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HYDRAULIC GEOMETRY OF ILLINOIS STREAMS

By JOHN B. STALL and YU-51 FOK

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HYDRAULIC GEOMETRY OF ILLINOIS STREAMS

by

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SUMMARY

A consistent pattern has been evaluated in which the width, depth, and velocity of flow in a stream change along the course of the stream with a constant frequency of discharge. These channel characteristics are termed *hydraulic geometry* and constitute an interdependent system which is described by a series of graphs having simple form, or by equations.

The data from 166 stream gaging stations in Illinois have been assembled and used to develop the parameters to define the hydraulic geometry of these streams. Results are presented as separate sets of equations for 18 river basins in Illinois. Stream characteristics are related to frequency of discharge and to drainage area as independent variables.

Stream velocities computed from hydraulic geometry equations check favorably with actual stream velocities measured by time-of-travel in streams determined by using dye tracers. These equations are used to predict the average depth and velocity of flow at problem locations on the stream where no measurements are available. This allows computation of the reoxygenation capacity of the stream at the problem location, and will be valuable for many purposes in water resources development.

INTRODUCTION

The Illinois State Water Survey has carried out since 1895 a continuing program of research and evaluation of the water resources of Illinois. This program has dealt extensively with the amount and mineral quality of the ground-water, surface water, and atmospheric water of the state.

The physical geography of Illinois has been described by Leighton and others (1948) and is shown in figure 1. Similar information is available for the entire United States from Fenneman (1938, 1946) , Raisz (1957), and Hunt (1967). In Illinois various important hydrologic phenomena have been shown to be associated with the physiographic divisions of the state. For example, Mitchell (1954) showed that physiographic divisions were important in explaining the variations in flood hydrology for the various regions of the state. The low flows of Illinois streams were shown by Stall (1964) to be associated with physiographic divisions.

In a pioneering paper Leopold and Maddock (1953) showed that channel characteristics of natural streams constitute an interdependent system which can be described by a series of graphs having simple geometric form. Such a system was termed the *hydraulic geometry* of the stream system. The authors showed how the nature of a particular river system can thus be described quantitatively in terms of the slopes and intercepts of the lines of such a series of graphs.

Some of the important hydraulic characteristics of a stream channel are the depth, width, and velocity of flow. These factors at a particular stream cross section can be related to the amount of stream flow (or discharge) by the simple power function:

$$W = a\,Q^b \qquad (1)$$

$$D = c\,Q^f \qquad (2)$$

$$V = k\,Q^m \qquad (3)$$

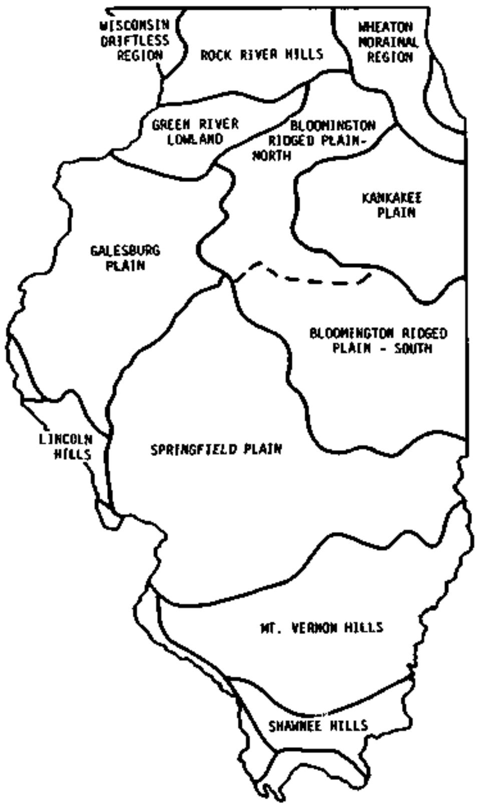


Figure 1. Principal physiographic divisions of Illinois

where W = width, D = mean depth, V = mean velocity, Q = discharge, and a, b, c, f, k, and m are numerical constants.

Leopold and Maddock (1953) showed that these functions, derived for a number of cross sections along the course of a stream, differ only in the numerical values of the coefficients and exponents. These relationships, when plotted on graphs, are greatly similar and consistent, even for stream systems quite different in physiographic setting. There appears to be a consistent pattern in which the width, depth, and velocity of flow in a stream change, along the course of the stream, for a constant frequency of discharge.

Objectives and Scope

The objectives of this research project are: 1) to determine whethe or not Illinois stream systems tend to adjust themselves to a consistent pattern which can be evaluated quantitatively by the concepts of hydraulic geometry using discharge measurement data from U. S. Geological Survey stream gaging stations; 2) to evaluate, if possible, the hydraulic geometry of the major stream systems of Illinois and to express the interrelationships involved by means of a set of equations; 3) to explore within Illinois the variations in the hydraulic geometry of streams for the various physiographic divisions of the state; 4) to show how hydraulic geometry relations for a stream system might be used to estimate the channel characteristics at a location within the stream system where no actual measurements are available; 5) to examine the reliability of the developed equations by means of data from other sources; and 6) to give examples of the application of hydraulic geometry equations.

Hydraulic geometry relations have been developed for 18 river basins. Table 1 shows the total size of these river basins and the number of gaging stations in each. The map in figure 2 shows the locations of the 18 major river basins which were studied. It is noted that the Rock River extends well into the state of Wisconsin, and that the Galena, Fox, and Des Plaines River Basins extend slightly into Wisconsin. Also, the Kankakee River Basin extends well into Indiana, a little beyond South Bend.

Table 1. Eighteen River Basins For Which Hydraulic Geometry Equations Have Been Developed

River basin	Approx. total drainage area (sq mi)	Number of stream gaging stations
Rock	10,720	28
Galena	210	5
Fox	2,600	7
Des Plaines	1,370	8
Kankakee	5,280	24
Vermi1ion (I11inois R. Basin)	1,315	4
Mackinaw	1,173	9
Henderson Creek	602	5
Spoon	1,890	4
La Moine	1,380	3
Sny	743	6
Sangamon	5,452	18
Kaskaskia	5,8TB	13
Vermi1ion (Wabash R. Basin)	1,440	7
Embarras	2,374	8
Little Wabash	3,212	4
Big Muddy	2,323	9
Biq Bay Creek	226	4

These basins were selected since they were large enough in size to allow the hydraulic geometry pattern to be evident, and each contained at least three gaging stations providing a sufficient amount of data to allow a reasonable evaluation of the hydraulic geometry pattern. For those parts of the state not included in a basin (figure 2), some generalized relations have been developed largely from available data and similarities to one or more of the 18 basins studied.

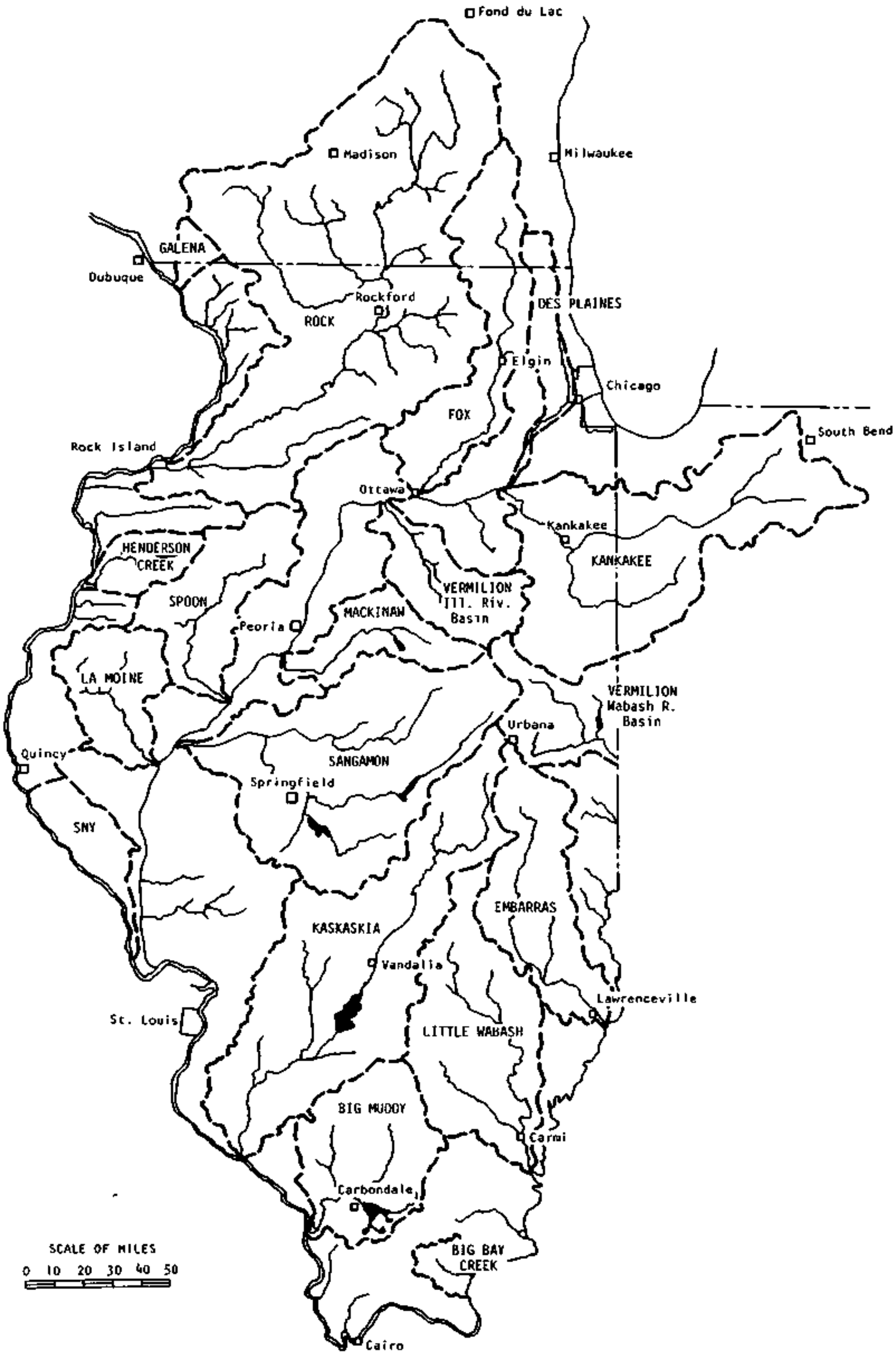


Figure 2. Location of 18 river basins in Illinois for which hydraulic geometry relations are developed

Acknowledgment

The research described in this report has been carried out by the authors as a part of their regular work at the Illinois State Water Survey under the direction of Harman F. Smith, Head of the Hydrology Section, and William C. Ackermann, Chief. Brief time-of-travel studies for the Vermilion River were carried out by Thomas A. Butts, Assistant Hydrologist, under the direction of Ralph L. Evans, Head of the Water Quality Section of the State Water Survey at Peoria.

Half of the cost of this project has been financed under the matching funds program of the Office of Water Resources Research, U. S. Department of Interior, under Grant 14-01-0001-1021 to the Water Resources Center of the University of Illinois. Dr. Ben B. Ewing, Director of the Water Resources Center, has been helpful to the authors in carrying out and reporting these research results.

William D. Mitchell, District Chief for Illinois, U. S. Geological Survey, and his staff members aided greatly in providing basic data to the authors for use in this report. Charles L. R. Holt, Jr., USGS District Chief for Wisconsin, provided the data from gaging stations in Wisconsin, and Malcolm D. Hale, District Chief for Indiana, provided data for the upper Kankakee Basin in Indiana. Mr. Hale and his staff also made available a great wealth of results from their time-of-travel studies on the White River in Indiana, as explained in the report. These time-of-travel data contributed greatly to the report and are heartily acknowledged. Luna B. Leopold, Research Scientist of the USGS in Washington, conferred with the authors in the latter stages of this research. His advice in the generalization of the results was very helpful.

The State Division of Waterways, John C. Guillou, Chief Waterway Engineer, cooperated in this study by making available about 200 detailed cross sections of the Embarras River surveyed as a part of their flood study. Gene Wertepnay, Engineer, discussed these data with the authors and provided the cross sections requested.

Assisting in this project were Robert A. Miller, Assistant Hydrologist, a University of Illinois graduate student working under a research assistantship provided by this project; and Dean C. Bernett, Douglas W. Hiestand, Stephen L. Leming, Ronald H. Mass, William E. Witzig, and Vincent C. Wroblewski, all undergraduate students. Other Water Survey personnel assisting in this project were John W. Brother, Jr., Chief Draftsman, who prepared the illustrations, and Mrs. J. Loreena Ivens, Technical Editor, who aided in editing the final report.

DATA

Stream Gaging Stations

All of the data used in this report were collected in the field by personnel of the U. S. Geological Survey as a part of their regular continuing program of streamflow measurement. Stream gaging data are available from about 200 locations on Illinois streams; about 150 of these stations are currently in operation. As a part of this work, discharge measurements are made regularly at each active station. Discharge measurements are usually made by wading the stream or by lowering a current meter into the stream from a bridge. Velocities are measured at a number of vertical sections. These are used to construct a cross section of the stream at this discharge and to provide a value of total discharge at the time the measurement is taken.

Figure 3 shows the stream gaging installation on Goose Creek near DeLand, Illinois, located within the Sangamon River Basin. It was operated for an 8-year period during 1951-1959. The picture shows the stream bed, stream banks, the stilling well and tower of the stream gage, and the bridge from which discharge measurements were taken. This station is typical of many of the stream gaging installations in Illinois.

Figure 3. Stream gaging installation on Goose Creek near DeLand, Illinois.

In order to illustrate the shape of a stream channel cross section at various discharges, a composite of cross sections has been compiled for figure 4. Here are shown the channel cross sections for five different rates of discharge, as determined by discharge measurements made at the stream gaging station on Sugar Creek near Hartsburg, Illinois, also located in the Sangamon River Basin.

Field records of discharge measurements made by the USGS were recorded on their Form 9-207; from 30 to 500 measurements were available per station. Data available for each measurement were the width *W*, channel cross-sectional area *A*, average velocity *V*, and the discharge *Q*. The average depth *D* was computed from *D* = *A/W* with *D* being defined as the hydraulic depth as described by Chow (1959).

The first step in depicting graphically channel conditions for a reach of stream represented by a particular stream gaging station was the plotting of station hydraulic rating curves. Relationships of *Q* to *A*, *W*, *D*, and *V* are plotted on log -log paper. For example, as shown in figure 5, data from a gaging station at Sugar Creek near Hartsburg were used with *A*, *W*, *D*, and *V* on the vertical scale and *Q* on the horizontal scale. Curves were drawn to fit the data. The points are scattered because of the variation of local stream conditions- over the recording periodof 21 years. However, the general pattern of the curves is evident. Much care was given to insure the consistent agreement among the curves so that if a vertical section is taken at a given discharge, the values depicted from these curves will satisfy two physical laws: the product of width and depth equals area, *WD* = *A*, and the area times velocity equals the given discharge, *AV* = *Q*.

The vertical dashed line in figure 5 indicates a discharge of 469 cubic feet per second (cfs), the flow that occurs 10 percent of the days. As discussed later, the relationships developed in this project were limited to flows at or below this 10 percent duration. Consequently, the relationships derived in this project are based primarily on the shape of the curves to the left of the dashed line in figure 5 That part of the graph to the right of the dashed line merely shows the consistency of the shape of these hydraulic rating curves for the higher discharges.

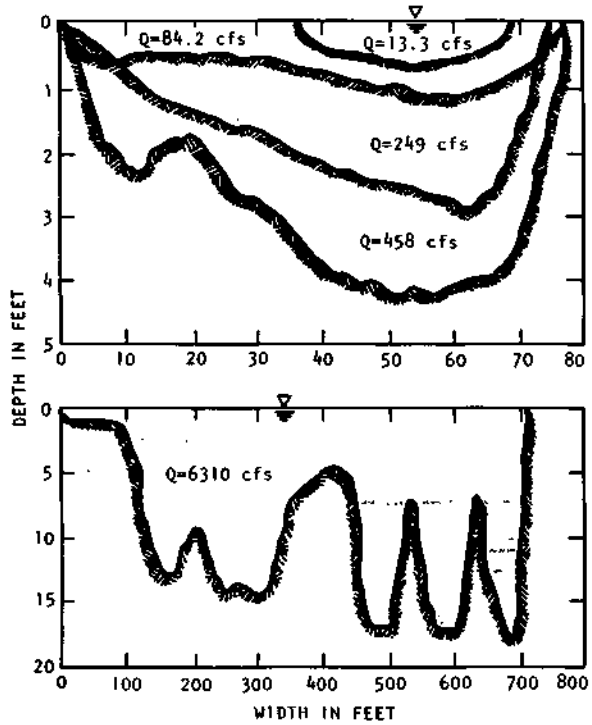


Figure 4. Stream channel cross sections at various stages of flow of Sugar Creek near Hartsburg

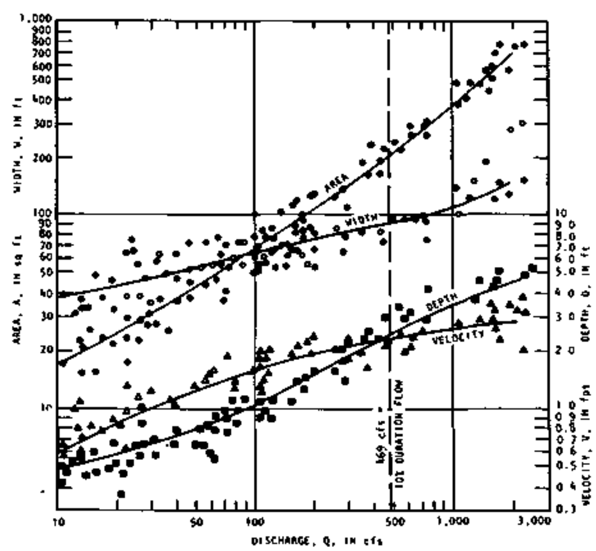


Figure 5. Hydraulic rating curves for Sugar Creek near Hartsburg

Representativeness of Gaging Station Cross Sections

In selecting a location along the course of a stream for a gaging station many factors are considered. A site is often selected where the flow is rather restricted for easier flow measurement. Because of this practice, the question arises as to whether the stream cross sections at these gaging stations are really typical of the stream in general. Because the hydraulic properties of the stream as measured at these gaging stations are being used in this study to generalize the hydraulic geometry of the entire stream system, it is of critical importance that these properties be typical.

This matter was evaluated by using a mass of special-purpose field data on river cross sections for the Embarras River. In an extensive field survey and study by the Illinois Division of Waterways (1963), about 1090 complete cross sections were taken along the 202-mile course of this river. The map of the Embarras River Basin in figure 6 indicates the location of six stream gaging stations on the main stem of the river, and shows by the intersecting short line-segments the location of the 76 special cross sections which were obtained from the Illinois Division of Waterways and studied.

This study to determine hydraulic geometry relations was confined to flows occurring at or below a frequency of 10 percent of the days per year (F = 0.10). For flows this large, in virtually all cases, flow was confined to the stream channel; over-bank flow did not occur. For the Embarras River (figure 6) at the 10 percent flow rate, the cross-sectional area A was less than 2000 square feet for the Lawrenceville gage and less than 1000 square feet for the other five upstream gages.

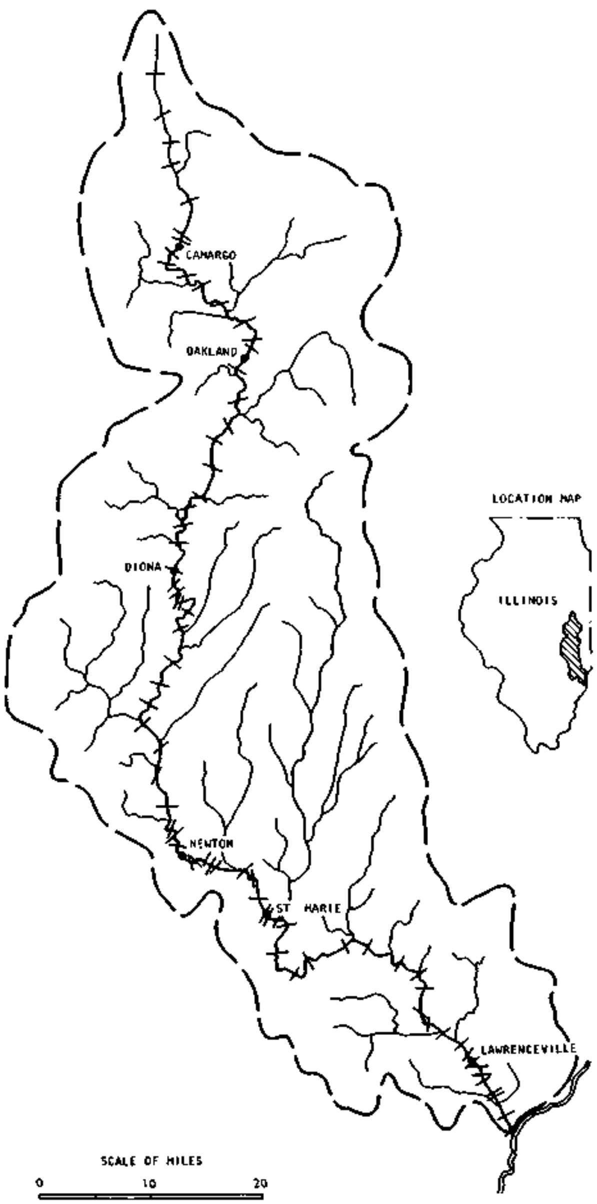


Figure 6. Map of Embarras Basin (Dots show location of stream gaging stations on main stem; line segments show locations of 76 cross sections)

The 76 cross sections along the entire course of the Embarras (figure 6) are considered to be an excellent sampling of the shape of the channel. Each of these 76 cross sections was carefully measured with a planimeter, and graphs were plotted of stage-area relations. These graphs represent the change of shape of the river channel with elevation. Inspection of these 76 graphs indicated a gradual change in the shape from the mouth of the Embarras River near Lawrenceville upstream 202 miles to the head of the river.

Comparison of these channel shapes in the region of two stream gaging stations on the Embarras is illustrated in figures 7 and 8. The seven solid curves in figure 7 indicate the change of the channel shape as measured at various river stations, from 9.6 miles downstream to 4.5 miles upstream from the stream gage at St. Marie. The dashed curve in figure 7 shows the change of the channel shape at St. Marie. The similarity of shapes of the solid and dashed curves indicates that the gaging station is providing a reasonable sampling of the channel shape for the reach in which it is located.

In figure 8 is a similar comparison between the channel shapes for the Embarras River at the Oakland gage (figure 6) and at five other cross sections from 4.8 miles downstream to 10.3 miles upstream. Although the curve shapes are not identical, they are similar enough to indicate the reliability of the Oakland gage measurements in reflecting the channel shape for this entire reach.

From this exploration of these special cross sections it is concluded that, generally, the data from measurements at stream gaging stations provide reasonable and typical information on the hydraulic characteristics of a stream reach. This is true for within-the-bank flows as used in this project. For over-bank flows, further study would be needed before such a conclusion could be made.

METHODS OF ANALYSIS

Comparison of Time

A basic method for evaluating the flow variability in a stream system is by means of flow duration data. For each gaging station a/flow_dujation curve was developed. The assumption made by Leopold and Maddock (1953) that discharges at various points along a stream which have the same frequency of occurrence provide a reasonable basis for comparison can be used to study the discharge of a given frequency at all gaging stations. Here the frequency of occurrence is the time in percent of total period in which the daily discharges were equalled or exceeded.

Investigation of the period of record for the stream gaging stations in Illinois revealed that for 97 stations, or about half of those available, continuous flow records were available for the 15-year period, 1950-1964. Judgment indicated this to be a period generally representative of flow variability in Illinois. For these 97 stations, duration curves of daily flows were prepared by traditional methods. For the remaining gaging stations, standard practice was to utilize the available record during the period of 1950-1964 and to draw short-period flow duration curves for the period of

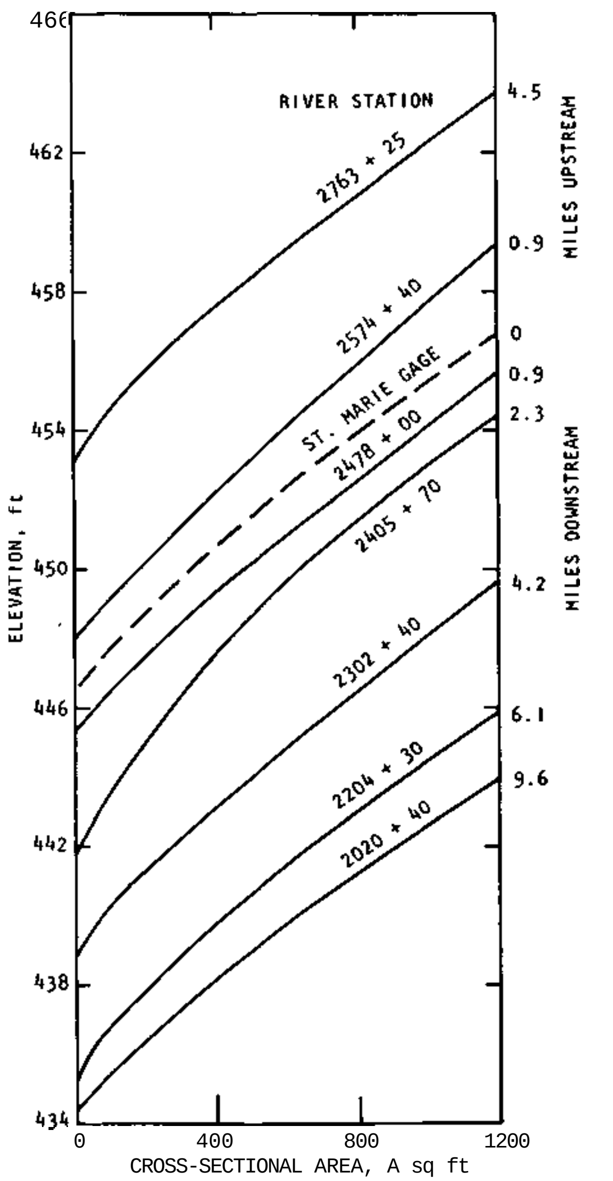


Figure 7. Relation of elevation to cross-sectional area for the Embarras River at the St. Marie gaging station, and at seven other cross sections located upstream and downstream

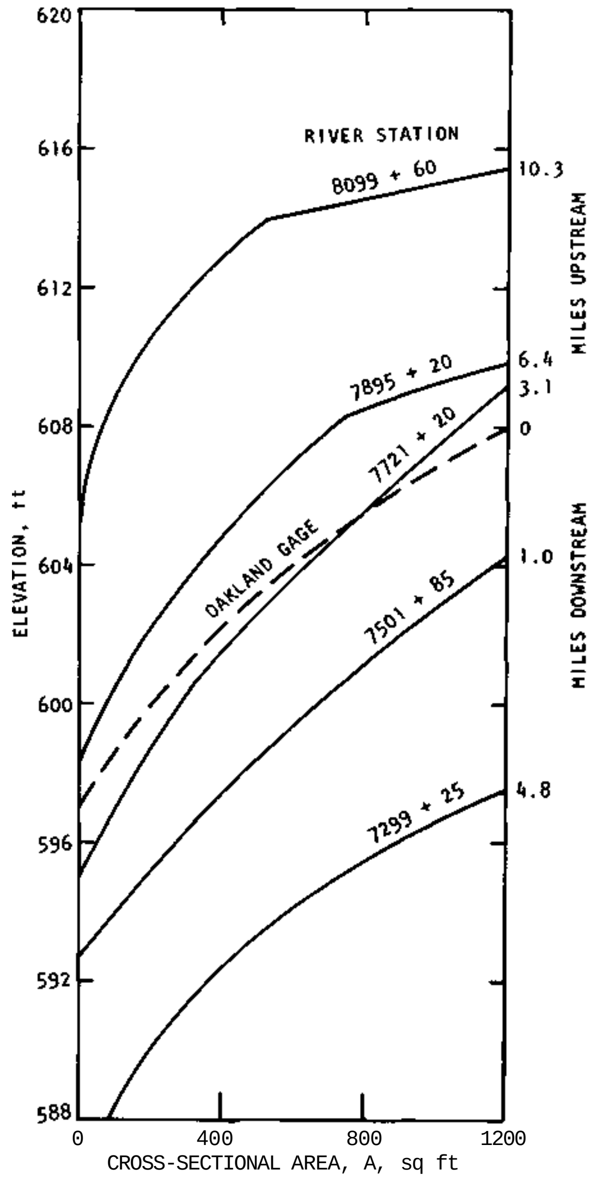


Figure 8. Relation of elevation to cross-sectional area for the Embarras River at the Oakland gaging station, and at five other cross sections located upstream and downstream

concurrent record at the short-record station and at one of the 97 stations. These shorter period flow duration curves were then used to synthesize a flow duration curve believed to be applicable to the standard 15-year period. The synthesis of these curves was carried out by methodology described by Mitchell (1957) and given in more detail by Searcy (1960).

The flow duration data used for Wisconsin gages were drawn from published results by Young (1965) and from additional flow duration data for later years provided by the USGS. No standard period of record was adopted. Although this deviates from the standard practice used within Illinois, it is believed to provide a reasonable comparability and was a great time-saver.

Flow duration data used from gages in Indiana/were drawn from the published data of the Indiana Stream Pollution Control Board (1962). Additional flow duration data for later years were provided as computer outputs from the USGS in Indiana. Again, these flow duration data were not adjusted to a common period of record, but it was felt the data provide a reasonable comparison of results.

The flow duration curves used in this study were drawn on log-normal probability paper. From these curves the flows for various frequencies were read and used. However, it was found that if the flow duration curves were plotted on semilog paper as shown in figure 9 the discharge *Q* can be expressed in terms of the frequency of occurrence *F* by

$$\ln Q = \alpha - \beta F \quad (4)$$

where a and β are empirical constants evaluated for the range between *F* = 0.10 and *F* = 0.90. This equation is the basic mathematical model used in this project to relate the discharge and the frequency of occurrence in a stream system. The four linear flow duration curves shown in figure 9 represent 4 of the 18 stream gaging stations in the Sangamon River Basin. Because the flow frequency is included as one of the variables in the mathematical model of this study, this model can be readily expanded into a more complex stochastic model when better information on the probability of occurrence of flows in a time series is available.

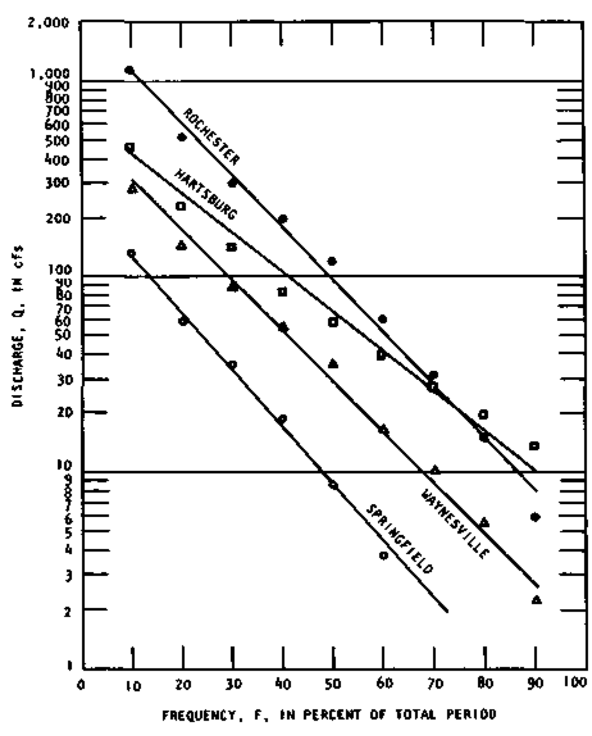


Figure 9. Flow duration curves of daily discharge, 1950-1964

Comparison of Place

Horton (1945) in a pioneering and comprehensive study of quantitative geomorphology described a consistent pattern under which stream systems develop and to which they continually adjust. (We showed that the number of streams, the length of streams, and the slope of streams were all related consistently to stream order throughout any existing stream system. Later revisions to the Horton stream-ordering system were made by Strahler (1957, 1964). Because of the inherent flexibility and soundness of the Strahler system of stream ordering, it has been used in this report. It provides a means of evaluating numerically the structure of a stream system, and has been used to allow a place comparison for hydraulic characteristics at various locations within a stream system.

Several writers have illustrated that the Horton-Strahler system of stream orders and the associated laws of stream development have hydrologic implications. Wong (1963) developed an empirical equation using multivariate

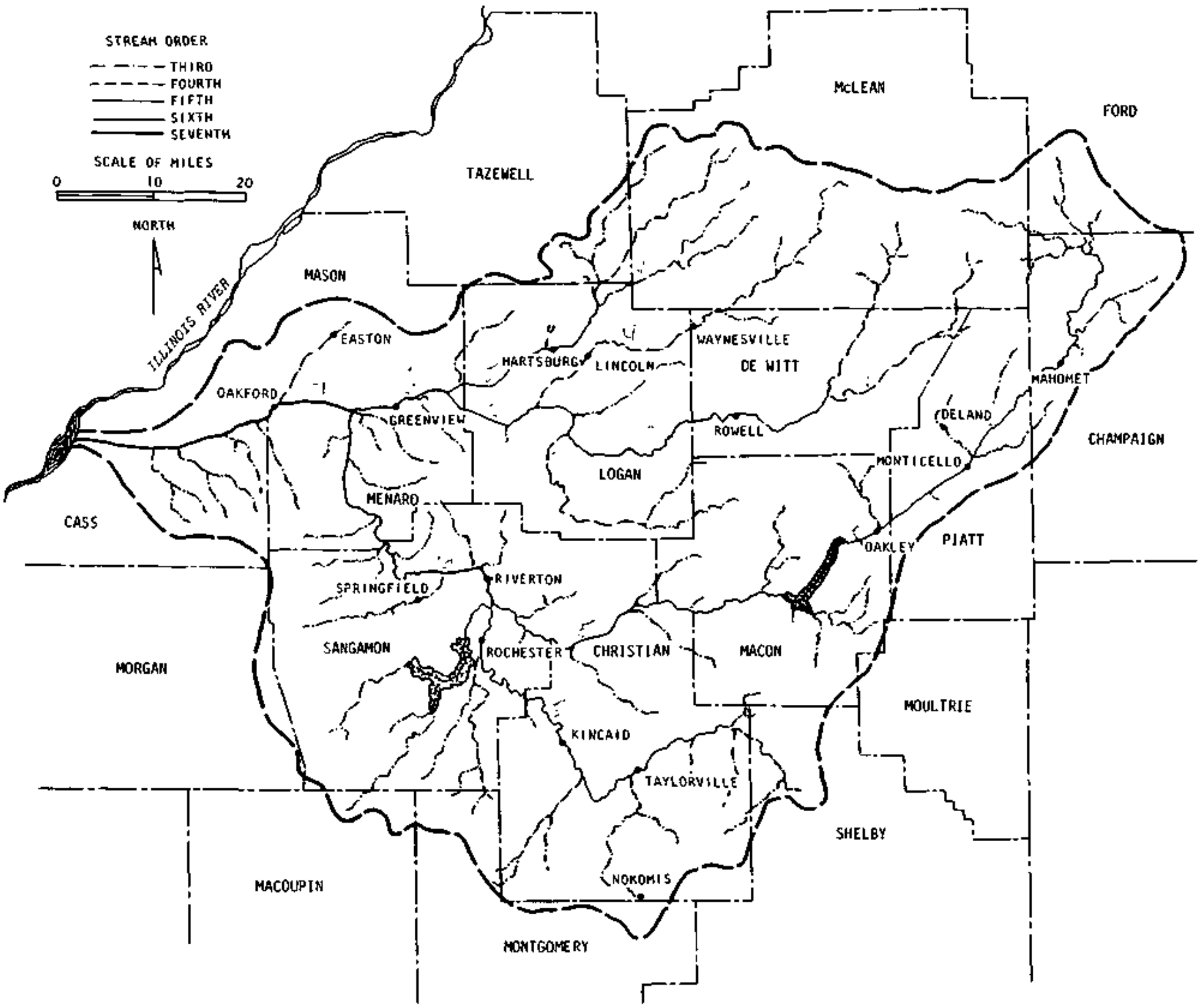


Figure 10. Stream system, gaging stations, and stream orders in Sangamon River Basin

analysis in which he showed that the mean annual floods in 90 basins in New England were associated mathematically with two parameters. One was an indicator of basin size (from Horton's first and second laws) and the other was an indicator of basin slope (Horton's third law). Wong used a multivariate mathematical model and explained the variance of flood flows to a coefficient of determination R^2 of 0.80.

Studies of reservoir sedimentation in Illinois by Stall and Bartelli (1959) confirmed the applicability of the Horton relationships on some Illinois basins. They also showed the importance of such channel factors in explaining sediment movement. For 20 watersheds in the Springfield Plain physiographic division of west-central Illinois, as shown in figure 1, the mean slope of the third-order streams was found to be an important quantitative measure in explaining the sediment delivered to a reservoir. Writings by Roehl (1963) and Miller (1965) also discussed the promise which morphological factors offer in understanding sedimentation. The concept of stream-ordering systems has also been discussed authoritatively by Rzhanitsyn (1960).

To illustrate the availability and use of hydraulic and stream-order data, examples will be given for the Sangamon River Basin in central Illinois, shown in figure 10. Here is shown the stream system, the location of the 18 stream gaging stations which provided data for this study, and the stream orders.

The standard 15-minute topographic maps published by the USGS were used for ordering the stream systems. The scale of these maps is about 1 inch = 1 mile. According to Strahler (1957) the visible unbranched streams shown on the topographic maps in blue were defined as the first-order streams. Where two first-order streams join, a second-order stream begins, and so forth.

After the whole stream system was ordered, the number of streams in each order were totaled. The length and slopes of streams of the third-order through the sixth-order were measured and averaged. Figure 11 shows the linear relationships of stream number, average length, and average slope to stream order on a semilog paper. These relationships prove that Horton's law of stream numbers, law of stream length, and law of stream slopes are applicable to Illinois stream systems.

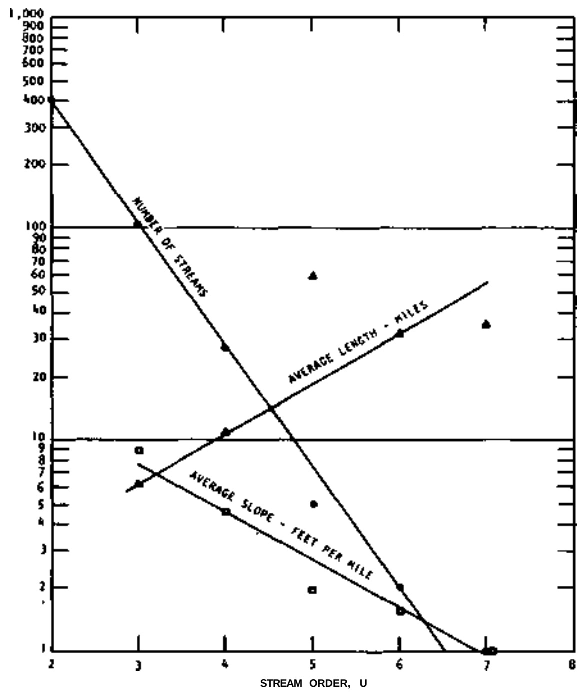


Figure 11. Stream morphology factors and stream order

The concept has been reported by Shreve (1967) that the first-order streams of a stream system constitute the *building blocks of the* system, and that the development of the higher order streams follows a random process. Thi concept has also been reported by Hirsch (1962) and Strahler (1964).

Proportional Stream Order

According to Horton (1945), the stream order is defined as integers only. Under this definition, the formation of a stream system is described in a discontinuous manner. In other words, the development of a stream system has been described at points of junctions only, and the variation within a segment of stream of the same order has been neglected. Therefore it was found useful to modify the stream-ordering system to include not only integers as used by Horton, but also rational numbers. Proportional stream order *U*, at a point *x*, which is located in a given stream segment of order *U.*, is defined as

$$U = U_i + U_x \tag{5}$$

where

$$U_x = \frac{N_{1x} - N_{1s}}{N_{1e} - N_{1s}} \tag{6}$$

and N_{1x} = total number of first-order streams above point *x*; N_{1s} – total number of first-order streams above the starting point of the stream of order U_i in question; and N_{1e} = total number of first-order streams above the ending point of the stream of order U_i When N_{1e} = N_{1s}, the value of U_x is defined as zero. When N_{1X} = N_{1e}, U_x is defined as 0.99 to maintain consistency. This proportional stream order can be used to determine the relative position of a point on a given stream in the Horton-Strahler system. With this concept, the stream measurements obtained from various gaging stations within a river basin can be studied more systematically than is possible by the Horton-Strahler system alone.

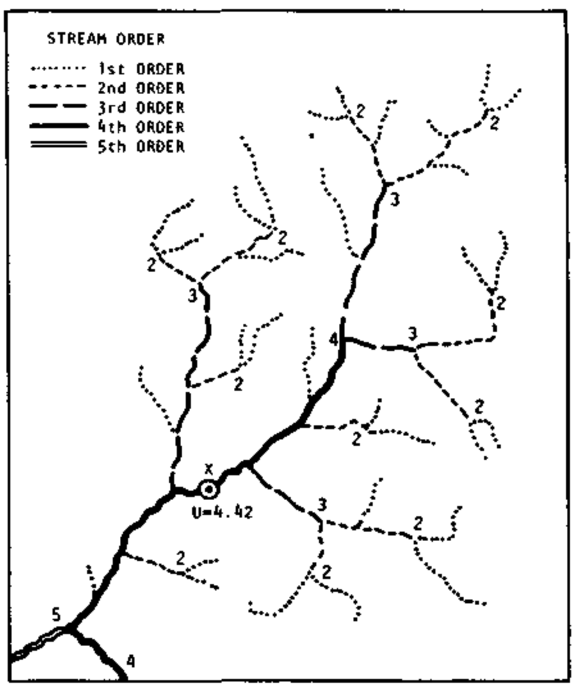


Figure 12. Determination of the proportional stream order at point x *of a fourth-order stream*

Figure 12 shows an example of how the proportional stream order is determined. Point *x* is located on a fourth-order stream. The number of first-order streams above the start of this fourth-order stream is 12, so N = 12. The number of first-order streams above point *x* is 20, so N_{1x} = 20. The number of first-order streams above the lower end of this fourth-order stream is 31, so $N_{1e} = 31$. Therefore, the proportional stream order was found to be 4.42 using equations 5 and 6.

The first-order stream with its contributing drainage area can be considered as the building block or unit cell of a watershed. Therefore, the first-order streams themselves, of a given map, cannot be proportioned using equations 5 and 6. The relationship between the proportional stream order and the drainage area will be discussed later.

Hydraulic Geometry Factors

The association of discharge to frequency of occurrence has been shown in equation 4. With the development of proportional stream order described, it was found that equation 4 can be extended into a linear multiple regression model such as

$$\ln Q = \alpha - \beta F + \gamma U \qquad (7)$$

in which Y is an empirical constant. The discharge is the product of cross-sectional area and velocity, $Q=AV$; and cross-sectional area equals width times depth, $A = WD$. Therefore, four additional equations that have forms similar to equation 7 can also be developed relating the hydraulic geometry factors A, V, W, and D to the proportional stream order and frequency of occurrence. A set of these five equations is shown later. The relationships between these five equations are:

$$\ln Q = \ln A + \ln V \qquad (8)$$

$$\ln A = \ln W + \ln D \qquad (9)$$

Discharge measurement records and flow duration frequency curves were available for 18 stream gaging stations within the watershed of the Sangamon River (figure 10). The hydraulic geometry information measured at the gaging station at Sugar Creek near Hartsburg can be utilized as an example. First, the discharges can be obtained for a set of selected frequencies by using the flow duration curve. Secondly, the values of the hydraulic geometry factors can be determined from figure 5 for the given discharges. Table 2 shows the values of A, V, W, and D to the corresponding values of F and Q that are depicted in figure 5.

Equations

Eighteen tables similar to table 2 were prepared in this study for the Sangamon River Basin. After the proportional stream orders for these 18 stream gaging stations were determined, the values of hydraulic geometry factors of all gaging stations were plotted on semilog papers, as shown in figure 13. and the corresponding equations evaluated. Figure 13 shows data for stream order and for discharges with the same frequency of occurrence F, for these 18 gaging stations. The curves were fitted as straight lines on the semilog plot.

Table 2. Hydraulic Geometry Factors Related To Flow Frequency For Sugar Creek Near Hartsburg

Flow frequency F (%)	Discharge Q (cfs)	Cross-sect. area A (sq ft)	Velocity V (fps)	Width W (ft)	Mean depth D (ft)
90	14.1	20.0	0.70	38	0.52
80	20.1	23.8	0.85	41.5	0.58
70	27.5	28.4	0.96	45	0.64
60	40.2	35.0	1.12	49	0.73
50	60.3	45.5	1.34	54	0.85
40	83.8	59.0	1.48	60	0.96
30	144	82.0	1.70	66	1.24
20	234	117	1.95	74	1.65
10	469	192	2.30	86	2.38

The discharges at 10-percent frequency (uppermost line) give a better fit with stream order than do the other two lines at 50- and 90-percent frequencies. This suggests that the higher flow rates exert more hydraulic effect on the geometry of the stream system.

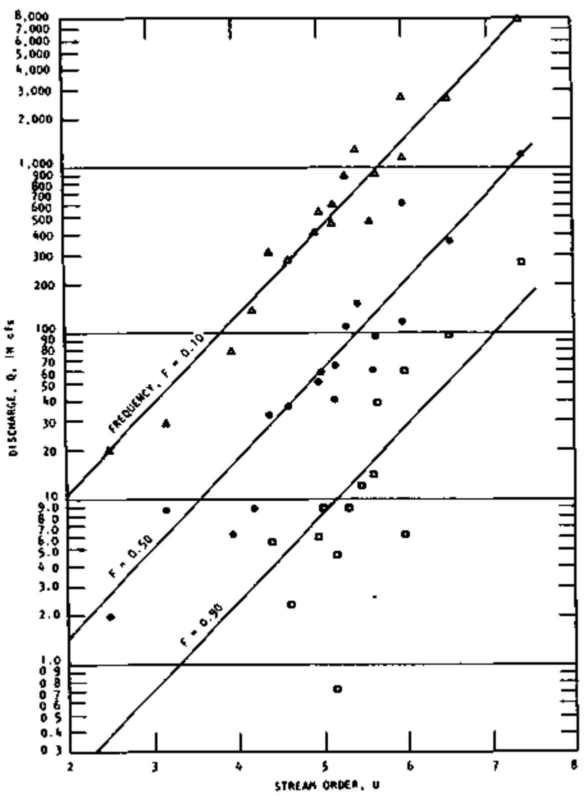


Figure 13. Discharge as related to proportional stream order and frequency of occurrence

Similarly, the associations of the channel cross-sectional area, velocity, width, and depth to proportional stream order were also made for the Sangamon Basin, and a set of five hydraulic geometry equations were developed. These are:

$$\ln Q = 0.39 - 4.93 F + 1.23 U \quad (10)$$

$$\ln A = 1.46 - 3.98 F + 0.92 U \quad (11)$$

$$\ln V = -1.07 - 0.95 F + 0.31 U \quad (12)$$

$$\ln W = 1.49 - 1.70 F + 0.61 U \quad (13)$$

$$\ln D = -0.03 - 2.28 F + 0.31 U \quad (14)$$

in which, *Q* is in cubic feet per second, *A* in square feet, *V* in feet per second, *W* and *D* in feet. The designation In signifies a logarithm to the base *e.*

The frequency of occurrence *F* in equations 10 to 14 is limited from 10- to 90-percent of the days. These hydraulic geometry equations are expected to give best results at higher flow rates as shown in figure 13. The development of these equations is closely related to the *laws* of stream morphology published by Horton (1945); these equations can be considered as an extension of Horton's work into the realm of hydraulic geometry of streams.

Relation of Drainage Area to Hydraulic Geometry Factors

The use of Horton-Strahler stream order numbers to designate the various parts of the stream system is relatively new in hydrology. It is well known however that, progressing downstream through a stream system, the drainage area increases. It was found that stream order can be related to drainage area by

$$\ln A_d = p + q\,U \tag{15}$$

where p and q are empirical constants. For example, plotting the drainage area A_d in square miles against the proportional stream order U of each gaging station on semilog paper shows the relationship between A_d and U for the Sangamon Basin to be

$$\ln A_d = -0.25 + 1.19\,U \tag{16}$$

Figure 14 shows the relationship between A_d and U for the Sangamon. The straight line depicts equation 16. Therefore, the drainage area A_d can replace the proportional stream order U in equations 10 to 14.

To enable the reader of this report to convert the drainage area back to the proportional stream order, the relations of drainage area to stream order for the 18 basins are listed in table 3.

Solving for U from equation 16 and substituting the result into equations 10 to 14 produces a set of five equations which represents the relation of drainage area to hydraulic geometry factors for the Sangamon Basin.

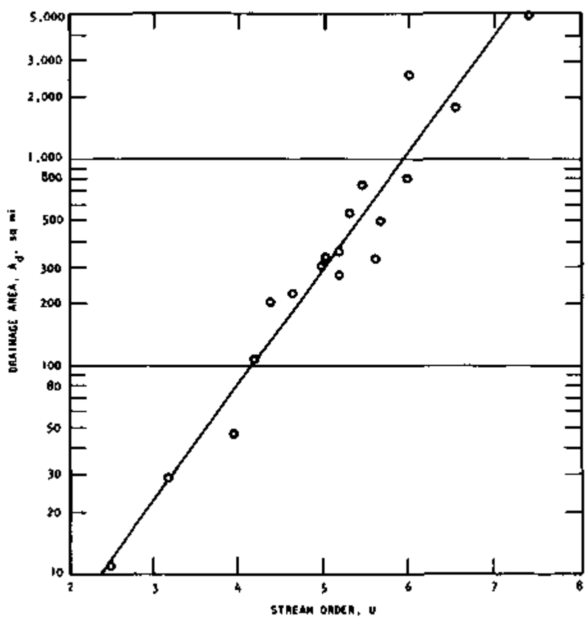


Figure 14. Drainage area related to stream order for Sangamon River Basin

The five equations are:

$$\mathrm{In}\ Q = 0.65 - A.93\ F + 1.03\ \mathrm{In}\ A_d \qquad (17)$$

$$\mathrm{In}\ A = 1.66 - 3\text{-}98\ F + 0.77\ \mathrm{In}\ A_d \qquad (18)$$

$$\mathrm{In}\ V = -1.01 - 0.95\ F + 0.26\ \mathrm{In}\ A_d \qquad (19)$$

$$\mathrm{In}\ W = 1.62 - 1.70\ F + 0.51\ \mathrm{In}\ A_d \qquad (20)$$

$$\mathrm{In}\ D = 0.04 - 2.28\ F + 0.26\ \mathrm{In}\ A_d \qquad (21)$$

Table 3. Drainage Area Relation to Stream Order for 18 Basins

General form of equation:

$$\mathrm{In}\ A_d = p + q\ U$$

where

A_d = drainage area in square miles
U = proportional stream order
In denotes a natural logarithm
p and q are empirical constants given below

Basin	p.	q
Rock	-1.81	1.41
Galena	-1.33	1.15
Fox	+0.29	1.07
Des Plaines	-3.89	1.74
Kankakee	-2.75	1.52
Vermilion (Illinois R. Basin)	+0.63	1.03
Mackinaw	-1.39	1.34
Henderson	-1.16	1.21
Spoon	-0.28	1.10
La Moine	-1.31	1.45
Sny	-3.35	1.70
Sangamon	-0.25	1.19
Kaskaskia	-1.68	1.47
Vermilion (Wabash R. Basin)	-1.67	1.48
Embarras	+0.14	1.14
Little Wabash	-1.30	1.39
Big Muddy	-2.95	1.52
Big Bay	-1.94	1.34

Equations 17 to 21, showing the hydraulic geometry factors, are power functions of the drainage area. For example, equation 17 can be rewritten as

$$Q = e^{(0.65 - 4.93 F)} A_d^{1.03} \tag{22}$$

where e = the base of the natural logarithms. Equation 22 is a numerical example of the power functional relationship between the discharge Q and the drainage area A_d. The general equation for such a relationship can be obtained by solving equation 7 in terms of equation I5, that is

$$Q = e^{(\alpha - \frac{\gamma p}{q} - \beta F)} A_d^{\frac{\gamma}{q}} \tag{23}$$

Because α, β, γ, p, and q are empirical constants, the independent variables in equation 23 are F and A_d. Therefore, when $A_d = 1$, equation 23 is reduced to the same form as equation 4. On the other hand, when F has a given value between 0.1 and 0.9, equation 23 becomes a simple power function with a form similar to the *simple flood formulas* as classified by Chow (1962).

Again, using equations 17 to 21 as an example, In A_d can be solved from equation 17 and the result substituted into equations 18 to 21, to obtain a set of four equations which represent the relation of discharge Q to hydraulic factors *W*, *D*, *V*, and *A* for the Sangamon Basin in simple power functions

$$W = e^{(1.30 + 0.74 F)} Q^{0.50} \tag{24}$$

$$D = e^{(-0.13 - 1.03 F)} Q^{0.25} \tag{25}$$

$$V = e^{(-1.17 + 0.29 F)} Q^{0.25} \tag{26}$$

$$A = e^{(1.17 - 0.29 F)} Q^{0.75} \tag{27}$$

It should be noted here that equations *2k* to 27 can also be obtained using equations 10 to *\k*.

Equations *2k* to 26 have the same forms as equations 1 to 3, respectively. The former were derived by the application of proportional stream order or drainage area for the stream system analysis. Therefore, equations *2k* to 26 can be considered as a rational verification of the work of Leopold and Maddock (1953) represented by equations 1 to 3.

In the morphological analysis of stream systems, the Horton-Strahler stream-ordering system has been used extensively. The validity of this ordering system has been shown time and again by such writers as Leopold and Miller (1956), Hack (1957), Morisawa (1959), and Broscoe (1959). However, for the hydrologic analysis or engineering application, the drainage area has been considered as a more important variable affecting the runoff from a given basin.

For application purposes, 18 sets of hydraulic geometry equations which have the same forms as equations 17 to 21 are listed in table 4 as the resulting equations for 18 river basins of Illinois. The reasons for publishing the equations using the drainage area A_d instead of the proportional stream order U are: 1) drainage area maps are readily available, 2) determining a drainage area is easier than determining a proportional stream order, and 3) the measurement of a drainage area is more accurate than that of the stream order because the stream orders were determined from the USGS 15-minutes topographic maps and maps of other sizes would change the orders.

Table 4. Hydraulic Geometry Equations for 18 River Basins

Description of Units

Q = discharge in cfs
A = cross-sectional area in sq ft
V = average velocity in fps
W = width of stream at the surface in ft
D = average depth of stream in ft
A_d = drainage area in sq mi
F = frequency in percent of days, as a decimal
In denotes that all logarithms are natural logarithms to the base e = 2.713

Rock River

$$\ln Q = 0.24 - 3.50\,F + 1.03\,\ln A_d$$
$$\ln A = 0.04 - 2.00\,F + 0.90\,\ln A_d$$
$$\ln V = 0.20 - 1.50\,F + 0.13\,\ln A_d$$
$$\ln W = 0.66 - 0.80\,F + 0.64\,\ln A_d$$
$$\ln D = -0.62 - 1.20\,F + 0.26\,\ln A_d$$

Galena River

$$\ln Q = 0.13 - 2.27\,F + 0.96\,\ln A_d$$
$$\ln A = 0.19 - 1.46\,F + 0.90\,\ln A_d$$
$$\ln V = -0.06 - 0.81\,F + 0.06\,\ln A_d$$
$$\ln W = 0.76 - 0.81\,F + 0.69\,\ln A_d$$
$$\ln D = -0.57 - 0.65\,F + 0.21\,\ln A_d$$

Fox River

$$\ln Q = -0.24 - 3.33\,F + 1.13\,\ln A_d$$
$$\ln A = -0.35 - 1.94\,F + 0.97\,\ln A_d$$
$$\ln V = 0.11 - 1.39\,F + 0.16\,\ln A_d$$
$$\ln W = 0.56 - 0.39\,F + 0.64\,\ln A_d$$
$$\ln D = -0.91 - 1.55\,F + 0.33\,\ln A_d$$

Des Plaines River

$$\ln Q = 1.78 - 4.98\,F + 0.90\,\ln A_d$$
$$\ln A = 1.52 - 3.67\,F + 0.82\,\ln A_d$$
$$\ln V = 0.26 - 1.31\,F + 0.08\,\ln A_d$$
$$\ln W = 1.56 - 1.62\,F + 0.60\,\ln A_d$$
$$\ln D = -0.04 - 2.05\,F + 0.22\,\ln A_d$$

Kankakee River

$$\ln Q = 1.41 - 5.12\,F + 0.96\,\ln A_d$$
$$\ln A = 1.79 - 3.93\,F + 0.79\,\ln A_d$$
$$\ln V = -0.38 - 1.19\,F + 0.17\,\ln A_d$$
$$\ln W = 2.12 - 1.68\,F + 0.45\,\ln A_d$$
$$\ln D = -0.33 - 2.25\,F + 0.34\,\ln A_d$$

Vermilion River (Illinois River Basin)

$$\ln Q = 0.97 - 6.28\,F + 1.01\,\ln A_d$$
$$\ln A = 1.17 - 4.09\,F + 0.84\,\ln A_d$$
$$\ln V = -0.20 - 2.19\,F + 0.17\,\ln A_d$$
$$\ln W = 1.99 - 1.38\,F + 0.49\,\ln A_d$$
$$\ln D = -0.82 - 2.71\,F + 0.35\,\ln A_d$$

Table 4 (Continued)

Mackinaw River

$$\ln Q = 1.39 - 7.52\,F + 1.00 \ln A_d$$
$$\ln A = 1.01 - 5.26\,F + 0.91 \ln A_d$$
$$\ln V = 0.38 - 2.26\,F + 0.09 \ln A_d$$
$$\ln W = 1.55 - 2.13\,F + 0.56 \ln A_d$$
$$\ln D = -0.54 - 3.13\,F + 0.35 \ln A_d$$

Henderson Creek

$$\ln Q = 1.44 - 5.00\,F + 0.89 \ln A_d$$
$$\ln A = 0.86 - 3.24\,F + 0.88 \ln A_d$$
$$\ln V = 0.58 - 1.76\,F + 0.01 \ln A_d$$
$$\ln W = 2.46 - 0.72\,F + 0.27 \ln A_d$$
$$\ln D = -1.60 - 2.52\,F + 0.61 \ln A_d$$

Spoon River

$$\ln Q = 0.86 - 4.82\,F + 1.00 \ln A_d$$
$$\ln A = 0.34 - 3.19\,F + 0.92 \ln A_d$$
$$\ln V = 0.52 - 1.63\,F + 0.08 \ln A_d$$
$$\ln W = 1.61 - 1.14\,F + 0.45 \ln A_d$$
$$\ln D = -1.27 - 2.05\,F + 0.47 \ln A_d$$

La Moine River

$$\ln Q = 1.03 - 5.60\,F + 0.92 \ln A_d$$
$$\ln A = 1.16 - 4.44\,F + 0.81 \ln A_d$$
$$\ln V = -0.13 - 1.16\,F + 0.11 \ln A_d$$
$$\ln W = 1.54 - 1.33\,F + 0.45 \ln A_d$$
$$\ln D = -0.38 - 3.11\,F + 0.36 \ln A_d$$

Sny River

$$\ln Q = -2.27 - 5.87\,F + 1.63 \ln A_d$$
$$\ln A = -0.98 - 4.81\,F + 1.24 \ln A_d$$
$$\ln V = -1.29 - 1.06\,F + 0.39 \ln A_d$$
$$\ln W = 1.11 - 2.39\,F + 0.67 \ln A_d$$
$$\ln D = -2.09 - 2.42\,F + 0.57 \ln A_d$$

Sangamon River

$$\ln Q = 0.65 - 4.93\,F + 1.03 \ln A_d$$
$$\ln A = 1.66 - 3.98\,F + 0.77 \ln A_d$$
$$\ln V = -1.01 - 0.95\,F + 0.26 \ln A_d$$
$$\ln W = 1.62 - 1.70\,F + 0.51 \ln A_d$$
$$\ln D = 0.04 - 2.28\,F + 0.26 \ln A_d$$

Kaskaskia River

$$\ln Q = 0.95 - 5.88\,F + 1.02 \ln A_d$$
$$\ln A = 1.21 - 4.60\,F + 0.88 \ln A_d$$
$$\ln V = -0.26 - 1.28\,F + 0.14 \ln A_d$$
$$\ln W = 1.47 - 1.39\,F + 0.50 \ln A_d$$
$$\ln D = -0.26 - 3.21\,F + 0.38 \ln A_d$$

Vermilion River (Wabash River Basin)

$$\ln Q = 1.11 - 4.96\,F + 0.98 \ln A_d$$
$$\ln A = 1.92 - 2.76\,F + 0.69 \ln A_d$$
$$\ln V = -0.81 - 2.20\,F + 0.29 \ln A_d$$
$$\ln W = 2.49 - 0.75\,F + 0.37 \ln A_d$$
$$\ln D = -0.57 - 2.01\,F + 0.32 \ln A_d$$

Embarras River

$$\ln Q = 0.04 - 5.61\,F + 1.17 \ln A_d$$
$$\ln A = 0.96 - 3.99\,F + 0.91 \ln A_d$$
$$\ln V = -0.92 - 1.62\,F + 0.26 \ln A_d$$
$$\ln W = 1.00 - 1.45\,F + 0.58 \ln A_d$$
$$\ln D = -0.04 - 2.54\,F + 0.33 \ln A_d$$

Little Wabash River

$$\ln Q = 1.91 - 7.90\,F + 0.96 \ln A_d$$
$$\ln A = 3.29 - 6.72\,F + 0.66 \ln A_d$$
$$\ln V = -1.38 - 1.18\,F + 0.30 \ln A_d$$
$$\ln W = 2.88 - 2.68\,F + 0.34 \ln A_d$$
$$\ln D = 0.41 - 4.04\,F + 0.32 \ln A_d$$

Big Muddy River

$$\ln Q = 1.26 - 8.50\,F + 1.09 \ln A_d$$
$$\ln A = 2.01 - 7.03\,F + 0.91 \ln A_d$$
$$\ln V = -0.75 - 1.47\,F + 0.18 \ln A_d$$
$$\ln W = 2.57 - 3.14\,F + 0.44 \ln A_d$$
$$\ln D = -0.56 - 3.89\,F + 0.47 \ln A_d$$

Big Bay Creek

$$\ln Q = 1.48 - 7.90\,F + 1.05 \ln A_d$$
$$\ln A = 2.01 - 7.49\,F + 0.91 \ln A_d$$
$$\ln V = -0.53 - 0.41\,F + 0.14 \ln A_d$$
$$\ln W = 2.45 - 3.54\,F + 0.40 \ln A_d$$
$$\ln D = -0.44 - 3.95\,F + 0.51 \ln A_d$$

RESULTS

Equations and Graphs

Equations 17 to 21 for the Sangamon River Basin exemplify the primary results of this study. This set of five equations shows quantitative relations between discharge, cross-sectional area, stream depth, width, and velocity for various drainage areas within the basin. These equations can be used to compute a generalized value for any of these parameters anywhere within the basin. These results are presented graphically in figure 15, in which parts A through E give representation of the five equations 17 to 21. In each part of figure 15 one of the stream system parameters is associated with drainage area and frequency of occurrence of a particular discharge.

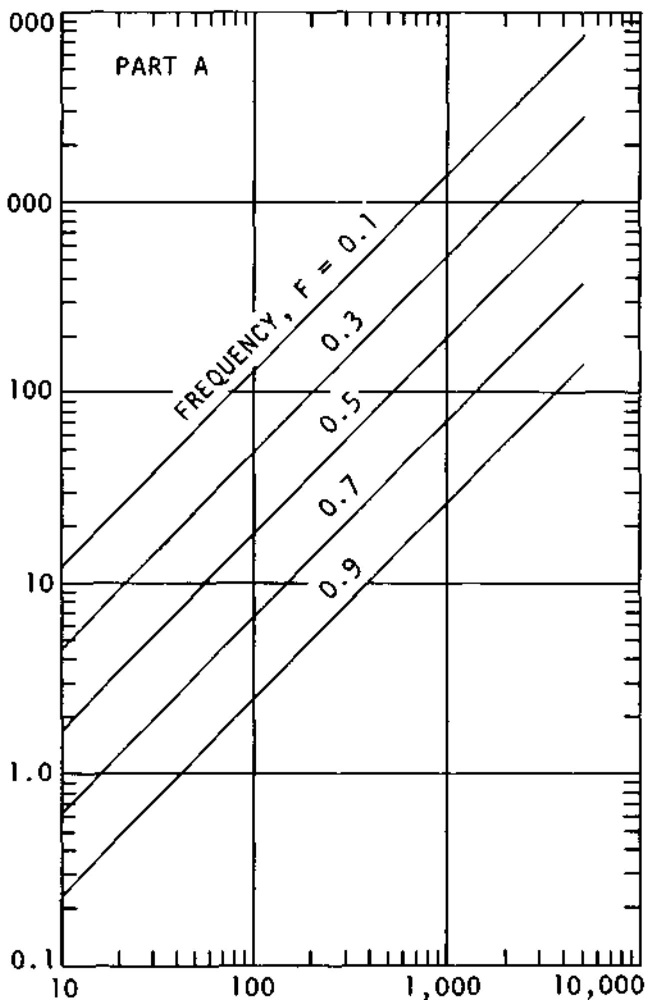


Figure 15 (Part A). Hydraulic geometry results for Sangamon River Basin

The type of information which can be read from the graphs in figure 15 is illustrated as follows.

Suppose it is desirable to know, within the Sangamon River Basin, what the stream-channel characteristics might be at a drainage area of 100 square miles during a discharge which occurs 50 percent of the time. By reading the graph in part A of figure 15, it is noted that, at a drainage area of 100 square miles and at a frequency of F = 0.5, the discharge is about 18 cubic feet per second. Part B of figure 15 shows that for a drainage area of 100 square miles and at a frequency of F = 0.5, the cross-sectional area of the stream is about 21 square feet. Part C shows that, under such conditions, the mean velocity would be about 0.7 feet per second. Part D then indicates that the average depth under such conditions would be about 1.1 feet, and part E that the stream width under such conditions is about 21 feet. Thus, the use of these five graphs makes it possible to obtain a general idea as to the stream characteristics anywhere within the Sangamon River Basin, based

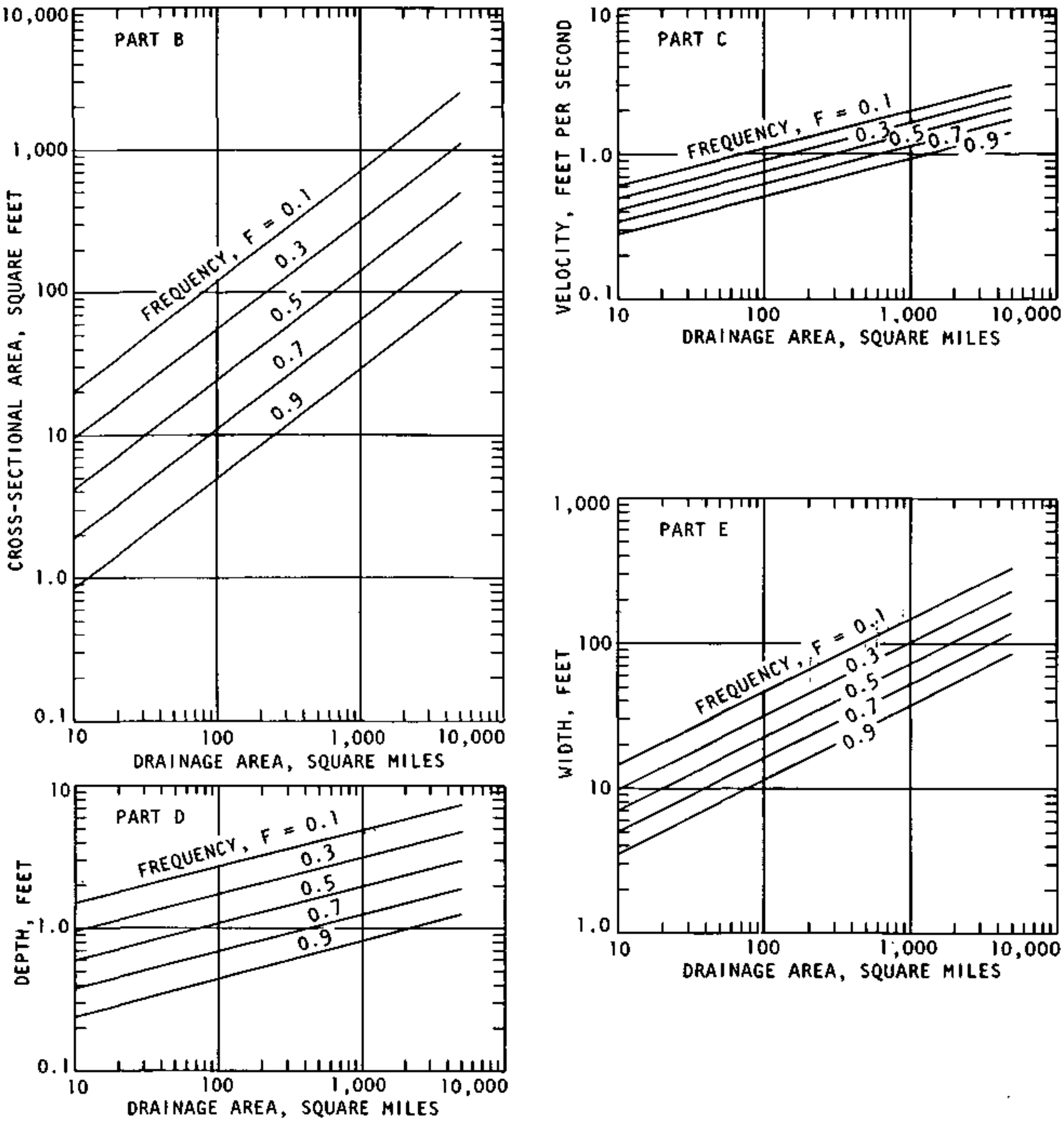


Figure 15 (Parts B-E). Hydraulic geometry results for Sangamon River Basin

upon the consistent patterns of hydraulic geometry as presented in these graphs and by equations 17 to 21.

The complete results of this study for the 18 basins in Illinois are presented in equation form in table 4 *(page 20).* A set of five equations, similar to equations 17 to 21 for the Sangamon River Basin, presents the hydraulic geometry for each of the 18 river basins (figure 2). In each case it is possible to obtain a solution by using the set of five equations as given in table 4. It would also be possible to plot these relations graphically for any of the 18 basins by the use of these equations, as was done in figure 15 for the Sangamon Basin.

The use of equations 17 to 21, or any of the equations in table 4, will be shown numerically by example. Suppose it is desired to know the cross-sectional area of a stream in the Sangamon River Basin at a location where the drainage area is 10 square miles, at a flow which occurs 90 percent of the days per year. Then A_d = 10 square miles and F = 0.90, and the governing expression is equation 18:

$$\ln A = 1.66 - 3.98 F + 0.77 \ln A_d$$

so,

$$\ln A = 1.66 - 3.98 (0.9) + 0.77 \ln (10)$$

$$= 1.66 - 3.581 + 0.77 (2.303)$$

$$= 1.66 - 3.581 + 1.772$$

$$\ln A = -0.149$$

and

$$A = 0.86 \text{ square feet.}$$

Results are not available for all of the stream basins in Illinois, only for the 18 basins shown in figure 2. As described earlier, sufficient data were not available in the remaining basins to describe meaningful hydraulic geometry equations, that is, two or less gaging stations were available. If the reader desires to obtain results for one of the basins for which equations are not available, the authors suggest that a solution be made for an adjoining basin or basins for which data are available. Use of these solutions, and judgment, would provide a general answer.

Horton-Strahler Parameters

One of the major efforts of this study was the determination of the Horton-Strahler stream orders for each of the 18 basins studied. After order numbers were assigned, counts were made of the number of stream segments of each order, the average length of the streams of each order, and the average slope for each order. As described earlier, the graph in figure 11 shows how these Horton-Strahler stream parameters follow Horton's various laws. The data in figure 11 are for the Sangamon River Basin.

The measurement and determination of these stream system parameters is believed to be a valuable part of the results of this study. Table 5 gives the complete tabulation of the Horton-Strahler stream system parameters for

Table 5. Horton-Strahler Stream System Parameters for 18 River Basins

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
Rook River Basin			
2	1566		
3	380	4.4	14.8
4	92	9.6	6.8
5	20	23.5	2.3
6	4	74.6	1.6
7	1	167	1.13
Galena River Basin			
2	64		
3	16	3.7	28.7
4	4	3.7	16.7
5	1	38.1	6.5
Fox River Basin			
2	285		
3	70	5.0	10.1
4	17	6.9	6.0
5	5	18.6	3.8
6	1	141	2.1
Des Plaines River Basin			
1	250		
2	73		
3	19	4.3	9.3
4	5	10.6	5.6
5	1	75.9	1.17

Strearr order	Number Of streams	Average length (mi)	Average slope (ft/mi)
Kankakee River Basin			
2	1365		
3	317	3.2	5.4
4	82	5.3	4.0
5	20	12.5	2.0
6	3	37.3	1.35
7	1	272	0.82
Vermilion River (Illinois R. Basin)			
1	494	1.6	
2	119	2.6	
3	31	6.9	8.6
4	8	8.3	4.1
5	2	19.4	2.7
6	1	47.9	2.9
Mackinaw River Basin			
2	140		
3	32	5.5	12.1
4	9	6.1	8.3
5	2	22.5	2.4
6	1	63.4	3.0
Henderson Creek Basin			
1	478		
2	120		
3	24	4.9	13.6
4	6	10.8	6.4
5	2	9.5	3.2
6	1	22.9	1.9

Table 5 (Continued)

Spoon River Basin

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
2	334		
3	80	3.8	14.3
4	20	7.3	7.7
5	5	15.5	3.9
6	1	123	1.46

La Moine River Basin

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
2	200		
3	42	5.9	15.9
4	10	8.2	6.5
5	2	36.0	2.9
6	1	21.6	1.32

Sny River Basin

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
2	196		
3	43	3,3	19.8
4	9	10.1	10.1
5	2	27.7	1.9
6	1	5.1	0.4

Sangamon River Basin

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
2	424		
3	103	6.2	9.0
4	26	10.8	4.6
5	5	59.7	1.9
6	2	31.7	1.6
7	1	34.6	1.01

Kaskaskia Basin

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
2	847		
3	200	3.9	10.9
4	47	12.3	5.1
5	13	15.5	3.0
6	3	54.1	1.22
7	1	90.0	0.48

Vermilion River (Wabash R. Basin)

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
2	118		
3	25	6.1	10.6
4	7	14.3	6.9
5	2	35.4	4.0
6	1	11.3*	1.77*

*Considered only to Indiana state line and not full length

Embarras River Basin

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
2	367		
3	85	4.1	10.5
4	18	8.1	5.7
5	4	22.7	3.2
6	1	127	1.28

Little Wabash River

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
1 1			
2	460		
3	106	4.2	8.8
4	27	9.9	4.5
5	4	49.3	2.1
6	2	30.2	0.50
7	1	33.1	0.18

Big Muddy River Basin

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
2	546		
3	136	2.9	15.4
4	30	8.0	5.1
5	9	19.2	2.4
6	3	30.7	1.15
7	1	52.3	0.17

Big Bay Creek Basin

Stream order	Number of streams	Average length (mi)	Average slope (ft/mi)
1	182		
2	44		
3	13	3.5	20.6
4	2	17.6	5.9
5	1	29.2	1.6

each of the 18 river basins studied. The stream order, the number of streams, the average length in miles, and the average slope in feet per mile are shown for each basin.

Statewide Generalization

In addition to the hydraulic geometry results presented separately for 18 basins in table 4, it is possible to use the same mathematical model and fitting methods to produce a set of composite hydraulic geometry equations for the state of Illinois as a whole. This has been done and the results are shown in table 6. Here equations 28 to 32 are the statewide composite equations written in terms of the frequency of flow F and proportional stream order U. Equations 33 to 36 are statewide equations solved for the proportional stream order, so that the resulting equations are written in terms of frequency and discharge only. The form of equations 34 to 36 is similar to that given in equations 1, 2, and 3 as written by Leopold and Maddock (1953).

Equation 37 in table 6 shows the statewide generalized relation between drainage area and proportional stream order. Equations 38 to 42 are the complete generalized composite statewide hydraulic geometry equations written in terms of drainage area. These are directly comparable to the equations given in table 4, although equations 38 to 42 are a generalized set, applicable only generally to the entire state of Illinois.

Table 6. Composite Hydraulic Geometry Equations for the State of Illinois

	Units	Equation number
$\ln Q = -0.844 - 5.22\,F + 1.417\,U$	cfs	(28)
$\ln A = -0.414 - 3.41\,F + 1.19\,U$	sq ft	(29)
$\ln V = -0.430 - 1.81\,F + 0.227\,U$	fps	(30)
$\ln W = 1.018 - 1.85\,F + 0.68\,U$	ft	(31)
ln D = -}.h31 – 1.56 F + 0.51 U	ft	(32)
$A = e^{(0.295 + 0.97\,F)}\,Q^{0.84}$	sq ft	(33)
$V = e^{(-0.295 - 0.97\,F)}\,Q^{0.16}$	fps	(34)
$W = e^{(1.423 + 0.66\,F)}\,Q^{0.48}$	ft	(35)
$D = e^{(-1.128 + 0.31\,F)}\,Q^{0.36}$	ft	(36)
$\ln A_d = -2.05 + 1.44\,U$	sq mi	(37)
$\ln Q = 1.176 - 5.22\,F + 0.984 \ln A_d$	cfs	(38)
$\ln A = 1.279 - 3.41\,F + 0.826 \ln A_d$	sq ft	(39)
$\ln V = -0.103 - 1.81\,F + 0.158 \ln A_d$	fps	(40)
$\ln W = 1.986 - 1.85\,F + 0.472 \ln A_d$	ft	(41)
$\ln D = -0.707 - 1.56\,F + 0.354 \ln A_d$	ft	(42)

For actual use in Illinois it is recommended that the equations from table 4 be used for the appropriate river basin, since for any of the 18 basins, the basin equations are considerably more precise. The statewide equations have value as a means of comparing the general Illinois results with those of other researchers throughout the United States.

Table 7. Values of Hydraulic Geometry Exponents and Coefficients For 18 Illinois Basins, Compared With Other Results

	Exponents				Coefficients (F=0.1)			
	width	depth	velocity	area	width	depth	velocity	area
Basin	*b*	*f*	*m*	*b+f*	*a*	*c*	*k*	*ac*
Rock	0.62	0.26	0.12	0.88	1.92	0.49	1.07	0.93
Galena	0.71	0.22	0.07	0.93	2.10	0.54	0.88	1.13
Fox	0.57	0.29	0.14	0,86	2.33	0.41	1.05	0.96
Des Plaines	6.67	0.24	0.09	0.91	1.71	0.58	1.02	0.98
Kankakee	0.47	0.35	0.18	0.82	4.56	0.42	0.52	1.92
Vermilion (Illinois R. Basin)	0.48	0.35	0.17	0.83	5.38	0.30	0.62	1.61
Mackinaw	0.56	0.35	0.09	0.91	2.65	0.34	1.11	0.90
Henderson Creek	0.30	0.69	0.01	0.99	8.16	0.08	1.51	0.66
Spoon	0.45	0.47	0.08	0.92	3.76	0.19	1.39	0.72
La Moine	0.49	0.39	0.12	0.88	3.24	0.42	0.74	1.36
Sny	0.41	0.35	0.24	0.76	7.70	0.26	0.49	2.02
Sangamon	0.50	0.25	0.25	0.75	3.94	0.79	0.32	3.12
Kaskaskia	0.50	0.37	0.13	0.87	3.17	0.49	0.64	1.55
Vermilion (Wabash R. Basin)	0.33	0.37	0.30	0.70	8.84	0.38	0.30	3.36
Embarras	0.50	0.28	0.22	0.78	3.06	0.86	0.38	2.64
Little Wabash	0.36	0.33	0.31	0.69	9.13	0.69	0.16	6.32
Big Muddy	0.40	0.44	0.16	0.84	8.16	0.32	0.38	2.62
Big Bay Creek	0.38	0.49	0.13	0.87	6.23	0.31	0.52	1.93
Mean	0.48	0.36	0.16	0.84	4.78	0.44	0.73	1.93
Standard deviation	0.11	0.11	0.08	0.08	2.52	0.20	0.38	1.33
White River, Ind.	0.45	0.46	0.09	0.91	6.02	0.13	1.24	0.80
Midwest Rivers (Leopold & Maddock 1953)	0.50	0.40	0.10	0.90				
Theoretical (Leopold & Langbein 1962)	0.55	0.36	0.09	0.91				

Exponents and Coefficients

The general form for hydraulic geometry equations as conceived by Leopold and Maddock (1953) has been presented earlier in equations 1, 2, and 3. These related width, depth, and velocity of the stream to the discharge through the use of the three exponents *b*, *f*, and *m*. The statewide equations 34, 35, and 36 in table 6 are written in the form directly comparable with the Leopold and Maddock equations.

In order that these Illinois results may be compared directly with theoretical values and with values available elsewhere, table 7 has been prepared. Table 7 lists the individual values of hydraulic geometry exponents and coefficients for the 18 Illinois basins, the average values of these Illinois results, and the values from other published results. For the Illinois basins, the mean value of the width exponent is $b = 0.48$, the mean value of the depth exponent is $f = 0.36$, and the mean value of the velocity exponent is $m = 0.16$. These three values were obtained by averaging the 18 individual values of *b*, *f*, and *m*, respectively, in table 7. It is to be noted, however, that these average values check exactly with those determined for the state of Illinois on a generalized basis as shown in equations 34, 35, and 36.

Shown also in table 7 is the exponent for cross-sectional area which is equal to the sum $b + f$. The average of this cross-sectional area exponent is $b + f = 0.84$. This average value also checks exactly with the exponent for cross-sectional area given in equation 33.

One of the values of the 18 separate listings of the exponents *b*, *f*, *m*, and $b + f$ in table 7 is that these numbers show the variability which is inherent in these exponents. As a means of comparing these results with others available, table 7 shows the exponent values that were determined for the White River in Indiana as a subsidiary part of this study described later. Also shown are values published by Leopold and Maddock (1953) for some other midwestern rivers, and the theoretical values of these exponents as given by Leopold and Langbein (1962). As can be seen, the average values for Illinois basins check very closely with those available for the White River and midwestern rivers, as well as with the theoretical values.

The hydraulic geometry equations in their general form as given in equations 1, 2, and 3 also contain the coefficients *a*, *c*, and *k*. The variation of these coefficients is not as meaningful as it is for the exponents just described. However, the 18 sets of individual coefficients derived for the equations in Illinois have been included in table *1*. These coefficients, determined at a frequency of $F = 0.10$, represent the intercept of the fitted equation at a value of discharge $Q = 1.0$ cfs. Because this is a very low discharge, the comparison and variability of these coefficients are believed to be of general interest only.

The relations governed by equations 1, 2, and 3 are meaningful to describe the interrelation of these variables for an entire stream system. These equations can also be used, as described by Leopold and Maddock (1953), to describe the interrelation of these factors at a particular station. For example, the curves in figure 5 are hydraulic rating curves for the specific stream gaging station located on Sugar Creek near Hartsburg in the Sangamon River Basin. Using the form of equations 1, 2, and 3 makes it possible to determine the coefficients and exponents for this station. Because our studies in Illinois have been carried out only for discharges which occur between durations of 10 and 90 percent of the days of the year, only that portion of the curves to the left of the dashed line at 469 cfs in figure 5 were used to fit equations 1, 2, and 3 to the curves.

Table 8 gives the exponents *b*, *f*, and *m* for equations 1, 2, and 3 which were used to fit the hydraulic rating curves for the 18 individual stream gaging stations within the Sangamon River Basin. In table 8 the entry for the stream gaging station on Sugar Creek near Hartsburg, which was illustrated in figure 5, shows that the width exponent has a value of $b = 0.24$, the depth exponent $f = 0.43$, and the velocity exponent $m = 0.33$. These exponents represent the slope of the curves in figure 5. Inspection of table 8 makes it possible to see the variation in these station values of exponents in the

Table 8. Station Values of Exponents in Hydraulic Geometry of the Sangamon River Basin and Midwest

Gaging station	Stream order	Exponents: width *b*	depth	velocity *m*	area *b+f*
Nokomis	2.50	0.30	0.35	0.35	0.65
Easton	3.17	0.17	0.43	0.40	0.60
DeLand	3.95	0.25	0.39	0.36	0.64
Springfield	4.20	0.29	0.39	0.32	0.68
Cornland	4.38	0.18	0.55	0.27	0.73
Waynesvi1le	4.62	0.34	0.38	0.28	0.72
Lincoln	4.95	0.28	0.52	0.20	0.80
Rowe11	5.02	0.32	0.51	0.17	0.83
Taylorvi1le	5.15	0.49	0.36	0.15	0.85
Mahomet	5.17	0.28	0.52	0.20	0.80
Montieello	5.30	0.28	0.53	0.19	0.81
Oakley	5.44	0.36	0.35	0.29	0.71
Hartsburg	5.61	0.24	0.43	0.33	0.67
Kincaid	5.67	0.31	0.47	0.22	0.78
Rochester	5.98	0.32	0.51	0.17	0.83
Riverton	6.00	0.45	0.28	0.27	0.73
Greenview	6.54	0.22	0.54	0.24	0.76
Oakford	7.40	0.13	0.59	0.28	0.72
Mean		0.29	0.45	0.26	0.74
Standard deviiation		0.09	0.09	0.07	0.07
Midwest Rivers (Leopold & Maddock 1953)		0.26	0.40	0.34	0.66

hydraulic geometry equations throughout the basin. Table 8 also shows the mean values of these basin exponents and similar station values derived for other Midwest rivers. It can be seen that the Illinois values are comparable with those of the other rivers of the Midwest.

Areal Variation in Stream Parameters

To provide an overall frame of reference for considering the variation in the various hydraulic geometry stream system parameters for the various parts of the state, it was decided to inspect the variation of these parameters under a prescribed set of standard conditions. For this purpose a drainage area of 100 square miles and a discharge equivalent to that equalled or exceeded 10 percent of the days each year, $F = 0.10$, were selected. A drainage area of 100 square miles is a medium-sized drainage area, large enough that the stream system parameters can be well determined and yet not so large as to suffer from lack of comparability. A discharge equalled or exceeded 10 percent of the days is believed to best represent a *channel-forming* discharge.

Earlier writers have described the concept of a *characteristic discharge* as being that which is most important in carving the stream channel. As described earlier and shown in figure 13, this study has revealed that the discharges which were exceeded at a frequency of 10 percent of the days seemed to be best correlated with discharge and stream order. In the fitting of equations for all of the other parameters it was noticed that invariably the 10 percent frequency curves seemed to fit the mathematical model best. For this reason it is believed that the 10 percent frequency discharge conditions are strongly related to the channel characteristics.

Table 9 and figure 16 present some of the important values of stream parameters in Illinois for standard conditions of a drainage area of 100 square miles and a 10 percent frequency discharge. Figure 16a shows that the mean annual discharge from a 100-square-mile basin increases from 55 cfs in northwestern Illinois to 130 cfs in extreme southern Illinois. The variation in the 10 percent frequency discharge (figure 16b) varies from 100 cfs in northern Illinois to 250 cfs in southern Illinois, displaying the same general pattern as the mean annual discharge. Figure 16 also shows the variations in mean velocity, cross-sectional area, width, and depth. In figure 16 it should be noted that the maps are consistent with each other. That is, for any location in Illinois, discharge is equal to mean velocity times cross-sectional area, $Q = AV$, and cross-sectional area is equal to width times the mean depth, $A = WD$.

For each of the 18 basins studied, table 10 shows the stream order and average channel slope for a basin having a drainage area of 100 square miles. The variation in this channel slope is shown in figure 17.

More complete information on the variation in the hydraulics of the stream was provided by making a computation of the hydraulic roughness, using the

Manning equation:

$$n = \frac{1.486}{V} D^{0.67} S^{0.5}$$

where n = hydraulic roughness coefficient; V = mean velocity, fps; D = mean depth, ft; and S = hydraulic slope as a decimal. Table 10 shows the solutions for the Manning roughness coefficient for each of the 18 basins. The variation in the Manning roughness coefficient is plotted graphically on a map in figure 18. Chow (1959, Chapter 5) provides a general treatment of hydraulic roughness of streams.

Areal Variation in Exponents

The values of the exponents shown in table 7 are essentially the slopes of the plotted graphs of hydraulic geometry factors, such as the slope of the lines shown in figure 13. Written in the form of equations 1, 2, and 3 and equations 33, 34, 35, and 36, the exponents in table 7 represent the rate of change of a particular stream parameter with a change in discharge.

Table 9. Stream Hydraulic Factors For 18 River Basins

(Drainage area = 100 sq mi; discharge of F = 0.1)

Basin	Discharge Q (cfs)	Cross-sectional area A (sq ft)	Mean velocity V (fps)	Width W (ft)	Mean depth D (ft)
Rock	103	54	1.9	34	1.6
Galena	75	66	1.2	45	1.4
Fox	103	51	2.0	32	1.6
Des Plaines	225	138	1.6	64	2.2
Kankakee	204	155	1.3	56	2.7
Vermilion (Illinois R. Basin)	148	104	1.4	60	1.7
Mackinaw	190	106	1.8	50	2.1
Henderson Creek	154	99	1.6	38	2.6
Spoon	145	70	2.1	36	2.0
La Moine	111	86	1.3	32	2.6
Sny	105	70	1.5	52	1.3
Sangamon	134	122	1.1	45	2.7
Kaskaskia	156	122	1.3	38	3.2
Vermilion (Wabash R. Basin)	170	124	1.4	62	2.0
Embarras	316	230	1.4	53	4.4
Little Wabash	254	287	0.9	66	4.4
Big Muddy	228	243	0.9	73	3.4
Big Bay Creek	250	232	1.1	51	4.6

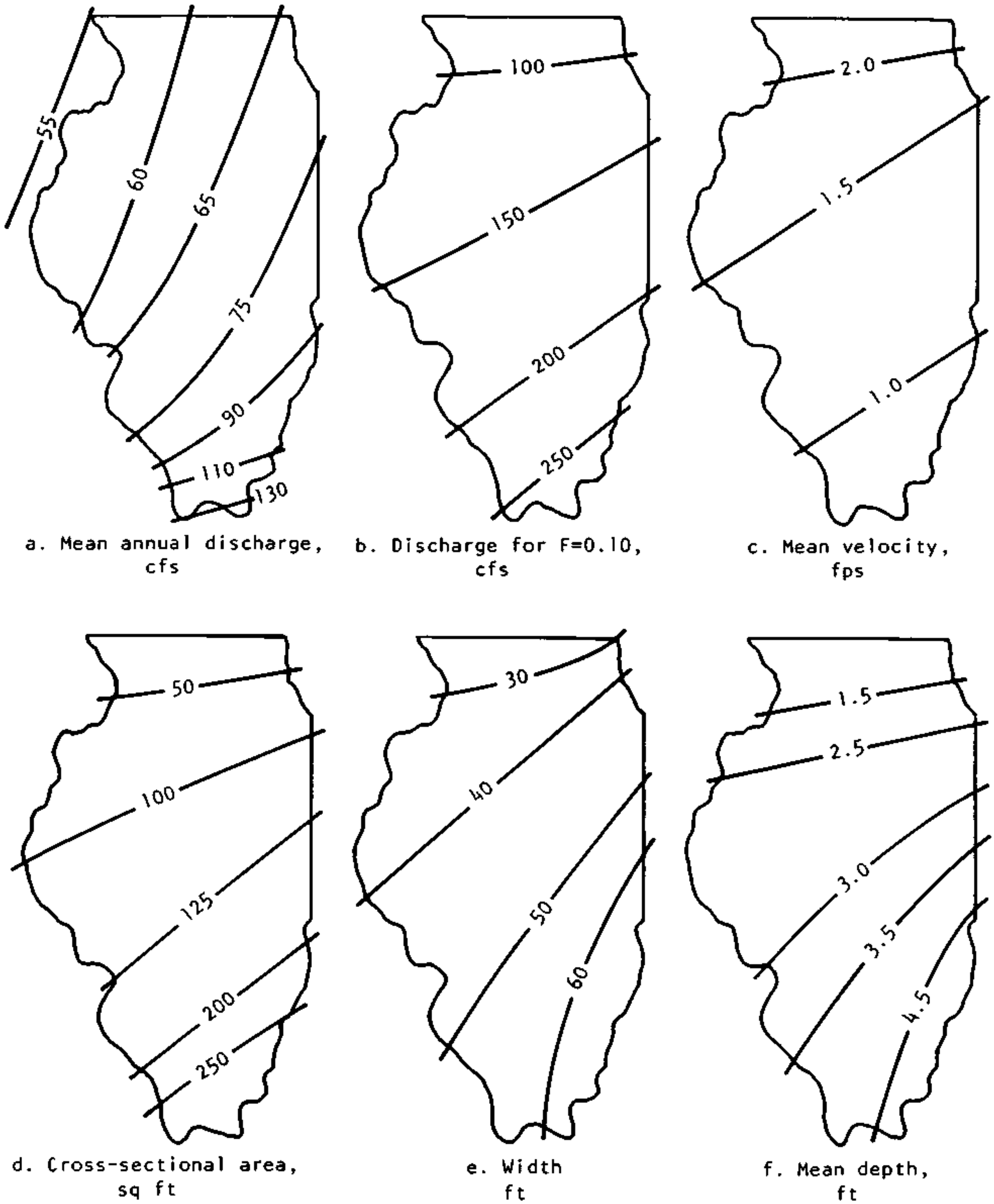


Figure 16. Areal variation in stream parameters for a drainage area of 100 square miles, at a discharge equalled or exceeded 10 percent of the days

The variation of these exponents throughout Illinois was explored by plotting the exponents in table 7 for *b*, *f*, *b* + *f*, and *m*, as shown on the maps in figure 19. It is to be noted that the map for cross-sectional area exponent *b* + *f* can be obtained at any point by adding the width exponent *b* and the depth exponent *f* as read from figures 19a and b. Thus these maps are internally consistent. The areal variations in these exponents are deemed to be reasonable; no general conclusion is signified.

Field Check of Velocity Results

One objective of this research project was to search for and to evaluate quantitatively the consistent pattern in which the hydraulic factors of streams develop. The approach to quantifying this hydraulic geometry system has been almost completely theoretical. The results of the study are exemplified by the five hydraulic geometry equations presented as equations 17 to 21 of this report. Because of the theoretical nature of this entire study, it was felt desirable to search for some way to provide an actual field check of the results of a particular set of hydraulic geometry equations. After looking over these equations, it was realized that equation 19, which relates

Table 10. Stream Slope and Hydraulic Roughness for 18 River Basins

(Drainage area = 100 sq mi; discharge of *F* = 0.1)

Basin	Stream order *U* for 100 sq mi	Stream slope (*ft/mi*)	Stream slope (*ft/ft*)	Manning's roughness coefficient, n
Rock	4.55	4.50	.00085	.031
Galena	5.16	6.40	.00121	.056
Fox	4.03	5.90	.00102	.032
Des Plaines	4.88	1.35	.00026	.024
Kankakee	4.84	2.35	.00045	.047
Vermilion (Illinois R. Basin)	3.86	5.10	.00096	.046
Mackinaw	4.48	5.00	.00095	.044
Henderson Creek	4.77	4.00	.00076	.051
Spoon	4.53	5.20	.00098	.035
La Moine	4.08	6.10	.00115	.074
Sny	5.27	1.24	.00023	.018
Sangamon	4.08	4.55	.00086	.077
Kaskaskia	4.96	2.75	.00052	.056
Vermilion (Wabash R. Basin)	4.24	5.80	.00110	.058
Embarras	3.92	7.20	.00136	.098
Little Wabash	4.25	3.50	.00066	.115
Big Muddy	4.97	2.60	.00049	.078
Big Bay Creek	4.88	1.95	.00037	.072

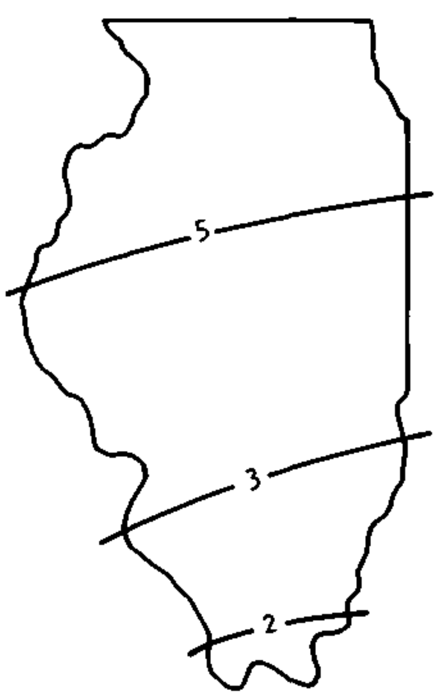


Figure 17. Variation in channel slope, in feet per mile, for a drainage area of 100 square miles

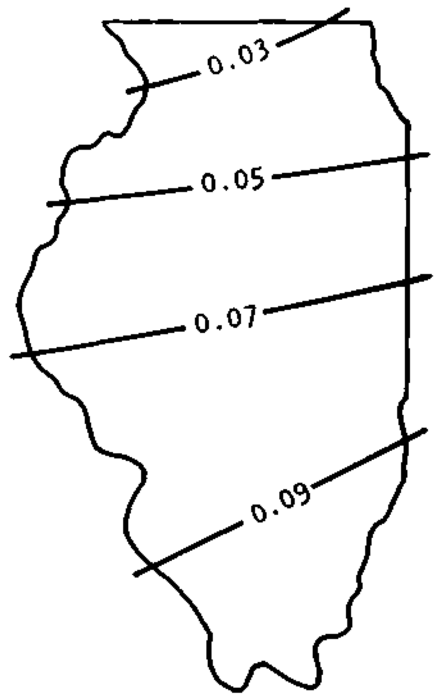
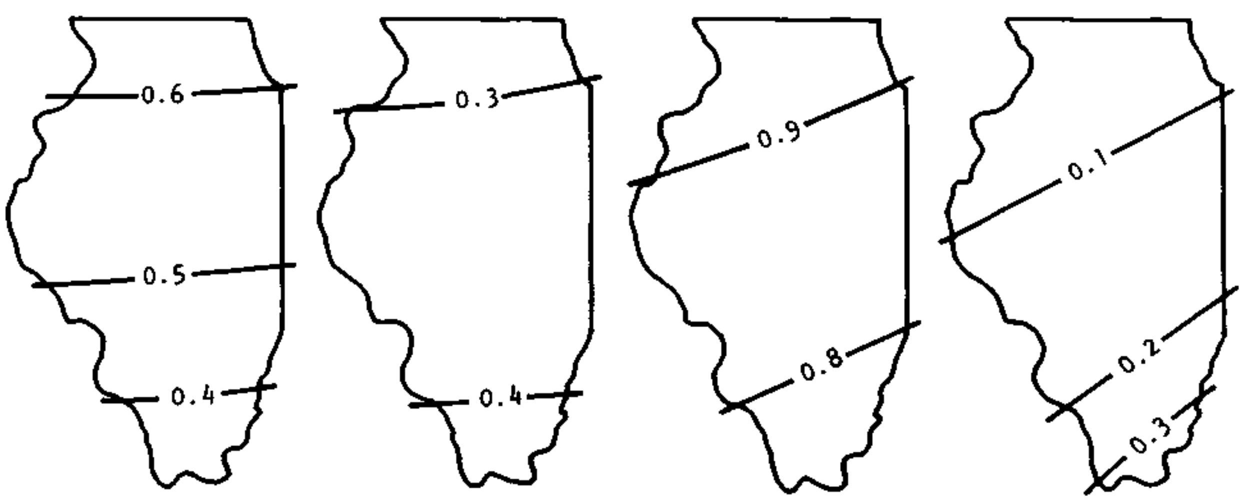


Figure 18. Variation in Manning roughness coefficient n at a discharge equalled or exceeded 10 percent of days, for a drainage area of 100 square miles



a. Width exponent, b

b. Depth exponent, f

c. Cross-sectional area exponent, b+f

d. Velocity exponent, m

Figure 19. Areal variation in hydraulic geometry exponents

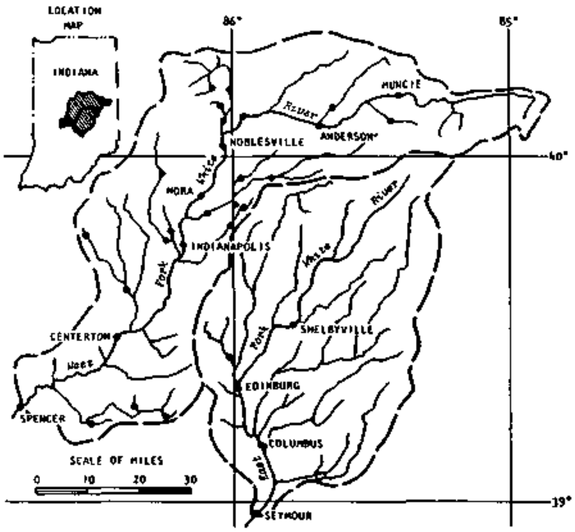


Figure 20. White River Basin in Indiana on which time-of-travel measurements were available from dye-tracer studies (Dots show stream gaging stations)

average stream velocity to the frequency of discharge and the drainage area, could be subjected to a field test. This equation was checked by using field measurements of time-of-travel to provide an actual value for the average stream velocity through a reach of stream.

The time-of-travel of contaminants in streams is a matter which is being investigated in the United States. Fluorescent dyes *are* now used extensively to measure travel time in streams and to solve particular water resource problems (Buchanan, 1964). The use of dye tracers for the field measurements of time-of-travel in a stream is now considered as an accepted practice as described by Wilson (1968).

Considerable data on travel times in streams were available on the White River Basin in Indiana, shown in figure 20. For the 72-mile reach of the West Fork of the White River between Muncie and Spencer (figure 20) engineers of the U. S. Geological Survey had made a number of determinations of travel times by using fluorescent dye tracers during the period 1965-1967. These travel-time measurements had been made at both high flows and low flows for the entire reach of the West Fork. In the basin of the West Fork upstream from Spencer, stream gaging information was available at 28 locations as shown in figure 20. These data were obtained and utilized as a part of this project to determine a set of hydraulic geometry equations for this basin, carried out in the same manner as the work on the 18 basins in Illinois. A set of hydraulic geometry equations was developed, applicable to the West Fork of the White River Basin in Indiana, as follows:

$$\ln Q = 1.34 - 4.62 F + 1.02 \ln A_d \quad (43)$$

$$\ln A = 0.89 - 2.97 F + 0.93 \ln A_d \quad (44)$$

$$\ln V = 0.45 - 1.65 F + 0.09 \ln A_d \quad (45)$$

$$\ln W = 2.35 - 1.64 F + 0.46 \ln A_d \quad (46)$$

$$\ln D = -1.46 - 1.33 F + 0.47 \ln A_d \quad (47)$$

Equation 45 can be used at a given flow frequency and drainage area, that is, at a specific point of a stream, to obtain the average stream velocity. The average velocity for a stream segment can be calculated by using the computed velocities at the upstream and downstream ends of the segment. This type of calculation was made for each of a number of reaches on the main stem of the West Fork of the White River. In each case the computed velocities were converted to time-of-travel and compared with actual measured time-of-travel determined by dye tracers. The results are shown in figure 21.

At high flows the measured and computed travel times are very close (figure 21). At low flows the computed travel times are considerably lower than flows actually measured. This is partly due to channel dams on the river which store and retard water movement, a man-made condition. The one large break in the low-flow measured travel time in figure 21 is for the reach between Nora and Indianapolis, where two channel dams are known to be present and to retard travel times at low flows.

The results shown in figure 21 are considered to provide evidence of the validity and utility of the results of hydraulic geometry computations. They illustrate one way in which hydraulic geometry results can be used to extend, expand, or generalize measurements of travel time.

An additional test of the validity of computed stream velocities was made. As a part of the same observational program in Indiana, the USGS had made a considerable number of measurements of velocities, by time-of-travel studies, on the East Fork of the White River, shown in figure 20. Because of the nearness and general similarity of the East Fork and West Fork Basins, the velocity equation derived for the West Fork, equation 45, was used to compute what the velocities might be in the East Fork. The comparison of measured and computed travel times for the East Fork of the White River is shown in figure 22. The two curves are reasonably close.

This single check seems to confirm the idea that hydraulic geometry considerations may be able to provide an estimate of travel time in any stream reach without any measurements of dye tracers if hydraulic geometry equations are available from a reasonably similar hydrologic region. Such estimates are considered to be highly desirable results because they can contribute considerable understanding of a stream system even in the absence of actual measurements.

Reaeration Capacity of a Stream

The relations and equations developed in this report make it possible to compute a reasonable value for such stream parameters as cross-sectional area, velocity, depth, and width for a particular known discharge or frequency of discharge and for a particular drainage area within a stream system. Sanitary engineers are often faced with the problem of computing the capacity of a stream to assimilate wastes at a particular point for which the physical characteristics of the stream are unknown. Since hydraulic geometry relations provide estimates of stream depth and velocity at any point within a stream system, they allow an exploration of the variation in the assimilative capacity of a stream.

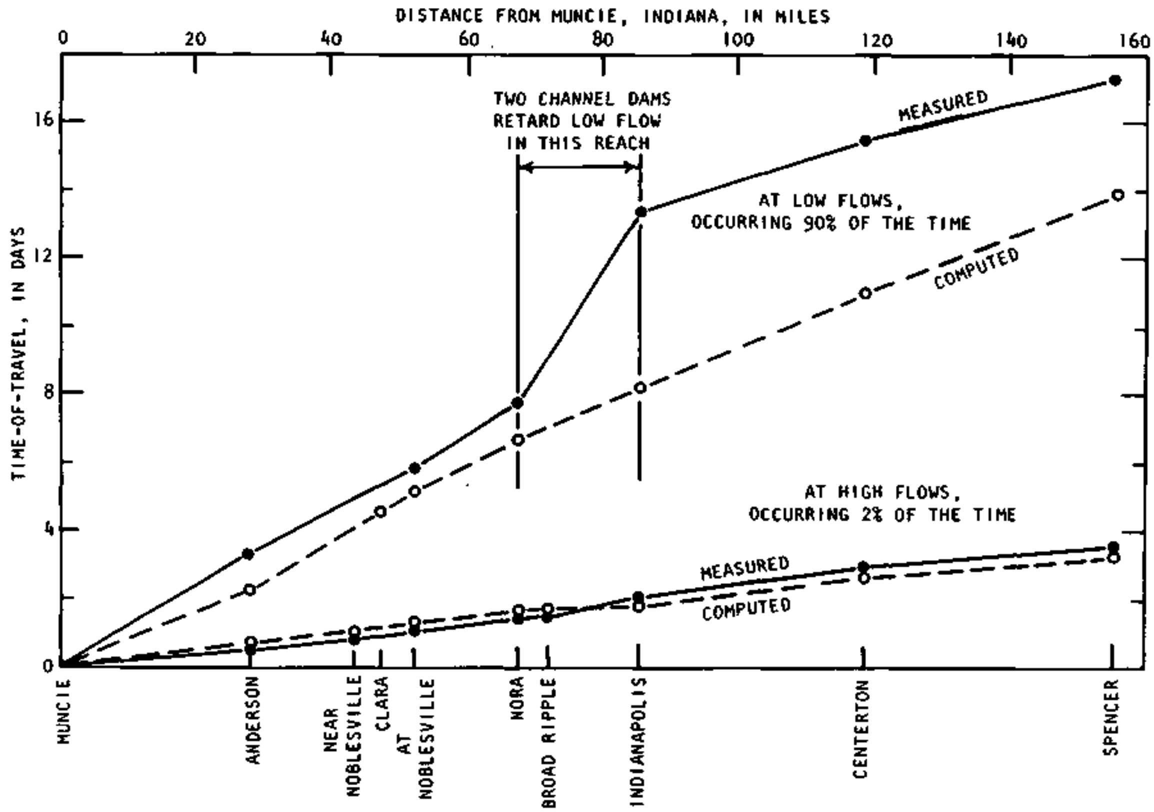


Figure 21. Measured and computed time-of-travel from Muncie to Spencer, Indiana, in the West Fork of the White River

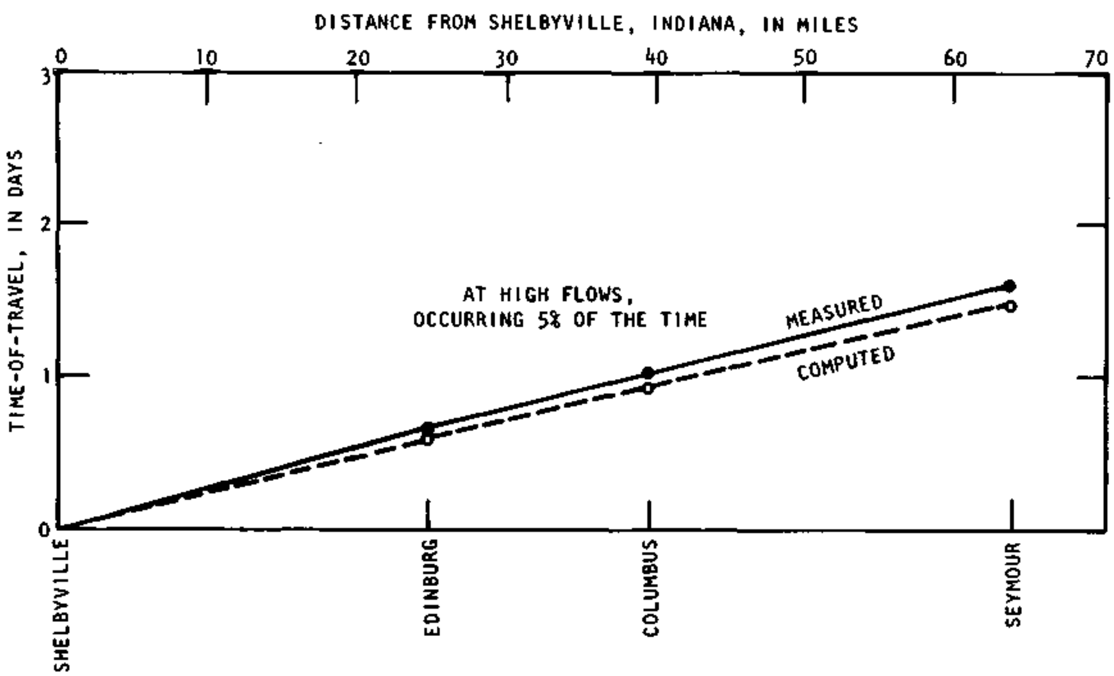


Figure 22. Measured and computed time-of-travel from Shelbyville to Seymour, Indiana, in the East Fork of the White River

As a stream flows, oxygen is transferred from the air into the stream. The rate at which the stream absorbs oxygen as given by Langbein and Durum (1967) is

$$\frac{dc}{dt} = k_2 (c_s - c) \tag{48}$$

where

$\frac{dc}{dt}$ = the rate a stream absorbs oxygen from the air
c = concentration of oxygen in the water in milligrams per liter
c_s = concentration when saturated with oxygen at the prevailing temperature
t = time in days
k_2 = reaeration coefficient per day

Exhaustive studies carried out on the Ohio River by Streeter and Phelps (1925) developed some understanding as to the means by which oxygen is absorbed by water. This study provided a deoxygenation coefficient k_1 and a reoxygenation coefficient k_2 to govern this process. Although the process is complicated by many local factors and by temperature, the hydraulic character of the stream has a primary effect upon the reaeration coefficient k_2 Dobbins (1964) compared several important equations for k_2 and showed the most effective one to be that published by O'Connor and Dobbins (1958) as

$$k_2 = \frac{(D_m V)^{0.5}}{2.30\ D^{1.5}} \tag{49}$$

where

k_2 = reaeration coefficient of stream in milligrams per liter of water per day (to base 10)
D_m = molecular diffusivity of oxygen in water in square feet per day
V = mean velocity of stream in feet per day
D = mean depth of stream in feet

As can be seen from equation 49, k_2 is dependent upon the velocity to the 0.5 power and inversely proportional to the mean depth of the stream to the 1.5 power. Dobbins (1964) in equation 44 of his paper provides an expression for the computation of D_m, the molecular diffusivity. By means of this equation it can be shown that at 20C, D_m = 190 x 10"^{5}. By introducing this into equation 49 it is possible to provide

$$k_2 = 5.55 \frac{V^{0.5}}{D^{1.5}} \tag{50}$$

where V is the mean velocity in feet per second. The structure of this equation clearly shows the dominant importance of knowledge of the average depth and velocity of flow in a stream at a particular location in determining the

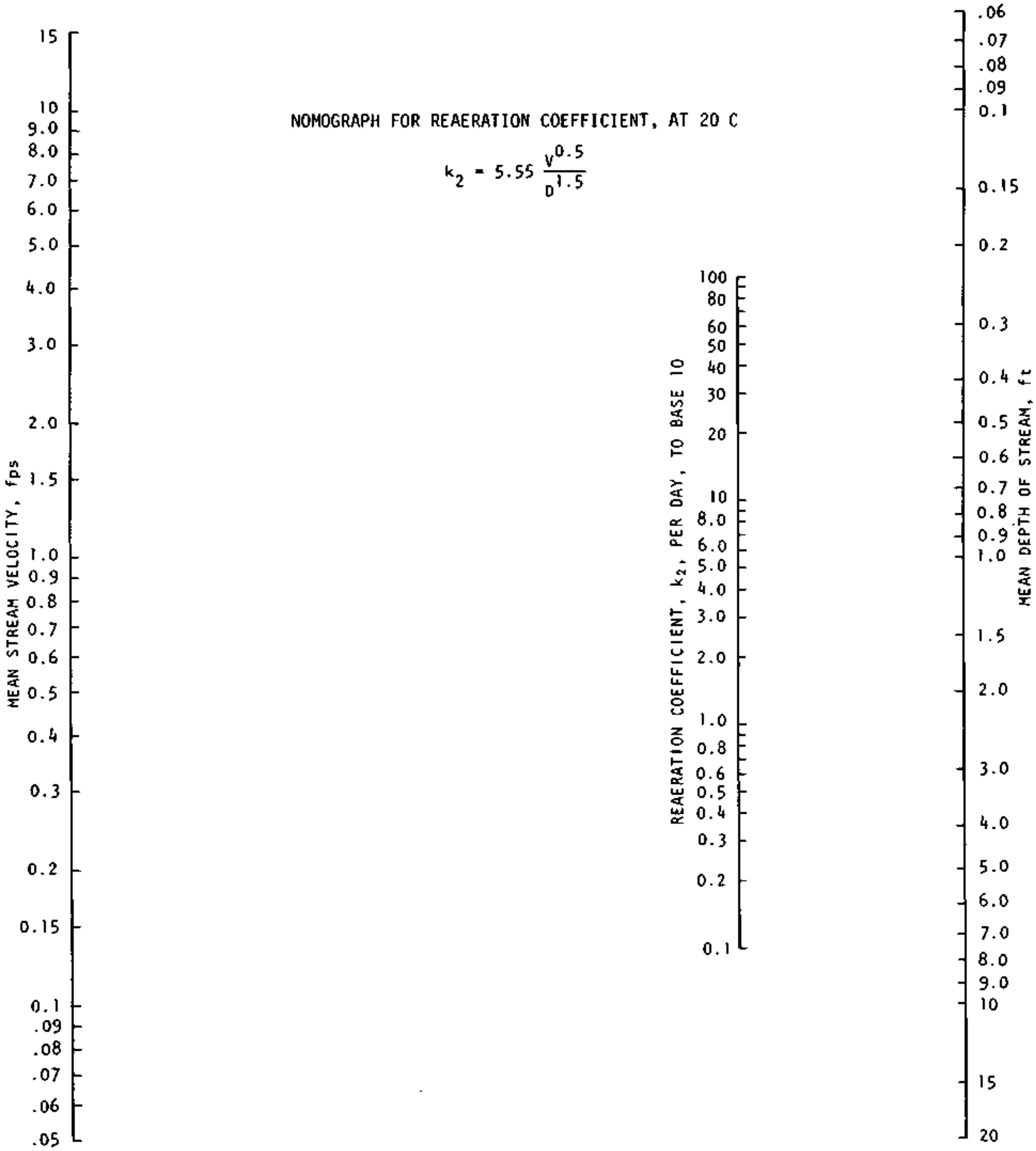


Figure 23. Nomograph for computing reaeration coefficient k_2 *at 20 C for a given mean stream velocity and mean depth*

capacity of the stream to reaerate itself, and thus the capacity of the stream to assimilate waste at this location. Figure 23 provides a nomograph which allows the solution of equation 50 for k_2 when mean stream depth and mean stream velocity *are* known.

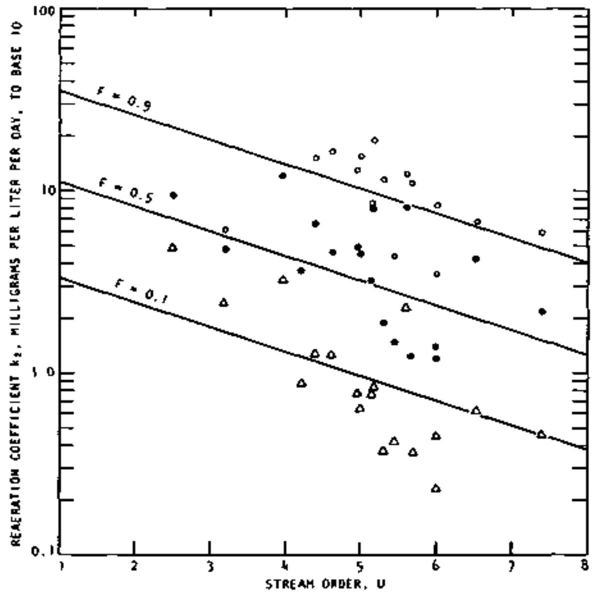


Figure 24. Variation of reaeration coefficient with, stream order U *and flow frequency* F *for the Sangamon River Basin*

For a given frequency of discharge, equation 28 (table 6) shows that, as a river flows downstream and the stream order increases, the discharge increases. Equation *34* shows that as the discharge increases in a downstream direction, the average velocity increases by the 0.16 power. Equation 36 similarly shows that as the discharge increases in the downstream direction the mean depth of stream increases as the 0.36 power. Because the depth of the stream has more importance in equation 50 than the velocity, it can be seen that, as the stream flows downstream the reaeration coefficient will decrease. This is generally due to the fact that the stream is deeper and particular elements of the water reach the surface less often. Figure *2k* shows the tendency of the reaeration coefficient k_2 to decrease in a downstream direction, that is, as the stream order increases. The lines in figure *2k* represent the values computed from equation 50 using velocity as computed from equation 34 and depth as computed from equation 36. Three curves are shown for frequencies *F* of 0.9, 0.5,and 0.1. The points plotted in figure *2k* are the individual values of k_2 from equation 50 computed at the actual 18 gaging stations within the Sangamon River Basin. These give an idea as to the variability of k_2.

To determine the total amount of oxygen which the stream can assimilate requires knowing the total amount of water in the stream, as well as the k_2 value. This computation can be accomplished by

$$A_c = [A\ L\ 5280]\ 2.3\ [k_2\ (3.12)\ 10^{-8}] \qquad (51)$$

where

A_c = assimilative capacity of a stream reach for each ppm of oxygen deficiency in tons per day
A = cross-sectional area of stream in square feet
L = length of stream reach in miles
k_2 = reaeration coefficient of stream in milligrams per liter of water per day (to base 10)

The variation in the total load in tons per day which can be assimilated by characteristic, or typical, streams within the various stream orders was

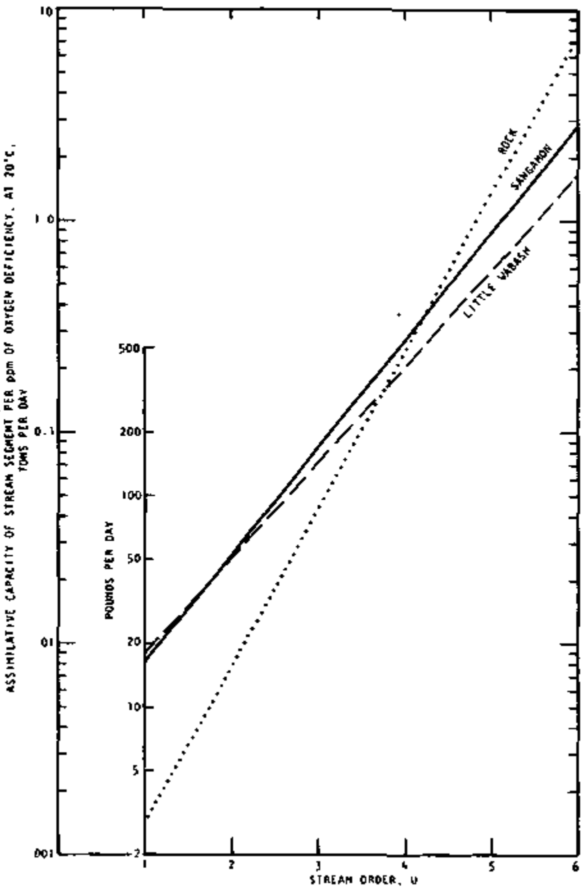


Figure 25. Relative assimilative capacity of stream segments of various stream order for three basins (discharge of F *= 0.90)*

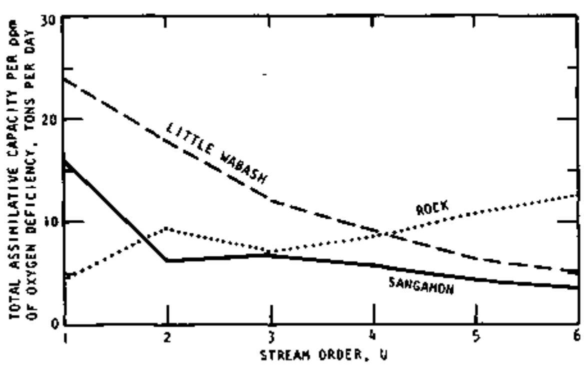


Figure 26. Total assimilative capacity of all streams of various stream order for three basins (discharge of F *= 0.50)*

explored by solving the hydraulic geometry equations for three basins in Illinois to provide the velocity, depth, cross-sectional area, and length as needed to solve equations 50 and 51. The results are given in figure 25 for the Rock River Basin in northern Illinois, the Sangamon Basin in central Illinois, and the Little Wabash Basin in southern Illinois. It can be seen that as the stream order increases, the total assimilative capacity of a typical stream segment increases greatly.

Horton's studies showed that the smaller stream orders were much more numerous than the larger stream orders. This tends to counteract somewhat the increasing assimilative capacity of the streams of larger orders shown in figure 25. For the same three river basins in Illinois, the assimilative capacity for each typical stream order was multiplied by the number of stream segments of that order as given in table 5. These results were then adjusted to represent a total drainage area of 3000 square miles, this size being selected for comparative purposes only. For such a selected 3000-square-mile basin within the Little Wabash, Rock, or Sangamon Basins, the total assimilative capacity of the stream in tons per day for various stream orders are shown in figure 26. For the Little Wabash and Sangamon Basins, the total assimilative capacity decreases with larger stream order, but for the Rock River the total assimilative capacity increases with stream order. This increase or decrease is believed to depend on the stream system itself.

The use of hydraulic geometry relations can aid considerably in the provision of a general estimate of the assimilative capacity of a stream at any particular point in Illinois. Such a general answer would be subject to a great number of local variations, but does provide an overall concept as to the resource available in terms of the total amount of waste that particular stream systems can accommodate at a particular point.

CONCLUSION

The study described here has resulted in the following conclusions.

1) Streams in the state of Illinois do tend to adjust their channels to a consistent pattern which has been evaluated by the concepts of hydraulic geometry.
2) Equations 17 to 21 and the sets of equations in table 4 adequately express the general hydraulic geometry relationships for major river basins in Illinois, and similar relationships for other basins can be inferred.
3) The numerical values of hydraulic geometry exponents for Illinois stream systems check nicely with published theoretical values, and with other published values for midwestern rivers.
4) For standard conditions which allow a valid comparison, the physical and hydraulic characteristics of Illinois stream systems vary throughout the state in a reasonable fashion, being generally consistent with physiographic differences.
5) Subject to local variations within a stream system, hydraulic geometry equations allow a general estimate to be made of channel characteristics at any location within the system.
6) Flow velocities for a reach of stream, computed from hydraulic geometry equations, have been proven to be valid *at high flows*, by measured time-of-travel using dye tracers; at low flows differences were considerable, partly due to man-made channel dams which were unaccounted for in hydraulic geometry computations.
7) The proportional stream ordering system introduced in this study is shown to be reasonable and valuable in extending the applicability of the Horton-Strahler stream morphology system. The use of the frequency of occurrence for time comparison of flows, and the proportional stream order for place comparison of flows within a stream system serves as a bridge to link the excellent works on stream hydraulic geometry done by Leopold and Maddock (1953) and stream morphology done by Horton (1945) and Strahler (1957).

The utility of hydraulic geometry relations has been illustrated by showing how these relations provide mean stream depth and velocity which allow estimates to be made of the reaeration capacity of the stream and the total capacity of the stream to assimilate wastes. Such estimates are derived from the generalized pattern to which the stream adjusts. Localized stream conditions could vary greatly from the pattern. However, in many additional areas of water resource planning it can be extremely valuable to have a general estimate of channel characteristics.

All results of this study have been derived from data obtained at 166 stream gaging stations of the U. S. Geological Survey within the state of

Illinois. The development of applicable sets of hydraulic geometry equations for the state is considered a major extension of these data. In 1967 the U. S. Geological Survey operated more than 8000 continuous-record stream gages on the United States mainland, as reported by Bue (1967). The majority of these stations have records longer than 15 years; some have records of 50 years. A highly meaningful expansion of the existing measurements at these gaging stations could be made using the hydraulic geometry concept. It is recommended that research personnel throughout the country pursue hydraulic geometry as a means of extending the knowledge available on stream-channel characteristics.

The book *Water Facts for the Nation's Future,* by Langbein and Hoyt (1959), is a systematic appraisal of the hydrologic data available in the United States. In addressing the topic of *better data for better decisions* in water resources development, these authors conclude that data programs have emphasized data collection to the neglect of advancing knowledge of basic principles. The authors of the present report propose that hydraulic geometry provides a set of consistent relations between stream parameters, through the use of which a wide and useful expansion of stream measurements has been made.

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