Proc. Indian natn. Sci. Acad., **63**, A, No. 4, 1997, pp. 287-314.

REVIEW ARTICLE

DYNAMIC BEHAVIOUR OF CRACKED STRUCTURES AND MACHINES

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(Received 11 December 1996; Revised 13 March 1997; Accepted 21 March 1997)

The review reported here covers the area of dynamics of cracked bodies-rotors, beams and columns, plates and shells. It also gives information on vibration-based techniques of crack detection. Unlike its counterparts in the past, it is broad based.

Key Words: Dynamics and Vibration; Cracked Rotors; Cracked Beams and Columns; Cracked Plates and Shells; Crack Identification

Survey - A General Introduction

The effect of crack on structures and machines has been a field of interest for quite long. In fact, the first work in the field appeared on the turn of the century. Inglis's[1] is the pioneering work which is devoted to analysis of stresses in notched elastic plates. It provided the necessary basis for the two papers of Griffith[2,3] which are outstanding in the history of deformable mechanics both for the idea they advanced and for the example used in presentation. A subsequent complete silence in research in this area has been noticed. Indeed so singly involved and so far ahead of time was Griffith that later progress in the field had to wait till the end of 1950s when Irwin[4] published his paper part of which is review and systematisation of existing work and the balance introduction of new working concepts. Irwin[4] and Williams[5] derived the elastic stress field at the crack tip by a power series assumption. Thus started the investigation into the field of conventional Fracture Mechanics. The subject became relevant especially to the Aerospace industry.

However, the study of vibration characteristics of a cracked structure has an independent origin. The two early reported works in the field are by Kirmser[6] and Thomson[7]. The papers studied the dynamic characteristics of a beam with local discontinuity in the form of a small slot. Since, the field of Fracture Mechanics was not well established at that time (Fracture Mechanics started in 1950s while the investigations were done in 1940s), a beam with different cross-sectional areas in different fields along the beam axis was considered. The idea of both the papers was to replace the slotted bar by a uniform bar with a modified load. For the next 20 years no essential advance could be noted until the end of 1960s. Meanwhile, the fundamentals of Fracture Mechanics were developed and since then a continuously increasing number of papers on the effects of crack in structure has been published. The relatively early works in this field was due to Schmerling and Hannon[8] and Dimarogonas[9]. However, such early work (e.g., Dimarogonas[9]) indicating the possible utility of vibration signals (/responses) as means for detecting crack initiation and growth was probably not available to the public. Thereafter, in and around the mid-1970s substantial research output along the said lines was projected in various conferences. Gasch[10], Henry and Okah-Avae,[11] and Mayes and Davies[12] are some of the leading few. It was only around 1980s papers relating to vibration aspects of cracked bodies started appearing in various international journals. Grabowski[13] and Inagaki *et al.*[14] are examples of a few early works. It needs also to be mentioned that Bently Corpn.[15-19] improved the understanding of cracked rotors by development of both demonstration rigs and useful crack detecting systems based on their own theoretical work and these papers have appeared in their inhouse journal *Orbit* and the stated references are examples of a few.

Presently, the literature available in vibration of cracked structures is quiet vast. Two books (Kramer[20] and Dimarogonas and Baipetis[21]) have devoted a complete chapter on vibration of cracked rotors. Besides two review papers by Wauer[22] and Gasch[23] covering limited aspects of the subject has been published. Further, a letter to the editor by Dimarogonas[24] has appeared where the author has reported the availability of a list of about 500 papers.

The first review on the subject by Wauer[22] contains 162 references. It deals with review of dynamics of cracked rotors including modelling of the crack. Since the objective of the review is to throw light on crack detection and modelling techniques used till 1990, the material presented includes cracked beams and columns. While the first 78 references are on cracks in rotors, the remaining deals with cracks on beams and columns. Although the review appears to be brief yet, the list of references appears to be quite exhaustive.

In the second review by Gasch[23], the author provides brief but comprehensive survey of the stability behaviour and forced vibration characteristics (due to crack and unbalance) of a rotating shaft (idealised as a Laval rotor). The treatment assumes that the vibration is small in comparison to the rotor sag under self weight and the author feels that the approach is valid even when rotating speed reaches unstable ranges and vibration begins to grow. The stiffness matrix for the cracked rotor which is nonlinear and time variant is split into two parts. The first represents the stiffness matrix of the uncracked shaft and the second represents the crack stiffness which is a function of the shaft displacement and rotation angle. The second part is small with respect to the first part but governs the stability of the shaft and also the forced vibration due to the crack. Assumption of weight dominance also allows the nonlinear differential equation to be transformed to a linear, time variant, periodic form. Concepts of breathing crack, open crack, closed crack and their relation to self weight deflection has also been mentioned. The crack model is a hinge model with single parameter where crack depth is small with respect to the rotor diameter. This approach has been found to trace the existence of crack with relative ease during run-up or run-down of the rotor and not during constant speed run. The review contains only 15 references and it appears that it is a survey of a possible methodology for analysis of cracked rotors and so is not a broad based literature survey.

The present literature survey is intended to be a broad based survey on the work reported in dynamics and vibration aspects of cracked structures, in general. It is broadly classified into the following heads -

(i) Cracked rotors
(ii) Cracked beam and columns
(iii) Crack identification
(iv) Cracked plate and shell
(v) Miscellaneous

Next, each of the heads listed above is discussed in detail.

Cracked Rotors

Inagaki *et al.*[25] have studied the transverse vibrations of a general rotor bearing system with a crack. They have discussed the steady state response to gravity and unbalance excitation and the natural frequencies of a system with open or open-close type crack. An iterative numerical method employing the transfer matrix approach has been used. The open-close type crack has been idealised as a step function of bending moment. The nonlinear equations of motion have been linearised using the Fourier expansion technique. Calculated values have been compared with experimental data. It has been concluded that rotor cracks create once/rev. and twice/rev. vibrations under gravity in certain phase relations. Unbalances, if large, affect open-close breathing of cracks. Lastly, the authors feel that static test methods are the final detection step of crack after the abnormality in the rotor has been established.

Mayes and Daview[26] have also studied the flexural vibrations of a rotor mounted on several bearings and containing a transverse crack. A method of solving the equations of motion for a general system has been presented. The advantage of the method lies in that the standard rotor dynamics programs may be used. An approximate method of estimating the reduction of section diameter required to model a crack for use in beam theory based on finite element has been given. The method yields acceptable results. Application to real life problems of 100 MW and 500 MW turbogenerator has been mentioned. However, accuracy reported is limited due to the truncation of the harmonic series and the crack opening and closing model. Davies and Mayes[27] have studied the problem as described by Mayes and Davies[26] but experimentally using a spin rig. It has been stated that except for very large cracks vibration behaviour has been found to be similar to that of a slotted shaft with additional excitation due to crack opening and closing. The dynamic stresses in the cracked system have been measured and the results show how the dynamic bending moment at the crack tip depends on the speed of rotation and crack depth. The experimental results obtained have been compared with the theoretical results of Mayes and Davies[26] and good agreement has been reported. This implies that the simple theory developed to calculate the dynamic bending moment is quite accurate. The results also indicate that the dynamic effect must be taken into account for Fracture Mechanics calculation of crack growth rate when the crack happens to be large.

While the previous works have treated the case of a crack on a rotor which grows transversely, Ichimonji and Watanabe[28] have presented a qualitative analysis of transverse vibration of a rotor system with a crack that grows at 45 degrees with the shaft axis. Based on the assumption that the bending stiffness changes synchronously with the opening/closing behaviour of the crack caused by the torsional vibration of the shaft, the equation of motion of a simple rotor system with a shaft having a slant crack has been represented by a differential equation with parametric excitation in the co-ordinate system rotating at the operating speed of the rotor. It has been shown that the steady state response of the cracked rotor system induced by imbalance will be containing a characteristic frequency which is a function of both the operating speed as well as the frequency of torsional vibration of the system. It has also been concluded that increasing the depth of the crack results in the system becoming unstable.

Ishida *et al.*[29] have investigated experimentally and theoretically the change in the resonance curve caused by the occurrence of a crack. The results show that the shape of the resonance curve happens to change due to the direction of the unbalance and an unstable region appears when the unbalance exists in the same direction as the crack. It has further been shown that this phenomenon is explained theoretically by either combining the characteristics of the unsymmetrical shaft and that of the symmetrical shaft or by considering rotating non-linear spring characteristics.

Gasch *et al.*[30] have provided a comparison of three different models of a cracked hollow shaft and its has shown that if the crack depth is not more than 50% of the shaft radius, the simple hinge model is very good for representation of the cyclic stiffness variation and for determination of stability limits. The same has been found to be true for solid shafts as well. However, for deeper cracks modifications are required. Besides comparing crack models, the model obtained the local crack flexibilities through the stress analysis of the residual problem using the theory of thin shells.

Wen and Wang[31] have used perturbation and numerical method (FEM) to analyse the response of shafts with cracks. Experiments have also been conducted for various situations. Correlation between analytical and experimental results has been found to be good. Besides the authors have well observed that the crack produces negligibly small decrease in critical speed, cracked shaft prominently shows second order harmonic and in the case of open-close crack higher order harmonics are also present.

Tamura *et al.*[32] have performed experiments on a rotor with imitation crack which simulates the characteristic of the practical transverse crack. Imitation cracks need be devised because it is then possible to vary the crack depth to any value without the risk of breakage. Such cracks are non-propagating. It was found that additional resonances at 1/2, 1/3, 1/4 and 1/5 of the critical speed were caused by the crack and gravity and it was also observed that unstable vibration occurred in and about 2/3 of the critical speed in case of large cracks.

Wauer[33] has dealt with the dynamics of cracked distributed parameter rotor. The model that has been used is a rotating Timoshenko shaft which is also flexible in extension and torsion. Two uniform fields have been connected by a local, complex, spring element with reduced stiffness and damping simulating an open crack. The modification describing a closed crack has been introduced and the open/close condition has been formulated. The boundary value problem has been derived. A special solution approach converts the geometric discontinuity to load discontinuity at the crack location. It is to be pointed out that the work stresses more on the mathematical procedures used than that on the representation of the results for a specific case.

Collins *et al.*[34] have also considered a rotating Timoshenko shaft with a transverse crack which opens and closes during motion. It has been represented by generalized forces and moments. Six coupled piece wise linear equations of motion have been integrated numerically after application of Galerkin's method. Time histories and frequency spectra have been compared for shafts with and without crack. Free vibration studies and responses to single axial impulse as well as periodic axial impulses have been presented. The last case according to the author appears to be an effective means for detecting cracks. It has also been noted that the maximum vertical displacement non-linearly increases with crack depth and crack produces coupling between the various displacements.

Suherman and Plaut[35] have studied the flexural torsional response of a cracked rotating shaft during passage through critical speed. The authors feel that it may be easier to detect a crack by observing the response when the shaft is accelerated or deccelerated through a critical speed at constant rate. A breathing crack has been considered and the effects of acceleration and decceleration rate, internal and external damping, crack location, mass, eccentricity and eccentricity angle has been studied.

Ref.[36-40] are works co-authored by Dimarogonas. Most of these papers have used the central theme that - if the stress intensity factor for a body with a specific crack configuration is known, the strain energy release function can be obtained and thereby it is possible to obtain the local flexibility due to the crack.

Dimarogonas and Massouros[36] have studied the influence of a circumferential crack on the torsional dynamic behaviour of a shaft. Results have shown that while the fundamental frequency strongly depends on the crack depth, higher frequencies have a weaker dependence. Experimental results have been reported to agree with theoretical prediction. It has been concluded that the change in dynamic response due to the crack is good enough to allow detection of the crack and estimation of its location and magnitude. The authors have proposed that continuous monitoring of shafts in service, especially for machines which have welded rotors is possible by natural frequency studies. Also it has been felt that stress intensity factor and strain energy rate can be obtained experimentally by natural frequency measurements.

Dimarogonas and Papadopoulos[37] have analysed the vibration of shaft in bending with both open and closing cracks. The response has been found to exhibit patterns similar to a shaft with dissimilar moment of inertia. The nonlinear behaviour of the closing crack, according to the authors, introduces characteristics of nonlinear systems. However, for many practical applications, the system may be considered bilinear as a consequence of which analytical solutions are possible. De-Laval rotor model has only been considered for simplicity. Since stress intensity factors are not available for transverse cracks on cylinders, the same have been found to be possible for plane strips. The analytically obtained values of flexibility have also been compared with experimental results. The most important conclusion from the investigation is the presence of near half critical speed which helps to identify a crack.

Papadopoulos and Dimarogonas[38] have considered both longitudinal and bending vibrations of a rotating shaft with an open crack. It has been observed that substantial coupling of the longitudinal and bending vibrations exists. The local flexibility due to the crack has been represented by a (6x6) matrix and the presence of the off-diagonal terms explains the cause of coupling in the various directions. However, shear has not been considered and three degrees of freedom viz two bending and one extension have been sued. Both undamped and damped vibrations have been studied. The authors feel that the coupling between longitudinal and bending vibration, over and above the occurrence of subcritical resonance, is an important clue for crack detection.

Papadopoulos and Dimarogonas[39] have performed stability analysis of cracked rotors in coupled vibration mode. A case of crack which opens and closes during rotation resulting in differential equation of motion with periodically varying stiffness has been considered. The solution to such a case has been obtained as a sum of harmonic functions of time. A method for determination of instability intervals of the first and second kind has also been presented. The authors feel that the analysis can be extended to various structures and diverse applications may be envisaged in the diagnosis of non-union healing of cracked bones, cracked welds in pressure vessels and in real time quality control of material production process.

The coupling of vibration modes of a clamped-free Timoshenko beam with circular cross-section has been studied by Papadopoulos and Dimarogonas[40]. Coupling has been found in all spectra obtained with harmonic sweeping excitation throughout the frequency range. The authors feel that the method is very sensitive even for small cracks such as cracks with depth of 6%. Application of the sweeping excitation to a rotating shaft is also feasible and infact, for electric motors or generators, it is even possible to apply a torsional sweeping excitation through the electric bus.

Sekhar and Prabhu[41-43] have studied dynamics of rotors using the crack model described by Papadopoulos and Dimarogonas[38] but by employing the finite element method. Sekhar and Prabhu[41] graphically have represented the fall of frequencies with crack depth for various positions of crack and different slenderness ratios of the shafts. It has been found that the fall in natural frequency due to crack has been appreciable in cases of shafts with low slenderness ratios. Sekhar and Prabhu[42] have studied the response and stress fluctuation of a cracked shaft. The steady state unbalance response has been obtained by neglecting the rotating flexibility. Results have shown that with increase of crack depth there is progressive reduction in natural frequencies and increase in the peak as well as split in the critical speed. Further the bending stress fluctuations developed in the shaft are related to backward whirling of rotors. An attempt has been by the authors to relate the fluctuations of the bending stress and the backward whirling due to crack as a possible diagnostic tool for crack growth monitoring. Sekhar and Prabhu[43] have undertaken a transient analysis of a cracked rotor while passing through the critical speed. Although the stiffness varies with time for a cracked shaft, during a small time step, the stiffness has been assumed to be constant and Houbolt's method has been used for obtaining the response. It has been found that with increase of crack depth, the critical speed region becomes wider and the vibration becomes more violent. It appears to the authors that rotors with large crack may not be able to pass through the critical speed. However, they have also pointed out the possibility of crack detection using detailed database on transient response.

Ref.[44-46] once again have used the concept of local flexibility near the crack tip to model the crack. Hamidi *et al.*[44] have used two mathematical models - one based on Rayleigh Ritz approach and the other on finite element strategy for the study of modal parameters. Both the models can account for rotation, variable cross section, multiple discs, multiple bearings and multiple cracks. The results of the models have been compared with an exact analytical solution for a simplified case. The vibration characteristics of a cracked Timoshenko shaft has been investigated by Rajab and Al-Sabeeh[45]. Analytical expression and graphical representation have been provided relating the crack depth and location on the shaft to changes in the first few natural frequencies. The authors have shown that crack depth and position for a single crack may be estimated by knowing changes in the first three natural frequencies and each additional crack requires knowledge of two or more higher frequencies. Huang *et al.*[46] have used the energy principle along with the assumed mode method to study the dynamic response of rotating shaft. The steady state responses and stability criteria also have been obtained. It has been pointed out that crack affects the dynamic response more significantly when it is located at a point where bending moment is high.

Lee *et al.*[47] have dealt with a rotor with switching crack. Instead of a finite element model, a simple rotor model has been used because the authors feel that the later model is still useful and relevant for understanding the vibrational behaviour of a cracked rotor since many rotors in the real world operate below the second flexural critical speed. A switching crack model having two different stiffnesses depending upon the crack status have been developed. The necessary conditions for crack opening/closing have been derived for a simple rotor with a switching crack. The validity of the crack model and crack opening/closing conditions have been provided through both numerical simulation and experiments and a crack identification scheme based on the analysis has been proposed.

Nelson and Nataraj[48] happen to be one of the early researchers in the dynamics of cracked rotors *using finite elements*. A transversely cracked rotor has been considered. The rotor has been modelled by using finite shaft elements and the crack modelled by a rotating stiffness variation which happens to be a function of the rotor's bending curvature at the crack location. This has been represented by a Fourier series expansion. A computer code has been developed and results compared with other existing analytical and experimental work.

Li Changhe *et al.*[49] have described the construction of general and reliable mathematical models of the dynamics of cracked shafts. The stiffness depends on the direction of bending moment at the cracked section. An analytic-experimental method leading to a numerical algorithm characterizing the crack model has been described. The crack model can be introduced naturally at the node of the finite element discretising the shaft. This happens to be undoubtedly a new and more generalised method of modelling the crack compared to the other methods which account for the crack by means of a special cracked element. The nonlinear differential equation resulting out of the model has been solved by a modified Newmark-Wilson method. Results obtained have been compared with existing results. The model appears to be a sound basis for further studies.

A rigorous hybrid displacement finite element has been developed by Chen and Wang[50] to deal with axisymmetric cracked solids subjected to static and dynamic torsional loading. Inter element compatibility conditions have been satisfied. The static and dynamic mode III stress intensity factors have been obtained. The need for using a hybrid singular crack tip element has been displayed. Numerical examples of Static analysis, Torsional vibration analysis and Dynamic analysis have also been presented. Coherence with existing literature has been found to be highly satisfactory. The error in Dimarogonas and Massouros[36] for torsional vibration analysis has also been remedied.

Ku and Chen[51] have studied the dynamic behaviour of a cantilevered shaft-disc system with flaws and subjected to axial periodic force. In the present analysis the effects of translational and rotary inertia, gyroscopic moment, bending and shear deformation have been considered. Four parameters have been employed to characterise the damaged zone - location, size, effective bending and shear stiffness of the damaged zone. Numerical results has shown how damage happens to alter regions of dynamic instability. With higher speed the system has been found to become unstable.

Papadopoulos[52,52a] has dealt with the torsional vibration of a rotor with a transverse crack. The crack has been modelled using a local flexibility matrix which has been calculated analytically and measured experimentally. Good agreement between the two methods has been found. The free vibration problem has been solved and the first three eigen values plotted with crack location and depth. The method may be used for the solution of any shaft system with crack using modified stiffness matrix for the cracked element.

Klompas[53] has mechanically simulated rotor joints by restoring moments in a manner distinctly different from the axisymmetric elastic hinge and has studied their effects on the dynamics of a complete turbo machine. A mechanism for locking subsynchronous whirl to a fractional frequency has been described and experimental observations discussed, sample analysis has shown the possibility of self sustained synchronous whirl due to preload asymmetry. Crack has been looked upon by the author as a special type of rotor joint.

The equations of motion for a simple rotor with a breathing crack has been derived in Jun et al.[54] based on Fracture Mechanics and the breathing crack model has been simplified to a switching crack model. By using the switching crack model, the conditions for crack opening and closing have been derived and can provide useful information for crack identification (crack depth and detection). It has been found that the vibration characteristics of a cracked rotor are best identifiable from the second horizontal component measured near to the second harmonic resonant speed.

Critical Review of this Section

The essence of the approach of the work in this section has been to determine the reduced stiffness at the cracked section which again happens to be a function of time. While Inagaki *et al.* have considered the bending stiffness at the cracked section as a function of bending moment, Mayes and Davies have considered it as a function of stress intensity factor, Ichimonji and Watanabe have described the change in the stiffness to be proportional to the torsional deflection of the shaft and Gasch *et al.* have proposed hinge model to represent a crack. Dimarogonas *et al.* have modelled a crack by determining its local flexibility due to the crack - a process much more rational than those of the other investigators. However, it needs to be pointed out that the approach belongs to the flexibility group of methods whereas the direct stiffness method happens to be more popular in solid mechanics. Last but not the least, the work by Nelson - Nataraj and LiChanghe *et al.* happen to be quite effective. While the former has used switching function to model the time varying stiffness, the later has included the crack at the node of the finite element - run approach not explored by other researchers. However, none of the two models appear to be popular for further research and undoubtedly the model due to Dimarogonas *et al.* has been widely used and is being used for crack modelling.

Cracked Beams and Columns

The problem of free vibration of uniform beams containing a local material damage has been studied by Josh and Madhusudan[55] with the aim of arriving at accurate closed form analytical expressions of the natural frequency for various homogeneous boundary conditions. Elementary beam theory has been used with the local material damage modelled as an effective reduction in Young's Modulus. The exact set of transcendental equations for the frequency parameter were obtained for various boundary conditions in terms of the damage parameters. It has been found that the nature of changes in frequency with respect to damage location is of the same type as the local curvature in the undamaged beams.

As has been stated earlier, coupling of bending and torsional vibration modes exist in cracked vibrating beams due to the off-diagonal terms in the local compliance matrix. Kikidis and Papadopoulos[56] investigated the influence of slenderness ratio of a beam with a cross-sectional open crack, on the dynamic behaviour of a beam. The effects of slenderness ratio on the beam response curves and the resonant frequencies of the system have been examined.

Longitudinal free vibrations of a beam with small axial rigidity, continuously restrained by imperfect elastic springs have been analyzed in Luongo[57]. It has been shown that small imperfections can localize the first modes of vibration in restricted region of the beam. System with single localized defects has been considered first. Periodic or nearly periodic imperfections have also been analyzed by Floquet theory and finally non-periodic imperfections numerically treated. It has been shown that when the exponential decay is strong the modes localize in the neighbourhood of each defect and so it is possible to limit oneself to a local analysis.

Ostachowitz and Krawczuk[58] have presented a method of analysis of the effect of two open cracks upon the frequency of vibration in a cantilever beam. Two types of cracks have been considered - double sided, occurring in the case of cyclic loadings and single sided which in principle occurs as a result of fluctuating loadings. It has also been assumed that the cracks occur in the first mode of fracture i.e., the opening mode. An algorithm and a numerical example have been given.

Works on vibration of cracked beams were made by Shen *et al.*[59-60] An approximate Galerkin solution to a 1-*d* cracked beam theory for the free bending motion of beams with pairs of symmetric open crack has been suggested.[59] Comparison functions chosen are mode shapes of the uncracked beam. Number of terms chosen have been determined by the convergence of natural frequencies and confirmed by studying the stress concentration profile near the crack. A 2-d finite element approach has also been proposed and used. Good agreement between the two have been reported. The equations of motion and boundary conditions for a uniform Bernoulli-Euler beam containing a single edge crack has been deduced by Shen *et al.*[60]. The stress concentration has been represented by introducing a crack function into the beams compatibility relations. a displacement function has been also introduced to modify the inplane displacement and slope near the crack. Both functions have been chosen so as to be maximum at the crack section and to decay exponentially along the beam' longitudinal direction. The equation of motion has been solved by Galerkin and Ritz procedure. Finite element method has also been used. The natural frequency obtained by using finite elements, classical methods and experiment have been found to match closely. Shen *et al.*[61] have dealt with single edge fatigue crack in uniform beams. The fatigue crack has been accounted for by a breathing crack model. A bilinear equation of motion for each vibration mode of a simply supported beam has been formulated by Galerkin procedure. The dynamic response of the bilinear equation under a concentrated forcing excitation has been calculated through numerical analysis. A clearly nonlinear behaviour has been found on time history and frequency spectrum for each vibration mode. The changes in the dynamic behaviour of cracked structures, according to the author, may be used for determining crack location and size.

Chen *et al.*[62-64] wrote on vibration of cracked rotating blades modelled as beams. Chen and Chen[64] have applied a finite element model for the study of vibration and stability of thick rotating beam with single edge crack. The effects of transverse shear deformation and rotary inertia have been taken into account. The model has been found to satisfy all the geometric and natural boundary conditions of a thick blade. The stability problems of a cracked rotating blade subjected to non-conservative follower forces has been considered. Crack has been found to alter the vibrational and stability characteristics of the system. In Chen and Chen[62], a similar investigation has been carried out but the system is instead subjected to follower moments and aerodynamic forces. Chen and Jeng[63] have added to the previous works the effect of pretwist and in all the cases significant effect of crack on dynamic characteristics has been noted.

Krawczuk[65] has presented a method of modelling and identification of crack location and magnitude in trusses. The damaged parts of the truss have been modelled by two node truss elements with an open, one edge transverse crack. The identification method of the crack parameters has been based on the measurement and analysis of time variation of the natural frequencies. Numerical tests have confirmed the possibility of identifying crack location and magnitude.

The finite element method with normal quadrilateral plate element has been used for the determination of bending compliance of hollow section beams with transverse cracks in Gounaris *et al.* (1991)[66]. Computed results have been compared with those from theory as well as experiments. The effect of such cracks on the dynamic response of channel section beams have been investigated. Such analysis has been stated to be of importance in identification of cracks in steel structures.

The use of simple models to study the structural dynamics and crack stability of a cracked cantilever beam subjected to impact has been demonstrated in Petroski[67]. A rigid perfectly plastic material model has been used and the *J*-integral and tearing modulus concepts employed in simple forms. Concise algebraic expressions happen to provide means for validating some of the assumptions made and they have been shown to be capable of capturing the silent phenomena involved in the problem.

Christides and Barr[68] did a fundamental work which has been subsequently used by others. The differential equations and the associated boundary conditions for nominally uniform Bernoulli-Euler beam containing one or more pairs of symmetric cracks have been derived. The reduction to one spatial dimension has been achieved using integrations over the cross section after plausible stress, strain, displacement and momentum fields have been chosen. The perturbation in stresses induced by the crack has been incorporated through a local function which assumes an exponential decay. Some experiments on beam containing cuts to simulate crack have been briefly described and the change in the first natural frequency matched closely by the predictions. The work of Christides and Barr[69] on non-circular section subjected to torsional vibration happens to be similar to ref[68].

A model for parametric excitation of a damaged bar has been developed in Parekh and Carlson[70]. Three parameters characterising the damaged region - location, size and amount of deterioration have been introduced to study the dynamic stability behaviour. It has been shown that a resonance type of response can be developed and can be expected to have deleterious effects on the integrity of the damaged bar.

Springer *et al.*[71] have presented a three element analytical model for a free-free longitudinally vibrating uniform beam containing a symmetric discontinuity. Based on this model, an analytical expression has been developed which relates the length of the section containing the damage to the depth of the discontinuity.

It thereby fixes the length of the element used to model this section of the beam. The expression has been verified experimentally and shown to be material independent.

A general flexibility matrix has been developed which expresses the local flexibility of a beam of rectangular cross section with a single edge crack in Anifantis and Dimarogonas[72]. The dominant terms in the matrix have been used to study the stability of the cracked column to follower and vertical loads. The kinetic method has been used and eigen curves developed to study the system stability. For follower type loads, the Beck column, stability charts have been given for several crack location and sizes. Flutter type of instability has been found to occur. For vertical loads, the same type of analysis has revealed divergent instability. It has been reported that cracks can make a system have flutter type instability for vertical loads.

A crack in a structure has been found to introduce a significant local flexibility which enhances the instability. Buckling of an edge - notched beam has been studied for isotropic and anisotropic composites in Nikpur[73]. The local compliance due to the presence of crack in an anisotropic medium has been formulated as a function of crack tip stress intensity factor and the elastic constants of the material. The effect of reducing rigidity on the load carrying capacity and the post - buckling behaviour has been discussed.

What crack location and orientation will result in a greatest likelihood of instability in a ductile cantilever beam subjected to dynamic forces acting transversely at the tip has been discussed in Petroski[74]. This has been found to be important for assessment of integrity of a cracked piping loop under pipe whirl or impact loading.

Petroski and Verma[75] have employed simple analytical models and experiments to demonstrate the structural response of a cantilever beam with a stable crack subjected to impact loading. The mode of plastic deformation has been shown to depend very strongly upon the size and location of the crack and the permanent damage suffered by a cracked beam has been found to be significantly different in magnitude and character from that of corresponding loaded uncracked beam.

Krawczuk and Ostachowicz[76] have investigated the effect of transverse on-edge non-propagating open crack on the natural frequencies of a graphite fibre reinforced polymide cantilever beam. Two models of crack have been presented - massless substitute spring model and cracked beam finite element model. The effect of various parameters upon change of natural frequency has been studied. It is noted for composites that volume fraction of the fibres and the angle are important influencing factors.

Various types of finite element models for cracked beams were provided Ostachowicz and others[77-79]. The beam has been modelled with triangular disc finite elements. The forced vibration of the beam and the effects of crack parameters on the vibrational behaviour have also been studied. Gounaris and Dimarogonas[78] have derived the stiffness matrix of a cracked prismatic beam element by using the compliance matrix of the element. The cracked element developed can be used with any standard structural analysis program. The dynamic response of a cracked cantilever beam to harmonic point force excitation has been evaluated. It has been found that resonant frequencies and vibration amplitude get affected considerably. Cawley and Ray[79] have compared the natural frequency changes produced by cracks and slots of various widths and depths using finite elements as well as experiments. It has been found that as the width of slot increases, the fall of frequency also increases. The authors feel that the importance of the work lies in its suggestion to correct results of artificially made slots to study behaviour of actual cracks.

Coupling of bending and longitudinal vibrations of a stationary cracked shaft with an open crack has been dealt with in Papadopoulos and Dimarogonas[80]. The crack has been modelled by a 2x2 local flexibility matrix the terms of which have been obtained analytically. Free vibration has been studied and the relation of the system eigen values and crack depth as function of slenderness ratio presented. Forced vibration has been also dealt with.

Longitudinal vibration of a cantilever bar with transverse crack has been investigated in Collins *et al.*[81]. Effect of crack location and compliance on the fundamental frequency have been determined. The case of forced vibration with distributed, longitudinal, harmonic force has also been studied. Results for breathing crack are compared with those for a crack which remains open as well as for uncracked beams.

Critical Review of this Section

The problem of vibration of cracked beam and column happens to be similar to that of cracked rotors in the sense that both may be treated as one dimensional members. While Shen *et al.* and Christides - Barr have accounted for the local stress induced due to the crack by the use of a function which is maximum at the crack tip but decays exponentially, Joshi and Madhusudan have used an effective reduction of Young's modulus to model the material damage and Ostachowiczk - Krawczuk have preferred to introduce point finite element. Dimarogonas *et al.* has modelled crack by using local compliance matrix - an approach similar to that used for modelling cracked rotors. It needs to be mentioned at this point that although the model proposed by Dimarogonas *et al.* is undoubtedly one of the approaches used, unlike the cracked rotor model, it is not the most popular. Last but not the least it needs to be stated that inspite of various models for describing vibration of cracked beams, there is no rigorous cracked beam vibration theory.

Crack Identification

Adams *et al.*[82] have discussed a method of non-destructively evaluating the integrity of structures and have applied it for one-dimensional structures. It has clearly been shown that vibration measurement made at a single station may be used along with the theoretical model to indicate the location and magnitude of a defect. Receptance analysis has also been employed. Agreement obtained between predicted and actual damage has been remarkable. The advantage of the method lies in the fact that one does not require to access the whole structure and the labour involved does not increase with increase in size of the structure. Besides the testing method has been reported to be simple and may possibly be automated. Cawley and Adams[83] have worked along similar lines with the exception that finite element analysis has been used instead of receptance analysis. So the method can easily be used for multi-dimensional structures-infact for all structures amenable to finite element analysis. The advantage of the method is that only one full analysis is required for a definite structure. It is also possible to monitor the growth of damage in a structure without the need to measure frequencies of the virgin structure.

Crack detection by modal analysis techniques has been a field of current interest. However, very little information on experimental results for beams are available. So validation of theoretical techniques/models is not possible. With this objective, Silva and Gomes[84] have performed an extensive dynamic analysis of free-free beams. The experimental techniques that have been used are described and results obtained for various locations and depths of cracks in a straight free-free beam have been presented. The authors observe that no information of this type is available for free-free beams.

A structural damage seems to alter the dynamic characteristics. The change is characterized by change in model and structural parameters. Pandey and Biswas[85] have judged the appropriateness of change in flexibility matrix (a structural parameter) for identifying and locating damage. The flexibility matrix has been shown to be easily and accurately estimable from the knowledge of the first few natural frequencies of the structure. Examples have been considered to elucidate the relevant points. Pandey and Biswas[86] proceed to demonstrate the effectiveness of the flexibility difference method discussed in an earlier paper[85] with detailed experiments. Experiments have been performed on two wide flanged I-beams. Damages in the beam have been introduced by cutting through the beam using a saw. Finally experiments have been repeated on a third beam to demonstrate repeatability. Experimental modal analysis techniques have been sued to estimate the natural frequencies and mode shapes for both intact and damaged beams. The authors report that using only the first three modes, it has been possible to locate a 13mm long saw cut in a 102mm wide flange. The method has been also successful in locating multiple damages. However, it has been found that measurements taken in the transverse direction are more sensitive than those taken in the vertical direction. The method does not require measurements to be taken at the damage location.

Akgun and Ju[87] have dealt with damage diagnosis in framed structures. The dynamic response of a planar frame composed of damped Bernoulli-Euler beams has been computed, with and without crack in the structure. The inert changes due to the crack have been investigated in relation to the crack location with the aim of developing a diagnosis tool. The optimum excitation location and frequency and the optimum location for response measurement have been determined for best diagnosis results. The effects of crack location and severity and of damping have been investigated. Damping has been accounted for by the complex Young's modulus. Damage i.e., crack has been modelled as a torsional spring. The authors find that the method has the capability of locating a crack within one or two cells of frame structure. Further, cracks at or near a joint are easily diagnosable but not cracks away from them.

Liang *et al.*[88] have presented a technique that enables the quantitative determination of the location and extent of crack induced damages in a beam structure. The technique requires the field measurement of the eigen frequencies of the inservice structural component which contains the damaged areas. The ratio $\frac{\Delta\omega_r}{\omega_r}$, where $\Delta\omega_r$ is the *r*th mode eigen frequency change due to crack and *r* is the *r*th mode eigen frequency of the uncracked structure, has been linked to the change of section modulus as well as the location of this change. The validity of the proposed method has been established by comparison with numerical simulation results as well as experimental results of various damage scenarios with different boundary conditions.

Nezu and Kidoguchi[89] have proposed measurement of mechanical impedance as a possible method for damage detection in passive systems. The mechanical impedance has been obtained by measuring vibration responses at several points by exciting a point on the structure. Change of impedance has shown the existence of the damage. The method has the advantage of giving information on the whole structure by excitation at a point without the need of overhauling for inspection since the excitation point is arbitrary. Examples of clamped rectangular plate with stiffners have been considered. Several interesting conclusions like - 'the most probable location for a damage is where the mechanical impedance fluctuates rapidly are drawn.'

Chondros and Dimarogonas[90] have used Rayleigh principle for an estimation of the change in the natural frequencies and modes of vibration of the structure if the crack geometry is known assuming the eigen value problem of the uncracked structure has been already solved. The method reduces computational effort for the full eigen solution of cracked structures and has given acceptable accuracy. It is extendable to higher modes. To demonstrate the change in dynamic behaviour of linear structures with crack depth, a shaft and a plane frame have been analysed for dynamic sensitivity to surface cracks.

Ju and Mimovich[91] have employed the modal frequency method to diagnose the fracture damage experimentally based on the analytical theory of the spring loaded 'fracture hinge'. The experiments performed have also measured the change in modal frequencies to locate the damage on the structure. With an accurate analytical model for the experimental samples, the location and extent of damages could be predicted quiet satisfactory. Only simple structures like beam have been considered.

Zimmerman and Kouk[92] have developed a decoupled damage location and extent algorithm. Both the algorithms have made use of an existing finite element model of the 'healthy' structure and a subset of the experimentally measured modal parameters of the 'damaged' structure. The decomposition of the damage detection into two problems has made the process not only computationally attractive but also permits engineering judgement to enter into the algorithm. The algorithms have been demonstrated using two different classes of structural models as well as experimental data. The effect of eigen pair measurement errors has also been discussed. Zimmerman *et al.*[93] appear to extend the work of Zimmerman and Kouk[92]. The minimum rank perturbation approach for structural health monitoring has been presented. The approach has used the mismatch between a pre-damage correlated model and the post-damage model properties to determine the location and extent of damage. The paper has focussed on several practical techniques. It has clearly concluded that while blind use of the algorithm is quite meaningless, its use along with engineering insight makes it a practical tool for real-world problems.

Gu *et al.*[94] have analysed the sensitivity of structural vibration parameters to fault. FRF which is sensitive to structural fault and capable of reflecting structural dynamic properties. The least square identification method, Kalman filtering method and the adaptive filtering method have been used to diagnose structural fault. The model test results indicate that structural fault is detectable even when the mode stiffness is reduced by 5%.

Soffker *et al.*[95] have established the relation between shaft cracks in turbo rotors and vibration effects measured in bearings by model based methods. A new concept has been presented based on the theory of disturbance rejection control, extended for nonlinear systems and applied on a turbo rotor. Simulations have shown the success of the method, especially for reconstructing disturbance forces as inner forces caused by the crack. Calculation of the relative crack compliance as the ratio of additional compliance caused by the crack and undamaged compliance, a clear statement about the opening and closing and therefore the existence of the crack and its depth has been possible. Theoretically it has been shown that it is possible to detect a crack with very small stiffness change corresponding to a crack depth of 5% of the radius of the rotor. The work of Soffker *et al.*[96] is along similar lines investigating the nonlinear and parametrically excited dynamics of a cracked rotor. To examine the system behaviour, phase plane plots, Poincare maps, time histories, power spectrum, greatest Lyapunov exponent have been presented. Nonlinear phenomenon has been treated in detail and as in Soffker *et al.*[95] crack detection methodology has been also given.

A comprehensive technology of on-line rotor crack detection and monitoring has been developed by Imam *et al.*[97]. The technique, based on vibration signature analysis approach, can detect incipient transverse rotor crack depth (1-2)% of the rotor diameter. The technique has been found to be applicable to all rotating machines. The technique is based on the analytical modelling of the dynamics of the system. The basic idea is that through the modelling approach, the crack symptoms can be determined in terms of the characteristic vibration signatures. These signatures have been used to diagnose the flaw in real life situations. A 3-d finite element crack model and a nonlinear rotor dynamics code have also been developed to accurately model a cracked rotor system. A new signature analysis technique has been used as well. The paper appears to be an authoritative work on the subject.

Swamidas and Chen[98] have performed modal analysis on a cantilevered plate with a small crack, using finite elements. Modal parameters were calculated against cracking in the plate. It has been found that surface crack in structure affects most of the modal parameters. Some of the most sensitive parameters are the difference of the strain model shapes and local strain frequency response functions. By monitoring the changes in the local strain frequency response functions and the difference between the strain model shapes, the location and severity of the crack that occurs in the structure can be determined.

A digital signal processing technique has been proposed by McFadden[99] for calculating the amplitude and phase demodulation of the tooth mehsing vibration of a gear from the signal average of the vibration with application to the early detection of local defects such as fatigue cracks. The importance of phase demodulation in the detection of defects has been confirmed.

A system for automatic inspection of rail road wheels using acoustic signatures has been proposed in Nagy *et al.*[100]. A discussion of the types of defects has been given. The main components of a system are then outlined. It is concluded that impact is the best way to excite the wheel into vibration due to the rich spectrum of resonance generated.

Ismail *et al.*[101] is an advanced investigation of the effect of crack closure on the frequency changes of cracked cantilevered beams. This closure has been found to cause the FRF to exhibit nonlinear characteristics. Study has been conducted by both computer simulation and experimental modal analysis. Crack flexibility for open and closed positions have been identified for several crack depths. The effect of static preloading on crack closure has also been studied. It is pointed out that relying on the drop in natural frequency alone, especially the higher modes may lead to serious underestimation of. the crack severity.

The problem of the influence of crack in a welded joint on the dynamic behaviour of a structural member has been discussed by Chondros and Dimarogonas[102]. Analytical and experimental investigations have given the relation between change in natural frequency of a cantilever beam and the crack depth appearing at the built-in edge. Measurement of the changes of natural frequency of the beam thus can give information for crack appearance and further more make possible estimation of its depth. The results have been used to determine the behaviour of beams with different boundary conditions and can be extended to structures with complicated geometries.

Measurement of flexural vibrations of a cantilever beam with rectangular cross section having a transverse surface crack extending uniformly along the width of the beam and associated analytical investigations have been reported by Rizos *et al.*[103] relating the measured vibration modes to the crack location and depth. From the measured amplitudes at two points of the structure vibrating at one of its natural modes, the respective vibration frequency and the dynamic response, it is observed that the crack location can be found and depth can be estimated with satisfactory accuracy.

An identification procedure to determine the crack characteristics (location and size) from dynamic measurement has been developed in Shen and Taylor[104]. This procedure is based on the minimization of either the mean square or maximum measure of difference between measured data and the corresponding predictions from computer model. The method has been tested for simulated damage in the form of one-side or symmetric cracks in simply supported Bernoulli-Euler beam.

The natural frequencies of a cantilever beam with an edge crack has been solved by using the finite element method and the results compared with experimental data by Qian *et al.*[105]. In order to consider the effect of crack closure, the modal parameters have been identified by means of an identification technique. Computational results have shown that the difference of the displacement response between beam and cracked beam due to the effect of crack closure has been reduced and the eigen frequency significantly affected. Finally a simple method for determining the crack position has been proposed.

Critical Review of this Section

The problem of crack detection is an inverse problem of dynamics. There exists various approaches for the solution of such problems. While some prepared to use drop in natural frequency (or fractional drop in natural frequency) directly as the criterion, others used it indirectly via the use of flexibility matrix and damage index. Some researchers have also concentrated upon the use of eigen parameter change coupled with update theory, effect of FRF and effect of strain mode shape and strain FRF as well as acoustic signatures. Conclusively, crack detection appears to be a field of active research. The work by Imam *et al.* is an example of a work successfully implemented in an industrial scenario.

Cracked Plates and Shells

Lee[106] has presented a simple numerical method based on the Rayleigh Ritz principle for predicting the fundamental frequency of an annular plate with an internal concentric crack. The method has been illustrated in the paper for annular plates with two edges simply supported and two edges clamped. It has been found to be applicable for annular plates with other boundary conditions. However, the author has found no suitable results in literature for comparison with his results.

Response of cracked cylindrical shells has been dealt with in Petroski and Glazik[107]. A flawless cylindrical shell deforms in an axisymmetric mode when subjected to uniform pressure, but the presence of material and geometric nonlinearities cause other deformation modes to appear. These additional modes induce bending deformation which get superimposed upon the basic breathing mode response and when such additional modes are due to the presence of crack they can have significant effect on the overall response of the shell. Such effects have been examined in this paper. The efficacy of a very simple model for predicting the static and dynamic deformation of cracked cylindrical shells have been clearly shown.

An elastic analysis has been presented for the tensile stretching and bending of a plate containing a surface crack penetrating through the thickness by Rice and Levy[108]. The treatment is approximate in that the two dimensional general plane stress and Kirchoff-Poisson plate bending theories have been employed with the cracked section represented as a continuous line spring. The spring has both stretching and bending resistance. Integral equation has been used for mathematical modelling. This has been solved numerically. The values of stress intensity factor has also been given. It appears that this work in not concerned with vibration effects of crack but with the fracture mechanics aspects.

Harmonic vibrations of a circular, cylindrical shell of rectangular planoform and with arbitrarily located crack has been investigated by Solecki and Forouhar[109]. The problem has been described by Donnel's equations and solved using triple finite Fourier transformation of discontinuous functions. Application of boundary conditions at the crack has led to a homogenous system of linear algebraic equations. Frequencies have been obtained from the characteristic equation resulting from the system. Numerical results for few cases have been provided.

Accurate finite element procedures have been developed for vibration of plates with significant membrane stresses by Mote[110]. Thermally stressed, rotating, circular plate with peripheral edge slots has been examined in detail and the influence of the slots upon critical rotational speed instability has been classified. In realistic environments slots can considerably increase critical speed through the reduction of the compressive thermal stresses and the introduction of the geometric asymmetry. Optimisation of critical instability speed has also been discussed.

Stahl and Keer[111] have dealt with eigen value problems of cracked rectangular plates. Vibration and buckling problems have been solved for a plate with a centrally located crack emanating from one edge. The problems have been formulated as dual series equations and reduced to Fredholm integral equations of the second kind. The singularity of the solution in each case has been isolated and treated analytically. Only plates whose four sides are simply supported has been considered.

The effect of matrix cracks on the frequency of unsymmetric, cross ply laminates has been investigated by Adali and Makins[112]. The cracks have been modelled as aligned slit cracks across the ply thickness and transverse to the laminate plane. The results have been given for antisymmetric and unsymmetric laminated plates. The effect of reduction in laminate stiffness has been studied. It has been observed that the reduction in frequency is local compared to the reduction in elastic modulus. However, only simply supported plates have been considered.

The effect of crack on the natural frequencies of a cracked plate has been studied in Ko[113] using 'CTRMEM' element in NASTRAN. Since, the said element has no capability of analyzing bending, only inplane modes could be studied. However, refinement near the crack tip has been employed to model the crack.

A method for the formation of the stiffness matrix of a rectangular plate finite element (with inplane forces) with a non-propagating, one-edge open crack has been presented by Krawczuk[114]. The crack has been modelled by an additional flexibility matrix, the terms of which have been calculated using fracture mechanics. The element may be used for the static and dynamic analysis of damaged structure. However, it appears, that the process is only applicable for inplane vibrations and examples chosen also use the element in the same mode.

Qian *et al.*[115] have determined the stress intensity factors of a plate with a through crack under bending, twisting and shearing. Using this, the stiffness matrix of a cracked element has been derived. The method has been used to obtain natural frequencies of a cracked plate with simply supported or cantilevered boundaries. Agreement with previous work, wherever available, has also been reported to be good.

Maruyama and Ichinomiya's[116] is an experimental work on natural frequency and mode shape determination of cracked plates with all edges clamped. The real time technique of time-averaged holographic interferometry has been applied. Effect of length, position and inclination angle of crack has also investigated. The experimentally obtained natural frequencies have been nondimentionalised and results are expressed graphically as function of dimensionless slit length and inclination angle.

Critical Review of this Section

Analysis of cracked structures in the form of annular plates, cylindrical shell, plate with considerable membrane stress, rectangular simply supported plate and laminated simply supported plate have been performed. Experimental investigation of clamped cracked plates have also been reported. Two attempts to analyse plate vibration has been made employing Dimarogonas model. While one considers only in-plane vibration, the other considers flexural vibration but the formulation is sufficiently involved. It appears that Dimarogonas approach is successful for treating rotors and beams. This does not seem to work well with plates and shells due to the difficulty in determination of local compliance matrix.

Miscellaneous

Ostachowicz[117,118] is concerned with the analysis of slip friction and dry friction effect inbeams. Response of beams to periodic excitation has been analysed. While Ostachowicz[117] has combined the Harmonic balance method with the finite element method, Ostachowicz[118] assumes the solution in the form of a Fourier series and applies the Harmonic balance method. The methods can be used for analysis of turbine and compressor blades and the author claims that they are more accurate than other traditional methods.

An efficient and quick numerical method for the computation of the natural frequency of a cracked Timoshenko beam in water has been given in Gounaris and Papazoglou[119]. The method has been based on one dimensional Timoshenko beam finite element for the structural part and on fluid boundary element for the fluid part in both coupled and uncoupled versions. Various boundary conditions have also been considered.

A detailed experimental work on a bridge has been performed by Salane and Baldwin[120]. The effect of high cycle fatigue on the dynamic modal properties of a full scale bridge test section has been examined in the said reference. Horizontal cracks were found to be formed in the webs of the twin bridge girders in the vicinity of the web-flange weld at or near the connections of the girders to the floor beams. Experimental data have been used to establish mode shapes and modal damping ratios during a 4 million cycle fatigue test. Damping ratios were found to vary from 1.09% to 0.53%. Modal stiffnesses were calculated based on the experimental mode shapes. There has been a modest correlation between the changes in stiffness and the observed cracking of the steel girders and the subsequent repairs.

A new experimental method for determination of stress intensity factory by using vibrations has been presented in Pye and Adams[121]. It is based upon measurements of natural frequency of a specimen before and after the crack is induced. Single edge notched rectangular and axisymmetric notched cylindrical specimens have been tested and the results compared with existing values. It has been shown that the new test will often be sufficiently accurate and is considerably easier to carry out than other experimental procedures.

Eigen values have been calculated for two dimensional elastic solids under antiplane strain vibration with oblique internal as well as boundary crack by Shen[122,123]. While Shen has used a perturbation solution in ref[122], in ref[123] he uses an analytical solution.

The concept of equivalent modulus to represent a damaged member has been given by Dimarogonas[124]. By using an equation given in the work (letter to the editor) and the values of dimensionless spring constant for the particular cross-section, the modulus of elasticity ratio ($E_{damaged}/E_{intact}$) may be computed for a definite structural element length. Thereafter any structural analysis method will yield the dynamic response of the damaged structure.

A simple model based on energy methods of applied mechanics has been developed and employed by Petroski[125] to study the effect of clapper location on the initiation and growth of cracks in bells. The origin of cracks in the Liberty bell and Big ben have been discussed and a simple model employed to explain why remedial measures on Big ben, though perhaps not optimal have prolonged its life.

Dentsoras and Dimarogonas[126-128] have dealt with dynamic crack arrest - a phenomenon related to the fatigue crack propagation in structures excited at resonance. Dentsoras and Dimarogonas[126,127] have modelled the crack as local flexibility. Application on a 4 node rotor with a circumferential crack has illustrated the dynamic crack arrest. The depth of the crack seems to determine the local stiffness introduced by the crack which in turn influences the dynamic response of the system under external excitation of constant amplitude at a resonant frequency. The propagation of crack introduces additional flexibility which causes gradual shift from resonance. The dynamic response has been found to be reduced and under certain circumstances becomes less than a threshold value characteristic of the material considered. This phenomenon, known as dynamic crack arrest, is drastically influenced by the loss coefficient of the material which is the main factor to determine the crack propagation rate at the initial stages of the crack growth when the loading of the cracked section becomes maximum. Dentsoras and Dimarogonas[128] is a work along similar lines but applied to longitudinal vibrations.

Conclusions and Observations from the Review

1. As pointed out by Dimarogonas24, vast amount of literature is available dealing with 'dynamic effects of crack' in structures and rotating machines. A substantial part of this appears in conference proceedings. However, due to unavoidable constraints, only informations available in acreeded

journals have mostly been reported.

2. From the survey presented, it is obvious that study of 'rotors' with cracks have been performed mostly extensively followed by studies relating to 'beam' and 'identification processes'.
3. Study of effects of crack relating to plates and shells has been scanty.
4. The possible reason for the emphasis on work relating to rotors and beams are due to their use in turbogenerators (power industry) and civil structures like highway bridges, port structures, offshore structures respectively. It needs to be specifically understood that other non-destructive techniques like industrial radiography and ultrasound are not convenient for such complicated structures in assembled condition and dismantling them for the purpose of maintenance and repair results in heavy loss in time and money. As such the scope of vibration aided fault diagnosis is wide.
5. The enormous amount of effort put in work relating to 'crack identification' is due to the need to set up generalized methods and approaches for prediction, detection and monitoring of damages in structures.
6. The authors feel that the reason for scanty literature in studying effects of crack in plates and shells is due to the complexity of the problem and the relative difficulty in extending one dimensional models to two dimensional structures. However, it should not be felt that scanty work is any indication of the absence of importance of the problem. This can clearly be understood from the fact that although cracks in shafts of turbines lead to failure, crack appearing in a single blade (of a turbine consisting of about a thousand blades) can also lead to a consequent failure of the machine.

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