REUSE OF FRACTIONAL WAVEFORM LIBRARIES FOR MIMO RADAR AND ELECTRONIC COUNTERMEASURES

C. Clemente[1], *C. Ilioudis*[1], *D. Gaglione*[1], *K. Thompson*[1], *S. Weiss*[1], *I. Proudler*[2], *J. J. Soraghan*[1]

[1]Centre for Excellence in Signal and Image Processing, University of Strathclyde, Glasgow, UK
[2] School of Electronic, Electrical and Systems Engineering, Loughborough University, Leicestershire, UK

ABSTRACT

A fundamental aspect in the hardware-software design of modern radar systems, for example MIMO or Low Probability of Intercept Radar, is to operate in electromagnetically crowded environments. Proper radar waveform design is central to effective solutions in such systems. In this paper cross-interference and waveform reuse for a set of waveform libraries based on the fractional Fourier transform are presented and analysed. The results demonstrate the potential of the novel libraries in increasing the number of available waveforms and for stealth transmissions.

Index Terms— MIMO Radar, Waveform Design, Fractional Fourier Transform, Electronic Countermeasures

1. INTRODUCTION

In the modern battlefield scenario radar systems typically operate in a congested electromagnetic environment and with severe constraints in terms of interference mitigation, frequency occupancy, security and performance. Coexistence of different systems for different applications, high accuracy in target detection, tracking and recognition, low probability of intercept, jamming and MIMO radar are all examples where a hostile environment, from an electromagnetic point of view, can cause dramatic consequences to the overall performance. In this scenario the selection of the most suitable waveform can play an important role. Fixed and adaptive radar waveform design has been widely investigated [1, 2], providing waveforms that can suit different applications. However each of them presents a trade-off between characteristics such as range resolution versus side lobe levels. The ability to form novel libraries of waveforms that are able to maintain the same or higher level of performance is of interest to the radar community and other related disciplines.[3].

In this paper the use of fractional Fourier Transform (FrFT) based phase coded waveforms is introduced to generate new families of waveform libraries.

The FrFT is a generalization of the Fourier transform and has already been applied in radar signal processing [4] and OFDM modulation [5] demonstrating the potential of this signal processing tool for various applications [6]. In our approach for the generation of novel radar waveform libraries the FrFT is applied to the waveforms (e.g. the code sequence). In this paper we analyse cross-interference and waveform reuse for the novel libraries of waveforms.

The remainder of the paper is organized as follows. Section 2 introduces the fractional Fourier transform, while in Section 3 the novel modulation approach and the signal model for the phase coded libraries are introduced. The analyses of cross-interference and waveform reuse are presented in Section 4, while Section 5 concludes the paper.

2. FRACTIONAL FOURIER TRANSFORM

Fourier transformation (FT) maps a one-dimensional time signal $x(t)$ into a one-dimensional frequency function $X(f)$, the signal spectrum. The Fourier transform operator can be visualized as a change in representation of the signal corresponding to a counter clockwise rotation of the axis by an angle $\pi/2$ in the time-frequency plane. Although the Fourier transform provides the spectral content of the signal, it fails to indicate the time location of the spectral components, which is of great importance when non-stationary or time-variant signals are considered. In order to describe and analyse such signals, time-frequency representations (TFRs) are used. A TFR maps a one-dimensional time signal into a two-dimensional function of time and frequency. The fractional Fourier transform belongs to the class of linear TFRs and was firstly introduced by 1980 by Namias [7]. Afterwards it was rediscovered in optics [8, 9] and introduced to the signal processing community by Almeida in 1994 [10]. The fractional Fourier transform, can be considered as a rotation by an arbitrary angle in the time-frequency plane or a decomposition of the signal in terms of chirps. It also called rotational Fourier transform or angular Fourier transform [11] and it serves as an orthonormal signal representation for chirp signals. The fractional Fourier transform is computed using the angle of rotation in the time-frequency plane as the fractional power of the ordinary Fourier transform. Letting $x(u)$ be an arbitrary signal of length U, its a^{th}-order discrete FrFT

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is defined as [6]:

$$\mathbf{X}_a[u] = \sum_{u'=-U/2}^{U/2} K_a[u,u']x[u'] \quad (1)$$

where a is the fractional transformation order (corresponding to a rotation angle $\theta = a\frac{\pi}{2}$ with $a \in \mathbf{R}$) and $K_a[u,u']$ is the FrFT kernel and is defined as [6]:

$$K_a[u,u'] = \begin{cases} A_0 \exp\{j\pi[(u^2+u'^2)\cot\theta - 2uu'\csc\theta]\} \\ \text{if } \theta \text{ is not a multiple of } \pi \\ \delta[u-u'] \text{ if } \theta \text{ is a multiple of } 2\pi \\ \delta[u+u'] \text{ if } \theta+\pi \text{ is a multiple of } 2\pi \end{cases} \quad (2)$$

where $A_0 = \frac{e^{j\frac{\theta}{2}}}{\sqrt{j\sin\theta}}$ and $j = \sqrt{-1}$.
Equation (2) shows that for angles that are not multiples of π, the computation of the FrFT corresponds to the following steps:
1-A product by a chirp;
2-A Fourier transform (scaled by $\csc\theta$);
3-Another product by a chirp;
4-A product by a complex amplitude factor.
In summary, the FrFT is an invertible linear transform, continuous in the angle θ, which satisfies the basic conditions for it to be meaningful as a rotation in the time-frequency plane.

3. FRFT BASED WAVEFORMS

In this section the new approach to obtain novel libraries of waveforms is introduced. The Fractional Fourier Transform introduced in Section 2 can be applied to common waveforms, such as phase modulated waveforms with different codes (e.g. Barker or P4 codes).
Let $s(n)$ be the canonical waveform (e.g. the traditional Barker 13 code) from which the Fractional Fourier transform library elements, $\mathbf{S}_{a_i}(u)$ $i = 1, ..., L$ are obtained, by applying (1). Thus we define a fractional waveform library as:

$$\mathbf{S} = [S_{a_1}[u], S_{a_2}[u], \ldots, S_{a_L}[u]] \quad (3)$$

where $a_i \in [0,1]$, and L represents the total number of waveforms populating the library. Note that for $a_i = 0$ the canonical waveform is obtained. The value of L depends on different aspects such as the original waveform used, waveform reuse, orthogonality requirements and applications.
In order to obtain the analytical representation of each element of the library, the cardinality of the waveform Ω (the number of chips used) and the number of samples per chip r must be introduced, from which the total digital signal length $N = r \times \Omega$ is obtained. Defining $\mathbf{c} = [c_1, c_2, \ldots, c_N]$ as the vector containing the amplitudes of N samples of the waveform before the modulation (e.g. ± 1 values), $s[n]$ can be written as:

$$s[n] = \sum_{k=1}^{N} c_k \delta[n-k] \quad (4)$$

The FrFT can now be applied to (4), and using the properties reported in [10]

$$S_a[u] = \sum_{k=1}^{N} c_k FrFT_a[\delta[n-k]] = \sqrt{\frac{1-j\cot\theta}{2\pi}} \sum_{k=1}^{N} c_k e^{j\frac{k^2+u^2}{2}\cot\theta - juk\csc\theta} \quad (5)$$

From (5) the l^{th}-element of $\mathbf{S}$ is the sum of N chirped functions weighted by the original waveform sequence and with modulation rate depending on a and k. An important remark is that the number of chirped components depends on N, thus by fixing the code cardinality Ω (e.g. the Barker 13 code), different waveforms can be obtained changing the chip sampling rate r.

4. WAVEFORM REUSE ANALYSIS

In this section the performance of the novel waveform libraries are analysed numerically in order to determine orthogonality properties between waveforms generated with the same **c** and different fractional order a. The analysis has been conducted for three values of $r = [50, 100, 200]$. In the analysis four code sequences have been selected to populate **c**, Barker 13, Costas 7, Frank 16 and P4 25 [1].
In order to analyse the orthogonality properties of the library we assume that two waveforms $S_a(u)$, with the same **c** and different values of a, are orthogonal if their cross-correlation is below the side-lobe level of the original sequence **c**. The side-lobe levels used in the analysis are reported in Table 1. In Figs. 1, 2 and 3, the SLL-thresholded maxima of the cross-correlations between the waveforms with different a and the same **c** are reported for the three choices of r. In all the analysed case the number of pairs of waveforms with cross-correlation below the threshold increases as r increases. This effect is due to the fact that for higher values of r the number of chirped components in (5) increases; the mismatching between waveforms of the same libraries modulated with different fractional orders becomes stronger leading to a higher

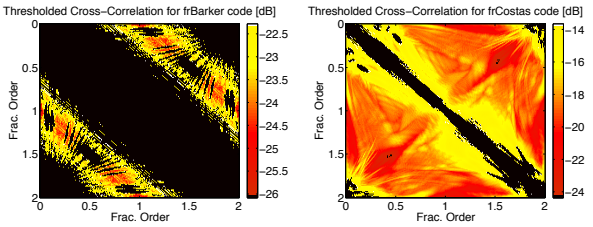
Sequence	SLL [dBs]
Barker 13	-22.28
Costas 7	-13.65
Frank 16	-21.07
P4 25	-22.22

Table 1. Side Lobe Levels used as thresholds in the orthogonality analysis.

Sequence	$r = 50$	$r = 100$	$r = 200$
Barker 13	0.77	0.40	0.22
Costas 7	0.25	0.14	0.08
Frank 16	0.28	0.15	0.08
P4 25	0.49	0.22	0.12

Table 2. Fractional order reuse interval for different values of r.

reuse factor. This framework can be implemented in a digital arbitrary waveform generator (AWG) present in modern radar systems, moreover the effect of the higher samples per chip rate r can be addressed using high speed D/A converters in order to obtain the same time duration of the signal. Waveforms obtained with Costas and P4 codes show good performance in terms of orthogonality with $r = 50$ as indicated in Fig. 1-b and Fig. 1-d, while higher values of r are required to achieve similar performance with Barker or Frank codes, as indicated in Fig. 2 and Fig. 3. A notable result is the level of cross-correlation achievable, which in some cases is below -35 dB.

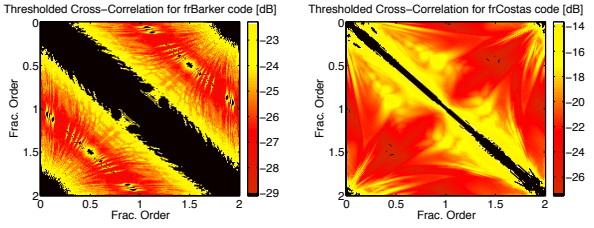


(a) Barker 13 (b) Costas 7

(c) Frank 16 (d) P4 25

Fig. 1. SLL-thresholded maxima of the cross-correlations between the waveforms with different a and the same $\mathbf{c}$ for $r = 50$. The cases above the thresholds are shown in black.

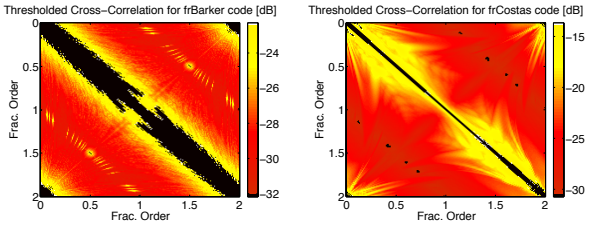
In Figs. 4, 5 and 6 the reuse interval (measured in fractional order - see below) quantified for each fractional order for the orthogonality matrices shown in Figs. 1, 2 and 3 are reported. The reuse intervals are estimated by selecting the first fractional order below the threshold for each value of a. These results show that the reuse interval becomes smaller for r increasing, meaning that the library $\mathbf{S}$ is composed of a higher number of orthogonal waveforms for r increasing. Costas and P4 codes are seen to provide higher reuse levels than Frank and Barker codes. In Table 2 the maxima (worst



(a) Barker 13 (b) Costas 7

(c) Frank 16 (d) P4 25

Fig. 2. SLL-thresholded maxima of the cross-correlations between the waveforms with different a and the same $\mathbf{c}$ for $r = 100$. The cases above the thresholds are shown in black.



(a) Barker 13 (b) Costas 7

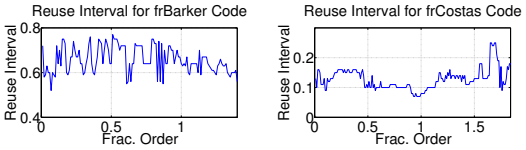
(c) Frank 16 (d) P4 25

Fig. 3. SLL-thresholded maxima of the cross-correlations between the waveforms with different a and the same $\mathbf{c}$ for $r = 200$. The cases above the thresholds are shown in black.

case) of the reuse intervals are summarized, showing that in some cases (reuse interval of 0.08) more than 10 orthogonal waveforms can be obtained for a given code sequence.

5. CONCLUSIONS

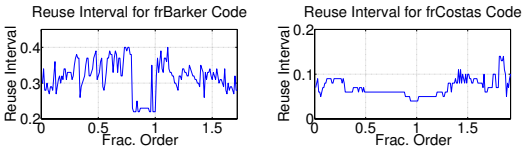
This paper presented novel radar waveform libraries based on the use of the fractional Fourier transform modulation. The orthogonality and reuse properties of these waveforms was



(a) Barker 13 (b) Costas 7

(c) Frank 16 (d) P4 25

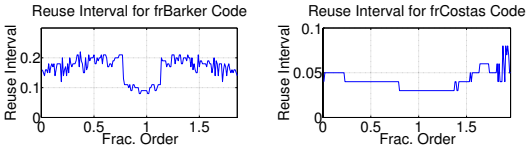
Fig. 4. Reuse Interval for the waveforms with different a and the same $\mathbf{c}$ for $r = 50$.



(a) Barker 13 (b) Costas 7

(c) Frank 16 (d) P4 25

Fig. 5. Reuse Interval for the waveforms with different a and the same $\mathbf{c}$ for $r = 100$.



(a) Barker 13 (b) Costas 7

(c) Frank 16 (d) P4 25

Fig. 6. Reuse Interval for the waveforms with different a and the same $\mathbf{c}$ for $r = 200$.

analysed. The novel libraries show a good level of orthogonality and reuse allowing possible applications in co-located and distributed MIMO radar, pulse agile radar and in electronic countermeasure such as Low Probability of Intercept Radar. For example a LPI Radar can use an arbitrary waveform provided by the novel libraries increasing its covertness due lack of knowledge from the interceptor of fractional order used by the transmitter.

6. REFERENCES

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