Design and Validation of a Hybrid Interactive Reference Point Method for Multi-Objective Optimization

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Some Backgrounds...

- Work is based on my Diploma Thesis at the Technical University, Dortmund (Germany) and Indian Institute of Technology, Kanpur (India) ...
- ...and focus on non-linear optimization

Publication

June 2008: M. Sathe, G. Rudolph, K. Deb: Design and Validation of a Hybrid Interactive Reference Point Method, IEEE CEC 2008, Hongkong.

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Real-World Problem: Car-Side Impact

- Car is subjected to a side-impact based on European Enhanced Vehicle-Safety Committee (EEVC) procedures
- Assignment: Minimize the damage to a car at side-impact

Real-World Problem: Car-Side Impact (cont.)

- Objectives:
  - Protection of the dummy
  - Minimize the weight of the car
  - Minimize the velocity of the B-Pillar

$\longrightarrow$ Balance between the weight and the safety performance

Real-World Problem: Car-Side Impact (cont.)

Objective Functions: linear + non-linear

$f_1(x_1,\dots,x_7) = \sum_{i=1}^{7} k_i x_i \longrightarrow$ Weight,
$f_2(x_2,x_3,x_4) = a_0 - a_1 x_4 - a_2 x_2 x_3 \longrightarrow$ Pubic Force,
$f_3(x_1,\dots,x_7) = a_3 x_1 x_2 + a_4 x_2 x_4 + a_6 x_3 x_7 + a_7 x_5 x_6 \longrightarrow$ Velocity of B-Pillar.

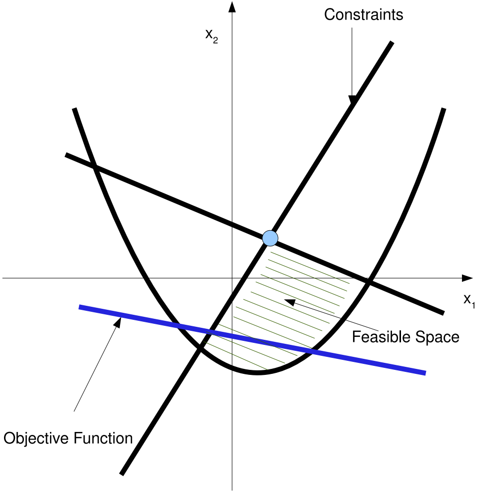
Constraints: non-linear

$g_1(x_2,x_3,x_4) = b_0 + b_1 x_2 x_4 + b_2 x_3 \longrightarrow$ Abdomen load,
$\dots$
$\dots$
$g_{10}(x_3,x_5,x_6,x_7) = b_{10} x_3 x_7 + b_{11} x_5 x_6 \longrightarrow$ Velocity of front door at B-Pillar.

Decision Variables: $x_1 - x_7$

$l_1 \leq x_1 \leq u_1 \longrightarrow$ Thickness of B-Pillar,
$\dots$
$l_7 \leq x_7 \leq u_7 \longrightarrow$ Thickness of roof rail.

Single-Objective Optimization



Multi-Objective Optimization

- At least two competitive objectives which are simultaneously to optimize
- Obtaining multiple incomparable solutions

MOOP

$$
\begin{array}{rll}
\text{optimize} & f_m(x) & m = 1, 2, \ldots, M, \\
\text{s.t.} & g_j(x) \leq 0 & j = 1, 2, \ldots, J, \\
& h_k(x) = 0 & k = 1, 2, \ldots, K, \\
& x_i^U \leq x_i \leq x_i^O & i = 1, 2, \ldots, n.
\end{array}
$$

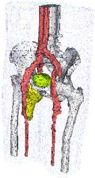
Multi-Objective Optimization (cont.)

Car-Side Impact

$$\begin{array}{lll} \min & f_m(x) & m = 1,2,3, \\ \text{s.t.} & g_j(x) \leq 0 & j = 1,2,\ldots,10, \\ & x_i^L \leq x_i \leq x_i^U & i = 1,2,\ldots,7. \end{array}$$

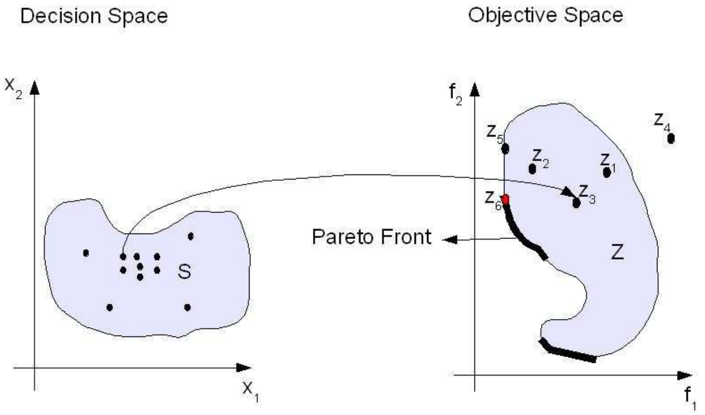
Hyperthermia Cancer Treatment Planning

$$\begin{array}{lll} \min & f_m(x) & m = 1,2, \\ \text{s.t.} & g_j(x) \leq 0 & j = 1,2,\ldots,10^6, \\ & x_i^L \leq x_i \leq x_i^U & i = 1,2,\ldots,23. \end{array}$$



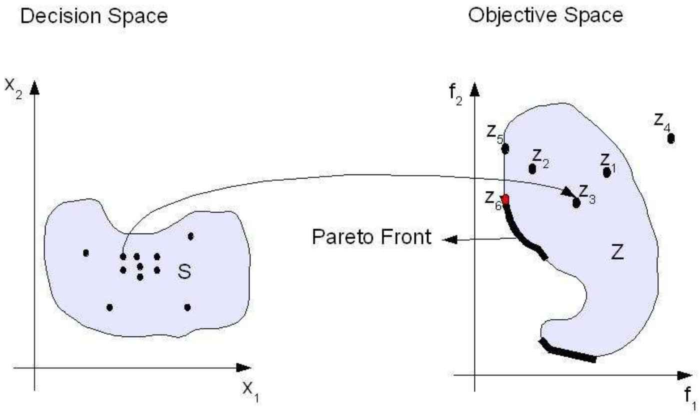
Matthias Christen, SNF Project (2007-2010): Nonconvex PDE-contrained optimization in Hyperthermia Cancer Treatment Planning.

Multi-Objective Optimization (cont.)



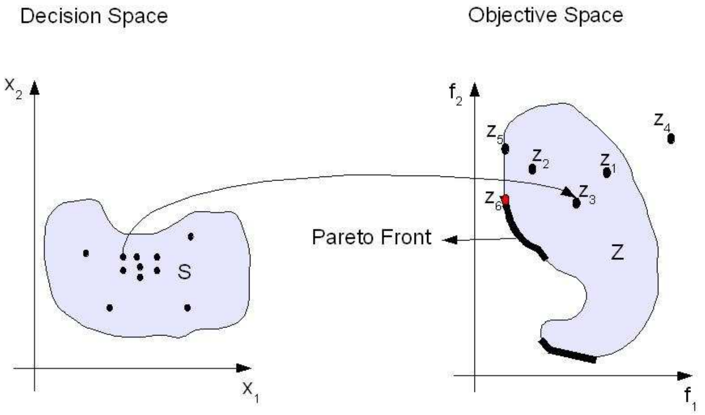
- Decision Space, Objective Function Space
- Goal to minimize f_1, f_2
- Evaluation function $p : S \longrightarrow Z$

Multi-Objective Optimization (cont.)



- Pareto Domination ($z_3 \preceq z_1$)
- Constraint Domination ($z_1 \preceq_c z_4$)
- Incomparable solutions ($z_2 \sim z_3$)

Multi-Objective Optimization (cont.)



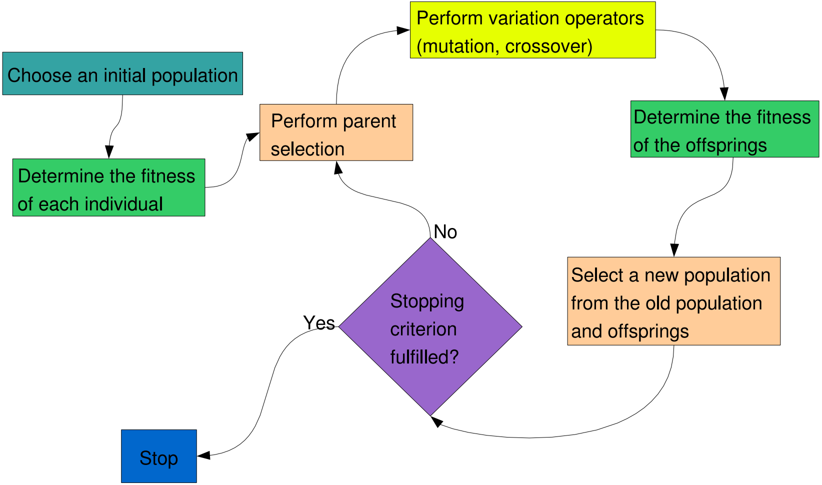
- Pareto Optimal (in S)
- Global Pareto Optimal Set (in S)
- Pareto Front (in Z)

Evolutionary Algorithms: Basics

- Random search heuristics which hopefully give a good approximation of the global optimum
- Applicable if deterministic methods do not find a solution in a reasonable time

Term	Interpretation
Individual	$x \in \mathbb{R}^n$ ($x \in \mathbb{B}^n$)
Mutation	Operates on exactly one individual ($x_i^{\text{mut}} = x_i + z_i$)
Population	Collection of individuals with a specified size
Crossover	Mix at least two individuals to create a new individual
Fitness	Evaluate each individual (often objective function)
Generation	Number of steps
Parents	Individuals from the old generation
Offsprings	Individuals created by variation operators from parents
Selection	Choose individuals from a population

Evolutionary Algorithms: General Outline

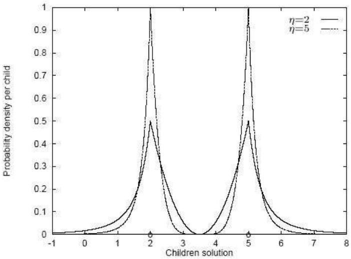


Evolutionary Algorithms: (1+1) - EA

Algorithm

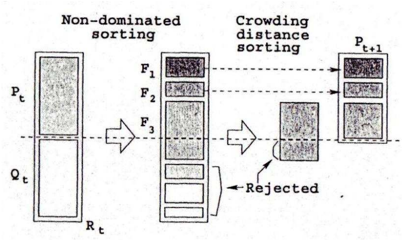
Choose $x_0 \in S$ randomly, $i = 0$.
while $i <$ maxGenerations

$y_i = \text{mut}_{\text{pol}}(x_i)$;
if $f(y_i) < f(x_i)$ then $x_{i+1} = y_i$
else $x_{i+1} = x_i$;
$i{+}{+}$;

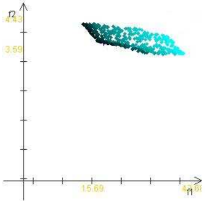


Evolutionary Algorithms for Multi-Objective Optimization

- State of the art EMOs: NSGA II, SPEA2, . . .



- Works very well on problems with two- and three-dimensional objective functions



Disadvantages: EMOs

- Calculate the approximated Pareto front takes some time with EMOs
- Posterori inclusion of DM $\longrightarrow$ Finding final solution difficult
- Challenging task by problems with more than three objectives

$\longrightarrow$ Interactive Algorithms

Interactive Algorithms

Basic Idea

- Include a user with the corresponding utility function
- Self-Exploration of the search space
- Feedback to current solutions
- Focus on regions of interest
- Goal: Satisfying the decision maker

- since 1960: Huge amount of classical interactive algorithms (Idea: Transformation of MOOP in SOOP)
- since 1993: Combination of classical methods with the field Computational Intelligence

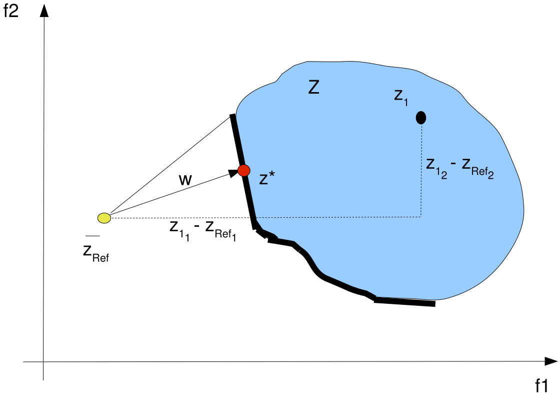
Interactive Reference Point Method: Algorithm

General Outline

1. Present information about the problem to the DM.
2. Ask the DM to specify a reference point.
3. Minimize an achievement function and obtain a Pareto optimal solution. Present the solution to the DM.
4. Calculate a number of k other solutions by minimizing a scalarizing function with perturbed reference points.
5. Present alternatives to the DM.
6. If the user is not satisfied, specify a new reference point.

Example: Scalarizing Function

$s(f(x), \overline{z}_{\text{Ref}}, w) =$
$\text{maximize}_{i=1}^{M}[w_i(f_i(x) - \overline{z}_{\text{Ref}_i})] + \rho \sum_{i=1}^{M}[w_i(f_i(x) - \overline{z}_{\text{Ref}_i})]$ with $\rho > 0$



Interactive Evolutionary Algorithms for Multi-Objective Optimization: Motivation

I-EMOs can

- …calculate many solutions during one run
  - User can choose some rough reference points
  - User obtains a better insight into the promised region
  - Focus on interesting trade-offs in the neighborhoods
- …cover several regions of interest
  - User can choose different preference information
- …deal with multi-objective problems (no transformation needed)
- …deal with non-smooth functions

- $(1+1)$-EA guides the user by focusing on small pieces of starting solutions

Hybrid Interactive Reference Point Method: Basic Idea

$(1+1)$ - EA

Select $x_0 \in S$ randomly, $i = 0$.
while $i <$ maxGenerations
  $y_i = \text{mut}_{\text{pol}}(x_i)$;
  if $f(y_i) < f(x_i)$
  then $x_{i+1} = y_i$
  else $x_{i+1} = x_i$;
  i++;

$(1+1)$ - EA + Scalarizing

Select $x_0 \in S$ randomly, $i = 0$.
while $i <$ maxGenerations
  $y_i = \text{mut}_{\text{pol}}(x_i)$;
  if $s(y_i, \overline{z}_i, w) < s(x_i, \overline{z}_i, w)$
  then $x_{i+1} = y_i$
  else $x_{i+1} = x_i$;
  i++;

where

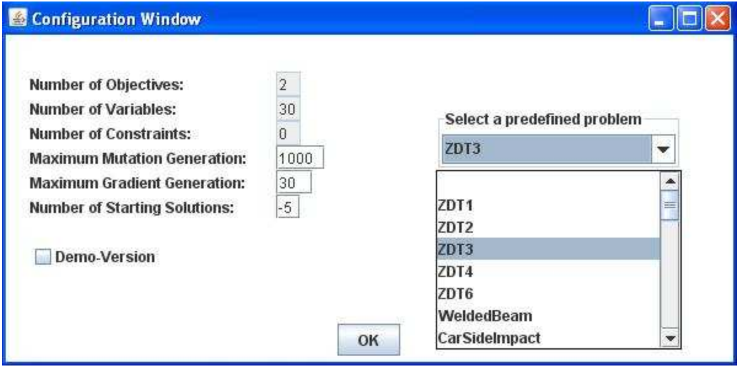
$$s(f(x), \overline{z}, w) = \text{maximize}_{i=1}^{M}[w_i(f_i(x) - \overline{z}_i)] + \rho \sum_{i=1}^{M}[w_i(f_i(x) - \overline{z}_i)] \text{ with } \rho > 0$$

Hybrid Interactive Reference Point Method

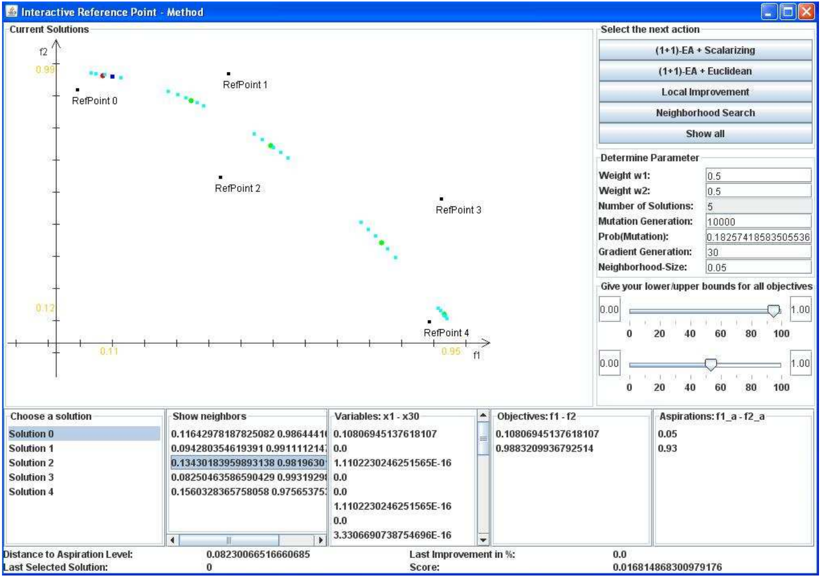
Hybrid Interactive Reference Point Algorithm

1. DM determines n reference points $\overline{z_i}$ with $i \in \{1, \ldots, n\}$.
2. Create n randomized and feasible starting points z_i.
3. While DM not satisfied with solution
   - Optimize with the $(1+1)$ - EA + Scalarizing
4. Possible local improvement with "Pareto descent method"
5. Calculate user-defined neighborhood

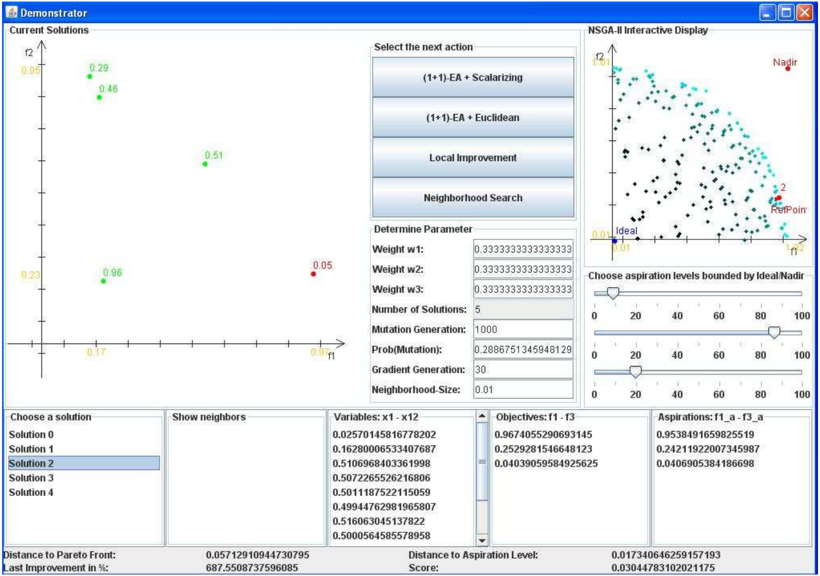
Configuration - Display



Interactive Reference Point - Display



Demonstrator - Display



Recall: Car-Side Impact

- Car is subjected to a side-impact based on European Enhanced Vehicle-Safety Committee (EEVC) procedures
- Assignment: Minimize the damage to a car at side-impact
- Objectives: Protection of the Dummy, Minimize the weight of the car, minimize the velocity of the B-Pillar
- An increase in dimension of the car parameters may improve the performance on the dummy but the increased weight of the car may have an adverse effect on the fuel economy

$\longrightarrow$ Balance between the weight and the safety performance

Case Study: Car-Side Impact

Video

Start: Car-Side Impact

Evaluation of the application

Criteria

- System generates Pareto optimal solutions
- System supports the DM to find a compromise solution
- System creates an insight into the Pareto front
- System takes per iteration a small amount of computation time
- System provides some information about solutions
- Communication between system and DM is simple

Summary

Summary

- Basics for Multi-Objective Optimization, Evolutionary Algorithms
- New Hybrid Interactive Reference Point Method
- Case Study: Car Side Impact

Research Field

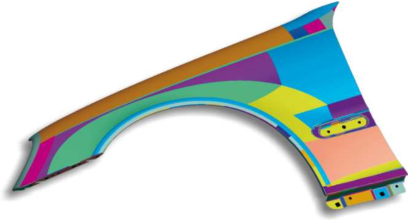
MOOP

$$\begin{array}{ll} \text{optimize} & f_m(x) \\ \text{s.t.} & g_j(x) \leq 0 \\ & h_k(x) = 0 \\ & x_i^U \leq x_i \leq x_i^O \end{array}$$

MOOP

$$\begin{array}{ll} \text{optimize} & f_m(x) \\ \text{s.t.} & g_j(x) \leq 0 \\ & h_k(x) = 0 \\ & x_i \quad \text{discrete} \end{array}$$

KTI-Project (2007 - 2010): Mixed-Integer Optimization in automobile sheet metal forming processes



Thank you for your attention !!!
Any questions ???