Renormalized Polyakov loop in the Fixed Scale Approach

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Introduction

- Polyakov loop $L(\vec{x})$ — Deconfinement Order Parameter (Spontaneous Breaking of $Z(N)$) (McLerran & Svetitsky, PRD 1981)

- One hopes to construct effective theories (Pisarski, PRD 2006) of L for investigations of deconfinement phase transitions and many models employ L.

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- One hopes to construct effective theories (Pisarski, PRD 2006) of L for investigations of deconfinement phase transitions and many models employ L.

- On an Euclidean $N_\sigma^3 \times N_\tau$ lattice $L(\vec{x})$ is defined at a site $\vec{x}$ as $L(\vec{x}) = \frac{1}{N_c} \,\mathrm{Tr}\, \Pi_{x_0=1}^{N_\tau} \, U^4(\vec{x}, x_0)$.

- No SSB on finite lattices/volumes. Usually one defines $\bar{L} = \sum_{\vec{x}} L(\vec{x})/N_\sigma^3$, and employs $\langle|\bar{L}|\rangle$, or its susceptibility, to locate the deonfinement phase transition.

- $\langle|\bar{L}|\rangle \to 0$ as 1/Volume in the confined phase, and $\langle|\bar{L}|\rangle \neq 0$ in the deconfined phase.

- But on the lattice, at fixed $T = 1/N_\tau a$, $L \rightarrow 0$ in the continuum limit of $a \rightarrow 0$ even in the deconfined phase.

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- Like any Wilson loop, Polyakov loop needs to be renormalized.

- More so, since as an order parameter it seeks to label phases by being zero or nonzero.

Earlier Work

♣ The physical interpretation of L as relate to the free energy of a single static quark offers a clue.

♠ The single quark free energy $F_b(N_\tau, a)$ is obtained from
$\ln\langle|\bar{L}|\rangle = -F_b(T)/T = -aN_\tau F_b(N_\tau, a)$.

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- Use of quark-antiquark (Polyakov loop) correlations (Kaczmarek et al. PLB 2002)
- Use of N_τ-grids and fits to L (Dumitru et al. PRD 2004)
- Use of renormalization group iteratively (S. Gupta et al. PRD 2008)

Fixed Scale Approach

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♡ Let β_c, corresponding to the position of the peak of the $|L|$-susceptibility for some fixed $N_{\tau,c}$, be the choice of the fixed scale a_c.

♠ Further, let it lie in the scaling region, then in the fixed scale approach $T/T_c = N_{\tau,c}/N_\tau$.

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♠ Further, let it lie in the scaling region, then in the fixed scale approach $T/T_c = N_{\tau,c}/N_\tau$.

♠ Write the single quark free energy as a sum of a would-be divergent and a regular contribution,

$F_b(T, a_c) = F(T, a_c) - A(a_c)$,

where A is the would-be divergent free energy in physical units.

♠ Since

$$\frac{T}{T_c} \ln\langle|\bar{L}|\rangle = -\frac{F(T, a_c)}{T_c} + \frac{A(a_c)}{T_c} \ ,$$

the free energy at any two different scales, a_{c1} and a_{c2}, differs by the *same* constant at all T.

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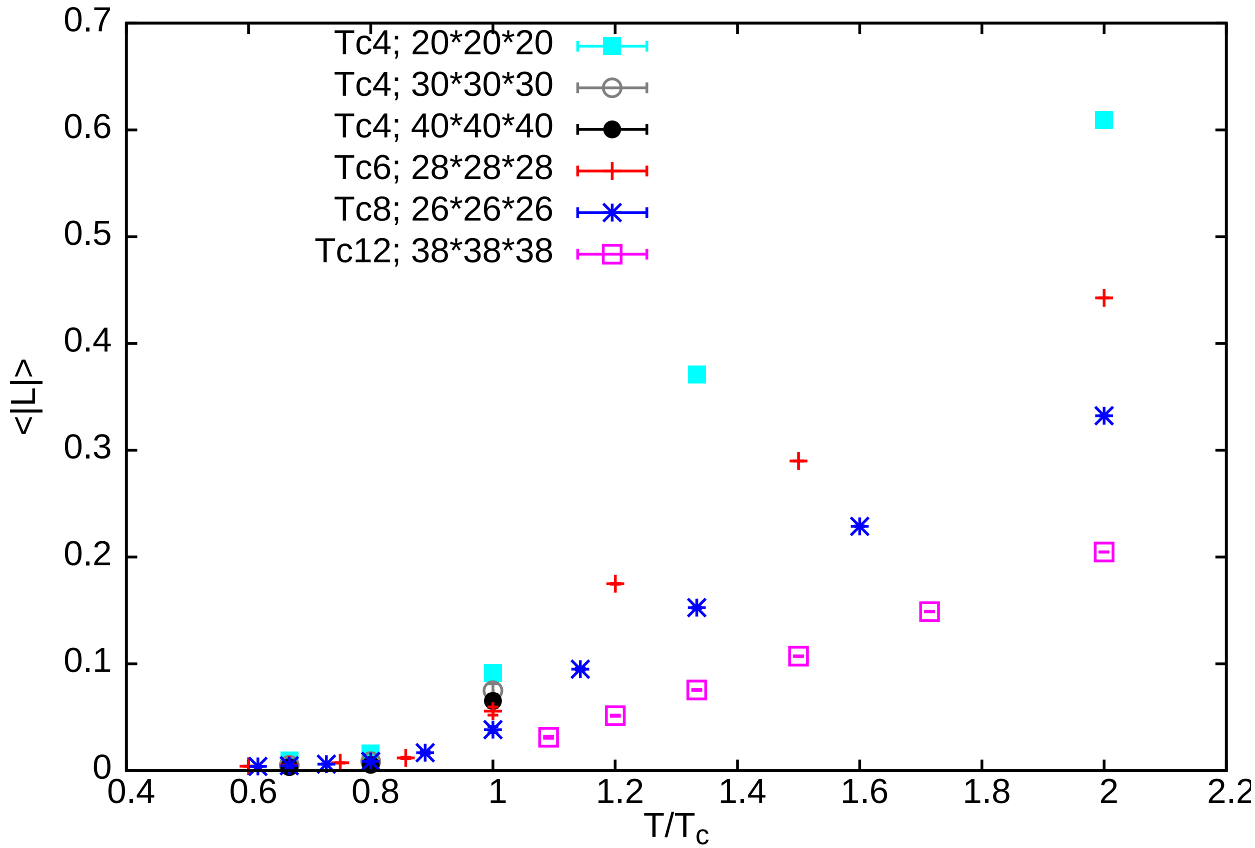
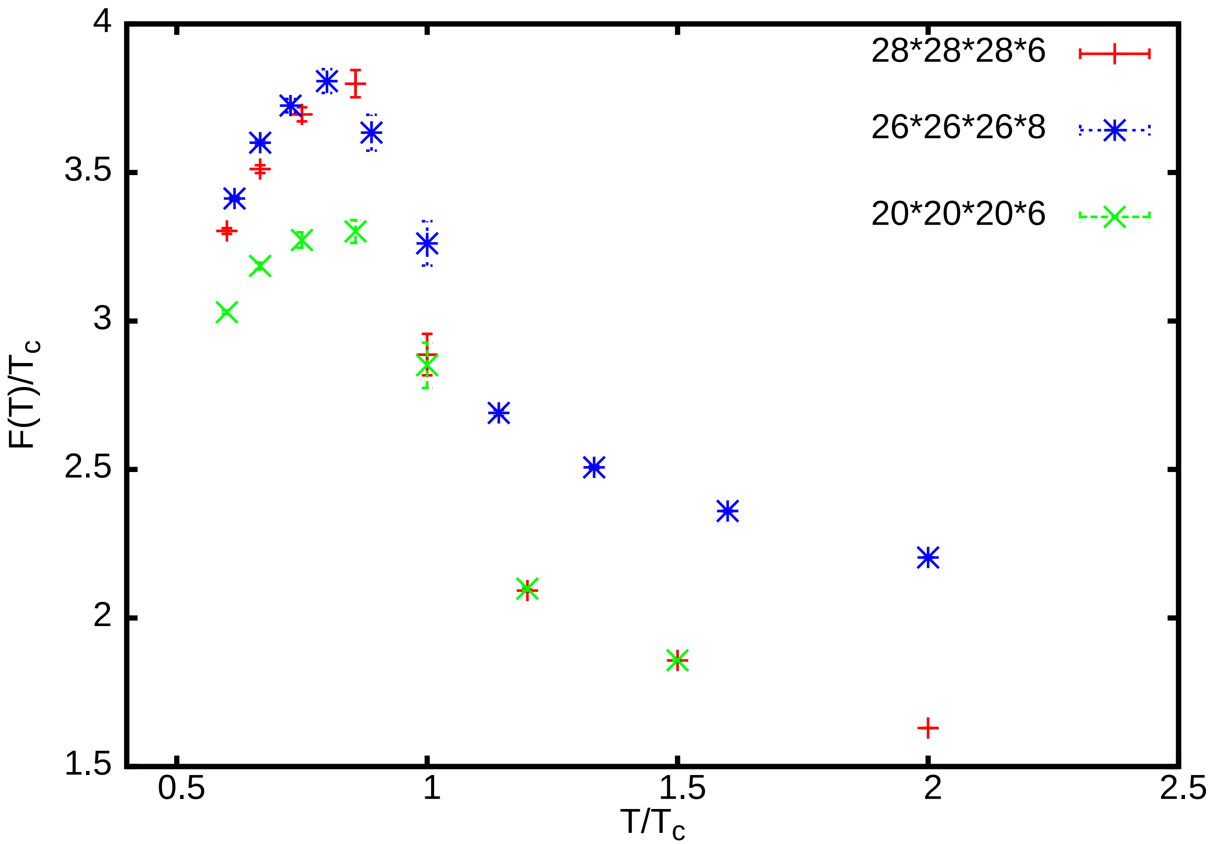
♣ In the following, I consider the simple case of $SU(2)$ to demonstrate how well it works. It should work similarly for any N_c or QCD.

♡ I employ the critical β for $N_\tau = 4$, 6, 8 and 12 from the table of Velytsky, IJMP C19, (2008), 1079, which agree with earlier results where available.

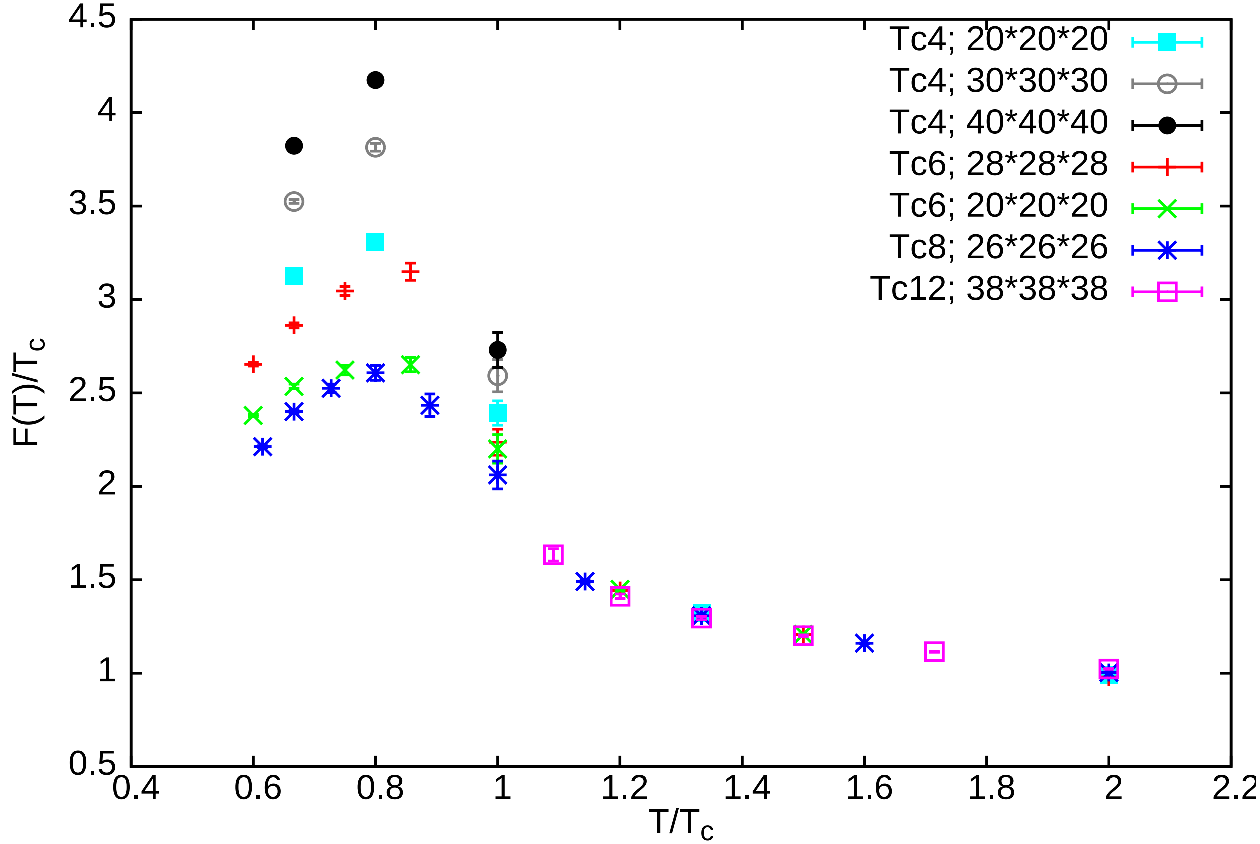
Results for $SU(2)$



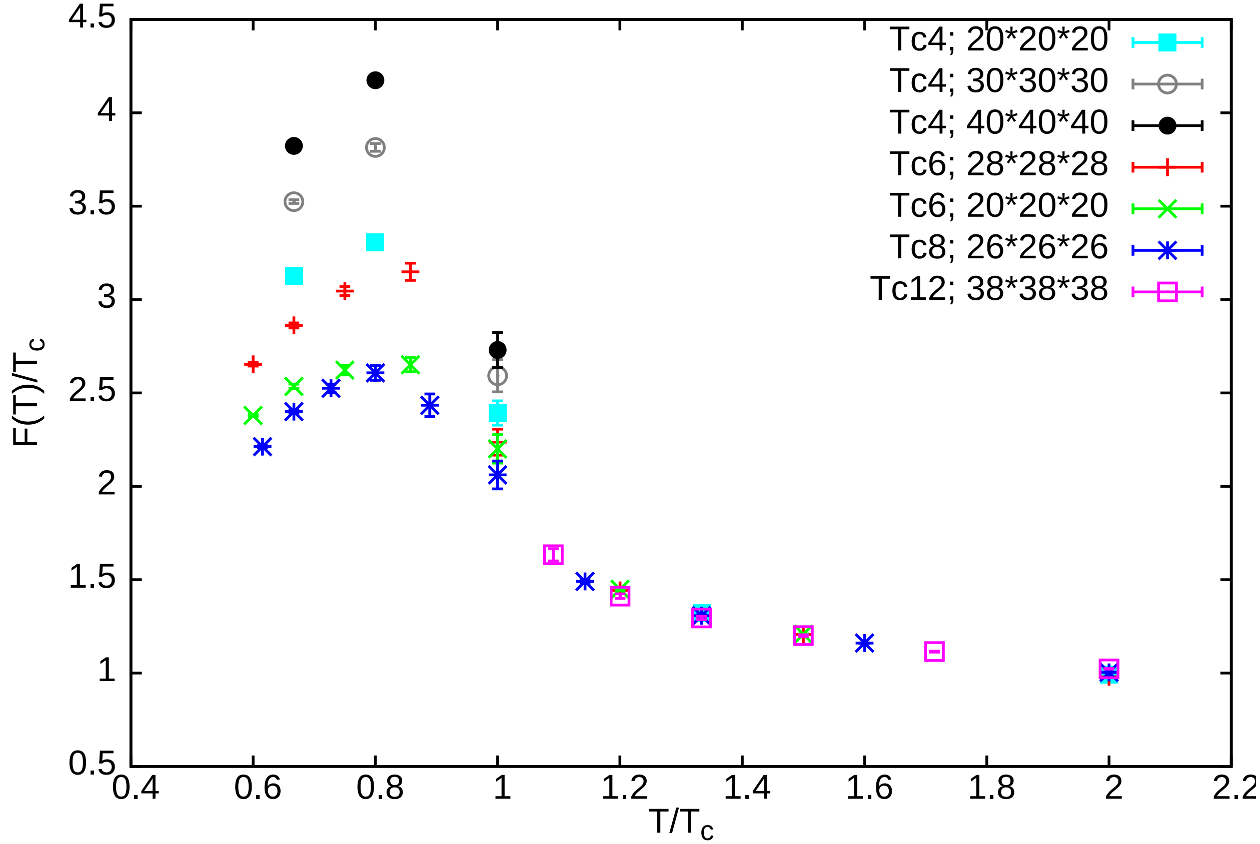
- 4 different scales : Tc4, Tc6, Tc8 and Tc12 with $a \to 0$ progressively. Increasing Spatial Volume leads to decrease in L for $T < T_c$.



- Illustrate for two scales : Different behaviour in T for the Free Energy. Shift F by $\Delta F(2T_c)$.

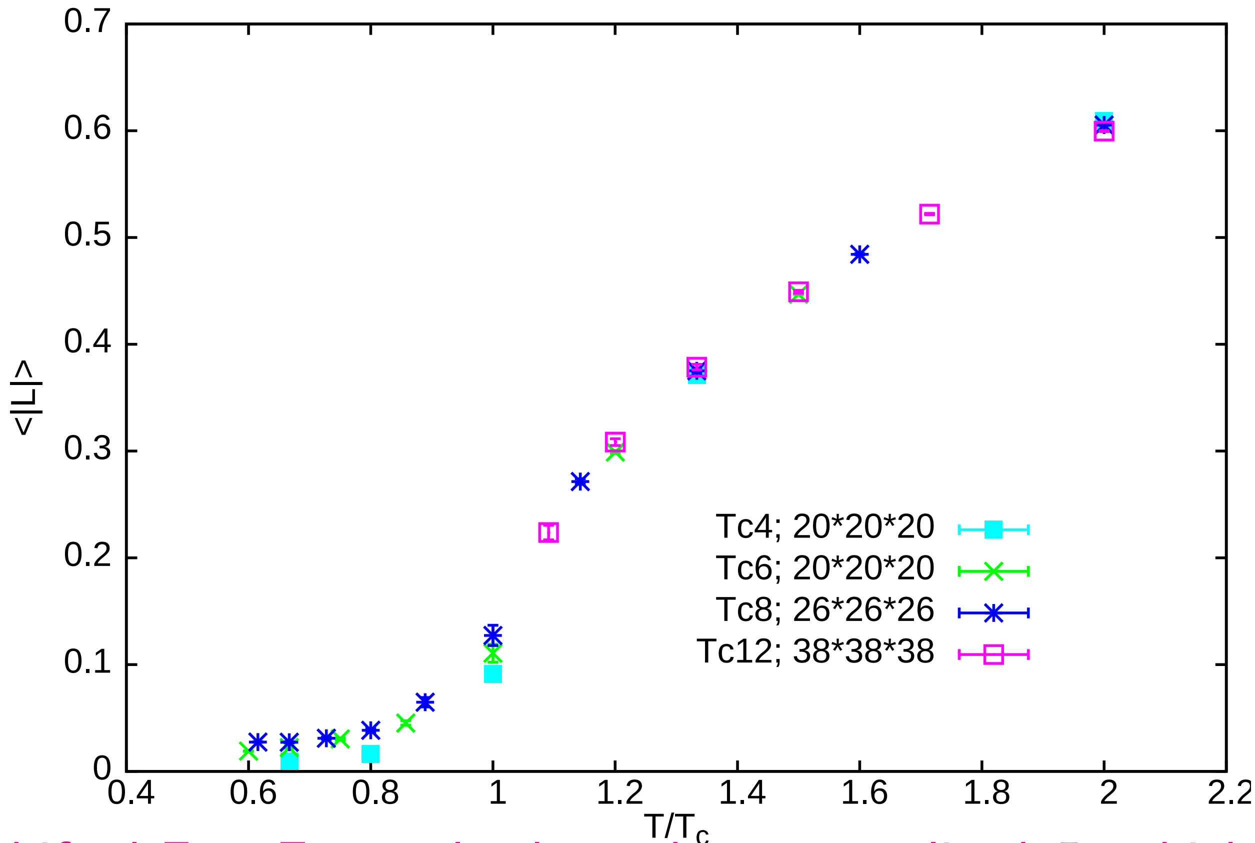


- Free Energy shifted by $\Delta F(2T_c)$ in each case: three constants for four scales.

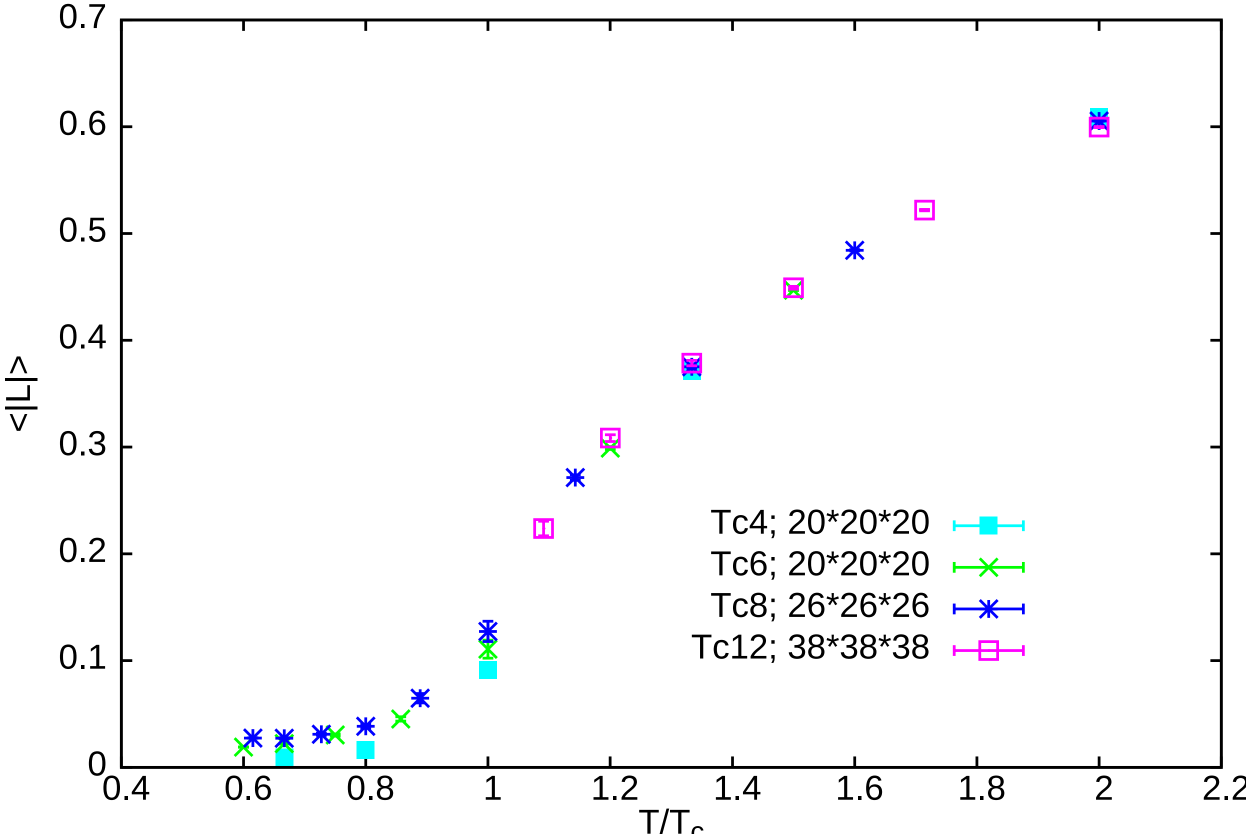


- Free Energy shifted by $\Delta F(2T_c)$ in each case: three constants for four scales.

- For $T \leq T_c$, F increases with the spatial volume but scale-independent.



- The shifted Free Energy leads to the renormalized L, which is independent of cut-off for $\beta \geq 2.2991$.



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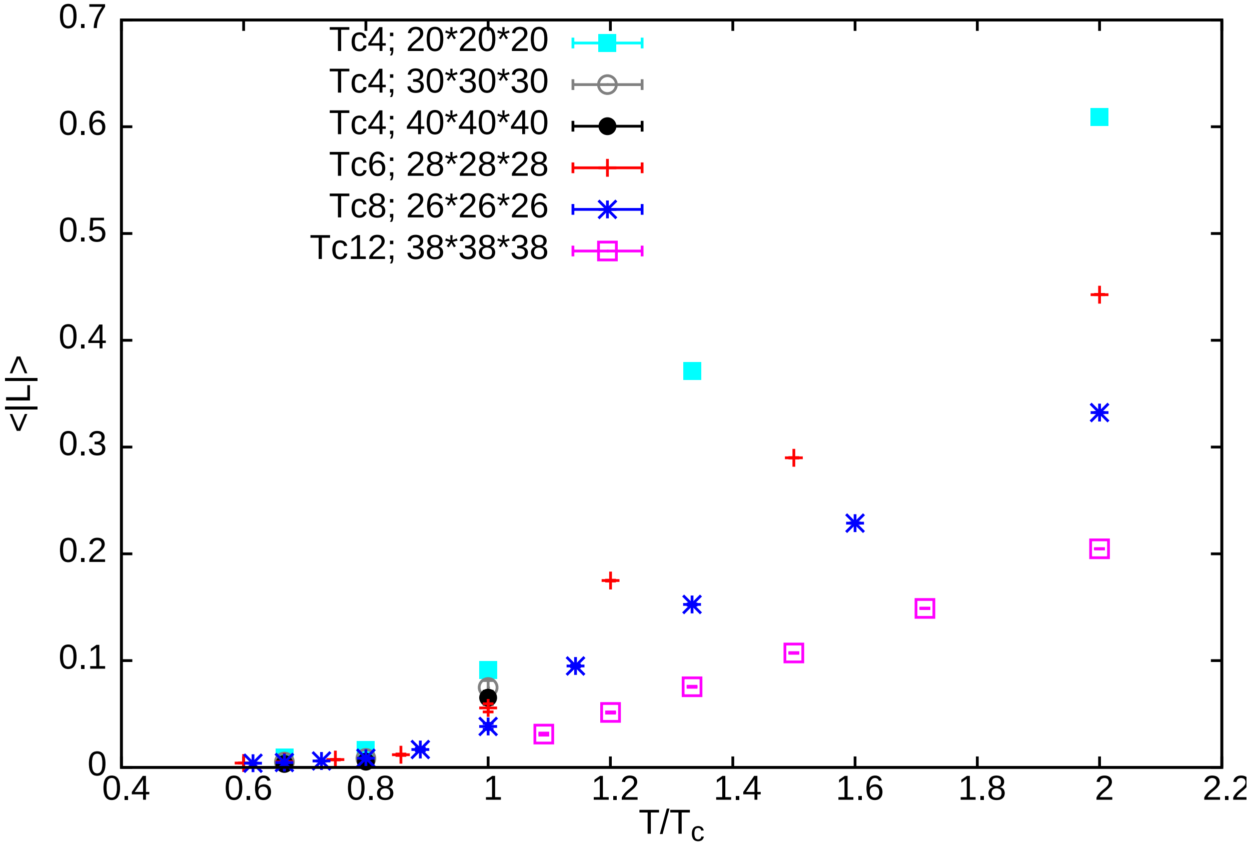
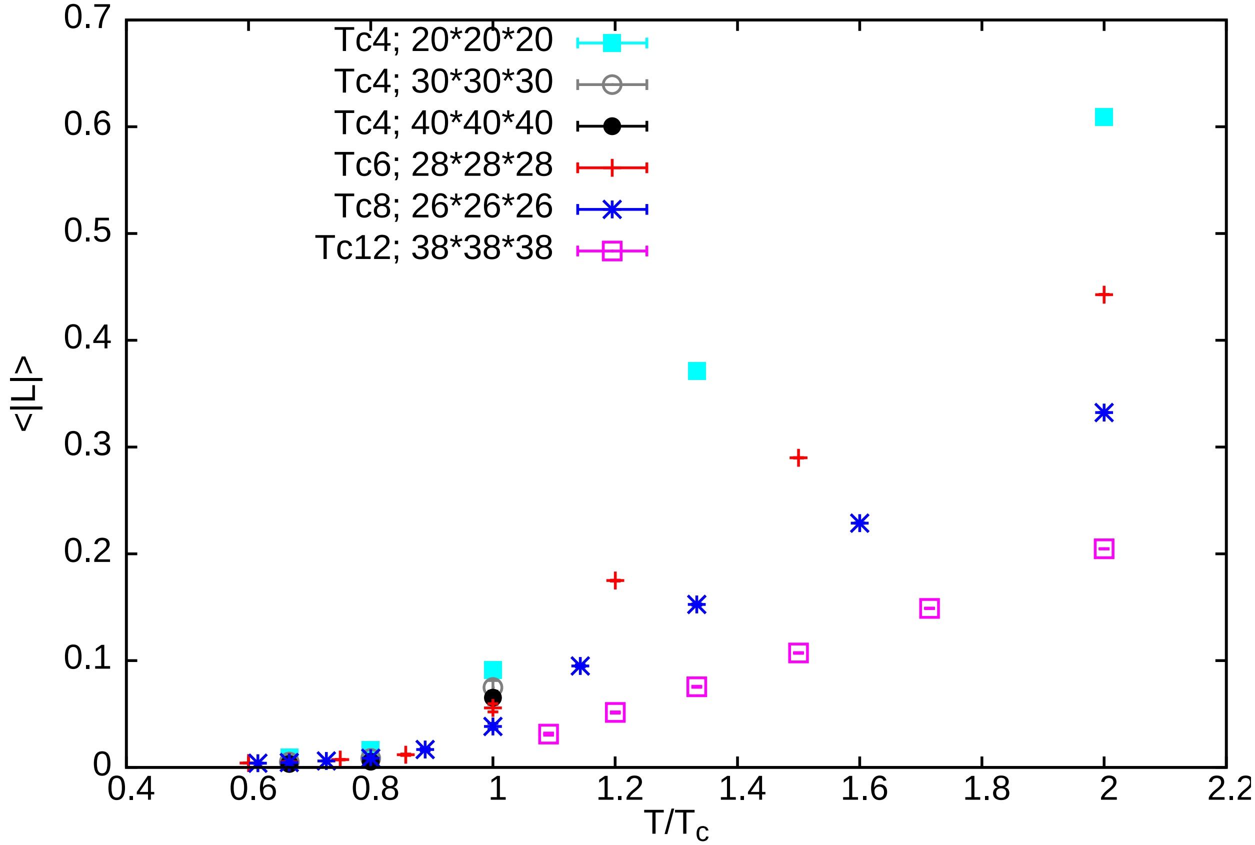


– I chose 3 constants to shift all the data to the Tc4 scale : The Tc6, Tc8, Tc12 results have simply jumped to their appropriate place on the $\langle |L| \rangle$ for it.



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- Does the renormalized L then climb to unity slowly?

- High Temperature Perturbation Theory (Gava-Jengo, PLB 1981) tells us that $L \longrightarrow 1$ from *above* at very large T : $L = 1 + C_3 g^3 + \mathcal{O}(g^4)$, where $c_3(N_c) > 0$ is a constant.

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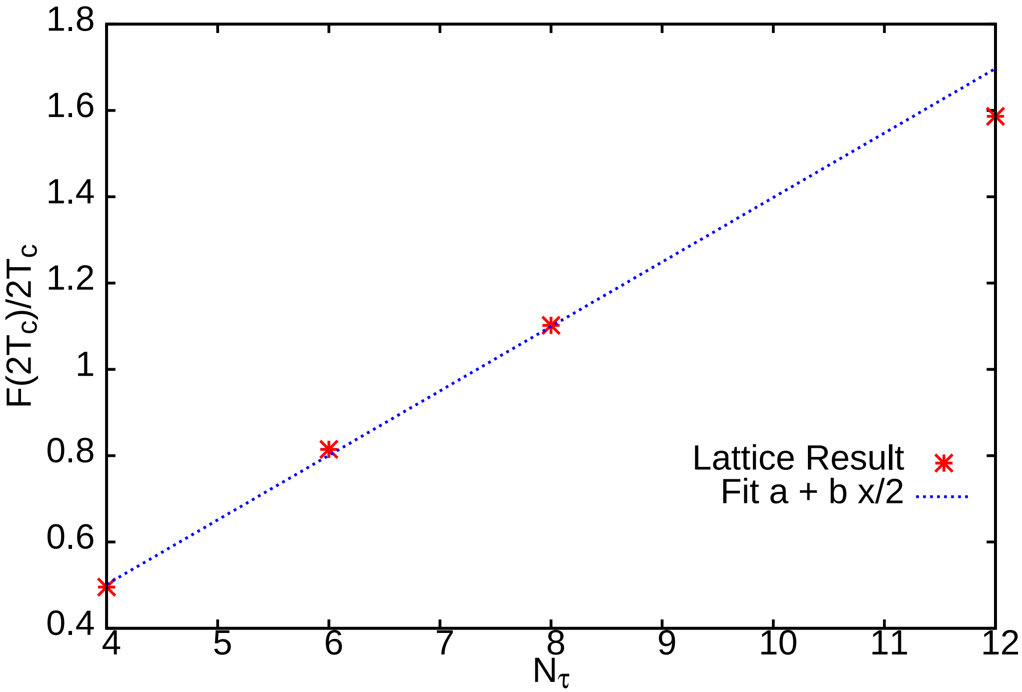
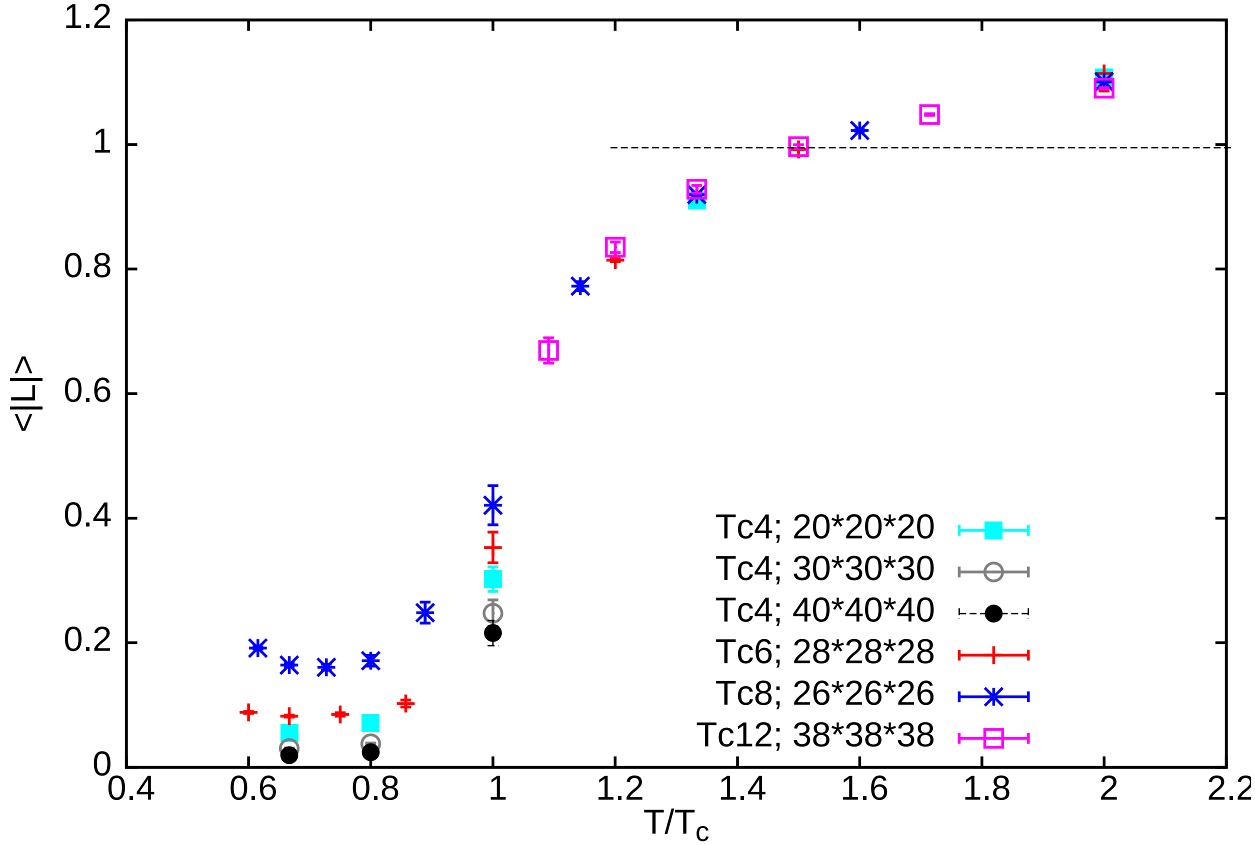
- In stead of shifts at $2T_c$ for varying scales, try a fit $-\ln\langle|\bar{L}_j|\rangle = F(2T_c)/2T_c + B \cdot N_{\tau j}/2$.



♣ Eliminating the B-dependent divergent term for the Tc4-scale in addition to the shifts, one has,



♠ L now does go to unity from above at large T. Large volumes, aspect ratio of ~ 10, needed for $L \simeq 0$ for low T.

Summary

- I showed that the fixed scale approach leads to a natural definition of a physical, N_τ-independent, order parameter which is defined in both the confined and the deconfined phases.

- It does not need two-point correlations, and works for even coarse lattices $(a \leq 1/4T_c)$.

Summary

- I showed that the fixed scale approach leads to a natural definition of a physical, N_τ-independent, order parameter which is defined in both the confined and the deconfined phases.

- It does not need two-point correlations, and works for even coarse lattices ($a \leq 1/4T_c$).

- The definition itself does not depend on any lattice artifacts or the lattice size in the deconfined phase.

- It displays the expected behaviour in both the phases, i.e., volume dependence in the low T-phase and approach to unity from above in high T-phase.