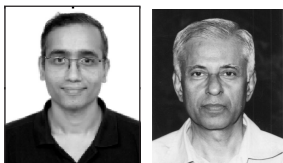


# The 2005 Nobel Prize in Physics: Optics

*Vasant Natarajan and N Mukunda*



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The 2005 Nobel Prize in Physics has been awarded in the area of optics, or more specifically in laser physics. One half of the prize (theory part) has been given to Roy Glauber of Harvard University “for his contribution to the quantum theory of optical coherence,” which became important soon after the invention of the laser. The second half of the prize (experimental part) is jointly awarded to two physicists, John Hall of the National Institute of Standards and Technology (NIST) in Boulder, USA, and Theodor Hänsch of the Max-Planck Institute for Quantum Optics in Garching, Germany. They have been cited “for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique.”

India has a rich tradition of research in optics dating back to the pioneering work of C V Raman (Nobel Prize for the Raman Effect in 1930). In the 1950’s there appeared S Pancharatnam’s fundamental studies on polarization optics in the course of which he discovered the geometric phase in its earliest form. Then in 1961 came the crystal optics work of G N Ramachandran and S Ramaseshan. Towards the end of the theory part of this article we will describe briefly the remarkable 1963 discovery of the Diagonal Coherent State Representation and the Optical Equivalence Theorem, central to quantum optics, by E C G Sudarshan working in the USA.

## 1. Quantum Theory of Optical Coherence

The understanding of the nature and properties of light has fascinated humankind for a very long time; its progress is an important part of the history of physics. It may be useful to very briefly remind the reader of some

### Keywords

Classical optical coherence, quantum electrodynamics, quantum optical coherence, laser-based precision spectroscopy, optical frequency comb.



of the more ‘recent’ events in this history, starting with the work of Maxwell in mid 19th century. With such a background, one can understand better the work for which the ‘theory part’ of the 2005 Physics Nobel has been given.

Maxwell<sup>1</sup> succeeded in uniting the laws of electricity and magnetism into a single theory, and then went on to show that light was an electromagnetic wave. Thus as a result of his work three previously separate fields of physics became one. Around the same time, the field of statistical mechanics, as the foundation for thermodynamics, was also being developed. Around 1900, however, it became clear that the combination of statistical ideas and the classical Maxwell description of electromagnetic radiation led to an impasse: it could not explain the experimental results concerning black body or thermal radiation, i.e. radiation in equilibrium with material bodies at a common temperature.

It was Planck’s solution of this problem that led to the birth of quantum theory in late 1900, the dawn of the 20th century (Nobel 1918). Planck’s work involved two steps: first, a mathematical interpolation amounting to inspired guess work that led to his famous radiation formula which fitted experiment beautifully; second, a derivation of this formula based on the hypothesis that (electrically charged) material oscillators could emit and absorb radiation energy only in discrete amounts or quanta. This was a revolutionary idea.

Each of the later advances in the understanding of light has been equally stunning. In 1905 Einstein was able to argue from the non-classical limit of Planck’s formula that radiation in its own nature has a lumpy or particle-like aspect, in contrast to the classical continuous Maxwell picture. He then presented an explanation of the photoelectric effect as one piece of evidence in support of his conclusions (Nobel 1921). A few years later in 1909 he studied the energy fluctuations of Planck radiation and deduced that radiation simultaneously possesses the seemingly contradictory, or dual, particle and wave properties. Then in 1916 he presented a startlingly new derivation of Planck’s law based on the processes of emission and absorption of radiation by matter, and also showed that light quanta – photons – carry momentum in addition to energy. In 1924, S N Bose gave yet another derivation of Planck’s law based on a deep understanding of the identity of light quanta; the work was immediately appreciated and taken further by Einstein. This series of events came to a triumphant conclusion with Dirac in 1927 showing how to apply the principles of the just discovered quantum mechanics to the classical Maxwell theory.

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<sup>1</sup> This year marks the 175th birth anniversary of Maxwell, and is being celebrated as Maxwell Year in Scotland. *Resonance* featured Maxwell in the May 2003 issue.



As this implies, the Dirac theory of the quantised electromagnetic field came *after* a satisfactory quantum mechanics for matter had been developed. The first steps here were (apart from Planck in 1900) again taken by Einstein in 1907, in his theory of specific heats; and then by Niels Bohr in 1913 with the theory of stationary electronic states in the hydrogen atom (Nobel 1922). (This Bohr theory was a vital component of Einstein's 1916 work on radiation). There followed what was later called the period of the Old Quantum Theory when Bohr's initial ideas were tried to be extended to more complex material systems. By about 1923 this effort ran into severe problems, and the situation was resolved only with the discovery of quantum mechanics by Heisenberg, Dirac and Schrödinger independently during 1925–26 (Nobels 1932, 1933).

Returning to radiation, after Dirac the theory of quantum electrodynamics – QED – was further developed by many leading physicists of that time including Heisenberg, Pauli, Peierls and Landau. However it was now found that when one went beyond the lowest level of approximation it was plagued by severe mathematical inconsistencies – the so-called problem of divergences. Calculations gave meaningless infinite answers for quantities which should have been finite. This was the situation through most of the 1930's and early 1940's, until the discovery of the method of renormalization independently by Tomonaga, Schwinger and Feynman (Nobel 1965), completed by around 1947. The impetus given to this effort by the experimental measurement of the Lamb shift (Nobel 1955) is emphasized in the second section of this article.

With the arrival of the renormalization procedure resulting in a finite QED, it became clear that our understanding of the fundamental nature of light and its interaction with matter had reached a level of completion. *All later work including what will be described below is within that framework.*

Meanwhile within the arena of classical optics many new developments had been taking place. They could be regarded as a completion of the earlier elementary treatments of diffraction and interference of classical wave amplitudes. It was realised that essentially all earlier classical optical effects could be described in terms of the two-point amplitude correlation function; and via this object the concepts of partial coherence and its propagation were brought into the field. (Analogous developments with regard to polarization of light had also taken place.) In this way the role of statistical methods in optics came to be much better appreciated. Some of the early names are those of Fritz Zernike (Nobel 1953), van Cittert, Blanc-Lapierre and Dumontet. From about the mid-1950's the whole subject



was developed in a systematic manner largely by Emil Wolf. After the invention of intensity interferometry by Hanbury Brown and Twiss in 1956, it became clear that it was necessary to go beyond the two-point amplitude correlation function (adequate to describe Young-type interference phenomena) to higher order correlation functions. Thus intensity correlations involve correlations among amplitudes at four space-time points, or a four-point function. Correlation functions of all higher orders came into play in the treatment by Mandel of the semi-classical photoelectron counting distribution formula. Here one has a fluctuating classical light beam falling on a photodetector, and one wishes to find the probabilities for various numbers of electrons to be emitted over a given time period. Then, from the experimentally measured statistical properties of the photoelectrons emitted, one obtains information on the statistical properties of the incident light beam.

Note the contrast to the original 1905 Einstein explanation of the photoelectric effect. In that treatment it was light which was regarded as possessing quantum features, and a quantum description of matter was still many years away. After the arrival of quantum mechanics for matter it became possible to account for the photoelectric effect in an alternative semiclassical manner – light can be treated as a statistical fluctuating classical quantity, while the electron is quantum mechanical. The key feature is that quantum ideas are needed *at least for one of* the two players in the process, light or electrons (ultimately of course for both in a completely satisfactory treatment). In any case, in Mandel's work the second of the above two viewpoints was adopted.

To give the reader some idea of the kinds of expressions and concepts involved in this development, we present in *Box 1* the definitions and interpretations of correlation functions in classical statistical optics. For simplicity we ignore the vector nature of the electric field and treat it as though it were a scalar. (We also omit reference to the magnetic field). The arguments  $x, y, \dots$  are combined spatial and time coordinates; and classical statistical averages are indicated by angular brackets. Note the separation of the real total electric field into two mutually conjugate parts, and the use of these parts in defining correlation functions and coherence. Again for simplicity only correlation functions with equal numbers of  $E^{(+)}$ 's and  $E^{(-)}$ 's are considered.

These two streams of work – the completion of QED and the growth of classical statistical optics – merged in the early 1960's and led to the quantum theory of optical coherence, more generally quantum optics, to which many basic contributions were made by R J Glauber. The invention of the laser by that time had



made it clear that there was a need to describe, *within the overall framework of QED*, states of electromagnetic radiation associated with arbitrary, in particular non-thermal, light beams. (The traditional uses of QED, in the realm of elementary particle physics, had only dealt with processes involving small numbers of photons – absorption and emission of single photons, scattering of a photon on an electron, and the like.) The physical principle underlying Glauber’s work, as foreshadowed in the Mandel treatment of photo electron counting, is that all conventional methods of light detection involve *absorption* of photons from the field being observed. (This is true even in the human and animal visual systems.) Building on this, Glauber was able to arrive at the most useful measure of (partial) coherence of the *quantised* electromagnetic field at the two-point level, and then to generalize it to correlation functions of all higher orders. This was a specific way to pass from the complete classical hierarchy of correlation functions of various orders – *Box 1* – to their quantum counterparts. In then defining and analysing the concepts of partial and of complete coherence, to some finite order

**Box 1. Classical Correlation Functions for Fluctuating Electric Fields**

Real classical electric field  $E(x) = E^{(+)}(x) + E^{(-)}(x)$ ,

$$E^{(+)}(x) = \text{complex positive frequency part,}$$

$$E^{(-)}(x) = E^{(+)}(x)^* = \text{complex negative frequency part.} \quad (1a)$$

Classical two-point correlation function = statistical average of product  $E^{(-)}(y)E^{(+)}(x)$  of two complex field amplitudes

$$= \langle E^{(-)}(y)E^{(+)}(x) \rangle : \quad (1b)$$

adequate to discuss intensity measurements ( $y = x$ ), Young type interference phenomena.

Classical four-point correlation function =

$$\langle E^{(-)}(y_1)E^{(-)}(y_2)E^{(+)}(x_1)E^{(+)}(x_2) \rangle : \quad (1c)$$

needed to discuss Hanbury Brown–Twiss intensity correlations ( $y_1 = x_1, y_2 = x_2$ ). Mandel’s semiclassical photo-electron counting distribution formula involves

$$\langle E^{(-)}(y_1) \cdots E^{(-)}(y_n)E^{(+)}(x_1) \cdots E^{(+)}(x_n) \rangle \quad (1d)$$

for all  $n$ , with  $y_1 = x_1, \dots, y_n = x_n$ .

Coherence of order  $2n$  holds if the expression (1d) factorises completely as  $V(y_1)^* \cdots V(y_n)^*V(x_1) \cdots V(x_n)$  for some field amplitude  $V$ .



or to all orders, he demonstrated the great usefulness of a special set of quantum states called *coherent states*. These states can be defined both for material oscillators and for the free radiation field. They had been discovered by Schrödinger in 1927, studied by von Neumann in 1930, and used in a specific context within QED by Bloch and Nordsieck in 1937. Glauber's work amounted to a rediscovery of their enormous usefulness in describing states of radiation in the complete quantum optics context.

Let us first describe briefly the quantum counterparts of the contents of *Box 1*, assembled in *Box 2*, and then turn to coherent states. It is of course out of place to attempt to give here anything like a complete resumé of the basic structures of quantum mechanics, much less of QED. We can do no better than make suggestive statements, and try to get across some basic ideas. For simplicity we use the same symbols  $E^{(\pm)}(x)$  in quantum theory as classically. In quantum theory, however, these are not complex valued numbers any more, but *operators* which act on *quantum state vectors*.  $E^{(+)}(x)$  is an operator which acting on a state annihilates or subtracts one photon;  $E^{(-)}(x)$  is the hermitian conjugate (replacement for the

**Box 2. Correlation Functions for Quantised Electric Field**

$$E(x) = E^{(+)}(x) + E^{(-)}(x), \text{ all field operators;}$$

$$E^{(+)} \text{ annihilates one photon, } E^{(-)} \text{ creates one photon.} \quad (2a)$$

Two-point correlation function adequate to describe intensity measurements by photon absorption, Young type interference:

$$G^{(1,1)}(x; y) = \text{Tr}\{\rho E^{(-)}(y)E^{(+)}(x)\},$$

$$\rho = \text{density operator of quantum state.} \quad (2b)$$

Four-point correlation function needed to describe Hanbury Brown–Twiss intensity correlations:

$$G^{(2,2)}(x_1, x_2; y_1, y_2) = \text{Tr}\{\rho E^{(-)}(y_1)E^{(-)}(y_2)E^{(+)}(x_1)E^{(+)}(x_2)\}. \quad (2c)$$

Higher order correlation functions:

$$G^{(n,n)}(x_1, \dots, x_n; y_1, \dots, y_n) = \text{Tr}\{\rho E^{(-)}(y_1) \dots E^{(-)}(y_n)E^{(+)}(x_1) \dots E^{(+)}(x_n)\}. \quad (2d)$$

Complete coherence  $\equiv$  for all  $n$ ,  $G^{(n,n)}(x_1, \dots; y_1, \dots) = V(y_1)^* \dots V(x_1) \dots$  for some  $V(x) \Rightarrow$  essentially,

$$\rho \text{ is a coherent state.}$$



classical complex conjugate) of  $E^{(+)}(x)$ , and acting on a state it creates or adds one photon.  $E^{(+)}$  and  $E^{(-)}$  *do not commute*. In the vacuum state there are no photons at all, so  $E^{(+)}$  applied to that state gives zero. States in quantum mechanics may be pure, describable by a single state vector or wave function  $\psi$ ; or mixed, namely an ensemble of several pure states  $\psi_1, \psi_2, \dots$ , each present with a corresponding probability  $p_1, p_2, \dots$ . In the latter case, the entire ensemble can be represented by what is called a density operator or density matrix  $\rho$ , this is the most general quantum state. The entries in *Box 2* can now be hopefully understood.

The symbol ‘Tr’ stands for ‘Trace’ and (along with the presence of  $\rho$ ) is the quantum counterpart of classical statistical averaging which was denoted in *Box 1* by angular brackets. One point to note with care is that in the definitions of  $G^{(1,1)}, G^{(2,2)}, \dots$  in *Box 2*, the  $E^{(-)}$  factors (creation operators) always stand to the left of the  $E^{(+)}$  factors (annihilation operators). This is the key feature of the Glauber definition – detection by absorption of photons – and we are not free to interchange the sequence of  $E^{(-)}$ ’s and  $E^{(+)}$ ’s since they do not commute. The last sentence in *Box 2* brings in the coherent states, so we describe them briefly at this point, aided by *Box 3*.

**Box 3. Coherent States of Single Mode Radiation Field**

States with definite number of photons :  $|n\rangle, n = 0, 1, 2, \dots$  . (3a)

For any complex number  $z$ :

Coherent state  $|z\rangle$  = superposition of states with definite photon numbers

$$= e^{-|z|^2/2} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} |n\rangle. \quad (3b)$$

Some important properties of coherent state  $|z\rangle$ :

Probability of finding  $n$  photons =  $e^{-|z|^2} \frac{|z|^{2n}}{n!}$  : Poisson distribution. (3c)

Mean number of photons = average of  $n = |z|^2$ . (3d)

Fluctuation in number of photons = average of  $n^2 - (\text{average of } n)^2$

$$= |z|^2$$

= mean : characteristic of Poisson distribution. (3e)



We limit ourselves to a single mode of the quantum radiation field, so all the photons have the same spatio-temporal characteristics. (The generalisations to several modes or to the entire field are straightforward). For example we may fix the frequency, propagation direction and polarization state for all of them, so only the photon number can vary. In quantum mechanical notation the state with exactly  $n$  photons is written as  $|n\rangle$ : for  $n = 0$  we have the vacuum or no photon state  $|0\rangle$ , for  $n = 1$  the one-photon state  $|1\rangle$ , and so on. Each of these is a pure state and they are mutually exclusive or orthogonal: if we know that there are exactly  $n$  photons present, we certainly know the total photon number is not  $n'$  for any  $n' \neq n$ . Given any set of pure states, we can multiply each one by some complex number and add them all up to get a new pure state. This is the fundamental Superposition Principle of quantum mechanics *which has no classical analogue*. For each complex number  $z$ , we can produce exactly one pure state using the expression given in (3b) of *Box 3*. This is the coherent state  $|z\rangle$  of the concerned mode. Thus a coherent state has a variable number of photons present, with probabilities given by a Poisson distribution, (3c) of *Box 3*. These states turn out to be quantum states as close as possible to classical field states in the sense that the unavoidable or inescapable uncertainty principle of quantum mechanics is barely obeyed. They also turn out to be as close to having a definite ‘phase’ – in contrast to a definite photon number – as is possible in the quantum framework.

We will conclude this part of our article by describing two crucial properties of coherent states, justifying their importance for quantum optics. Each coherent state  $|z\rangle$  is a pure state. Consider now a mixed state  $\rho$  in which all these  $|z\rangle$  are present with various probabilities, described by a ‘classical’ probability distribution  $\phi(z)$  over the complex plane. It then turns out that the particular definitions of the quantum optical correlation functions given in (2b), (2c), (2d) of *Box 2* combine with the very special properties of coherent states to lead to a remarkable result: *each quantum correlation function has the same form and the same value as the corresponding classical correlation function calculated for a suitably defined classical statistical state*. The key to this lies in two facts: the  $E^{(-)}$  factors always stand to the left of the  $E^{(+)}$  factors in the definitions of quantum correlation functions; and the  $E^{(+)}$  factors act very simply on coherent states  $|z\rangle$ . This brings out graphically the extreme appropriateness of coherent states in these problems.

Now we come to our final point. A general quantum state  $\rho$  can certainly not be reconstructed from the coherent states  $\{|z\rangle\}$  via a classical probability dis-





tribution  $\phi(z)$ , as was assumed in the previous paragraph. But in a remarkable and truly fundamental result it was shown in 1963 by E C G Sudarshan that *every* quantum state  $\rho$  can be *formally* regarded as a ‘generalized ensemble’ over the coherent states  $|z\rangle$ , except that  $\phi(z)$  may not be interpretable as a ‘classical’ probability distribution! This is known technically as the Diagonal Coherent State Representation and the Optical Equivalence Theorem. Referring to the highlighted phrase in the previous paragraph we can say that in case  $\phi(z)$  is not a true probability distribution, we have equivalence of forms but not of values for the two families of correlation functions, quantum and classical. For the most general quantum state  $\rho$ ,  $\phi(z)$  is not a function in any ordinary mathematical sense, but a singular quantity, a so-called distribution of a particular class that can be precisely characterized. This result is truly basic to the theory of quantum optics, as it is the only way in which we can exhibit the clear distinction between classical and quantum natures of optical fields. States displaying sub-Poissonian photon statistics or antibunching, so-called squeezing and Hanbury Brown–Twiss anticorrelations are all truly quantum in nature, and correspond to singular, or at least non-positive definite,  $\phi(z)$  in the Sudarshan classification. One can say that the need to allow  $\phi(z)$  to go beyond the collection of probability distributions in considering all quantum states shows why quantum and classical theories are radically different, the former overstepping the confines of the latter. In fact this is a recurring feature of attempts to express quantum mechanics in the language of classical physics – the range of quantum mechanical possibilities always overflows classical boundaries. It is extremely unfortunate that this result of Sudarshan has not received the credit and recognition that is its due. The interested reader may refer to the article, ‘On Sudarshan’s Diagonal Coherent State Representation’ by C L Mehta [2].

That apart, the reader would have appreciated all the developments that form the backdrop to the ‘theory part’ of the physics 2005 Nobel.

## 2. Optical Frequency Comb Technique

Lasers have impacted our lives in a countless number of ways. Today they are found everywhere, in computer hard disk drives, CD players, grocery store scanners, and in the surgeon’s kit. In research laboratories, almost everyone uses lasers for one reason or another. However, arguably the greatest impact of lasers in physics has been in high-resolution spectroscopy of atoms and molecules. To see this, consider how spectroscopy was done before the advent of lasers. You would use a high-energy light source to excite all the transitions in the system,



and then study the resulting emission “spectrum” as the atoms relaxed back to their ground states. This is like studying the modes of vibration of a box by hitting it with a sledgehammer and then separating the resulting sound into its different frequency components. A more gentle way of doing this would be to try and excite the system with a tuning fork of a given frequency. Then by changing the frequency of the tuning fork, one could build up the spectrum of the system. This is how you do laser spectroscopy with a tunable laser; you study the absorption of light by the atoms as you tune the laser frequency. When you come close to an atomic resonance, you build up a typical absorption curve with a characteristic width called the natural width.

In order to be able to do such high-resolution laser spectroscopy, two things have to be satisfied. First, the atomic resonance should not be artificially broadened. This can happen, for example, due to the Doppler effect in hot vapour, where the thermal velocity causes a frequency shift and broadens the line. Even with atoms at room temperature, the Doppler width can be 100 times the natural width, and can prevent closely-spaced levels from being resolved. The second requirement for high-resolution spectroscopy is that the tunable laser should have a narrow “linewidth”. The linewidth of the laser, or its frequency uncertainty, is like the width of the pen used to draw a curve on a sheet of paper. Obviously, you cannot draw a very fine curve if you have a broad pen.

It is in the above context that the Nobel citation mentions the work of the two laureates in laser-based precision spectroscopy. Their names are quite well known to anyone working in laser spectroscopy. In the early 1970s, Hänsch, then working at Stanford University with Arthur Schawlow (Nobel Prize for laser spectroscopy, 1981), pioneered the use of Doppler-free techniques such as saturation spectroscopy, particularly for spectroscopy in hydrogen. Around the same time, Hall developed many techniques to stabilize the frequency of lasers and reduce their linewidth. Today, two of the most popular techniques for laser stabilization are called the Hansch-Couillaud technique and the Pound-Drever-Hall technique, in honour of these scientists.

In 1976, Hall and coworkers used high-resolution laser spectroscopy in methane to observe for the first time the recoil-induced splitting of a line. In other words, when the molecule absorbs a photon of wavelength  $\lambda$ , the photon momentum  $h/\lambda$  imparts a recoil to the molecule. This recoil velocity results in a frequency shift due to the Doppler effect. But this is a small effect, about 2 kHz in a frequency of  $10^{14}$  Hz, and requires an extremely high resolving power. In the same year, Hänsch

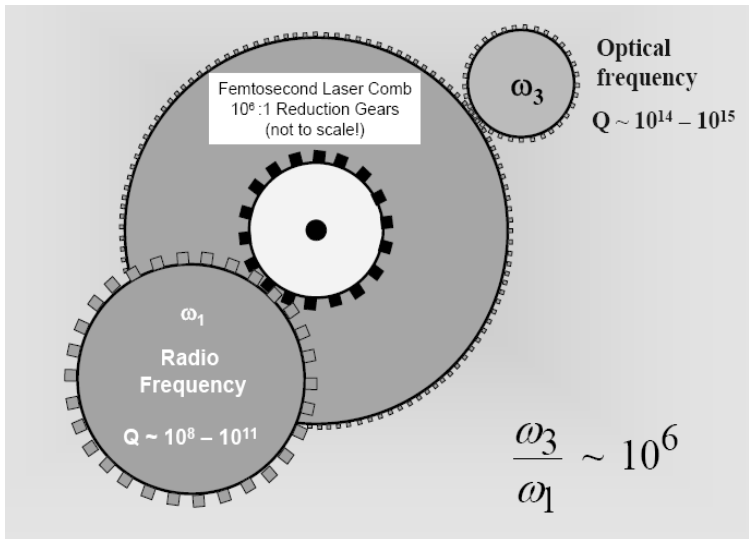


and Schawlow independently proposed that the momentum of laser photons could be used to cool atoms to very low temperatures, a technique that is now called “laser cooling”. The field of laser cooling has grown explosively in the last two decades, and two Nobel Prizes have been awarded; the first in 1997 for techniques of laser cooling (Chu, Cohen-Tannoudji, and Phillips), and the second in 2001 for using laser-cooled atoms to achieve Bose-Einstein condensation (Cornell, Ketterle, and Wieman).

But back to spectroscopy. Many advances in physics have been brought about by high-resolution spectroscopy of atoms. Indeed, one might argue that the most obvious manifestation of quantisation (or discreteness) at the atomic scale is the fact that atomic spectra show sharp spectral lines. The well known Fraunhofer lines were first observed in the solar spectrum as dark lines using a spectrometer that was “high-resolution” for its time. In the early part of the twentieth century, Niels Bohr (Nobel Prize 1922) was able to explain such discrete lines by postulating that an electron in an atom was allowed only certain quantised values of angular momentum. This led to the development of quantum mechanics as a theory in the atomic domain. Further measurements of atomic spectra at higher resolution revealed that many lines were actually doublets. A common example is the yellow light emitted by the ubiquitous sodium vapour lamp; it actually consists of two lines, called *D1* and *D2*, which can be resolved and measured in a high school laboratory today. The origin of this splitting is the interaction between two types of electronic angular momentum – orbital and spin. In 1928, Dirac (Nobel Prize 1933) wrote down his famous equation to describe the electron, which incorporated its spin angular momentum in a natural way. However, even the very successful Dirac theory predicted that the  $2S$  and  $2P$  states of hydrogen have the same energy. A precise measurement of these levels by Lamb (Nobel Prize 1955) showed that their energies are slightly different, which is now called the Lamb shift. The discovery of the Lamb shift led to the birth of quantum electrodynamics (QED), for which the Nobel Prize was awarded to Feynman, Schwinger, and Tomonaga in 1965.

We thus see that improvement in precision almost always leads to new discoveries in physics. In recent times, one atomic transition that has inspired many advances in high-resolution spectroscopy and optical frequency measurements is the  $1S-2S$  resonance in hydrogen, with a natural width of only 1 Hz. Measurement of the frequency of this transition is important as a test of QED and for the measurement of fundamental constants. However, the wavelength of this transition is 121 nm, corresponding to a frequency of  $2.5 \times 10^{15}$  Hz. Since the SI unit of time is defined





**Figure 1. Radio frequency to optical frequency link using a frequency comb.**

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in terms of the cesium radio-frequency transition at  $9.2 \times 10^9$  Hz, measuring the optical frequency with reference to the atomic clock requires spanning 6 orders of magnitude! You can think of this as having two shafts whose rotation speeds differ by a factor of 1 million, and you need to measure the ratio of their speeds accurately. If we use a belt arrangement to couple the two shafts, then there is a possibility of errors in the ratio measurement due to phase slip. Instead, one would like to couple them through a gearbox mechanism with the correct teeth ratio so that there is no possibility of slip (see *Figure 1*). This is precisely what is achieved by the frequency comb.

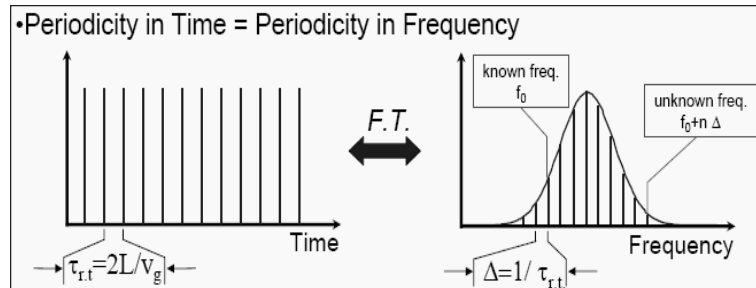
The basic idea of the comb technique is that periodicity in time implies periodicity in frequency. Thus, if you take a pulsed laser that produces a series of optical pulses at a fixed repetition rate, then the frequency spectrum of the laser will consist of a set of uniformly spaced peaks on either side of a central peak. The central peak is at the optical frequency within each laser pulse (carrier frequency), and the peaks on either side are spaced by the inverse of the repetition period (called sidebands). You can produce such a spectrum by putting the laser through a nonlinear medium such as a nonlinear fiber. The larger the nonlinearity, the more the number of sidebands. Around 1999, there was a major development in making nonlinear fibers; fibers with honeycomb microstructure were developed which had such extreme nonlinearity that the sidebands spanned almost an octave<sup>2</sup>. If you sent a pulsed laser (operating near 800 nm) through such a fiber, you would get a near continuum of sidebands spanning the entire

<sup>2</sup>Hall calls this development the dawn of a new epoch.



**Figure 2. Frequency measurement using a comb.  $\tau_{r,t}$  is the pulse repetition rate, and F.T. is Fourier transform.**

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visible spectrum. The series of uniformly spaced peaks stretching out over a large frequency range looks like the teeth of a comb, hence the name optical frequency comb. The beautiful part of the technique is that the comb spacing is determined solely by the repetition rate, thus by referencing the repetition rate to a cesium atomic clock, the comb spacing can be determined as precisely as possible. In 1999, Hänsch and coworkers showed that the comb spacing was uniform to 3 parts in  $10^{17}$ , even far out into the wings.

Thus the procedure to produce a frequency comb is now quite straightforward. One starts with a mode-locked, pulsed Ti:sapphire laser and sends its output through 20–30 cm of nonlinear fiber. The pulse repetition rate is referenced to an atomic clock, and determines the comb spacing. The carrier frequency is controlled independently, and determines the comb position. But how does one measure an optical frequency using this comb? This can be done in two ways. One way is to use a reference transition whose frequency  $f_0$  is previously known. We now adjust the comb spacing  $\Delta$  so that the reference frequency  $f_0$  lies on one peak, and the unknown frequency  $f$  lies on another peak that is  $n$  comb lines away, i.e.  $f = f_0 + n\Delta$  (see Figure 2)<sup>3</sup>. Thus by measuring  $n$ , the number of comb lines in between, and using our knowledge of  $f_0$  and  $\Delta$ , we can determine  $f$ . This was the method used by Hänsch in 1999 to determine the frequency of the  $D1$  line in cesium (at 895 nm). The measurement of this frequency can be related to the fine-structure constant  $\alpha$ , which is one of the most important constants in physics because it sets the scale for electromagnetic interactions and is a fundamental parameter in QED calculations.

However, the above method requires that we already know some optical frequency  $f_0$ . If we want to determine the absolute value of  $f$  solely in terms of the atomic clock, the scheme is slightly more complicated. In effect, we take two multiples

<sup>3</sup> It is not necessary that the comb peak aligns perfectly with the laser frequency. A small difference between the two can be measured easily since the beat signal will be at a sufficiently low frequency



(or harmonics) of the laser frequency, and use the uniform comb lines as a precise ruler to span this frequency difference. Let us say we align one peak to  $3.5f$ , and another peak that is  $n$  comb lines away to  $4f$ , then we have determined

$$4f - 3.5f = n\Delta,$$

which yields

$$f = 2n\Delta,$$

so that we have  $f$  in terms of the comb spacing. In 2000, Hänsch and coworkers used this method to determine the frequency of the hydrogen  $1S - 2S$  resonance with an unprecedented accuracy of 13 digits. This was the first time that a frequency comb was used to link a radio frequency to an optical frequency.

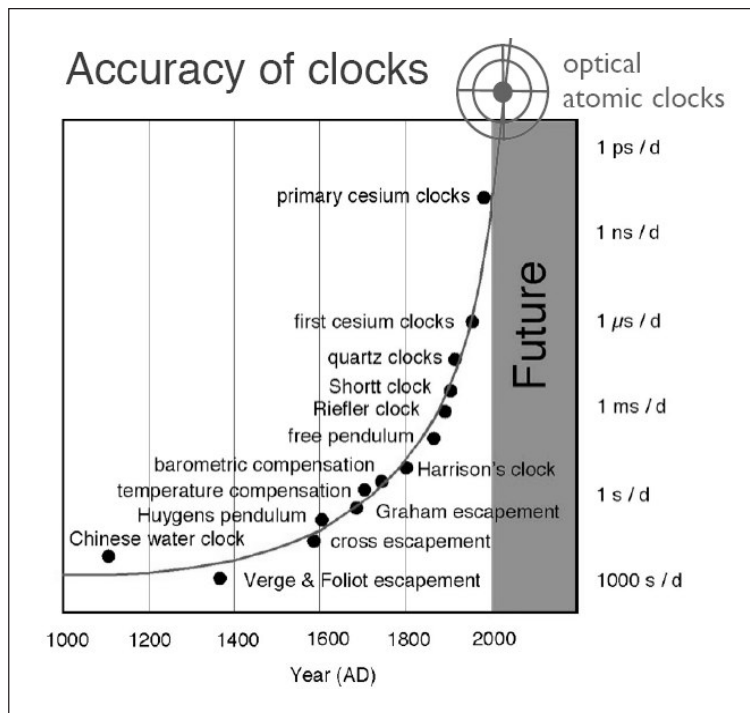
Currently, one of the most important questions in physics is whether fundamental constants of nature are really constant, or are changing with time. For example, is the fine-structure constant  $\alpha$  constant throughout the life of the universe or is it different in different epochs? Now, if you want to measure a very small rate of change  $\dot{\alpha}$  ( $= d\alpha/dt$ ), then you can do it in two ways. You can take a large  $dt$  so that the integrated change in  $\alpha$  is very large. This is what is done in astronomy, where looking at the light from a distant star is like looking back millions of years in time. You can then compare atomic spectra from distant stars to spectra taken in the laboratory today. Alternately, if you want to do a laboratory experiment to determine  $\dot{\alpha}$ , then you have no choice but to use a small  $dt$ . Therefore, you have to improve the accuracy of measuring  $\alpha$  so that even small changes become measurable. This is what has been done by Hänsch and his group. By measuring the hydrogen  $1S - 2S$  resonance over a few years, they have been able to put a limit on the variation of  $\alpha$ . Similar limits have been put by other groups using frequency-comb measurements of other optical transitions. The current limit on  $\dot{\alpha}/\alpha$  from both astronomy and atomic physics measurements is about  $10^{-15}$  per year.

In the last few years, several optical transitions have been measured using frequency combs. The primary motivation is to find a suitable candidate for an optical clock to replace the microwave transition used in the current definition. An optical clock will “tick” a million times faster, and will be inherently more accurate. However, since the cesium atomic clock has an accuracy of  $10^{-15}$ , one has to measure the candidate optical transition to this accuracy to make sure it is consistent with the current definition. The race is on to find the best candidate among several alternatives such as laser-cooled single ions in a trap, ultracold



**Figure 3. Improvement in the accuracy of clocks over the last millennium.**

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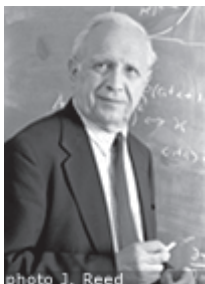
neutral atoms in an optical lattice, or molecules. As shown in *Figure 3*, the accuracy of clocks has increased by several orders of magnitude in recent times. The applications for more precise clocks of the future range from telecommunications and satellite navigation to fundamental physics issues such as measurement of pulsar periods, tests of general relativity, and variation of physical constants.

In concluding this article, one of the authors (VN) would like to switch to the first person singular and make some comments on the motivations that underlie work in experimental physics. I recently attended a small reception in honour of John Hall after he won the Nobel Prize. In his speech, he mentioned that the thing he enjoyed most about being at NIST was that the management allowed him complete freedom to play with the latest “toys and gadgets”, pleasures that he has carried from his childhood. I remember that, as a child, I too was fascinated by mechanical and electrical gadgets, and the precision with which they were engineered. I think many of us take to experimental research precisely for this reason, that it allows us to take our childhood pleasures of playing with toys into adulthood, and even make a living out of this enjoyment! I cannot think of a greater advertisement for the young readers of this journal to take up a fulfilling career in research.





## Nobel Laureates

photo J. Reed  
Roy Glauberphoto Sears P. Studio  
John Hallphoto F. M. Schmidt  
Theodor Hänsch

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## Suggested Reading

- [1] <http://nobelprize.org/physics/laureates/2005/info.pdf> and references therein.
- [2] C L Mehta, in *E C G Sudarshan: Selected Scientific Papers*, Ranjit Nair (Ed), Principia, Centre for Philosophy and Foundations of Science, New Delhi, 2006 (in press).

Errata: *Resonance*, Vol.11, No.2, February 2006.

**Page 90:** TIO – Solution to ‘On a Use of Normal Distribution’

Equations (10) and (11) should read as

$$B_n \approx \frac{1}{\sqrt{\pi}} \int_{-\pi/2}^{\pi/2} e^{-v^2} dv = \frac{\int_{-\pi/2}^{\pi/2} e^{-v^2} dv}{\int_{-\infty}^{\infty} e^{-v^2} dv}. \quad (10)$$

$$\frac{\int_{-\pi/2}^{\pi/2} e^{-v^2} dv}{\int_{-\infty}^{\infty} e^{-v^2} dv} \approx 0.97368, \quad (11)$$

**Page 99:** Classics – Suggested Reading [1] should read as

- [1] C E Shannon, A mathematical theory of communication, *Bell Sys. Tech. J.*, Vol.27, pp.379-423, July 1948.

