Deuchler & Weston

Transverse tests of I-beams & other structural shapes

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TRANSVERSE TESTS OF I-BEAMS AND OTHER STRUCTURAL SHAPES

BY

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AND

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THESIS

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June 1, 1910

This is to certify that the thesis prepared in the Department of Theoretical and Applied Mechanics by WALTER EDWARD DEUCHLER and FREDERICK WILLIAM WESTON entitled Transverse Tests of I-beams and Other Structural Shapes is approved by me as fulfilling this part of the requirements for the degree of Bachelor of Science in Civil Engineering.

H. F. Moore

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Introduction.

I-beams and built-up sections have long been used in structural work with but a slight experimental knowledge of their ultimate strengths and methods of failure. The elastic limits and ultimate strengths of such beams have been theoretically determined by the use of flexure formulas. Such strengths were considered developed when the extreme fiber stress as computed by the formula equaled the value for the elastic limit or ultimate stress as previously determined by tension tests on the material of the beam.

In recent experiments on deep I-beams, without lateral support, stresses have been obtained considerably less than those computed by the above method. This thesis, which consisted largely of tests on 8 inch I-beams, was undertaken to determine:

(1) Whether or not this variation from the usual theoretical values could be found

to exist in the smaller sizes of I-beams as well as the larger ones.

(2) The applicability of the most common flexure formulas and the validity of existing theories of flexure.

and to investigate:

(3) The effect of lateral support upon the elastic limit and maximum fiber stress developed in beams.

(4) The variation of the maximum fiber stress with the length of span of the beam.

Theory and Available Data

The amount of experimental data on transverse tests of I-beams is surprisingly small, and theoretical discussions of the subject are few in number. The available literature can be divided into two classes: first, such data as can be obtained from the various steel hand-books, and second, experimental data obtained for the purpose of determining the action of I-beams under transverse bending stresses.

Data of the first class are valuable in that they furnish the basis for determining the maximum permissible loads used in practice, and by comparison with the results of the beams tested, the validity of these data can be investigated for beams of the given dimensions. Theoretical discussions and data of the second class, with possibly two exceptions, are of little value for comparison with the results of this thesis, due to important differences in the testing, or the use of factors of which there is but little known at present.

Pencoyd Hand-Book.

Most of the steel hand-books provide for the reduction of the maximum allowable loads for beams with an increase in their lengths. The Pencoyd Hand-book states that by experiment it has been found that the allowable load should be reduced one-third when the distance between supports is eighty times the width of the flange. A larger reduction is subsequently recommended so that, with a length of seventy times the flange width, the load is but 50 percent of the maximum.

The following are the values recommended for the various lengths of beam:

TABLE I

BEAMS WITHOUT LATERAL SUPPORT

Length of beam	Proportion of tabular stress for greatest safe stress
20 times flange width	Whole tabular stress
30 " " "	$\frac{9}{10}$ " "
40 " " "	$\frac{8}{10}$ " "
50 " " "	$\frac{7}{10}$ " "
60 " " "	$\frac{6}{10}$ " "
70 " " "	$\frac{5}{10}$ " "

A. E. Guy.

Mr. Albert E. Guy performed a quite elaborate series of experiments on wooden test beams, from which tests he deduced laws for failure by buckling. These laws, save one, are not applicable to the tests of this thesis, and the value of the one that can be used is doubtful, due to mathematical inconsistencies in the deduction of the formula expressing it. This formula is

$$PL^2 = K$$

where P is the load, L the length of span, and K is a constant.

A. G. M. Michell.

Mr. A. G. M. Michell, in an article in the Philosophical Magazine, deduced a mathematical formula for the resistance of beams based on the theory that the failure of such beams may be due to a want of torsional rather than flexural rigidity. This formula will here be of little value for comparative purposes, due to the use of a factor representing the

torsional rigidity, the value of which is at present not available for the beams tested.

Marburg.

Professor Marburg of the University of Pennsylvania conducted a series of tests on Bethlehem Special and Standard I-beams to determine their relative strengths and to investigate the veracity of claims for superior economy of material in the first named beams. The Bethlehem Specials have a thinner web and a wider flange than have the Standard beams and are approximately ten per-cent lighter for the same section moduli. The experiments showed that the standard beams failed almost uniformly by buckling due to lateral weakness of the compression flange, while the failure of the Bethlehem Specials was caused by "a sudden and considerable twisting of the web,----- the beam as a whole assuming the form of a very flat letter S, the flange remaining comparatively straight."

It was also found that the maximum stress developed by the beams was much smaller than the

theoretical one. Since no side restraint was used in this testing, the question arose as to how much such restraint would increase the elastic limits and ultimate strengths of such beams.

Another important feature of these experiments was the variation in the character of the metal of the beams. Tension tests of specimens taken from the root of the flange showed uniformly low elastic limits, while pieces from the flange and web developed strengths more nearly equal to the average resistances expected.

Tests on beams centrally loaded gave moduli of rupture approximately 17 percent higher than those on beams loaded at the quarter points. This tends to verify the assumption that the latter conditions are the more severe and would suggest a reduction of the constant used in the transformation of the hand-book values, from 0.375 to 0.300.

H. D. Hess

Mr. H. D. Hess, in a paper presented before the Engineers' Club of Philadelphia, develops a formula showing the variation of the strengths of standard beams with their lengths. Rankine's column formula is used as a basis in deducing this expression, the force in the compression flange being assumed as acting along the axis of the flange. Due to the fact that this force varies from zero at the ends to a maximum at the center, a length of span is mathematically deduced such that the combination of the buckling effect due to this length and the stress occurring at its end sections gives a maximum fiber stress upon substitution in the formula.

This formula is deduced for uniform loading by Mr. Hess, but to obtain comparison with the experiments of this thesis the formula must be deduced for third point loadings. This is deduced as follows:

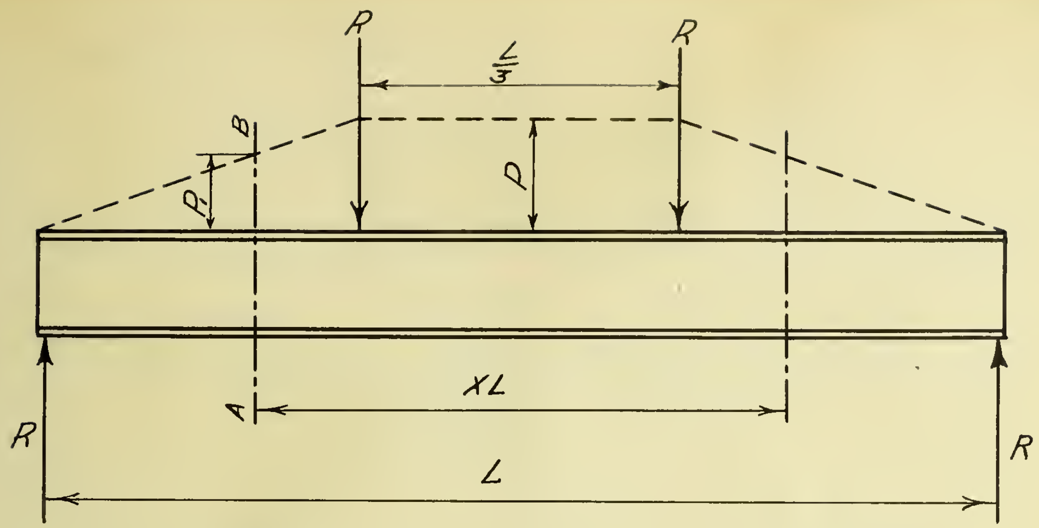


FIG. 1 DERIVATION OF HESS FORMULA FOR $\frac{1}{3}$ POINT LOADING

$$\frac{P_1}{P} = \frac{\frac{L}{2} - \frac{xL}{2}}{\frac{L}{3}}$$

$$P_1 = \frac{3}{2} P(1-x) \qquad (1.)$$

From Rankine's column formula

$$P_1 = \frac{P_c}{1+k\left(\frac{l}{r}\right)^2} \qquad (2)$$

here P_c = maximum stress in column due to P_1 as compressive load
P_1 = extreme fiber stress at any section A-B
k = constant depending upon formula used, material etc.
l = length of column considered.
r = radius of gyration of flange about axis at right angles to flange face

To find the length XL which when acted upon by a corresponding stress P_1 will produce a max. increase in the fiber-stress at the center.

$$P_1 + P_1 k\left(\frac{l}{r}\right)^2 = P_c$$

$$P_c - P_1 = P_1 k\left(\frac{l}{r}\right)^2 \qquad (3)$$

Combining (1) and (3) and $xL = l$

$$P_c - P_1 = \frac{3}{2} Pk (1-x) \left(\frac{xL}{r}\right)^2$$

$$= \frac{3}{2} Pk (x^2 - x^3) \left(\frac{L}{r}\right)^2 \qquad (4)$$

Differentiating, $\frac{d(P_c - P_1)}{dx} = 2 \cdot \frac{3}{2} Pk \left(\frac{L}{r}\right)^2 - \frac{3}{2} Pk \left(\frac{L}{r}\right)^2 3x = 0$

$$x = \frac{3Pk\left(\frac{L}{r}\right)^2}{\frac{9}{2} Pk \left(\frac{L}{r}\right)^2} \qquad x = \frac{2}{3} \qquad (5)$$

Total maximum fiber-stress at center of span

$$P_T = P_c - P_1 + P \qquad (6)$$

$$P_T = \frac{3}{2} Pk \times \frac{4}{27} \left(\frac{L}{r}\right)^2 + P$$ Combining (4), (5), and (6)

$$\frac{P_T}{P} = \frac{2}{9} k \left(\frac{L}{r}\right)^2 + 1$$ One flange resisting buckling

$$\frac{P_T}{P} = \frac{1}{9} k \left(\frac{L}{r}\right)^2 + 1$$ Both flanges resisting buckling

Since $k = \frac{4}{25000}$ (Mechanics of Materials - MERRIMAN)
and r for 8 in. 18 lb. I-Beam = 0.84 in.

$$\frac{P}{P_T} = \frac{1}{1 + 0.0000505 L^2}$$ Considering one flange acting

$$\frac{P}{P_T} = \frac{1}{1 + 0.0000253 L^2}$$ Considering both flanges acting

LITERATURE

GUY — Flexure of Beams.
(Van Nostrand)

MICHELL — Philosophical Magazine.
Vol. 48, p.p. 298 (Sept. 1899)

MARBURG — Proceedings of the American Society.
for Testing Materials. – (1909) - Vol. 9, p.p. 378

HESS — Proceedings of the Engineer's Club.
of Philadelphia - Vol. 26, p.p. 106 (April, 1909)

Materials, Apparatus, and Tests.

Seventeen tests in all were made. In eleven of these, standard I-beams were used. All these I-beams were of 8 inch depth; two were 25.50 lb. sections and the rest were 18.00 lb. sections. The other six tests were on built-up members of a discarded wrought-iron bridge furnished by the Wabash R. R. The Schedule of Beams, Table II, gives the lengths, sections, and other important data concerning the I-beams. The Schedule of Tests, Table III, gives an outline of the tests on the I-beams, including the sections, spans, and special methods used.

The span varied from five to twenty feet. The beams were tested under two conditions, with and without lateral support. In all cases the ends were unrestrained. Several methods of lateral support were used. In Test 3, for a 10 ft. span, two 8 inch, 18 lb., I-beams were hitched together by three standard cast-iron separators. In Test 13 the beam was prevented from moving laterally by means of blocks placed a-

gainst the edges of the flanges and fastened to the table of the testing machine. For the wrought-iron sections the effect of the removal of end stiffeners from the girders, and the latticing from posts, tested as beams, was investigated.

In all the tests, the loads were applied at the third points. For practically all the I-beam testing, an Olsen beam testing machine, 200,000 lb., capacity, was used. For these beams this machine was large enough since 56 000 lb. was the maximum load reached. The load was distributed to the proper points of application by means of a short beam of sufficient depth and section, bearing on rollers, spherical blocks, and narrow plates. The beams were supported at their ends by the ordinary rocking supporting blocks, spherical blocks and seats, or rollers and plates. Figs. 5, 6, 9, and 10 show the method of loading the beam.

Soon after the series of tests was commenced a system for applying the deflectometers and deformation indicating devices was developed. Fourteen instruments were used and read for each load. The readings were numbered from one to fourteen in the log-book. The lateral movements of the upper

and lower flanges were observed at or near the ends and quarter points by means of eight Ames dials. Fig. 5 shows the arrangement and numbering of these dials. Fig. 6 illustrates the method of supporting them. The linear longitudinal deformation of the upper and lower flanges, for a certain gauge length, was observed at the middle of the beam by four Ames dials, arranged and attached as shown in the figures. The deflections of the beams at the middle were obtained by using two wire-wound dials supported by clamps to two angle irons which rested on rods passing through the neutral axis directly above the axes of support.

The Ames dials were found to be unsatisfactory for measuring the lateral movements of the flange, because of their small range and the many readjustments required. For the last few tests a device was used which is illustrated in Fig. 7. It consisted of a radial pointer moved by a wire connected to the flange and passing over a small drum mounted on the axle of the pointer.

One thousand pounds was taken as the zero loading, and all zero readings were taken at that load. The load was applied to the beam repeatedly

and when removed, was always brought down to the zero loading. An effort was made to increase the load each time, in such a manner that the increments of increase in deflection would be equal. When it was thought that the load approached the ultimate, all the instruments were removed, and the beam tested to failure. A curve, plotted during the progress of the test, aided in judging the time at which to remove the instruments.

After the beams were tested, specimens for tension tests were cut from the least-stressed portion of the beams. Three test bars were taken from each beam, from the region shown in Fig. 2, and marked as shown. These specimens were tested for yield point and ultimate strength in tension. The results show that the steel was of good quality and there was no marked difference in the distribution of strength over the section.

TABLE II

SCHEDULE OF I-BEAMS TESTED.

Test Number	Depth in.	Weight lb.	Area sq. in.	$\frac{I}{c}$	Gross Length ft.	Remarks
1	8	18.00	5.33	14.2	11.0	Coated with a cement wash
2	8	18.00	5.33	14.2	11.0	Coated with white enamel
3	8	36.00	10.66	28.4	11.0	Two beams hitched together by three separators.
13	8	18.00	5.33	14.2	11.0	
9	8	18.00	5.33	14.2	5.5	
12	8	18.00	5.33	14.2	5.5	
14	8	18.00	5.33	14.2	8.0	
15	8	18.00	5.33	14.2	8.0	
6	8	18.00	5.33	14.2	21.0	
5	8	25.50	7.50	17.1	11.0	
7	8	25.50	7.50	17.1	11.0	

TABLE III

SCHEDULE OF TESTS ON I-BEAMS.

Test №	Section of I-Beam	Length of Span ft.	Conditions of Lateral Support	Ultimate Load lb	Method of Failure	Remarks
1	8 in. 18 lb.	10	None	27 000	Noticeable side-thrust	Beam was cement washed
2	8 in. 18 lb.	10	None	23 300	Failed by bending sidewise with slight twist of web. Web bent over in direction of lateral movement of center.	Beam was painted with white enamel.
3	2 - 8 in. 18 lb.	10	Two beams hitched together by three standard cast-iron separators	54 200	Bowed straight down	Painted with white enamel
13	8 in. 18 lb.	10	Flanges restrained laterally by blocks fastened to the table of the machine	29 100	Bowed straight down	600 000 lb. testing machine used. Load applied at ends, beam supported at third points. Upper flange in tension.
9	8 in - 18 lb.	5	None	54 550	Compression flange bowed. Web concaved. Local failure under load. Flange bent down.	100,000 lb. machine used
12	8 in. 18 lb.	5	None	53 740	Flange crippled. Web twisted.	
14	8 in. 18 lb.	7.5	None	34 500	Side wise buckling fol-	
15	8 in. 18 lb.	7.5	None	34 800	lowed by twist	
6	8 in. 18 lb.	20	None	10 000		
5	8 in. 25 lb.	10	None	33 830	Buckled and twisted	
7	8 in. 25 lb.	10	None	31 000		

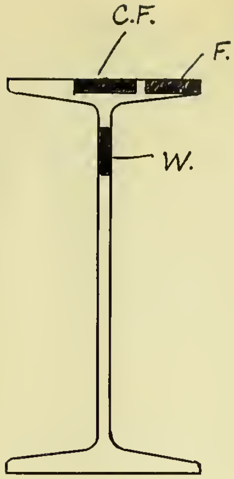


FIG. 2

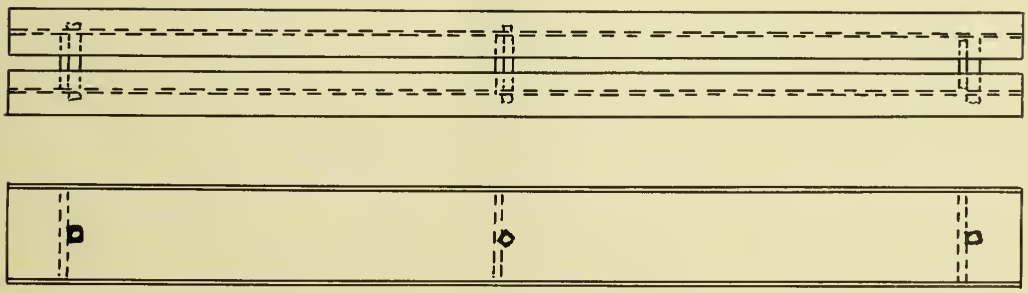


FIG. 3

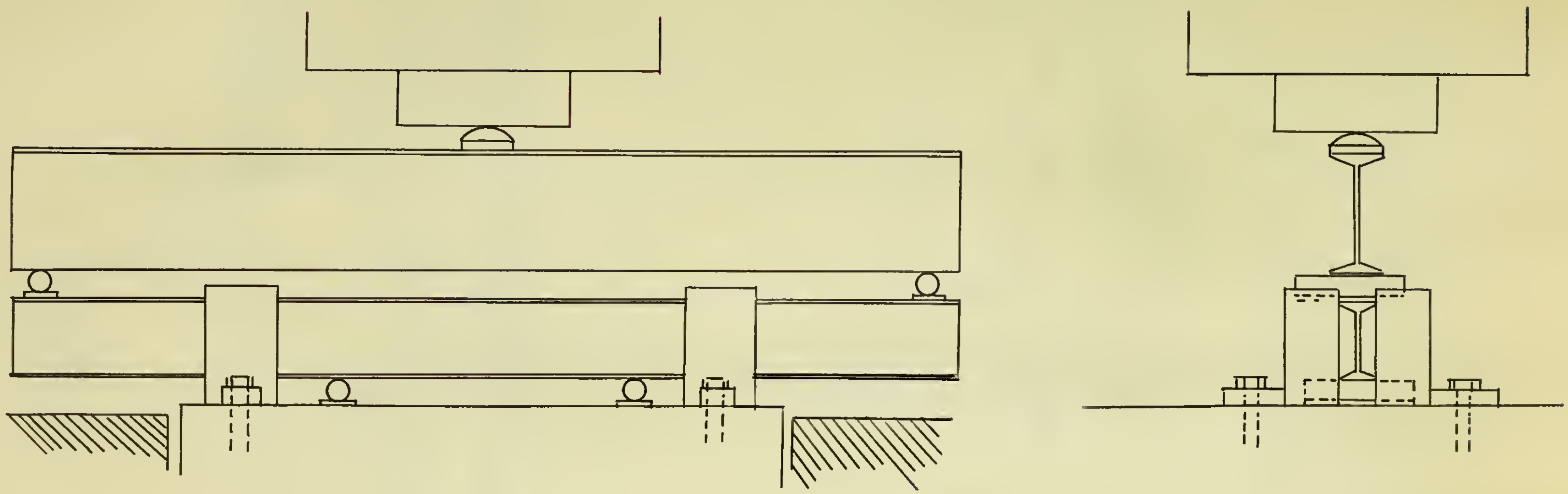


FIG. 4

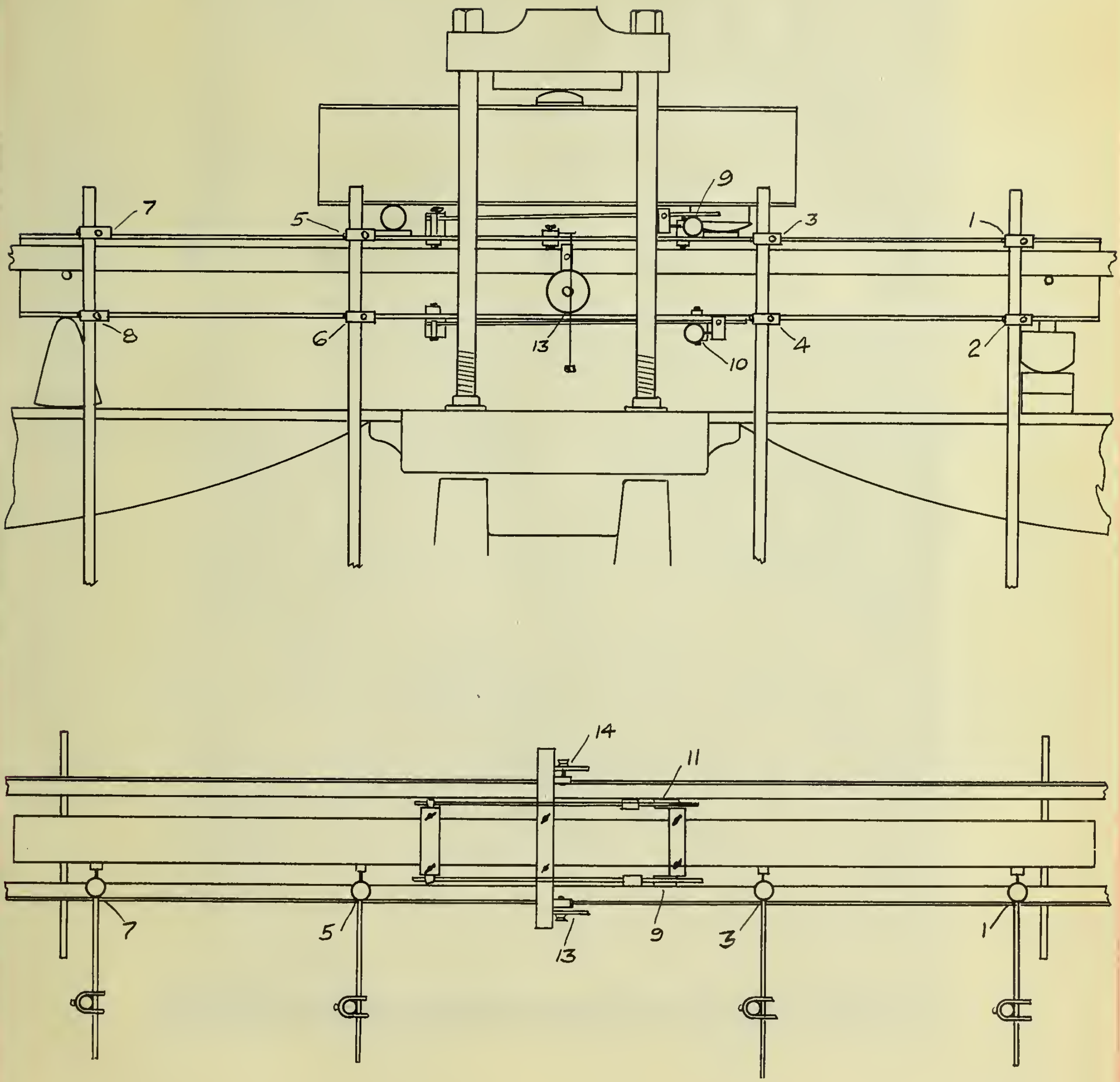


FIG. 5.

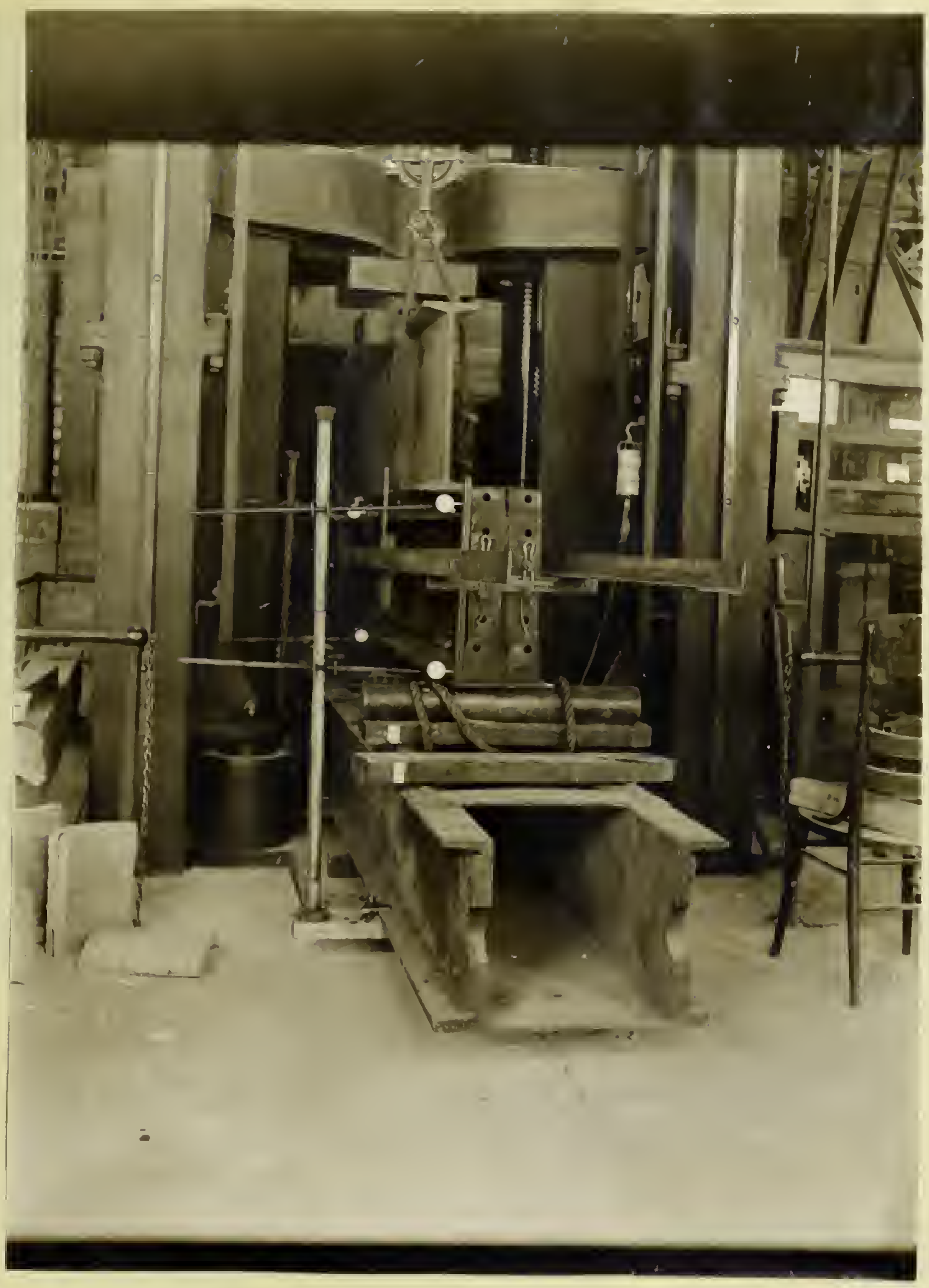


FIG. 6

FIG. 7

FIG. 8

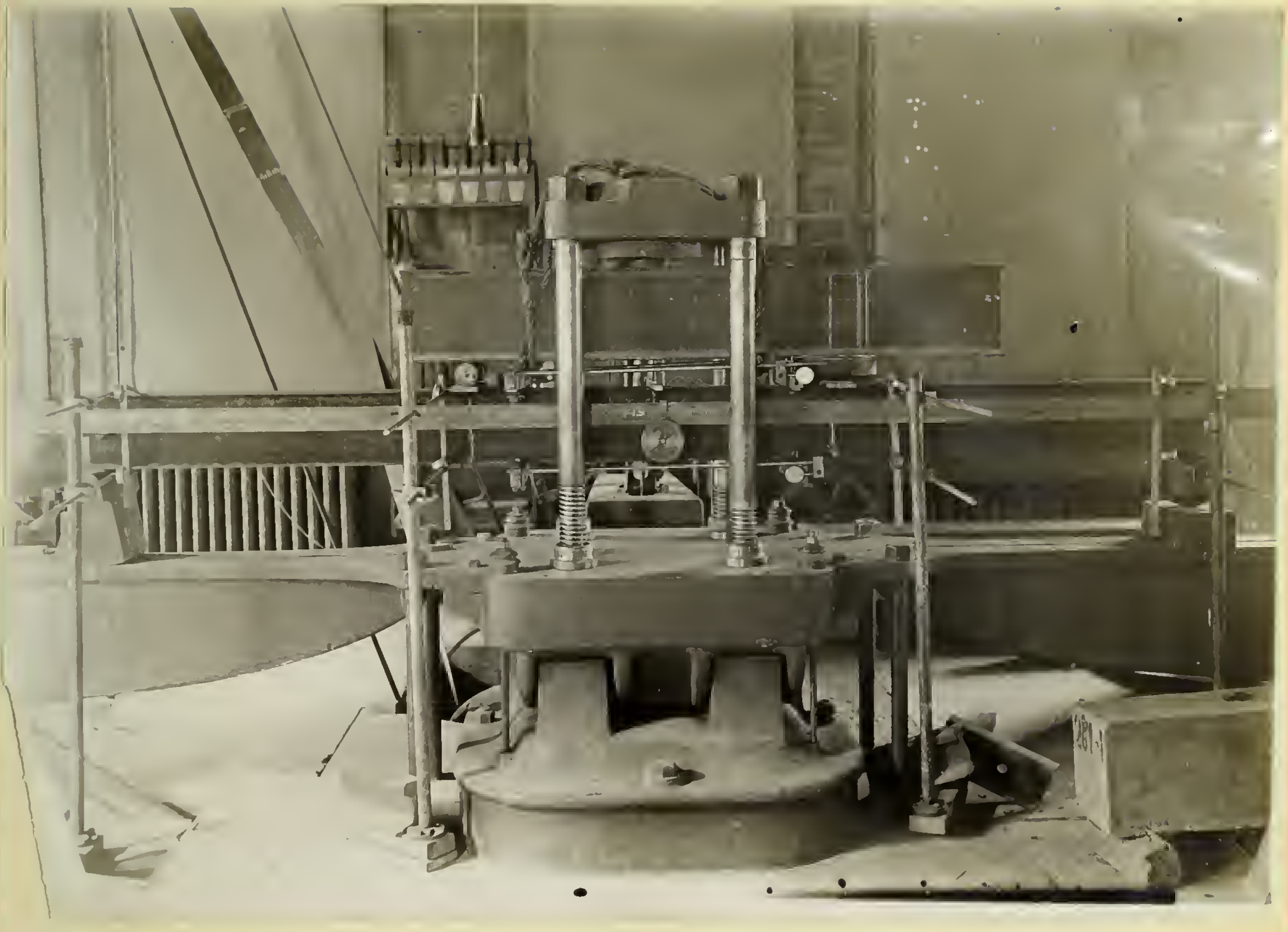


FIG. 9

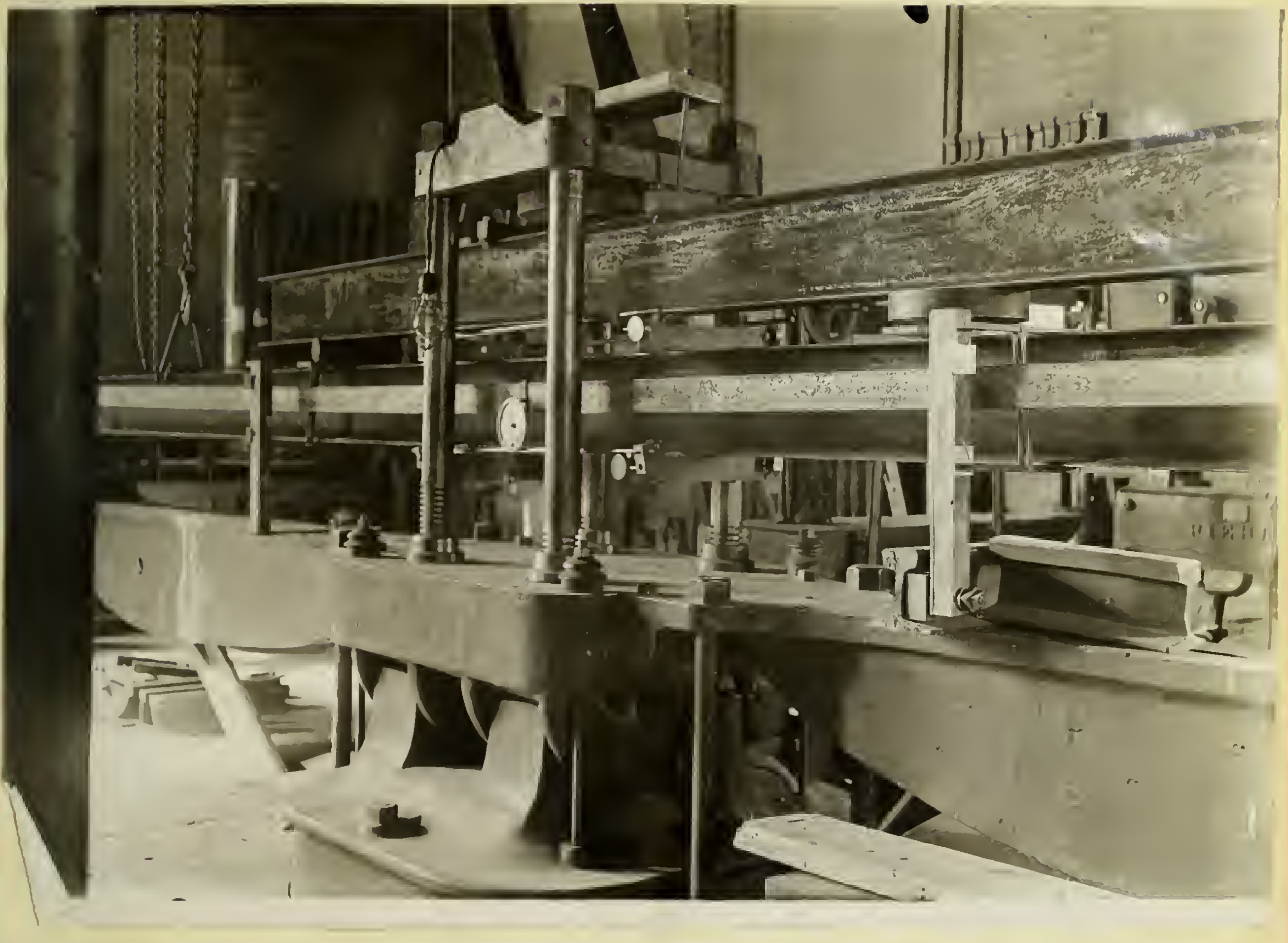


FIG. 10

TABLE IV

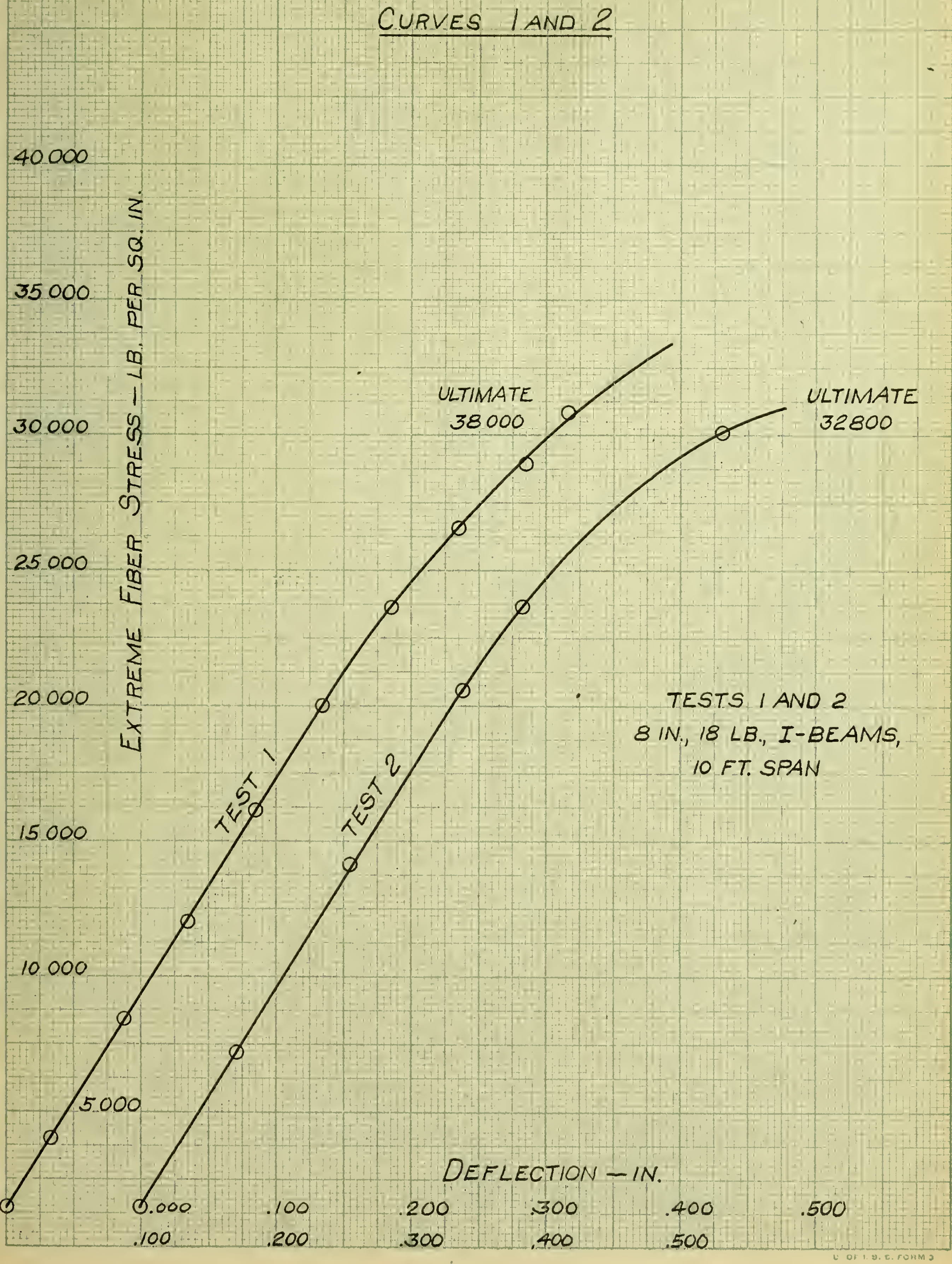
EXTREME FIBER STRESSES AS DETERMINED BY FORMULA $S = \frac{Mc}{I}$

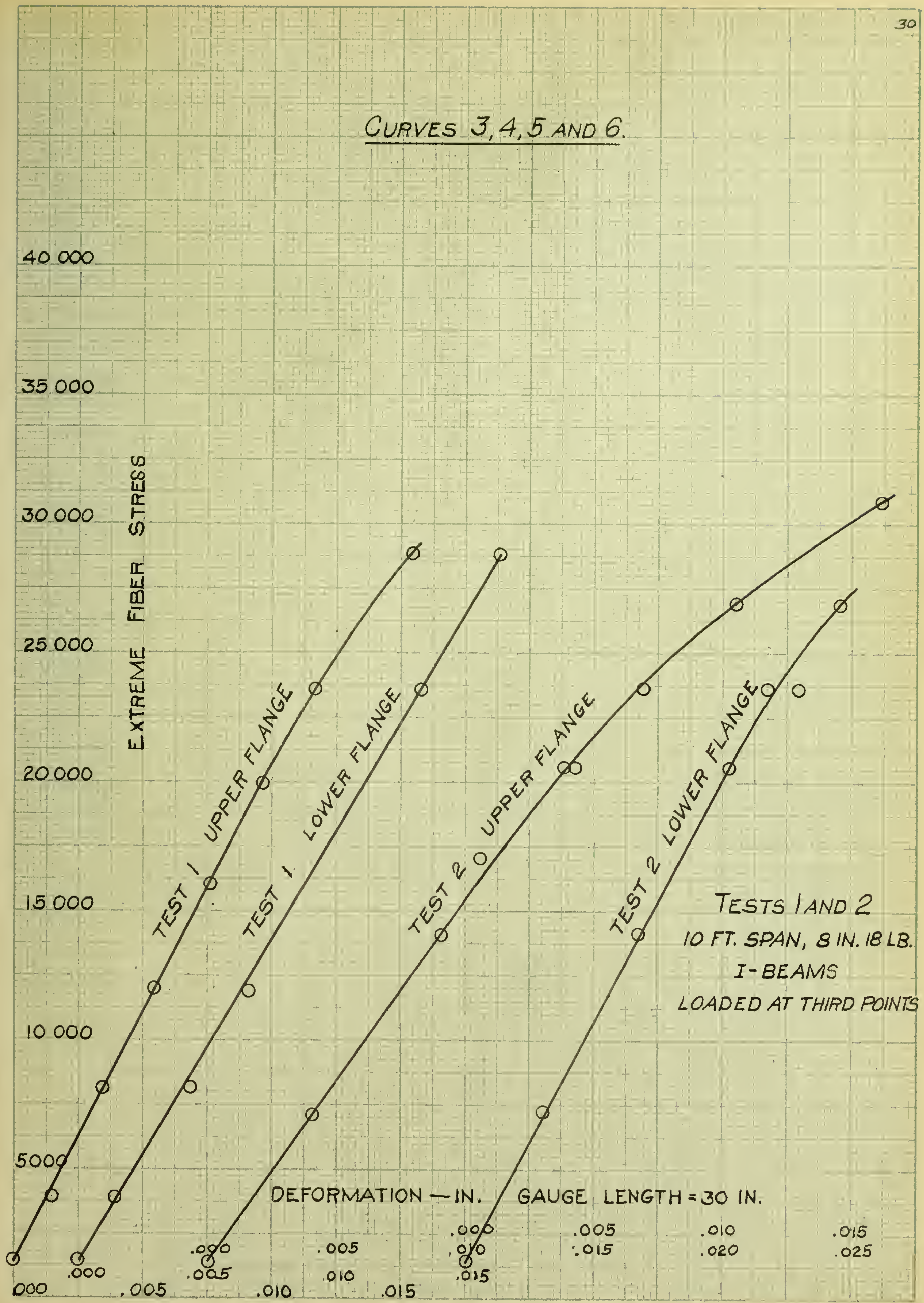
Test Nº	Span ft.	Section of I-Beam	Extreme fiber stress – lb. per sq. in.				Remarks
			Yield Point		Ultimate		
				Average		Average	
1	10	8in. 18lb.	22000	21500	38 000	35400	
2	10	8in. 18lb.	21000		32 800		
13	10	8in. 18lb.	35 500	35500	40000	40000	Web restrained laterally
3	10	2-8in. 18lb.	25000	25000	38 200	38 200	Two beams connected by three separators
9	5	8in. 18lb.	30000		38400	38120	
12	5	8in. 18lb.	Indefinite		37850		
14	7.5	8in. 18lb.	29000	28500	36 500	36650	
15	7.5	8in. 18lb.	28 000		36 800		
6	20	8in. 18lb.	21000	21000	28140	28140	
5	10	8in. 25.5lb.	15000	18500	39 800	38150	
7	10	8in. 25.5lb.	22000		36 500		

TABLE V

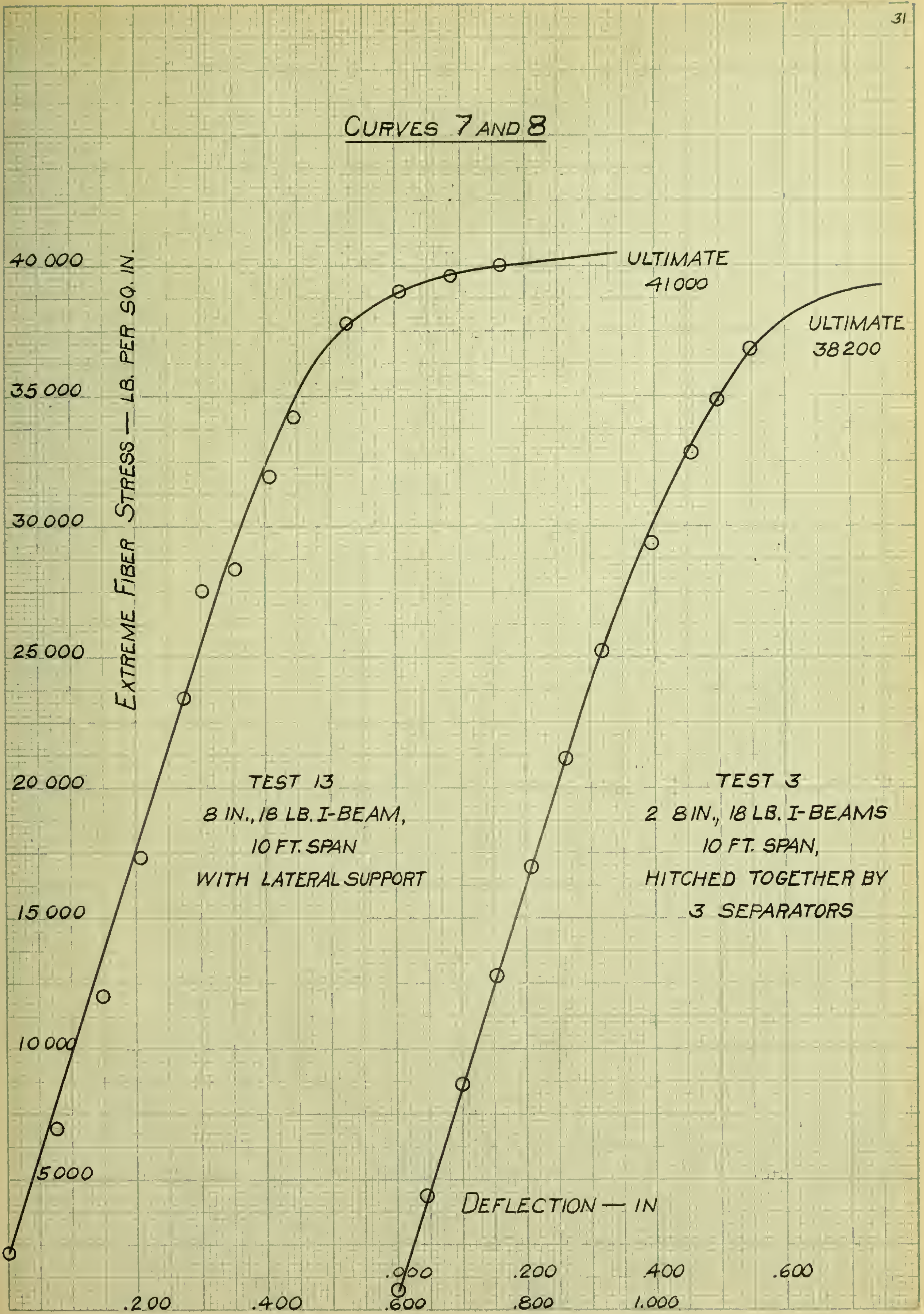
YIELD-POINTS AND ULTIMATE STRENGTHS AS DETERMINED BY TENSION TESTS

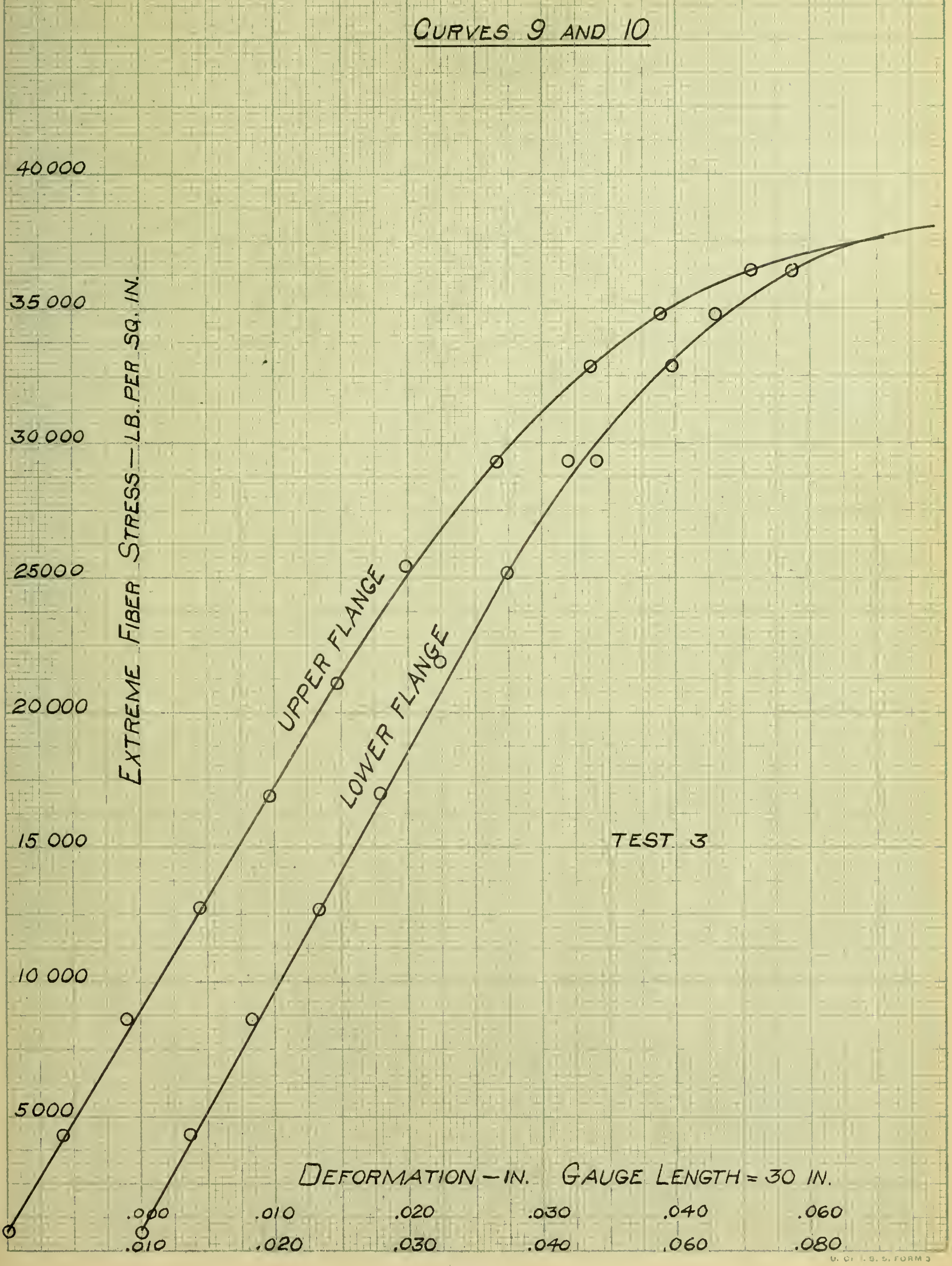
Mark	Stress - lb. per sq. in. Yield Point	Stress - lb. per sq. in. Ultimate	Remarks	Modulus of Rupture of Beam in bending
1 CF	32800	61200		38000
1 F	33800	60900		
1 W	39700	65900		
2 CF	33300	59700		32800
2 F	34000	58300		
2 W	35100	55100		
3 ACF	32000	58300	Two beams connected by three cast-iron separators	38200
3 AF	30200	51300		
3 AW	33100	56400		
3 BCF	32200	60200		
3 BF	34600	62700		
3 BW	39900	66500		
5 CF	38200	66000		39800
5 F	42200	69800		
5 W	44500	79800		
6 CF	36400	64300		28140
6 F	46200	64500		
6 W	39800	62000		
7 CF	31300	59000		36500
7 F	46800 ?	59300		
7 W	31900	61000		
9 CF	35800	58000		38400
9 F	37000	59300		
9 W	34400	55200		
12 CF	34800	58000		37840
12 F	37100	59100		
12 W	35200	55900		
13 CF	32000	58000		41000
13 F	34600	56500		
13 W	34700	57600		

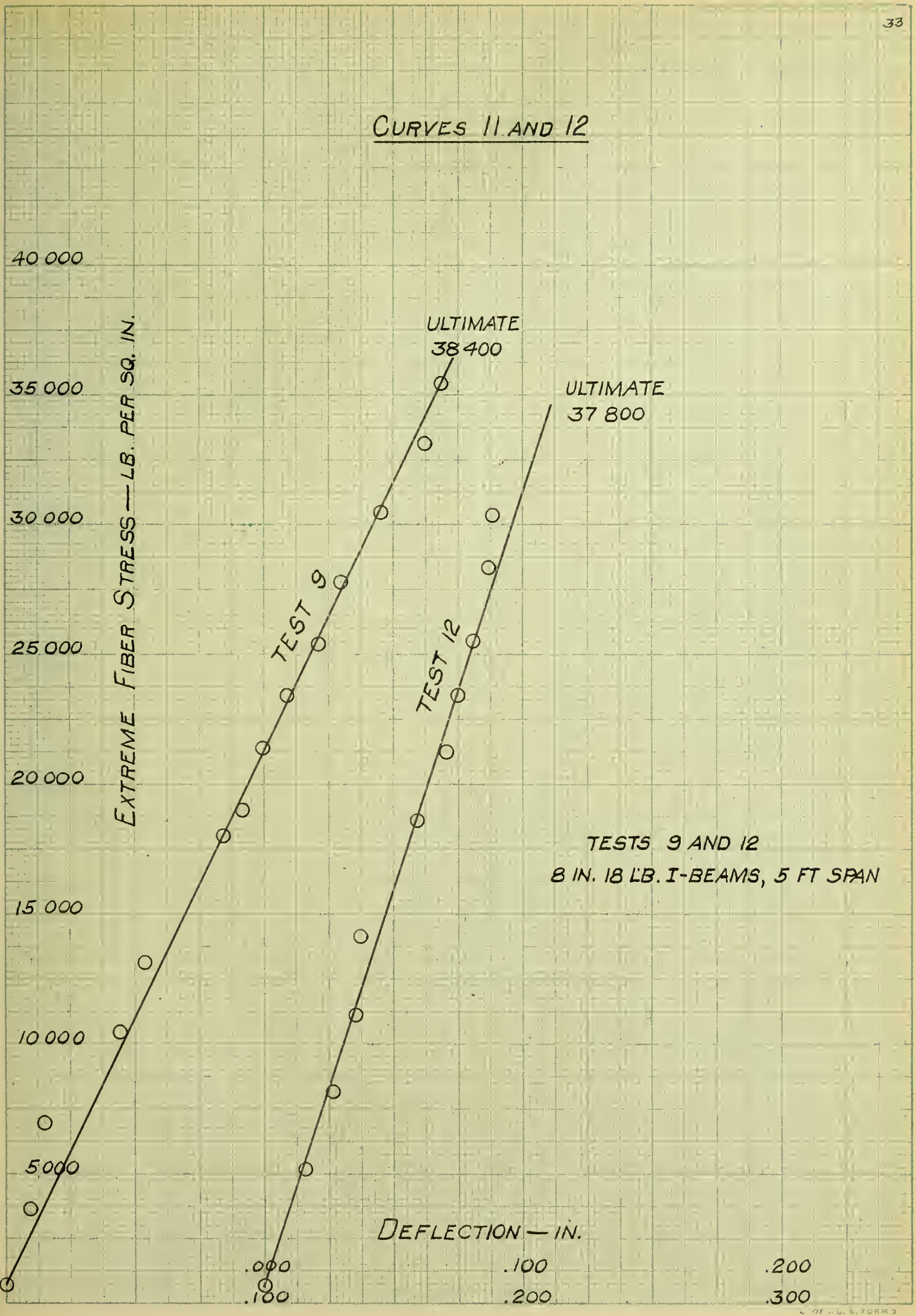

CURVES 1 AND 2
EXTREME FIBER STRESS — LB. PER SQ. IN.
40.000
35.000
30.000
25.000
20.000
15.000
10.000
5.000
ULTIMATE
38.000
ULTIMATE
32800
TEST 1
TEST 2
TESTS 1 AND 2
8 IN., 18 LB., I-BEAMS,
10 FT. SPAN
DEFLECTION — IN.
.000
.100
.200
.300
.400
.500
.100
.200
.300
.400
.500

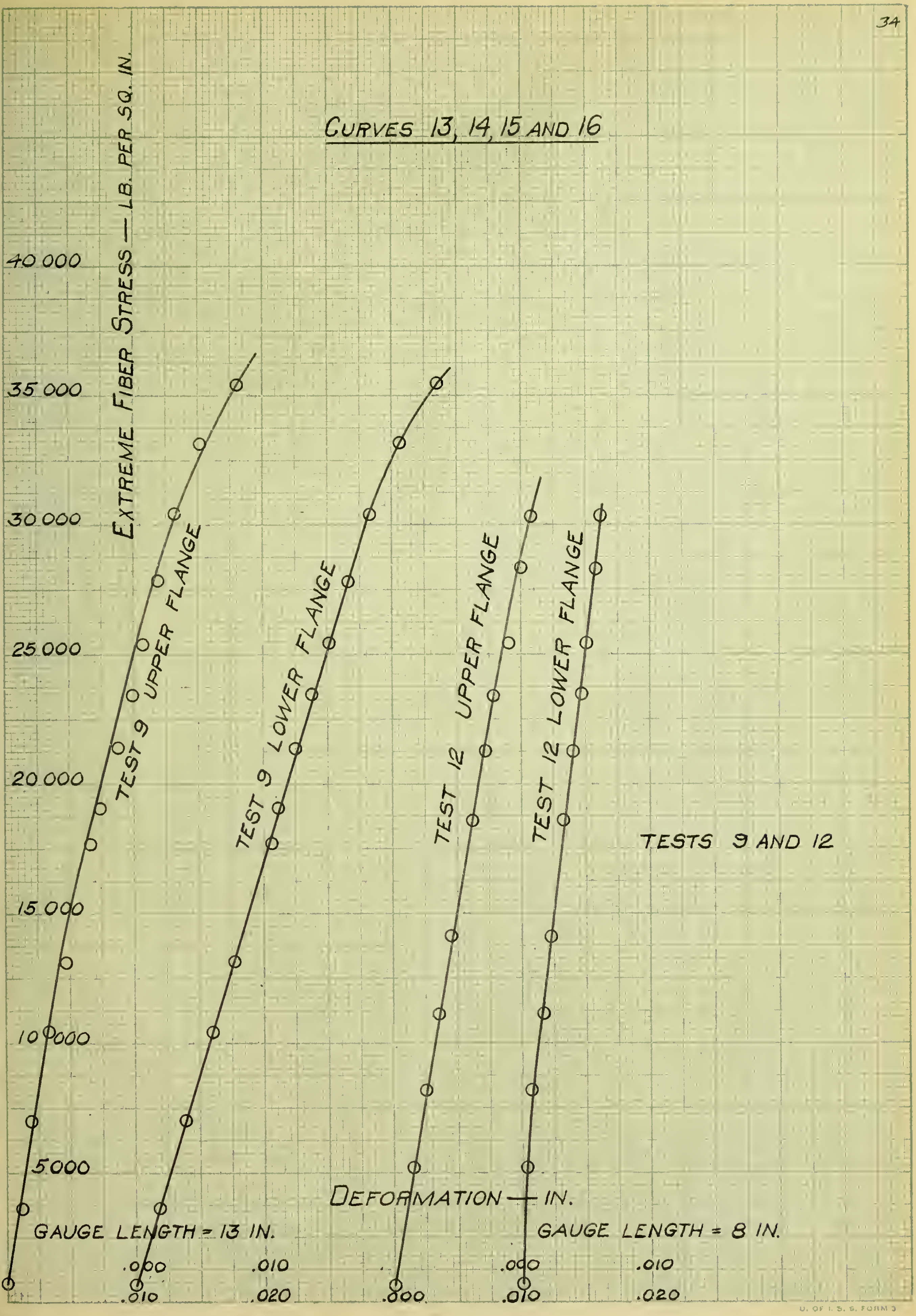


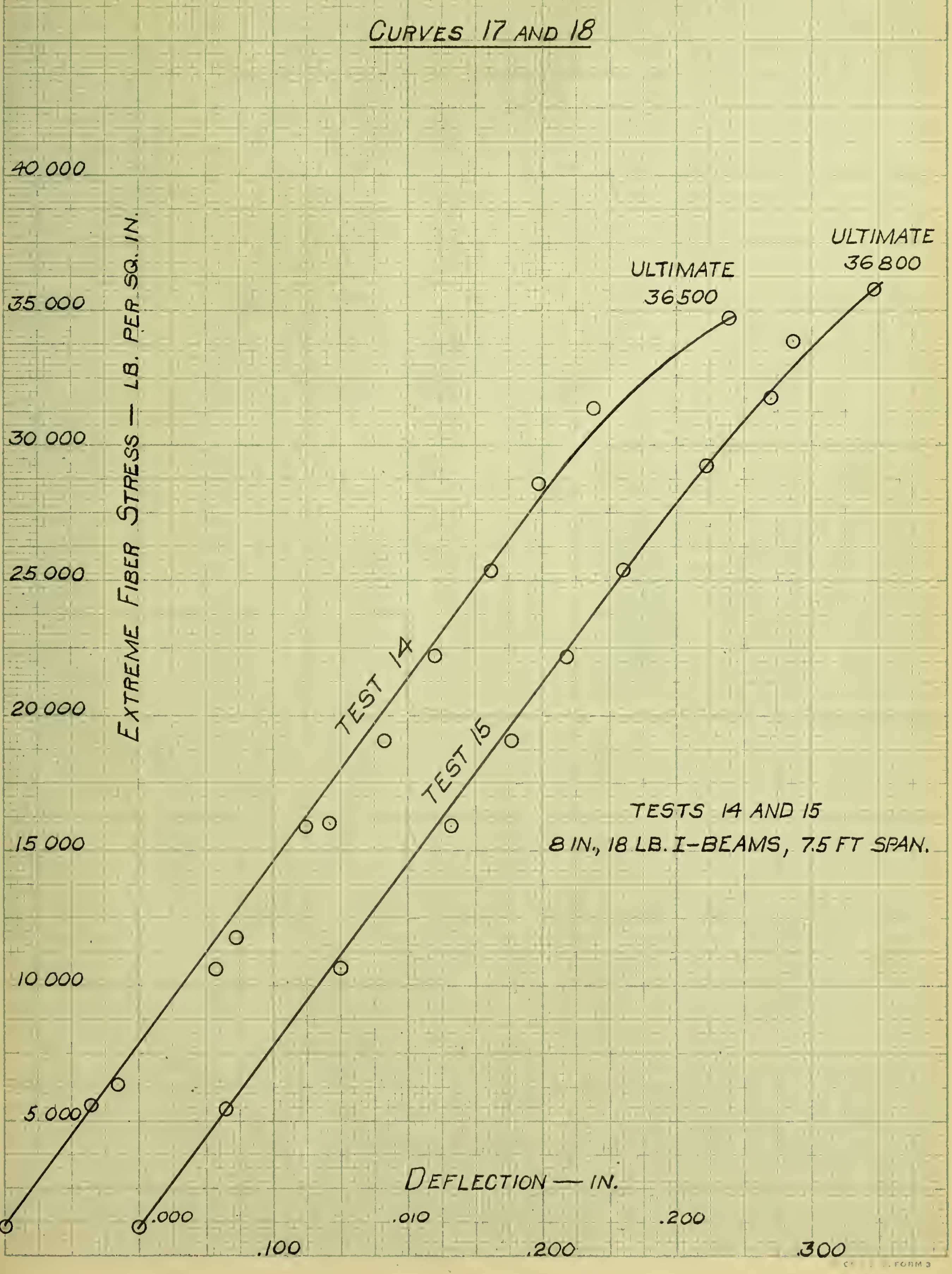
CURVES 3, 4, 5 AND 6.
40 000
35 000
30 000
25 000
20 000
15 000
10 000
5000
EXTREME FIBER STRESS
TEST 1 UPPER FLANGE
TEST 1. LOWER FLANGE
TEST 2 UPPER FLANGE
TEST 2 LOWER FLANGE
TESTS 1 AND 2
10 FT. SPAN, 8 IN. 18 LB.
I-BEAMS
LOADED AT THIRD POINTS
DEFORMATION — IN. GAUGE LENGTH = 30 IN.
.000 .005 .010 .015
.000 .005 .010 .015
.000 .005 .010 .015
.005 .010 .015 .020 .025


CURVES 7 AND 8
EXTREME FIBER STRESS — LB. PER SQ. IN.
40 000
35 000
30 000
25 000
20 000
15 000
10 000
5000
ULTIMATE
41000
ULTIMATE
38200
TEST 13
8 IN., 18 LB. I-BEAM,
10 FT. SPAN
WITH LATERAL SUPPORT
TEST 3
2 8 IN., 18 LB. I-BEAMS
10 FT. SPAN,
HITCHED TOGETHER BY
3 SEPARATORS
DEFLECTION — IN
.000
.200
.400
.600
.200
.400
.600
.800
1.000

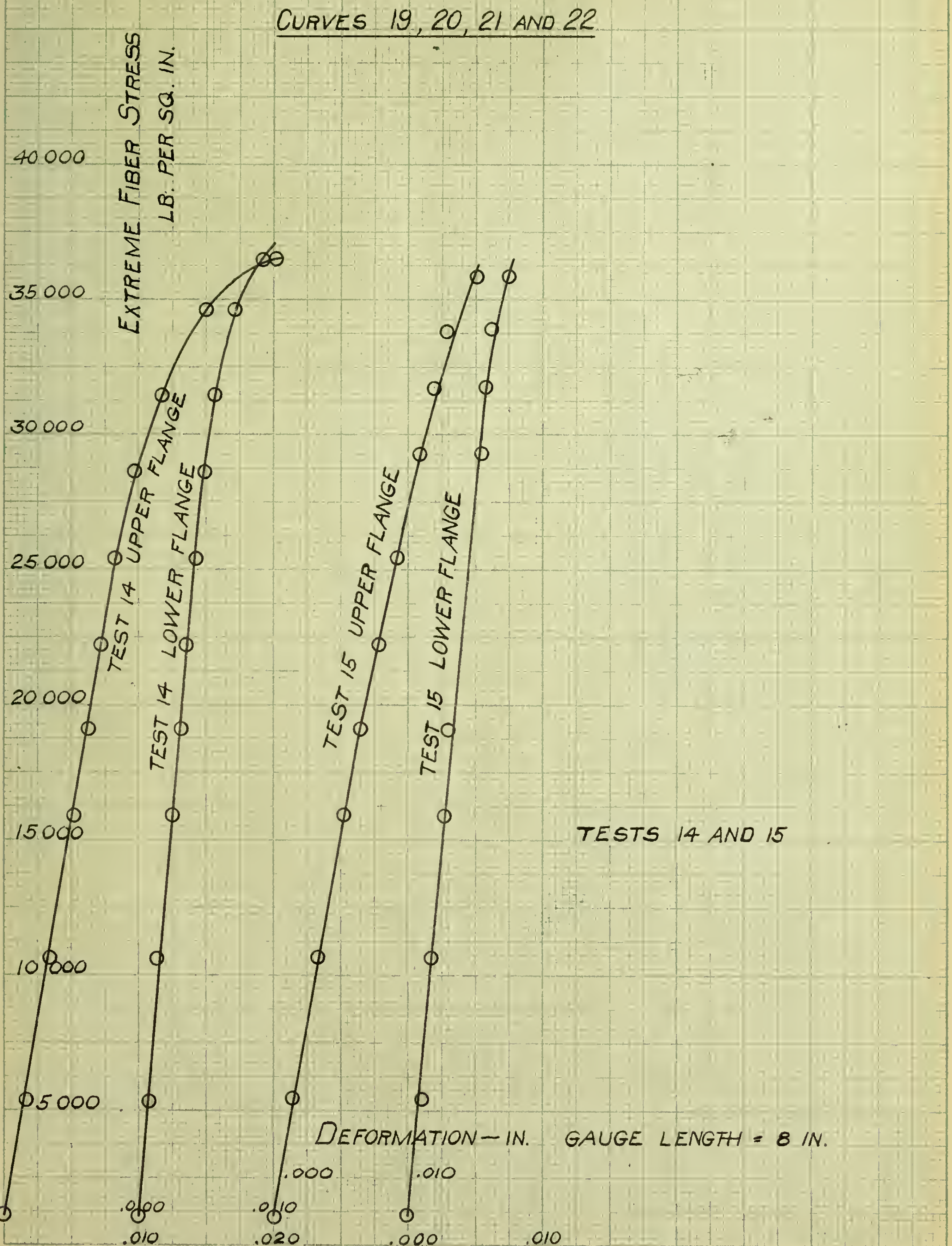

CURVES 9 AND 10
40 000
35 000
30 000
25000
20 000
15 000
10 000
5000
EXTREME FIBER STRESS—LB. PER SQ. IN.
UPPER FLANGE
LOWER FLANGE
TEST 3
DEFORMATION—IN. GAUGE LENGTH = 30 IN.
.000
.010
.020
.030
.040
.060
.010
.020
.030
.040
.060
.080

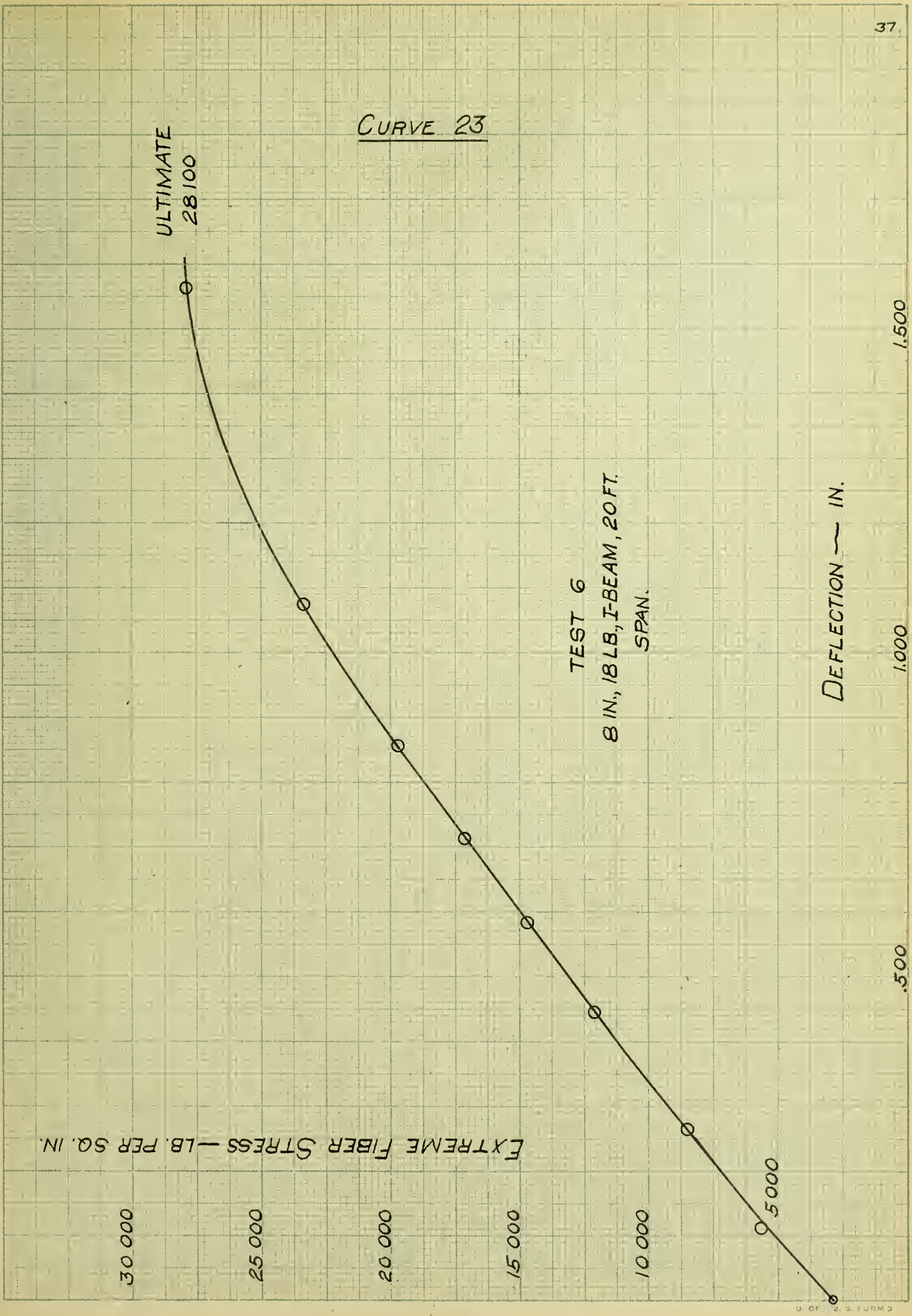

CURVES 11 AND 12
40 000
35 000
30 000
25 000
20 000
15 000
10 000
5 000
EXTREME FIBER STRESS—LB. PER SQ. IN.
ULTIMATE
38 400
ULTIMATE
37 800
TEST 9
TEST 12
TESTS 9 AND 12
8 IN. 18 LB. I-BEAMS, 5 FT SPAN
DEFLECTION—IN.
.000
.100
.200
.100
.200
.300

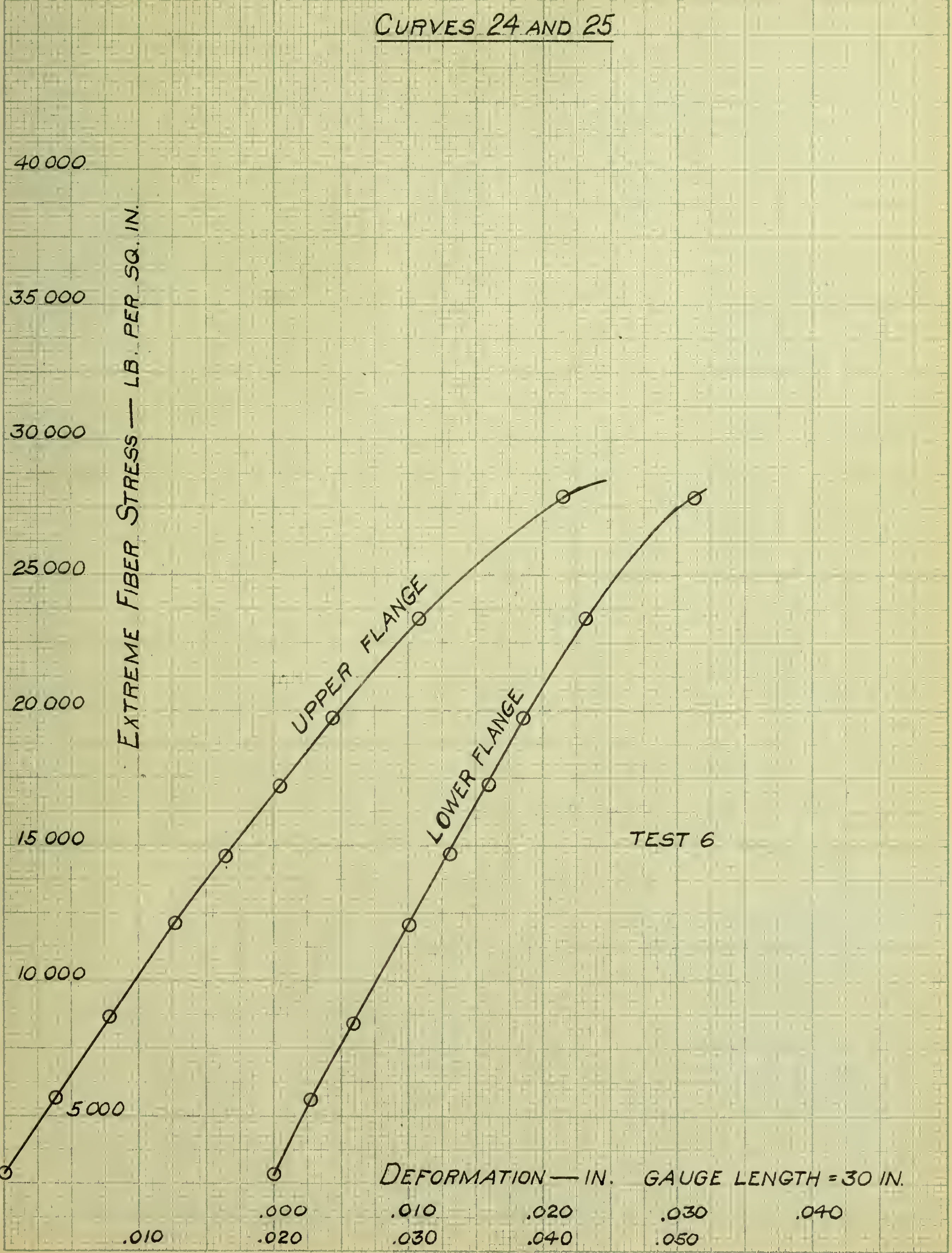

CURVES 13, 14, 15 AND 16
EXTREME FIBER STRESS — LB. PER SQ. IN.
40 000
35 000
30 000
25 000
20 000
15 000
10 000
5000
TEST 9 UPPER FLANGE
TEST 9 LOWER FLANGE
TEST 12 UPPER FLANGE
TEST 12 LOWER FLANGE
TESTS 9 AND 12
DEFORMATION — IN.
GAUGE LENGTH = 13 IN.
GAUGE LENGTH = 8 IN.
.000
.010
.010
.020
.000
.000
.010
.010
.020

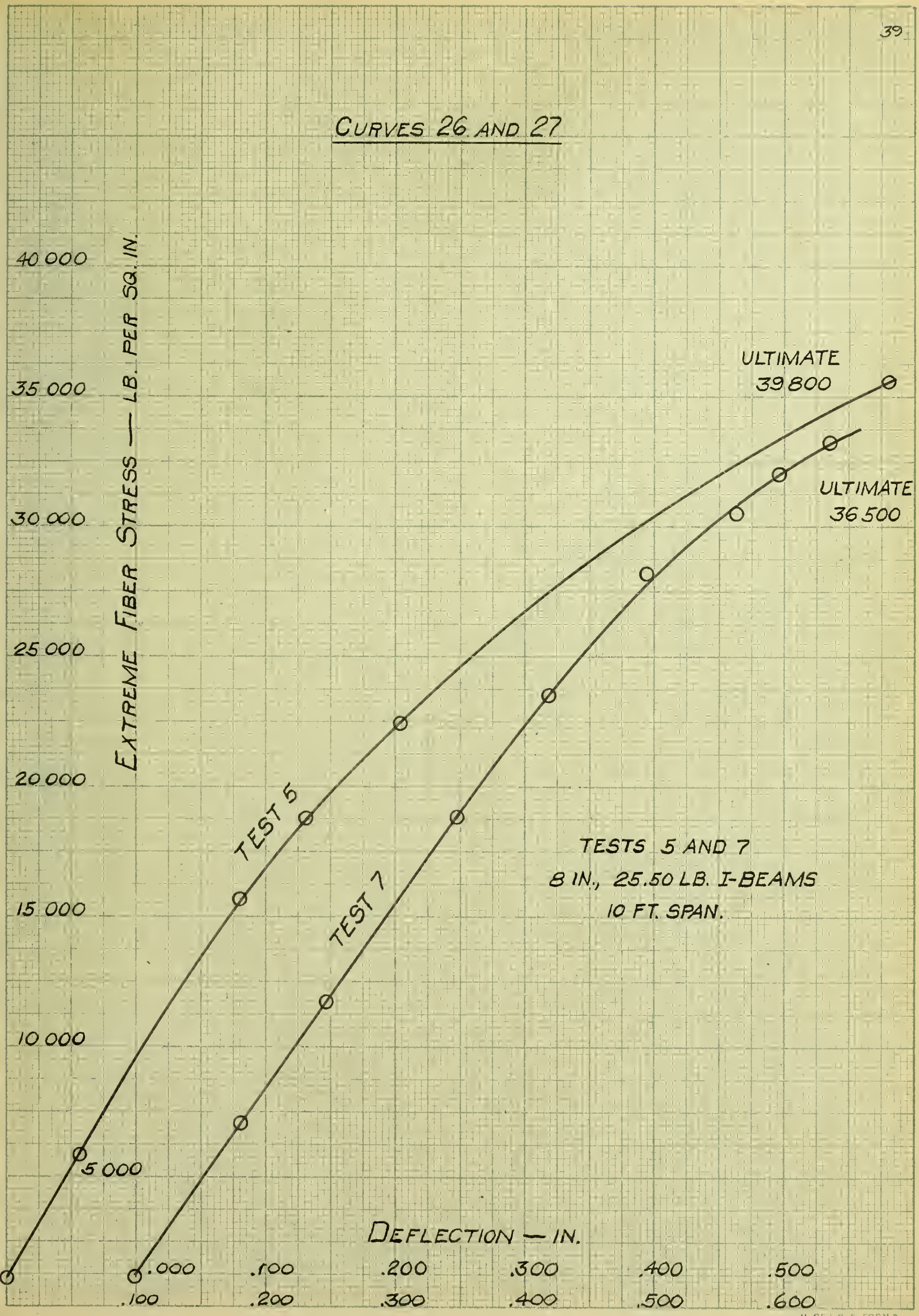


CURVES 17 AND 18
EXTREME FIBER STRESS — LB. PER SQ. IN.
40 000
35 000
30 000
25 000
20 000
15 000
10 000
5 000
ULTIMATE 36500
ULTIMATE 36800
TEST 14
TEST 15
TESTS 14 AND 15
8 IN., 18 LB. I-BEAMS, 7.5 FT SPAN.
DEFLECTION — IN.
.000
.010
.200
.100
.200
.300

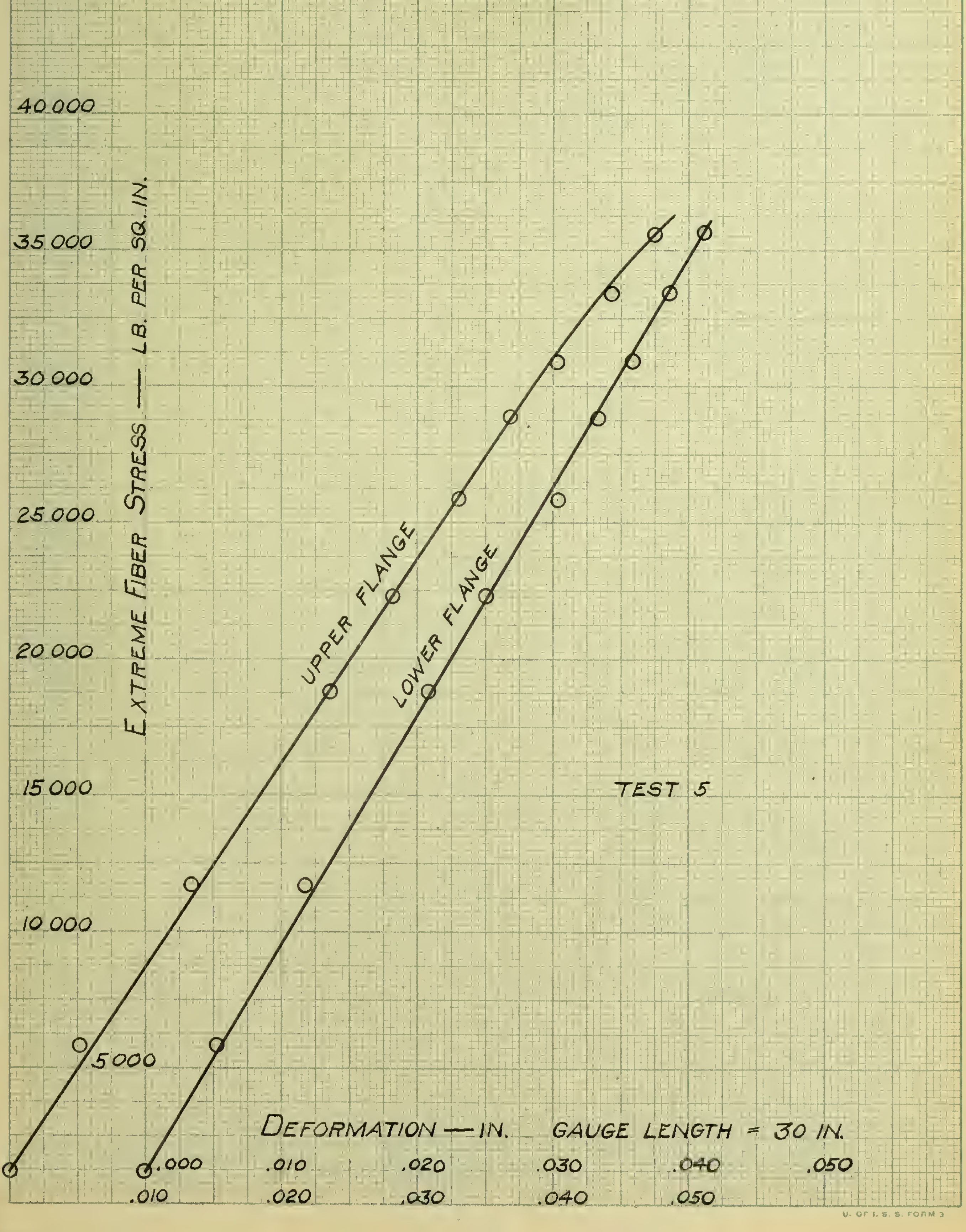

CURVES 19, 20, 21 AND 22
EXTREME FIBER STRESS
LB. PER SQ. IN.
40 000
35 000
30 000
25 000
20 000
15 000
10 000
5 000
TEST 14 UPPER FLANGE
TEST 14 LOWER FLANGE
TEST 15 UPPER FLANGE
TEST 15 LOWER FLANGE
TESTS 14 AND 15
DEFORMATION — IN. GAUGE LENGTH = 8 IN.
.000
.010
.000
.010
.010
.020
.000
.010

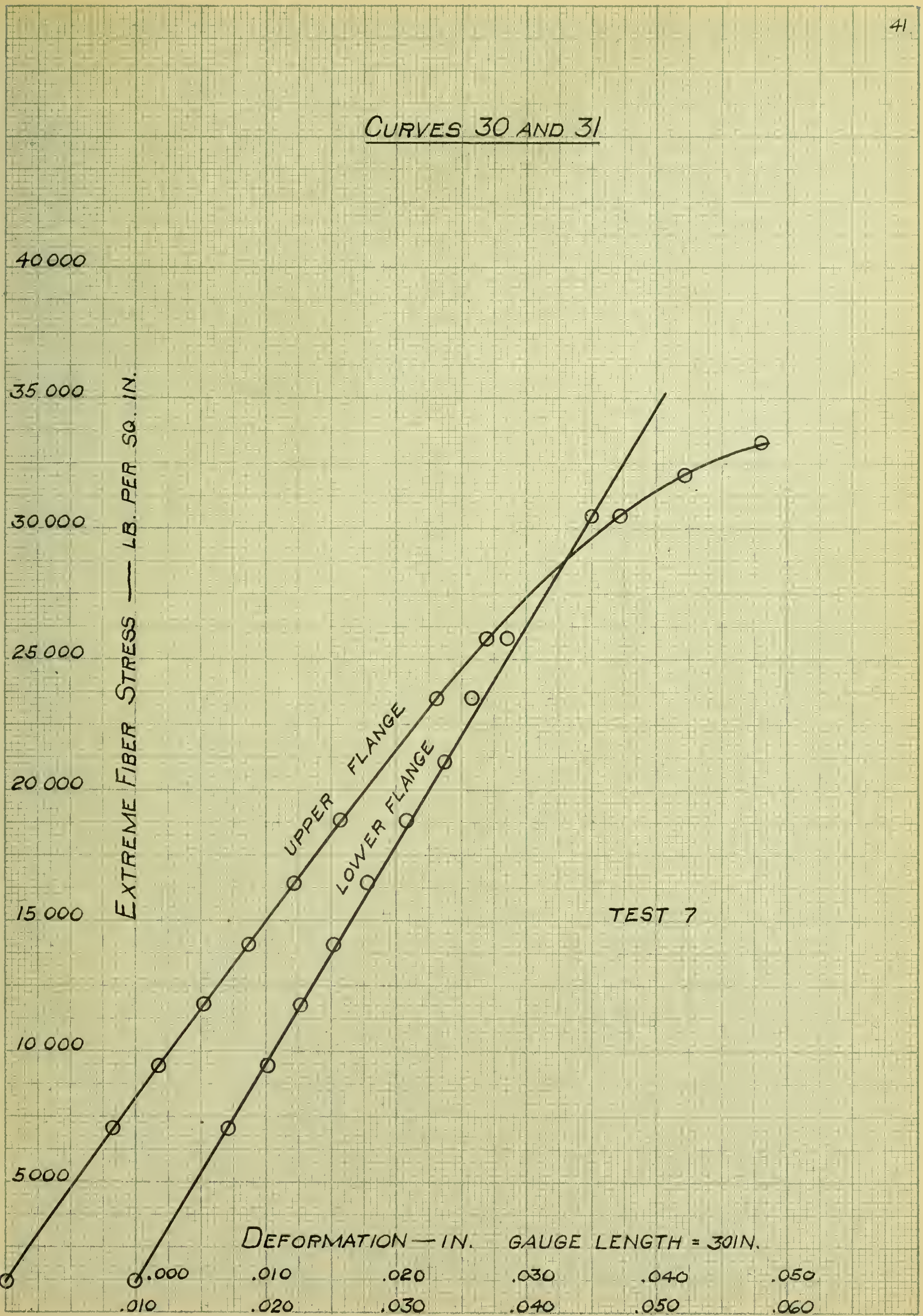

CURVE 23
ULTIMATE
28100
TEST 6
8 IN., 18 LB., I-BEAM, 20 FT.
SPAN.
DEFLECTION — IN.
1.500
1.000
.500
EXTREME FIBER STRESS — LB. PER SQ. IN.
30.000
25.000
20.000
15.000
10.000
5000

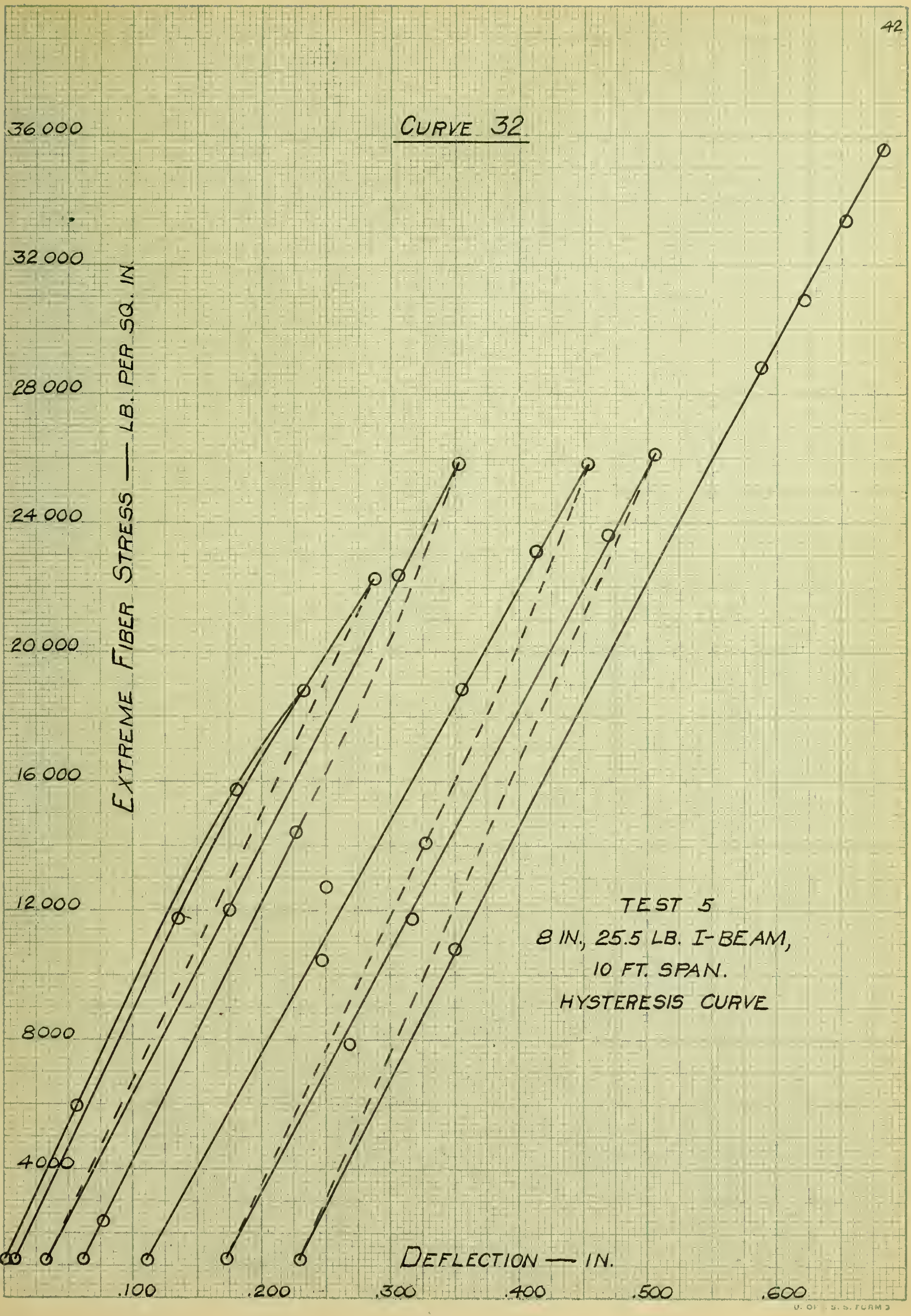

CURVES 24 AND 25
40 000
35 000
30 000
25 000
20 000
15 000
10 000
5 000
EXTREME FIBER STRESS — LB. PER SQ. IN.
UPPER FLANGE
LOWER FLANGE
TEST 6
DEFORMATION — IN. GAUGE LENGTH = 30 IN.
.000 .010 .020 .030 .040
.010 .020 .030 .040 .050


CURVES 26 AND 27
EXTREME FIBER STRESS — LB. PER SQ. IN.
40.000
35.000
30.000
25.000
20.000
15.000
10.000
5.000
ULTIMATE
39.800
ULTIMATE
36.500
TEST 5
TEST 7
TESTS 5 AND 7
8 IN., 25.50 LB. I-BEAMS
10 FT. SPAN.
DEFLECTION — IN.
.000
.100
.200
.300
.400
.500
.100
.200
.300
.400
.500
.600

CURVES 28 AND 29




CURVES 30 AND 31
40000
35000
30000
25000
20000
15000
10000
5000
EXTREME FIBER STRESS — LB. PER SQ. IN.
UPPER FLANGE
LOWER FLANGE
TEST 7
DEFORMATION — IN. GAUGE LENGTH = 30 IN.
.000
.010
.020
.030
.040
.050
.010
.020
.030
.040
.050
.060
U. OF I. S. S. FORM 3


CURVE 32
EXTREME FIBER STRESS — LB. PER SQ. IN.
36 000
32 000
28 000
24 000
20 000
16 000
12 000
8000
4000
DEFLECTION — IN.
.100
.200
.300
.400
.500
.600
TEST 5
8 IN., 25.5 LB. I-BEAM,
10 FT. SPAN.
HYSTERESIS CURVE

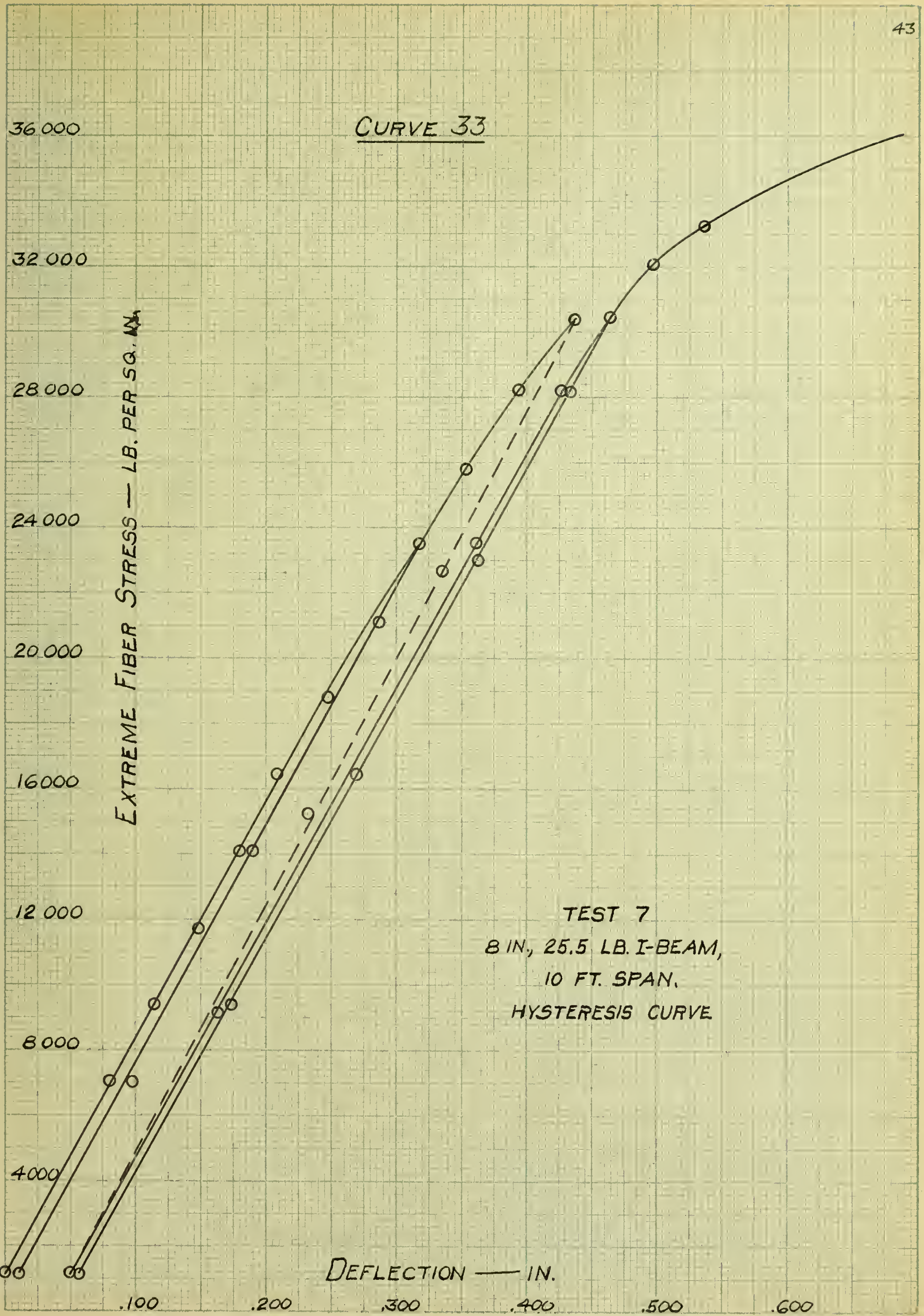

CURVE 33
36000
32000
28000
24000
20000
16000
12000
8000
4000
EXTREME FIBER STRESS — LB. PER SQ. IN.
DEFLECTION — IN.
.100
.200
.300
.400
.500
.600
TEST 7
8 IN., 25.5 LB. I-BEAM,
10 FT. SPAN.
HYSTERESIS CURVE

TABLE VI

VALUES OF HESS FORMULA FOR 8 IN.-18 LB. I-BEAMS $r = 0.84$ IN.

COMPRESSION FLANGE ONLY CARRYING LATERAL LOAD $\frac{P}{P_T} = \frac{1 \times 100}{1 + 0.0000505 L^2}$			BOTH COMPRESSION AND TENSION FLANGES CARRYING LATERAL LOAD $\frac{P}{P_T} = \frac{1 \times 100}{1 + 0.0000253 L^2}$		
L Length of span - ft.	$\frac{L}{r}$	Values of $\frac{P}{P_T}$ in percent	L Length of span in ft.	$\frac{L}{r}$	Values of $\frac{P}{P_T}$ in percent.
1	$\frac{1 \times 12}{.84} = 14.3$	$\frac{100}{1 + 0.0000505 \times (1 \times 12)^2} = 99.2$	2	$\frac{2 \times 12}{.84} = 28.6$	$\frac{100}{1 + 0.0000253 \times (2 \times 12)^2} = 98.6$
2.5	35.7	95.7	3.5	50	95.8
5	71.4	84.5	5	71.4	91.6
7.5	107	71.0	8	114	81.0
10	143	57.9	12	171	65.6
15	212	38.2	16	229	51.8
20	285	25.6	20	285	40.8

TABLE VII

DATA FOR CURVE BASED ON THE
*CARNEGIE HAND-BOOK VALUES

L Length of span		$\frac{L}{r}$	Percentage of Max. load allowable
No. of times the flange width	Inches	$r = 0.84$	
Under 20	—	—	100
20	80	95.3	100
30	120	143	90
40	160	190.5	80
50	200	238	70
60	240	286	60
70	280	333	50

* The same data are given in the Pencoyd Hand-Book

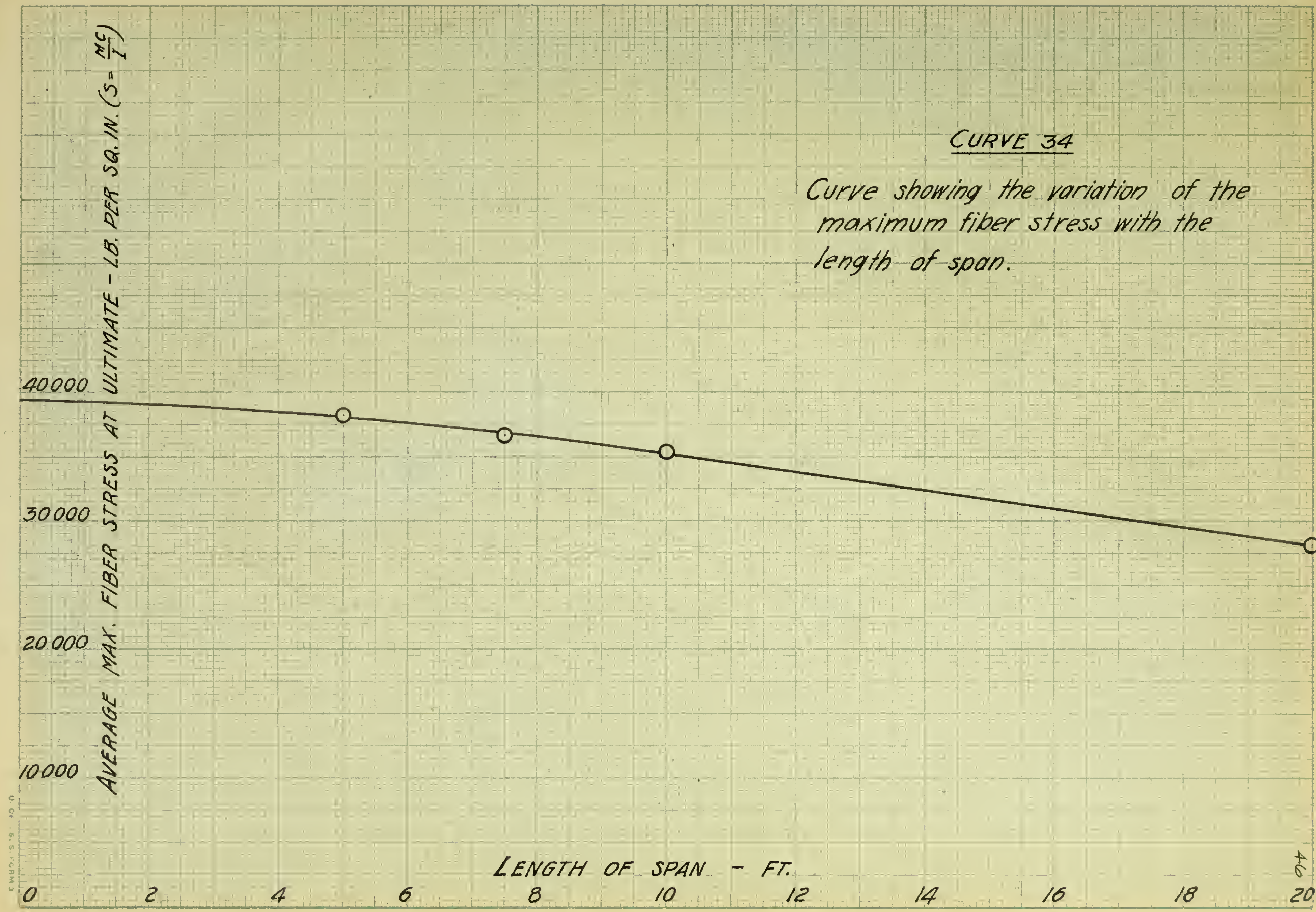
TABLE VIII

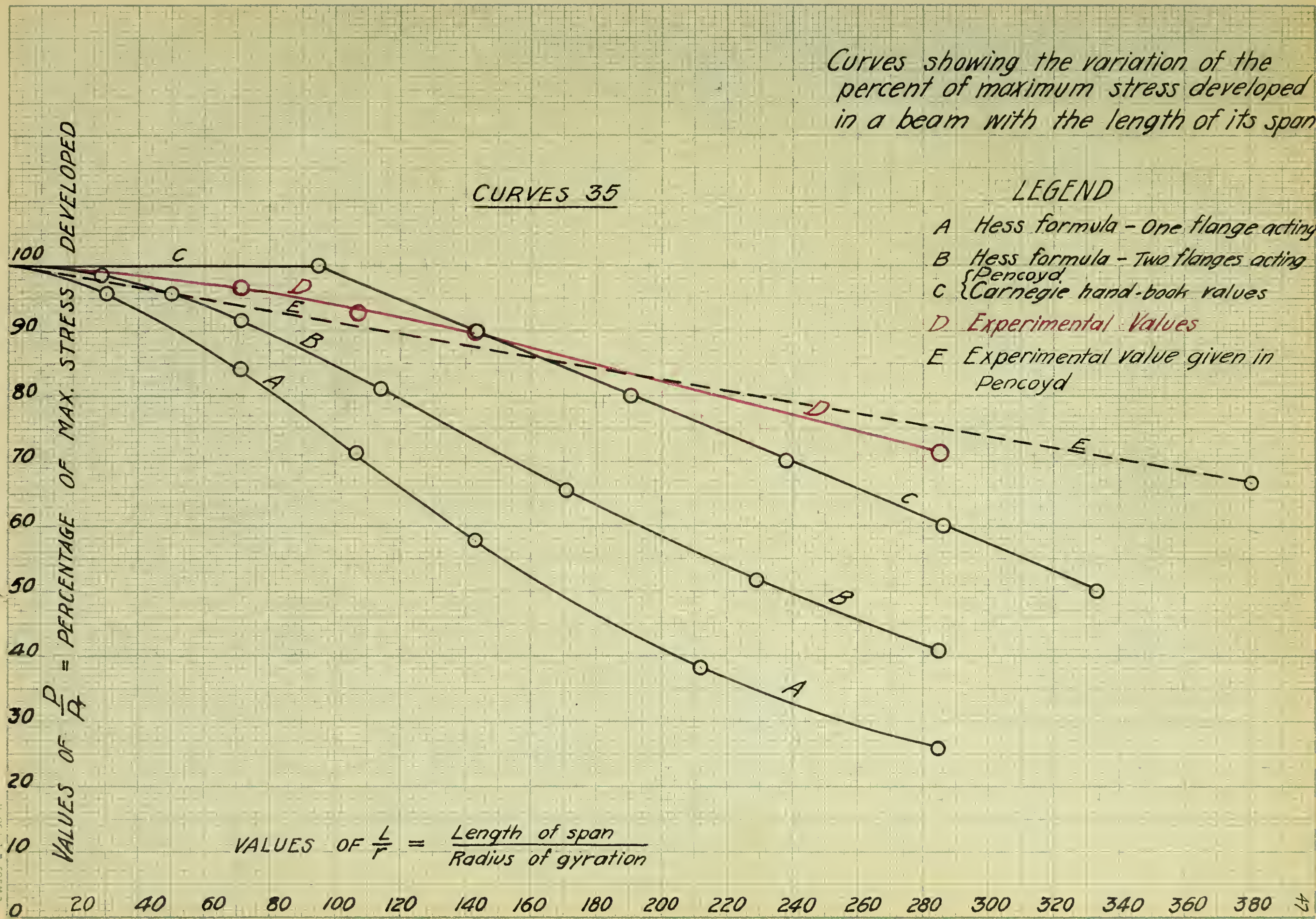
DATA FOR CURVE TAKEN FROM EXPERIMENTS

L Length of span - in	M_A Average of Max. stresses developed. lb/sq in	$\frac{100 M_A}{*M_P}$ M_A expressed in percent of M_P	$\frac{L}{r}$ $r = 0.84$
60	38120	96.4	71.4
90	36650	92.8	107
120	35400	89.7	143
240	28140	71.2	285

*M_P = Max. stress possible = 39500 lb. per sq. in.

See curve № 34

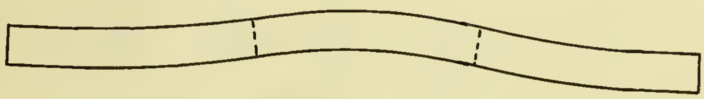

CURVE 34
Curve showing the variation of the maximum fiber stress with the length of span.
AVERAGE MAX. FIBER STRESS AT ULTIMATE - LB. PER SQ. IN. $(S = \frac{MC}{I})$
40000
30000
20000
10000
LENGTH OF SPAN - FT.
0
2
4
6
8
10
12
14
16
18
20
46



CURVES 35
Curves showing the variation of the percent of maximum stress developed in a beam with the length of its span
LEGEND
A Hess formula - One flange acting
B Hess formula - Two flanges acting
C {Pencoyd {Carnegie hand-book values
D Experimental Values
E Experimental value given in Pencoyd
VALUES OF $\frac{p}{A}$ = PERCENTAGE OF MAX. STRESS DEVELOPED
VALUES OF $\frac{L}{r}$ = $\frac{\text{Length of span}}{\text{Radius of gyration}}$
0 10 20 30 40 50 60 70 80 90 100
20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380
A B C D E

Discussion of Results

The general method of failure of the eight inch I-beams was by bowing or buckling of the compression flange rather than a twisting of the web. The top view of a beam failing in this manner would be as follows:



This indicates that to develope the greatest strength possible the flange is not large enough for the web of the beam. Marburg found this same fact to be true for the larger sizes of standard I-beams.

On both tests of the built-up wrought-iron sections, failure occurred by twisting of the web, an example of which is shown in Fig. 8. The strength of such sections would probably be considerably increased by the use of stiffeners. The two tests made on built-up sections are insufficient to establish anything definite in this connection

Curves 32 and 33 show the effect of repeated application of loads slightly beyond the elastic limits of the beams. Curve 32 is exceptional in that the

amount of the set does not decrease appreciably with the repetition of the load. In all cases the effect of removing the load after it had passed the elastic limit of the beam was to raise that elastic limit, the amount of increase diminishing with the number of repetitions.

In all cases where lateral restraint was used the beams developed a modulus of rupture somewhat higher than that of similar beams without the lateral support. The restrained I-beams showed increases of 2,800 lb. and 4,600 lb. over the average values, which represent values that are probably very close to the maximum obtainable. This is further shown by the fact that these maximum fiber stresses agree very closely with the maximum possible stress as determined by Curve 34.

The experimental curve, showing the variation of the maximum stress developed in a beam with the length of its span, agrees qualitatively with the theoretical ones in that this stress decreases as the distance between supports becomes greater. The curves representing the Hess formula, which is based upon the Rankine column formula, show that for eight inch standard I-beams at least the conditions of the

theory are too severe. For a value of $\frac{l}{r} = 200$, the curve representing the assumption that one flange only resists the lateral bending forces gives a percentage of the maximum allowable load only half as large as the one found by experiment. While these formulas would be safe to use in design they do not promote economy of material.

The dotted curve shows a reduction in the maximum stress of $33\frac{1}{3}$ percent when the length of beam is eighty times its flange width. This value is given by the Pencoyd Hand-book, and is there stated to represent the results of numerous experiments. This curve is seen to agree very closely with that determined by the experiments of this thesis. If the Pencoyd tests were made with central loadings this agreement would be still more marked, for third-point loading imposes a more severe condition. due to the greater length of span through which the maximum moment acts. Marburg's tests showed that the load sustained with quarter-point loading is about 16 percent less than that for central loading.

The values recommended by the Pencoyd Hand-book are slightly large for spans up to ten feet, while for longer spans they are well within the limits

set by the experimental data. This addition to the factor of safety for the long spans is probably used to provide for the vibratory stresses occurring in such beams.

Although enough tests to fully establish the experimental curve have not been made, and an attempt to deduce a law from the existing data would be premature, the curve, nevertheless, presents quite clearly the variation in the maximum strengths developed that should be expected with changes in the length of span. A more elaborate investigation should produce a formula more satisfactory than any of the existing ones.

A study of Table V brings out quite an interesting and important fact. The modulus of rupture developed by a beam is in no case much larger than the yield point of the material as shown by the tension tests. For three of the beams this maximum fiber stress was found to be lower than the yield point. These facts indicate that the yield in tension is a good criterion for determining the modulus of rupture of a beam. In other words, the allowable extreme fiber stress in a beam should be chosen with reference to the yield point of the

material rather than its ultimate tensile strength. The results of these tests show that beams which are commonly designed with 64000 lb. and 16000 lb. as ultimate and working stresses, respectively, are not likely to have factors of safety greater than 2 or $2\frac{1}{2}$.

Conclusions.

(1) The effect of lateral support is to slightly increase the strength of a beam.

(2) The Hess formula for one flange alone resisting buckling is too severe. There is provided a considerable factor of safety when both flanges are considered as resisting buckling.

(3) The maximum fiber stress developed by a beam decreases with the length of its span.

(4) The modulus of rupture of at least the smaller sizes of I-beams is but little greater than the yield point of the material as determined in tension.

(5) For designing beams, it would be well to regard the yield point of the material in tension as the ultimate strength of such material.