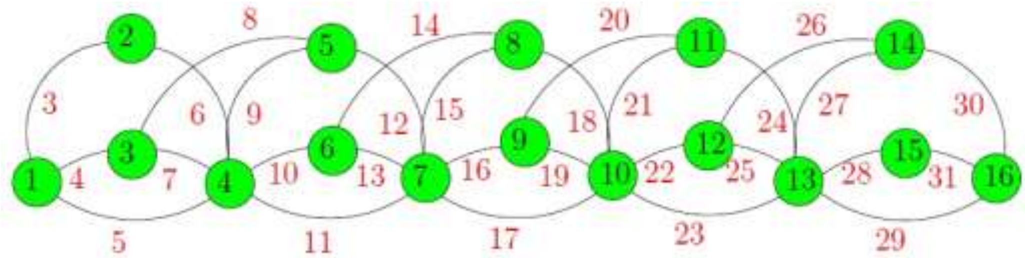

Figure 2: Vertex labeling (3,1)-edge antimagic base on Sw_5

Theorem 1 *If* $n \geq 2$ *then the graph* Sw_n *has a super* $(9n + 3, 0)$*-edge-antimagic total labeling and a super* $(3n + 5, 2)$ *−edge-antimagic total labeling.*

Proof. *Case 1.* for $d = 0$

The *edge* label of Sw_n for $d = 0$ are:

$f_2(x_i z_i) \quad = 9n - 6i + 6, for\ 1 \leq i \leq n$

$f_2(y_i z_i) \quad = 9n - 6i + 5, for\ 1 \leq i \leq n$

$f_2(z_i z_{i+1}) = 9n - 6i + 4, for\ 1 \leq i \leq n$

$f_2(x_i z_{i+1}) = 9n - 6i + 3\ , for\ 1 \leq i \leq n$

$f_2(y_i z_{i+1}) = 9n - 6i + 2, for\ 1 \leq i \leq n$

$f_2(y_i x_{i+1}) = 9n - 6i + 1, for\ 1 \leq i \leq n - 1$

We have proved that the vertex labeling f_1 is a (3, 1)-edge antimagic vertex labeling. With respect to Lemma 1, by completing the edge labels $p + 1, p + 2, \ldots, p + q$, we are able to extend labeling f_1 to a super $(a, 0)$-edge-antimagic total labeling.

We can find the total labeling Wf_2 with summing $wf_1 = wf_2$ with edge label f_2. It is not difficult to see that the set $Wf_2 = \{9n + 3, 9n + 3, \ldots, 9n + 3\}$ contains an arithmetic sequence with the first term $9n + 3$ and common difference 0. Thus f_2 is a super $(9n + 3, 0)$-edge-antimagic total labeling.

Proof. *Case 2.* For $d = 2$

If $f_2(z)$ is edge label of Sw_n for $d = 0$, and $f_3(z)$ is edge label of Sw_n for $d = 2$, we can determine:

$$f_3(s) = 2|p| + |q| + 1 - f_2(s)$$
$$= 2(3n+1) + (6n-1) + 1 - f_2(s)$$
$$= 12n + 2 - f_2(s)$$

Then, if we subtitute the edge label of $d = 0$ to the formula, so we get the edge label of $d = 2$:

$f_3(x_i z_i) = 3n + 6i - 4, for\ 1 \le i \le n$

$f_3(y_i z_i) = 3n + 6i - 3, for\ 1 \le i \le n$

$f_3(z_i z_{i+1}) = 3n + 6i - 2, for\ 1 \le i \le n$

$f_3(x_i z_{i+1}) = 3n + 6i - 1, for\ 1 \le i \le n$

$f_3(y_i z_{i+1}) = 3n + 6i, for\ 1 \le i \le n$

$f_3(y_i x_{i+1}) = 3n + 6i + 1, for\ 1 \le i \le n - 1$

The total labeling f_3 is a bijective function from $V(Sw_n) \cup E(Sw_n)$. The edge-weights of Sw_n, under the labeling f_2. □

$Wf_3(x_i z_i) = 3n + 12i - 7, for\ 1 \le i \le n$

$Wf_3(y_i z_i) = 3n + 12i - 5, for\ 1 \le i \le n$

$Wf_3(z_i z_{i+1}) = 3n + 12i - 3, for\ 1 \le i \le n$

$Wf_3(x_i z_{i+1}) = 3n + 12i - 1, for\ 1 \le i \le n$

$Wf_3(y_i z_{i+1}) = 3n + 12i + 1, for\ 1 \le i \le n$

$Wf_3(y_i x_{i+1}) = 3n + 12i + 3, for\ 1 \le i \le n - 1$

From the first and second case we can conclude that if $n \ge 2$ then the graph Sw_n has a super $(9n+3, 0)$-edge-antimagic total labeling and a super $(3n+5, 2)$-edge antimagic total labeling, for $n \ge 2$.

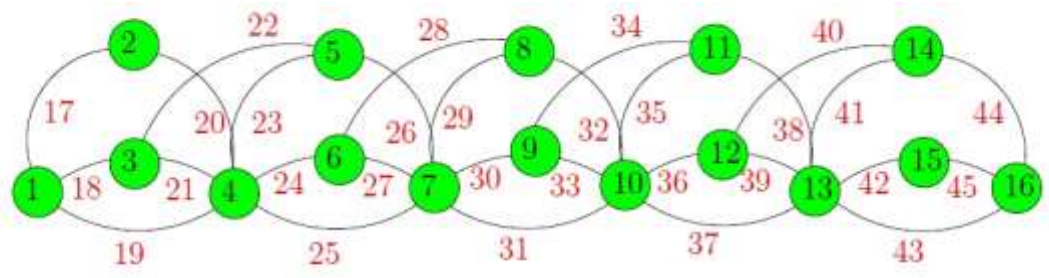

Figure 3: SEATL Silkworm graph Sw_5 for $d = 2$

Theorem 2 *If* $n \geq 2$ *then the graph* Sw_n *has a super* $(6n + 4, 1)$*-edge-antimagic total labeling.*

Proof. let us define the vertex labeling of Silkworm graph Sw_n as $f_4(x_i z_i) = f_1(x_i z_i), f_4(y_i z_i) = f_1(y_i z_i), f_4(z_i z_{i+1}) = f_1(z_i z_{i+1}), f_4(x_i z_{i+1}) = f_1(x_i z_{i+1}),$ $f_4(y_i z_{i+1}) = f_1(y_i z_{i+1})$, and $f_4(y_i x_{i+1}) = f_1(y_i x_{i+1})$ then the edge labeling f_4 for super $(a, 1)$-edge total labeling can be defined as follow:

$f_4(x_i z_i) \quad = 6n - 3i + 4, for\ 1 \leq i \leq n$

$f_4(y_i z_i) \quad = 6n - 3i + 3, for\ 1 \leq i \leq n$

$f_4(z_i z_{i+1}) = 6n - 3i + 2, for\ 1 \leq i \leq n$

$f_4(x_i z_{i+1}) = 9n - 3i + 3\ , for\ 1 \leq i \leq n$

$f_4(y_i z_{i+1}) = 9n - 3i + 2, for\ 1 \leq i \leq n$

$f_4(y_i x_{i+1}) = 9n - 3i + 1, for\ 1 \leq i \leq n - 1$

The edge-weight of Sw_n under the labeling f_4 can be determine by adding the edge-weight of EAVL and the edge labeling by f_4. Namely $Wf_4 = wf_1 + f_4$. We can have the following:

$Wf_4(x_i z_i) \quad = 6n + 3i + 1, for\ 1 \leq i \leq n$

$Wf_4(y_i z_i) \quad = 6n + 3i + 2, for\ 1 \leq i \leq n$

$Wf_4(z_i z_{i+1}) = 6n + 3i + 3, for\ 1 \leq i \leq n$

$Wf_4(x_i z_{i+1}) = 9n + 3i + 1, for\ 1 \leq i \leq n$

$Wf_4(y_i z_{i+1}) = 9n + 3i + 2, for\ 1 \leq i \leq n$

$Wf_4(y_i x_{i+1}) = 9n + 3i + 3, for\ 1 \leq i \leq n - 1$

We can state all the above edge-weights in a set $Wf_4 = \{6n+4, 6n+5, \ldots, 12n+2\}$.It can be seen that the edge-weights for a consecutive arithmetic sequence. The Silkworm graph Sw_n admit a super $(6n+4, 1)$-edge-antimagic total labeling. □

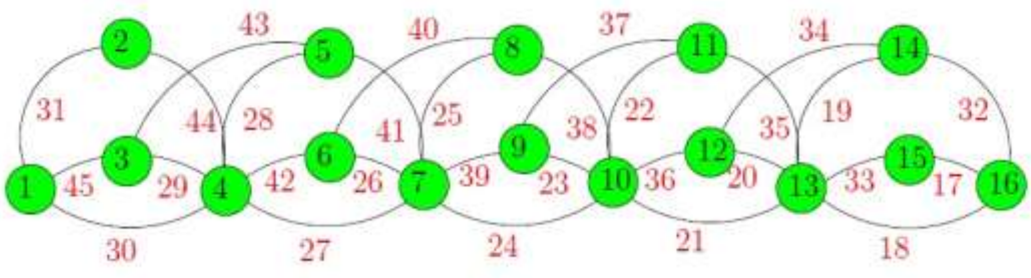

Figure 4: SEATL Silkworm graph Sw_5 for $d = 1$

CONCLUSION

Finally, we can conclude that the graph Sw_n admit a super (a,d)-edge antimagic total labeling for all feasible d and $n \geq 2$.

REFERENCES

A. Kotzig and A. Rosa, Magic valuations of finite graphs, *Canad. Math. Bull.* 13 (1970), 451–461.

Dafik, Alfin Fajriatin, Kunti Miladiyah. 2012. *Super antimagicness of a Well Defined Graph*. (Saintifika, vol.14 No 1 hal 106-118).

Griffin, C. 2012. *Graph Theory*. United Stated: Creative Commons Attribution Noncommercial-Share.

J. A. Galian. 2009. *A Dynamic Survey of Graph Labelling*. [serial on line]. http://www. combinatorics.org/Surveys/ds6.pdf. [17 Agustus 2010]

J. A. Gallian. 2013. *A Dinamic Survey Of Graph Labeling*.Jember: Gallian Survey.124-128.

J. Baugh, Richard. 2009. *Discrete Mathematics, seventh edition*. New Jersey: Pearson Education, Inc.

K.A. Sugeng, M. Miller and M. Baca, Super edge-antimagic total labelings, *Utilitas Math.*, 71 (2006), 131-141.

Lee, Ming-ju.2013. *On Super (a,1)-edge Antimagic Total Labelings Of Subdifition Of Stars* .Miaoli: Jen-Teh Junior Collage Of Madicine.1-10.

M. Bača, Y. Lin, M. Miller and R. Simanjuntak, New constructions of magic and antimagic graph labelings, *Utilitas Math.* 60 (2001), 229–239.

R.M. Figueroa-Centeno, R. Ichishima, F.A. Muntaner-Batle, The place of super edge-magic labelings among ather classes of labelings, Discrete Mathematics, 231 (2001), 153-168.

R. Simanjuntak, F. Bertault and M. Miller, Two new (a, d)-antimagic graph labelings, *Proc. of Eleventh Australasian Workshop on Combinatorial Algorithms*- (2000), 179–189.