Modification of the equivalent photon approximation (EPA) for 'resolved' photon processes.[1]

Manuel Drees[2]

University of Wisconsin, Dept. of Physics, 1150 University Avenue, Madison, WI 53706, U.S.A.

and

Rohini M. Godbole[3]

INFN–Laboratori Nazionali di Frascati, P.O.Box 13, I-00044, Frascati (Roma), Italy

Abstract

We propose a modification of the equivalent photon approximation (EPA) for processes which involve the parton content of the photon, to take into account the suppression of the photonic parton fluxes due to the virtuality of the photon. We present simple, physically motivated ansätze to model this suppression and show that even though the parton content of the electron no longer factorizes into an electron flux function and photon structure function, it is still possible to express it as a single integral. We also show that for the TRISTAN experiments its effect can be numerically of the same size as that of the NLO corrections. Further, we discuss a possible measurement at HERA, which can provide an experimental handle on the effect we model through our ansätze.

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[1]Talk presented by R.M. Godbole at Photon'95, Sheffield, UK, April 8-13,1995.
[2]Heisenberg Fellow
[3]Permanent address: Physics Department, University of Bombay, Vidyanagari, Bombay - 400 098, India

Studies of jet production in $\gamma\gamma$ (e^+e^-) collisions at TRISTAN [1] and LEP [2] and in γp (ep) collisions at HERA [3] have yielded clear evidence for hard scattering from partons in the photon or the so called 'resolved' processes [4]. Having confirmed the existence of these contributions to jet production, the next step is to use them to get further information [5] about the parton content of the photon, especially $g^\gamma(x, Q^2)$, about which very little direct information is available so far. To that end one needs to address the question of uncertainties in the theoretical predictions of these cross-sections. This implies that the approximations made in the calculations need to be improved. In this short note we study the issue of improvement of one of these approximations.

Theoretical calculations for the e^+e^- and ep processes are usually done in the framework of the Weizsäcker–Williams (WW) approximation also alternatively called the equivalent photon approximation (EPA) [6, 7]. In this approximation the cross–section for a process $e + X \rightarrow e + X'$ where a γ is exchanged in the t/u channel, is given in terms of the cross–section for the process $\gamma + X \rightarrow X'$ (for an *on-shell* γ) and the flux factor $f_{\gamma|e}(z)$ for a photon to carry energy fraction z of the e. For example, the cross–section for jet production in ep collisions (with $X = p$ and $X' = $ jets) can be written as

$$\frac{d\sigma}{dp_T}(ep \rightarrow \text{jets}) = \int_{z_{\min}}^{1} f_{\gamma|e}(z)\, \frac{d\sigma}{dp_T}(\gamma p \rightarrow \text{jets})|_{\hat{s}=s\cdot z}\, dz\ , \tag{1}$$

where s is the squared centre–of–mass energy of the ep system, and z_{min} is the minimum energy fraction that the γ has to carry in order for the process to be kinematically possible. This approximation is valid only if (i) the major contribution to the cross–section for the full process comes from the region where the virtuality of the photons P^2 is small compared to Q^2, where Q^2 is the typical scale characterising the process (say p_T^2 in the present case), and (ii) the contribution of longitudinal photons to the cross–section is very small. In this approximation, neglecting any P^2 dependence that the cross–section $\gamma + p \rightarrow$ jets may have, the flux factor $f_{\gamma|e}$ is given by [6],

$$f_{\gamma|e}(z) = \int_{P^2_{\min}}^{P^2_{\max}} \tilde{f}(z)\, \frac{dP^2}{P^2}\ -\ \frac{\alpha}{2\pi} m_e^2 \cdot z \left[\frac{1}{P^2_{\max}} - \frac{1}{P^2_{\min}}\right] \tag{2a}$$

$$= \ln\left(\frac{P^2_{\max}}{P^2_{\min}}\right)\, \tilde{f}(z)\ -\ f^{\text{rest}}(z). \tag{2b}$$

Here f^{rest} is simply the second term in the first line of Eq. (2) and $\tilde{f}(z)$ is just the usual WW splitting function. In Eq. (2) the kinematic values for the limits $P^2_{\max}$, $P^2_{\min}$ on virtuality *viz.* $P^2_{\max,\text{kin}}$, $P^2_{\min,\text{kin}}$ are determined by the (anti)tagging conditions in a particular experiment. It is also clear from the conditions of the validity of the approximations that the upper limit on the virtuality has to be less than $\mathcal{O}(Q^2)$. Taking this into account one has for $P^2_{\max}$ and $P^2_{\min}$ in Eq. (2)

$$P^2_{\max} = \min\,(P^2_{\max,\text{kin}}, \kappa Q^2);\quad P^2_{\min} = P^2_{\min,\text{kin}}. \tag{3}$$

κ is a number $\sim \mathcal{O}(1)$ whose proper value can be determined [6, 7] in processes where the γ directly participates in the hard process, by comparing the results of the exact calculation with that given by Eq. (1). For 'resolved' processes such an estimation of κ is not available. However, since the contribution from the large P^2 region to these processes is suppressed

very strongly the exact value of κ is not important, except for realising that if $P^2 > Q^2$ the concept of partons 'in' the photon will not make much sense at all. We therefore set $\kappa = 1$.

The important point is that for 'resolved' processes, the approximation of neglecting the P^2 dependence of $\frac{d\sigma}{dp_T}(\gamma + p \rightarrow \text{jets})$ is not quite correct as it involves the parton content of the photon which can have an additional P^2 dependence which is not simply of the type P^2/Q^2 as there is an additional scale Λ^2 in this case. The 'resolved' contribution to this process is normally given by

$$\frac{d\sigma}{dp_T}(ep \rightarrow \text{jets}) = \sum_{P_i} \int dy \, dx_2 \, f_{P_i|e}(y, Q^2) \, f_{P_2|p}(x_2, Q^2) \frac{d\hat{\sigma}}{dp_T}(P_1 + P_2 \rightarrow P_3 + P_4), \qquad (4)$$

where

$$f_{P_i|e}(y, Q^2) = \int_y^1 \frac{dx}{x} f_{\gamma|e}\left(\frac{y}{x}\right) f_{P_i|\gamma}(x, Q^2), \qquad (5)$$

and $f_{\gamma|e}$ is given by Eq. (2b). To take into account the effect of the virtuality of the photon on its parton content and the resultant modification of the EPA, we generalise Eq. (5) (using Eq. (2a)) as,

$$f_{P_i|e}(y, Q^2) = \int_y^1 \frac{dx}{x} \left[\tilde{f}\left(\frac{y}{x}\right) \int_{P^2_{\min}}^{P^2_{\max}} \frac{dP^2}{P^2} f_{P_i|\gamma}(x, Q^2, P^2) + f^{\text{rest}}\left(\frac{y}{x}\right) f_{P_i|\gamma}(x, Q^2, 0) \right]. \qquad (6)$$

Since f^{rest} is non–negligible only for $P^2_{\min} \ll \Lambda^2$, in the second term we can drop the P^2 dependence of the parton density in the photon and they are thus the same as those measured in the DIS experiments. For simplicity we will omit the second term in our subsequent expressions, although it will be taken into account in our numerical results.

The real question now concerns the P^2 dependence of $f_{P_i|\gamma}(x, Q^2, P^2)$. For $Q^2 \gg P^2 \gg \Lambda^2$ this can be computed rigorously in perturbative QCD (pQCD) [8, 9]. These calculations [9] tell us that the parton densities in a virtual γ simply approach the QPM predictions as $P^2 \rightarrow Q^2$, while the P^2 dependence vanishes altogether if $P^2 \ll \Lambda^2$. For the region $P^2 \simeq \Lambda^2$, however, we do not have any information from these calculations. These calculations [9] also show that the gluon density in the virtual photon is more suppressed than the quark densities. This is also reasonable as the gluon in the photon arises from radiation off a quark, so the further away the photon (hence the quark) from the mass–shell, the more suppressed will be the gluon densities. Thus we have for large Q^2

$$f_{q_i|\gamma}(x, Q^2, P^2) = q^\gamma(x, Q^2) \qquad P^2 \ll \Lambda^2$$

$$= c_q^{\text{QPM}} \ln \frac{Q^2}{P^2} \qquad P^2 \gg \Lambda^2 \qquad (7a)$$

$$f_{g|\gamma}(x, Q^2, P^2) = g^\gamma(x, Q^2) \qquad P^2 \ll \Lambda^2$$

$$\propto \ln^2 \frac{Q^2}{P^2} \qquad P^2 \gg \Lambda^2. \qquad (7b)$$

Here $q^\gamma(x, Q^2)$ and $g^\gamma(x, Q^2)$ are just the quark and gluon densities in an on–shell γ. Using this information as guideline we can propose ansätze for $f_{P_i|\gamma}(x, Q^2, P^2)$ which will inter-polate smoothly between the two abovementioned behaviours. Because of the different P^2

dependence of the quark and gluon densities in the photon at large P^2 we have to treat the two separately. In modelling $f_{P_i|\gamma}(x, Q^2, P^2)$ for the intermediate region $P^2 \simeq \Lambda^2$ we do not attempt to produce exactly either the P^2 dependence at a fixed x or the x dependence at a fixed P^2. We try instead to model the overall effect in terms of a single parameter P_c. The simplest ansatz is

$$f_{q|\gamma}{}^{(1)}(x, Q^2, P^2) = q^\gamma(x, Q^2), \qquad P^2 \leq P_c^2$$

$$= c_q(x, Q^2) \ln \frac{Q^2}{P^2}, \quad P^2 \geq P_c^2. \tag{8}$$

Here P_c is a free parameter of typical hadronic scale, and continuity at $P^2 = P_c^2$ determines $c_q(x, Q^2)$:

$$c_q(x, Q^2) = \frac{q^\gamma(x, Q^2)}{\ln(Q^2/P_c^2)}. \tag{9}$$

However, since this ansatz is motivated by the pQCD result, one can also alternatively have a parameter free ansatz where one uses instead the QPM prediction for c_q:

$$c_q(x, Q^2) = c_q^{QPM}(x) = 3\frac{\alpha}{2\pi}e_q^2 \left[x^2 + (1 - x)^2\right], \tag{10}$$

in Eq. (8), and then solve Eq. (9) for P_c. The P_c so calculated will in general depend on x and Q^2. The fact that the values of P_c so obtained are typically of hadronic scale gives us confidence in our ansatz. The simple ansatz of Eq. (8), however, has a kink when plotted as a function of $\ln P^2$. We can smooth this out by writing

$$f_{q|\gamma}{}^{(2)}(x, Q^2, P^2) = q^\gamma(x, Q^2)\frac{\ln \frac{Q^2 + P^2}{P^2 + P_c^2}}{\ln(1 + Q^2/P_c^2)}. \tag{11}$$

This ansatz now interpolates smoothly between the two limits $P^2 \to 0$ and $P^2 \to Q^2$. The nice feature of these two ansätze is that when we put these back in Eq. (6), we can once again write $f_{q|e}$ as

$$f_{q|e}(y) = \int_y^1 \frac{dx}{x}\tilde{f}\left(\frac{y}{x}\right)q^\gamma(x, Q^2)\mathcal{H}_q(Q^2, P^2, P_c^2, P_{\max}^2, P_{\min}^2), \tag{12}$$

where $\mathcal{H}_q$ is an analytic function [10]. In the case of ansatz of Eq. (11) this is possible only as an approximation, but the approximate analytical expression given in Eq. (17) of Ref. [10] is accurate to better than 2%. Thus even though we have lost the exact factorization of the density of partons in an electron into a photon flux factor and partonic densities in the photon, the expressions obtained by us are no more complicated than usual to use in a numerical calculation since we can still write $f_{P_i|e}$ as a single integral as in Eq. (5) before.

The ansätze for the gluon density $f_{g|\gamma}(x, Q^2, P^2)$ will have to take into account the stronger suppression due to virtuality implied by Eq. (7b). We do this by considering the diagram where γ splits into a $q\bar{q}$ pair and then the $q(\bar{q})$ of virtuality q_1^2 emits a gluon of virtuality $q_2^2 > q_1^2$, in the spirit of the backward showering algorithm. This gives:

$$f_{g|\gamma}(x, Q^2, P^2) \sim \int_{P^2}^{Q^2} \frac{dq_1^2}{q_1^2} \int_{q_1^2}^{Q^2} \frac{dq_2^2}{q_2^2} \ \alpha_s. \tag{13}$$

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The different ansätze now depend on the choice of scale of α_s which is constrained by the fact that in the limits of $P^2 \to 0$ and $P^2 \to Q^2$ the result must give the behaviour implied by Eq. (7b). This rules out q_1^2 as the choice of this scale, but both Q^2 and q_2^2 as the choice of the scale give acceptable behaviour of $f_{g|\gamma}(x, Q^2, P^2)$. This gives rise to two different ansätze similar to Eq. (8). The simplest is the former choice and in this case we get

$$f_{g|\gamma}^{(1a)}(x, Q^2, P^2) = g^\gamma(x, Q^2), \qquad P^2 \leq P_c^2$$

$$= c_G(x, Q^2) \frac{\ln^2(Q^2/P^2)}{\ln(Q^2/\Lambda^2)}, \quad P^2 \geq P_c^2, \tag{14}$$

Again the continuity of the ansatz at $P^2 = P_c^2$ gives us an equation for $c_g(x, Q^2)$ similar to Eq. (9). Here again, for the ansatz of Eq. (14) one can either choose P_c to be a free parameter or determine it by requiring that

$$c_g(x, Q^2) = c_g^{QPM}(x) = \frac{\alpha}{\pi} \frac{6}{33 - 2N_f} \sum_{q,\bar{q}} e_q^2 \left[\frac{4}{3} \left(\frac{1}{x} - x^2 \right) + 1 - x + 2(1 + x) \ln x \right]. \tag{15}$$

Here we have to remember that the dependence on Q^2 and P^2 has already been factored out in Eq. (14). If we take q_2^2 to be the scale of α_s in Eq. (13), we get

$$f_{g|\gamma}^{(1b)}(x, Q^2, P^2) = g^\gamma(x, Q^2), \quad P^2 \leq P_c^2$$

$$= c_G(x, Q^2) \left[\ln \frac{Q^2}{P^2} - \ln \frac{P^2}{\Lambda^2} \ln \left(\frac{\ln(Q^2/P^2)}{\ln(Q^2/\Lambda^2)} \right) \right], \quad P^2 \geq P_c^2 \tag{16}$$

Again both these ansätze suffer from the discontinuity in $\ln P^2$; as before this problem can be solved by writing an ansatz that smoothly interpolates between the entire P^2 region:

$$f_{g|\gamma}^{(2)}(x, Q^2, P^2) = g^\gamma(x, Q^2) \frac{\ln^2 \frac{Q^2 + P^2}{P^2 + P_c^2}}{\ln^2 \left(1 + \frac{Q^2}{P_c^2} \right)}. \tag{17}$$

As was the case with Eqs. (8) and (11), substituting Eqs. (14), (16) and (17), in Eq. (6) we can write the gluon density in the electron again as a single integral

$$f_{g|e}(y) = \int_y^1 \frac{dx}{x} \tilde{f} \left(\frac{y}{x} \right) q^\gamma(x, Q^2) \, \mathcal{H}_g(Q^2, P^2, P_c^2, P_{\max}^2, P_{\min}^2), \tag{18}$$

where the function $\mathcal{H}_g$ can be computed analytically as before [10]. Apart from the QPM version of the ansatz (1a) of Eq. (14), P_c is a free parameter; $P_{\max}^2$, $P_{\min}^2$ depend on the momentum fraction x of the photon carried by the electron, Q^2, as well as the (anti–)tagging conditions.

Now we are ready to give some numerical examples of the suppresions of the photonic parton densities at TRISTAN energies for the various ansätze given above. Fig. **??** shows the suppression expected for the u−quark density in a no–tag situation for TRISTAN energy for $Q^2 = 10 \text{ GeV}^2$ (which is the scale relevant for jet production in these experiments), for the GRV [11] parametrisation of $q^\gamma(x, Q^2)$ and $g^\gamma(x, Q^2)$. 'No–tag' means that $P_{\max,\text{kin}}^2 = s(1-z)$.

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The dotted line shows the suppression, w.r.t. the full unconstrained density given by Eq. (5) using Eq. (2b), from requiring $P^2 < Q^2$ as implied by Eq. (3). We see that this condition already suppresses the parton flux by about 20 % over most of the x range. At large values of x, $P^2_{\max,\mathrm{kin}}$ itself is small and hence requiring $P^2 < Q^2$ does not cause further suppression. The short and long dashed curves show the ansatz of Eq. (8) with fixed $P^2_c = 0.3$ GeV^2 and with P^2_c estimated from QPM, respectively. The solid line shows the case of fixed $P^2_c = 0.5$ GeV^2 for the smoothed ansatz of Eq. (11). We see that all the various ansätze predict a similar further suppression by about 10%. The fact that the QPM case and fixed P^2_c case give similar results is encouraging. The suppression is reasonably independent of the shape of $q^\gamma(x, Q^2)$ and hence is similar for the various parametrisations of $q^\gamma(x, Q^2)$. This suppression will become more severe with increasing Q^2, since in that case a larger fraction of the integral in Eq. (6) will come from the region $P^2 > P^2_c$ where the densities $f_{q|\gamma}$ are suppressed. For the gluons, in a no–tag situation, one finds on the average marginally less suppression, as compared to the quark case, coming solely from the dynamical constraint $P^2 < Q^2$, but considerably higher additional suppression due to virtuality effects, of about 12-15%.

In Fig. **??** we show the suppression expected for $f_{g|e}$ in an anti–tag situation with anti–tagging angle of 3.2° (for $z < 0.75$, as used by the TOPAZ collaboration [1]). Here we have used the DG [12] parametrisation of the photonic densities. We again show the ratio with the completely unconstrained densities. The dotted line shows effect of the dynamical constraint $P^2 < Q^2$. The different dashed, solid and dash–dotted lines show that the effects of virtuality cause a further suppression of as much as 10%, even in the small $x-$region, which contributes most to the cross–sections. For the quark case, in the anti–tag situation, the additional suppression coming from virtuality effects is only about 5%. It is clear that the suppression of double–resolved processes will be stronger as $f_{g|\gamma}$ and $f_{q|\gamma}$ are involved twice. In Fig. **??** we see indeed that the effect of virtuality on the theoretical predictions, for the rather large anti–tagging angle used by AMY [1], is considerable and therefore has to be included in the predictions.

Let us note in passing that for most HERA measurements of photoproduction cross–sections reported so far, these effects are completely irrelevant as they use a cut $P^2 < 0.1$ GeV^2. However, HERA experiments will soon have a small angle e tagger which will give events with $0.1 < P^2 < 1$ GeV^2. In this situation, at least for our ansätze with fixed P_c the suppression of parton densities is completely independent of x as $P^2_{\max}$ and $P^2_{\min}$ are fixed, and further this suppression decreases with Q^2. Taking the ratio of jet–events when the e is tagged in the small angle tagger to the ones which are tagged in the forward tagger, correcting for the known difference in the photon fluxes in the two cases, one should be able to see directly the suppression of the cross–sections due to the virtuality of the photons [10].

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