R. Dual Bimodules and PF Rings

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Kasch [K, Theorem 13.4.2] has shown that, for a two-sided Artinian ring R, the following conditions

(1) R is QF,

(2) For every primitive idempotent e, soc(eR) and soc(Re) are simple and in soc(R_R) resp. soc($_RR$) all simple right resp. left R-modules occur up to isomorphism,

(3) For every primitive idempotent e, soc(eR) and soc(Re) are simple and we have soc(R_R) = soc($_RR$), and

(4) There exists a permutation σ of $\{1, 2, \ldots, k\}$ so that for every $i = 1, 2, \ldots, k$ we have

$$soc(e_i R)_R \cong (\bar{e}_{\sigma(i)} \bar{R})_R \quad \text{and} \quad {}_R soc(Re_{\sigma(i)}) \cong_R (\bar{R}\bar{e}_i)$$

are equivalent. Then, generalizing this theorem, Anh[A, Corollary 2] has given that, if R is a ring such that $_RR$ and R_R are both linearly compact and finitely cogenerated, the conditions (2), (3) and (4) are equivalent to

(1') R is two-sided PF.

However, under the assumption that R is two-sided Artinian, QF rings are just dual rings. Hence, it seems to be natural to consider that each condition of (2), (3) and (4) characterizes the dual ring but not the PF ring. The aim of this note is to investigate this fact.

The detailed version of this note will be submitted in publication elsewhere.

References

[A] P. N. Anh, Characterization of two-sided PF rings, J. Algebra 141(1991). 316 – 320.

[K] F. Kasch, "Modules and Rings", Academic Press, London, New York (1982).