Graph-Theoretic Extension of the Matrix Model of an R&D Organization

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Abstract—In this paper the matrix model of an R&D organization developed by Dean [1] has been extended to organizations with mixed global objectives based on a graph-theoretic formulation. The extended model can be applied to problems like the maximization of R&D outputs of a number of organizations at the level of either a corporation or a country such that a specified growth in the overall research competence of the entire corporation or the country is maintained.

I. Introduction

A MATRIX model of an R&D organization has been developed by Dean [1]. This model, though developed initially for army operational requirements, has certain features that could be extended so as to be applicable to a wider class of organizational requirements. In this paper, the matrix model is extended and applied to a specific organizational structure represented by a corporation with a large number of laboratories funded by it or by a central funding agency like the Council (or Department) of Scientific and Industrial Research existing in several Commonwealth countries like Australia, India, and Pakistan.

Organizations like the above are governed by mixed global objectives combining short-range objectives, such as maximizing the macroeconomic parameters like the growth of capital of the corporation or the gross national product (GNP) of the country, and long-range objectives, such as increasing the R&D competence of the corporation or the country to which no interim value can be attached.

The extension of the matrix model developed in the following sections makes use of graph theory and provides a framework within which problems involving mixed global objectives can be treated.

II. Organizational Framework

The type of organization under study can be described in terms of the objectives and constraints imposed by a democratic system. The realization of these objectives subject to the constraints presupposes the definition of certain indices and the methods for evaluating them under conditions of uncertainty.

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A. Principal Objectives

The funding agency under consideration has the following short-term objectives.

1) The funding should have maximum impact on certain macroeconomic variables, e.g., GNP.
2) The laboratory to which the contract is awarded should be that in which adequate technological and scientific infrastructure for handling the contract is available.
3) The allocation of projects to the laboratories is made such that a defined cost function is optimized.

In addition, there exist the following long-range objectives.

1) The R&D competence of the corporation, or the country as a whole, should be increased as much as possible over the long term.
2) Depending upon the planned growth rate of different disciplines, as compared to their present rate, the funding is biased so as to meet planned targets.
3) In case the overall research competence of the recipient laboratory can be increased by strengthening one or more disciplines pursued by it, the funding should be biased to make this possible.

B. Principal Constraints

1) The contract for each project is awarded to only one laboratory. The selected laboratory can award subcontracts to other laboratories independently of the central funding agency.

2) One of the criteria of allocation is specified to be the past performance of the laboratories with respect to reliability of time schedules, resource utilization, strategies, and achievements.

3) The laboratories are autonomous. That is, they can specify their preferred list of projects from which selections may be made by the funding agency. Further, they can reject the award in case the resource allocated falls below their minimum expectation.

4) The laboratories are obligated to supply accurate information concerning the availability of talents, existing scientific and technological apparatus, and scientific and technological know-how.

5) The funding agency can not control the manner in

which funds are to be utilized once the contract is awarded.

6) The funding agency is required to supply complete details concerning its criteria for evaluating the past performance of the laboratories, the computer software package simulating the relevant mathematical model for allocation, and the results of computer runs to the laboratories for their scrutiny.

7) Each laboratory may have an upper bound on the number of preferred projects awarded to it by the central funding agency.

C. Indices Governing Allocation

For a quantitative method of realizing the stated objectives subject to the constraints outlined, a set of indices is defined below. General procedures for evaluating such indices will be discussed in Section II-D. We consider a typical allocation problem with a central funding agency, m laboratories bidding for n main projects, and p main disciplines whose growth have been emphasized by the funding agency. Further, q component projects are defined as those distinct projects that can be handled by distinct teams. They are realized by a breakdown of the main project, for example, a systems engineering analysis.

Past-Performance Index: The past-performance index enables the central funding agency to evaluate the capabilities of the laboratories on the basis of their past achievements and present capabilities. These indices, for the m laboratories, can be represented by the matrix $[\alpha i_1 i_2]_{1,m}$. The assumption of a single funding agency requires i_1 to be unity.

Preference Index: The preference index is a binary number taking a value 1 if a given laboratory prefers a given main project and the funding agency is willing to award the contract on this project and 0 otherwise. For m laboratories and n main projects these indices can be represented by the matrix $[\beta i_2\, i_3]_{m,n}$.

Correlation Index: The correlation index between a main project and a main discipline is the amount of increase in competence of the main discipline realized by the execution of the main project. For n main projects and p main disciplines, these indices can be represented by the matrix $[\gamma i_3\, i_4]_{n,p}$.

Discipline-Biasing Index: The discipline-biasing index is introduced for realizing the long-range objectives described in Section II-A. Let g_i and g_i' be, respectively, the actual and planned level of competence of the main discipline D_i. Let $b_i = (g_i' - g_i)/g_i'$. Then, b_i gives the bias that has to be provided to the correlation indices for coordinating the planned growths of various disciplines.

In due consideration of these requirements, $[\gamma' i_3\, i_4]_{n,p}$ is obtained by multiplying each column of $[\gamma i_3\, i_4]_{n,p}$ by the discipline-biasing indices $b_1, b_2, \cdots, b_n$, defined above.

Competence Index: The competence index measures the extent to which the financing of the component projects increases the competence of the country (or the corporation) in a main discipline. For p main disciplines and q component projects, these indices can be represented by the matrix $[\delta i_4\, i_5]_{p,q}$.

Availability Ratio: The availability ratio gives a measure of the extent to which a given laboratory meets the component project requirements a priori. For m laboratories and q component projects, these indices can be represented by the matrix $[\mu i_2\, i_5]_{m,q}$.

Requirement Ratio: The requirement ratio is the ratio between the investment required for the component project over and above the investment in facilities already made and the total investment in the component projects constituting the main project. This gives the relative importance of the component projects of a main project. A laboratory having more investment in a component project that is strongly correlated to a main project has a higher chance of wining a contract. For n main projects and q component projects, these indices can be represented by the matrix $[\epsilon i_3\, i_5]_{n,q}$.

D. Evaluation of Indices

Though the methods of evaluating the above indices are not the main objective of this paper, a brief discussion about them will be given here for the sake of completeness. Several authors [4]–[8] have already investigated useful methods of evaluation. Dean and Nishry [4] as well as Moore and Baker [5] have considered scoring models that can be applied either directly or with minor modifications in the evaluation of indices like the requirement ratio. The evaluation of correlation index, discipline-biasing index, and the competence index can be made by techniques like those developed by Harrold [6] and Souder [7]. During evaluation, uncertainty can be included by utilizing the suggestions made by Reichner [8], Eyring [9], and Dean and Nishry [4].

In addition to the concepts of measurement developed by the above authors, the evaluation of the past-performance index requires the following feature.

Past performance depends, for example, on the past reputation of a contractor in meeting the performance specifications of earlier projects as well as the time targets specified for them. Either of these can be quantified if the funding agency or other agencies accessible to it had a sufficient number of interactions with the contractor. For time target this evaluation will be, for the simplest case, as described below.

Let:

A_{ij} = specified time limit for the jth project allocated to the ith laboratory in the past;

B_{ij} = elapsed time for the completion of the jth project by the ith laboratory;

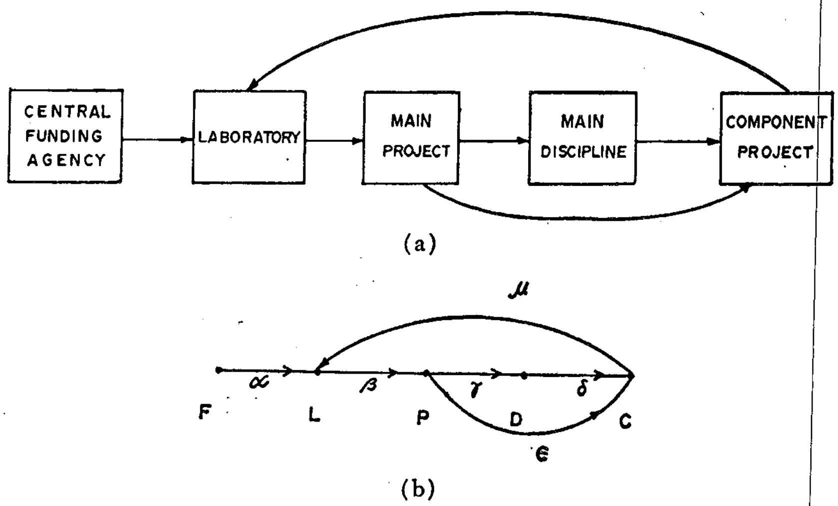

Fig. 1. (a) Block diagram. (b) Signal flow graph.

C_{ij} = criticality factor assigned to project j allocated to laboratory i by the funding agency. This is decided on the basis of the importance and urgency of the project.

Then,

$$a_i = \sum_j C_{ij} \left(\frac{A_{ij} - B_{ij}}{A_{ij}} \right)$$

gives a performance measure for the time criterion. Similarly, the performance-target index b_i can be evaluated. The sum $\alpha i = a_i + b_i$ gives the past-performance index for the ith laboratory.

III. Graph-Theoretic Evaluation of System Index

The allocation problem proposed in the foregoing section will be analyzed in this section from a graph-theoretic angle. Most of the established notions of graph theory, in general, and signal flowgraphs, in particular, in standard treatises [2], [3] are assumed here without elaboration. Fig. 1(a) gives the block diagram of the system under study and Fig. 1(b) gives the corresponding signal flow graph.

The network in Fig. 1(b) is a rooted, finite, directed, connected, labelled graph, the root being the starting vertex representing the funding agency. With each vertex (or node) is associated a vertex (or node) label indicating whether the vertex is the funding agency or a laboratory or a main project. With each branch is associated a branch label or function label representing an incidence relation for the vertices, e.g., a past-performance index, binary-preference index, correlation index, or competence index. Further, the network of Fig. 1(b) is a signal flow graph G (comprised of cascade and parallel paths) between the source node F and the sink node C.

In Section III-A it will be shown that the system index for the allocation problem is equal to the graph transmittance G of the signal flow graph, defined as the ratio of the signal at the dependent node C to the unit signal applied at the source node F. In Section III-B derivation of the graph transmittance G will be given.

A. System Index as a Transmittance

A linear system index standing for the entire R&D system represented by the graph should satisfy the following conditions.

1) It should, in general, be a function of all the edge weights (index variables) of the graph.

2) Two serial evaluations in the R&D organization complement each other. This requires that the system index for a degenerate system made up of two edges in series should increase proportionately if any one of the edge weights is multiplied by a constant. That is, the system index should be of the form $F = k i_1 i_2$ where k is a constant. This is illustrated in Fig. 2(a).

3) Two parallel evaluations in the R&D organization should supplement each other. This requires that the system index for a degenerate system made up of two edges in parallel should be additive. That is, the system index should be of the form $F = k_1 i_1' + k_2 i_2'$ where k_1 and k_2 are constants. This is illustrated in Fig. 2(b).

The validity of the above conditions is established by the following arguments.

The first condition is obvious. For the second and third conditions we note that, in general, an index could be a vector (or a matrix) of $m \times n$ dimensions. The dimensions of the matrix $[i_1]$ are related to the number of nodes of a particular category, e.g., the number of laboratories, the number of projects represented collectively by the nodes A, B, or C. If A is a set of m nodes and B of n, then the matrix $[i_1]$ is of dimension $m \times n$. Similarly if C has p nodes, the matrix $[i_2]$ is of dimension $n \times p$. The system matrix $[F]$ between A and C should have a dimension of $m \times p$. This is possible only by a multiplication of matrices that commute, so that $[F] = k\,[i_1] \times [i_2]$. On the other hand, for parallel edges, the dimension of the system index should be identical to each of the

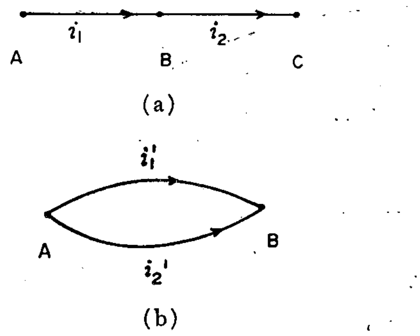

Fig. 2. (a) Degenerate serial system. (b) Degenerate parallel system.

edge matrices. This is possible only if the system index is of the form

$$F = k_1\,[i_1] + k_2\,[i_2].$$

The above validation for the matrices is also true for the degenerate case of a scalar edge weight.

All the above conditions governing the system index are satisfied by the transmittance of the graph.

B. Derivation of the Graph Transmittance G

The transmittance of any branch is the label for this particular branch as seen in the network of Fig. 1(b). G can be deduced by

1) analyzing the various cascade and parallel paths,
2) reducing and transforming the network by contraction (or node absorption) and substitution processes,
3) computing the graph transmittance in terms of relevant branch and path transmittances.

Such an analysis is given in the following steps. Let the scalar $\tau(P,Q)$, in general, denote the transmittance of the path with P as source node and Q as sink node. Then, referring to Fig. 1(b),

$$\tau(F,C) = \tau(F,L)\,\tau(L,C).$$

Further $\tau(L,C)$ can be expanded by contraction and substitution in terms of $\tau(L,P)$, $\tau(P,C)$, and $\tau(C,L)$. Thus,

$$\tau(F,C) = \tau(F,L)\,[\tau(L,P)\,\tau(P,C) + \tau(C,L)].$$

Similarly, $\tau(P,C)$ can be further expanded using contraction and substitution to give,

$$\tau(F,C) = \tau(F,L)\,[\tau(L,P)\,\{\tau(P,D)\,\tau(D,C) + \tau(P,C)\} + \tau(C,L)]. \quad (1)$$

Equation (1) can be written in general as

$$G = \alpha[\beta\,\{\gamma\delta + \epsilon\} + \mu], \quad (2)$$

where, α, β, γ, δ, ϵ, μ can be considered as scalars, vectors, or matrices. Typically, for the system under study we consider a single node of type F, m nodes of type L, n nodes of type P, p nodes of type D, and q nodes of type C. This results in an expanded signal flow graph, partly depicted in Fig. 3.

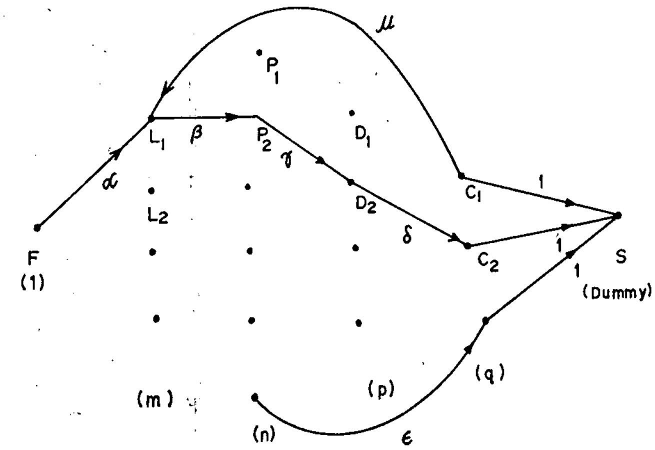

Fig. 3. Expanded signal flow graph.

Typical branch transmittances can be written as $\alpha_{i_1 i_2}$, $\beta_{i_2 i_3}$, $\gamma_{i_3 i_4}$, $\epsilon_{i_3 i_5}$, $\mu_{i_2 i_5}$ corresponding to the indices defined in Section II-C. Fig. 3 can be considered as the graphical analog for the allocation problem with a single funding agency, m laboratories, n main projects, p main disciplines, and q component projects. As a result of the foregoing analysis, the system index F can be expressed by a relation corresponding to (2) in terms of the matrices defined in Section II-C. Thus

$$F = [\alpha_{i_1 i_2}]_{1,m} \times [\,[\beta_{i_2 i_3}]_{m,n} \times [\,[\gamma_{i_3' i_4}]_{n,p} \times [\delta_{i_4 i_5}]_{p,q} + [\epsilon_{i_3 i_5}]_{n,q}] + [\mu_{i_2 i_5}]_{m,q}] \times [U_{ij}]_{q,1} \quad (3)$$

where $\times$ denotes matrix postmultiplication and U is a unit column vector.

Conclusion

A graph-theoretic approach has been described in the foregoing with specific reference to the problem of deriving a system index for the fund allocation under mixed global objectives. Though the derivations made are for a particular system, the concepts behind it are of more general application. The foregoing also serves to bridge the matrix approach of Dean with the well-established discipline of signal flow graphs.

In the expression for the system index, viz. (3), β_{ij} is a binary variable. Consequently, if k out of n projects are to be chosen for each of the m laboratories, the possible number of combinations is $(n_{c_k})^m$ from which the optimal allocation is required to be sifted. When n is large, the possible number of combinations becomes large. Hence, direct search methods will be futile. To cope with this, a decision-refinement method has been developed for arriving at the optimal allocation in realistic computational time. This forms the subject matter of a separate paper [10] because the concept of decision refinement is of more general applicability.

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