Bagby & Casey

Tests of Reinforced Concrete Beams:
Modulus of Elasticity for Various Mixtures

Civil Engineering

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TESTS OF REINFORCED CONCRETE BEAMS: MODULUS OF ELASTICITY FOR VARIOUS MIXTURES

BY

FRANCIS CYRUS BAGBY

AND

AUGUSTUS BACON CASEY

THESIS

FOR

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

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May 24, 1907.

This is to certify that the following thesis prepared under the direction of Professor A. N. Talbot, Head of the Department of Municipal and Sanitary Engineering, by

FRANCIS CYRUS BAGBY

AUGUSTUS BACON CASEY

entitled TESTS OF REINFORCED CONCRETE BEAMS: MODULUS OF ELASTICITY FOR VARIOUS MIXTURES

is accepted by me as fulfilling this part of the requirements for the Degree of Bachelor of Science in Civil Engineering.

Ira O. Baker

Head of Department of Civil Engineering

TESTS OF REINFORCED CONCRETE BEAMS:
MODULUS OF ELASTICITY FOR DIFFERENT MIXTURES.

Introduction.

During this era of the extensive and rapidly increasing use of reinforced concrete for nearly all kinds of construction, it is becoming essential that more be known concerning this material. Extensive tests have been carried on in the past few years on the various properties of concrete. It is the object of this thesis to determine the modulus of elasticity for various mixtures, as few, if any, tests have previously been made for this purpose. Throughout this work, by modulus of elasticity, is meant the relation which would exist between stress and deformation if the concrete compresses uniformly at the rate it compresses for the lower stresses. This is often called the "initial modulus of elasticity".

The modulus of elasticity has a direct bearing upon design in reinforced concrete construction. In the first place the position of the neutral axis depends, other conditions being the same, upon the modulus of elasticity of the concrete. (This is clearly brought out later in Part III.) The resisting moment in a beam may be taken as equal to the stress carried by the steel multiplied by the distance between the steel and the center of gravity of the compressive forces in the concrete. It is now seen that this last distance increases as the position of the neutral

axis rises;and thus, with the same stress in the steel, the resisting moment is greater for higher positions of the neutral axis. In other words, upon the modulus of elasticity of concrete depends the position of the neutral axis, and the position of the neutral axis is an important element in the design of reinforced concrete structures.

So, as it is seen that the modulus of elasticity of concrete is an important factor of reinforced concrete design, it is hoped that these results will give a fair idea of what can be expected of concrete in that respect, and also, what effect various mixtures have upon this property.

Six different mixtures were tested, using reinforced concrete beams with plain reinforcement, and of standard dimensions. At least two beams of each mixture were tested. All beams were sixty days old and were made under uniform conditions.

The following division of the work will be made. I - Theory and Available Data, II - Materials, III - Observed Data and Discussion of Results, IV - Conclusions, and V - Original Readings and Curves.

I. THEORY AND AVAILABLE DATA.

Modulus of Elasticity is a term which has been used very loosely in connection with reinforced concrete. It is generally defined as the ratio of the unit stress to the unit deformation within the elastic limit of the material. In materials having the property of proportionality of stress and deformation, the modulus of elasticity is constant, but in materials having a variable stress-deformation relation, like concrete, this will not be true. However it is important that a definite expression for this be used, so the name "Initial Modulus of Elasticity," as used by Prof. Talbot in Bulletin No.4 of the University of Illinois Engineering Experiment Station, is adopted. This expresses the relation which would exist between stress and deformation if the concrete compressed uniformly at the rate it compresses for the lower stresses.

The calculation of the position of the neutral axis and of the deformations at the extreme fibers was based upon the assumption that a plane section before bending remains a plane section after bending. This work was done graphically from the observed readings of the extensometers and the known positions of the rollers with respect to the beam. The deformation for unit of length was calculated by dividing the total deformation by the distance between contact points. These deformations used refer to the zero or initial position of the beam under its own weight and that of the loading apparatus at the time the load was first applied. In all calculations .002 was assumed as the ulti-

mate unit deformation for concrete. This, of course, should not hold true for all mixtures, but it was found that, assuming this value as .0016, very little difference was made in the modulus of elasticity, and therefore .002 was used throughout.

In Bulletin No.4 of the University of Illinois Engineering Experiment Station, the following formula is deduced:

$$k = \sqrt{\frac{2pn}{1-\frac{1}{3}q} + \frac{p^2n^2}{(1-\frac{1}{3}q)^2}} - pn$$

where:

k = ratio of distance between compression face and neutral axis to distance "d".

p = percent of reinforcement.

$n = \frac{E_s}{E_c}$ = the ratio of the modulus of elasticity of steel to that of concrete.

$q = \frac{\varepsilon_c}{\varepsilon_c'}$ = ratio of deformation existing in most remote fiber to ultimate deformation.

From this formula values of "k" were computed for ε_c = .0005, making $q = \frac{1}{4}$, "n" being changed so that values of "k" were obtained for n = 10, 12, 15, 20 and 25.(See Table No.1) These values were plotted on a diagram as shown on next page. Thus we have a relation between the position of the neutral axis and the modulus of elasticity. The modulus of elasticity of steel was taken at 30,000,000 lb.per sq.in. In this manner the modulus was obtained with ε_c = .001 or $q = \frac{1}{2}$.

Stress in steel:-

The stress in the steel was found in two ways,

First: $S = E_s\varepsilon_s$, or the modulus of elasticity of the steel times the deformation in the steel,

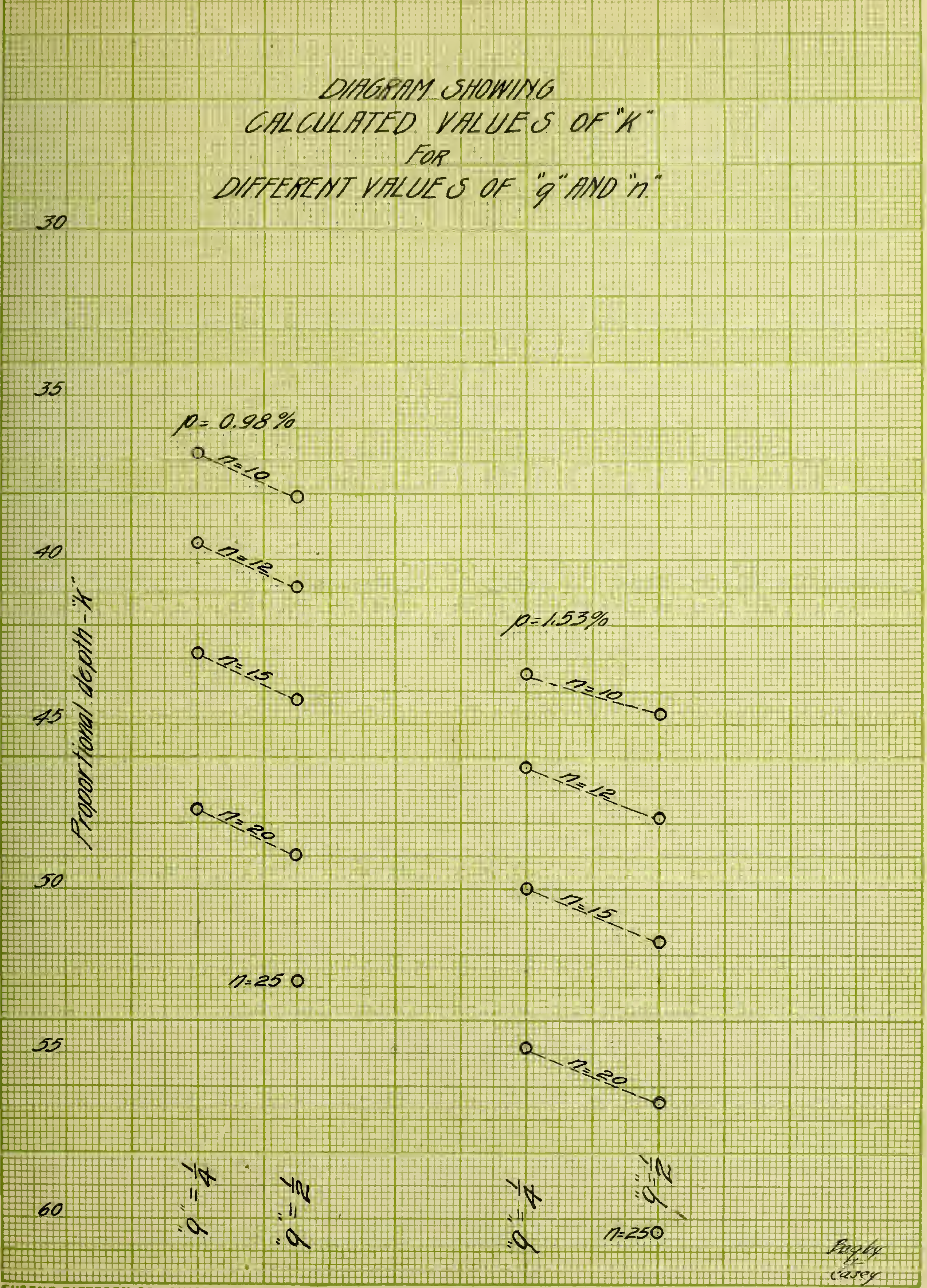


DIAGRAM SHOWING
CALCULATED VALUES OF "K"
FOR
DIFFERENT VALUES OF "q" AND "n"
30
35
40
45
50
55
60
Proportional depth - "K"
p = 0.98 %
n = 10
n = 12
n = 15
n = 20
n = 25
p = 1.53 %
n = 10
n = 12
n = 15
n = 20
n = 25
"q" = 1/4
"q" = 1/2
"q" = 1/4
"q" = 1/2
Bagby & Casey

Table No.1.

Values of "k" for different values of "n"

n	p = .98		p = 1.53	
	q = 1/4	q = 1/2	q = 1/4	q = 1/2
10	.3680	.3814	.4350	.4476
12	.3950	.4085	.4635	.4788
15	.4290	.4432	.5000	.5162
20	.4760	.4898	.5490	.5651
25	----	.5284	----	.6040

where

S = tensile stress in steel,

E_s = modulus of elasticity of steel,

ε_s = unit deformation in steel.

Second: - Resisting Moment,

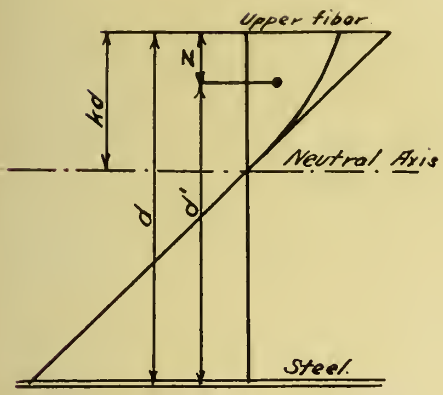


Fig. 1

Stress and deformation distribution.

The internal resisting moment of the beam must equal the external moment, to insure equilibrium. The internal moment "M" equals the stress in the steel, "S" times the moment arm. The moment arm of the internal couple is equal to the distance d'.

$$d' = d(1-.35k), \text{ when } q = \frac{1}{2} .$$

See Bulletin No.4 of the University of Illinois Engineering Experiment Station, page 17.

Therefore, the internal moment,

$$M' = S\ d(1-.35k)$$

where M' = the internal moment,

S = stress in the steel,

d = effective depth of the beam,

k = ratio of distance between compression face and neutral axis to distance "d".

The external moment $= 4 \times 12 \times \frac{W}{2}$

$$M = 24W$$

$$M = M'$$

$$\therefore\ 24W = S\ d(1-.35k)$$

$$S = \frac{24W}{d(1-.35k)}$$

For unit stress in the steel this is divided by the area of the steel.

Vertical Shear:-

As many of the beams failed in diagonal tension, the vertical shear was figured for each beam failing in this manner.

As taken from Bulletin No.4, heretofore mentioned, the shear in lb.per sq.in. $= \frac{V}{bd'}$,

where V = the total vertical shear at the given section,

b = width of the beam,

d' = distance of the reinforcement from the center of gravity of the compressive stresses.

Available Data:-

As before stated, very little data could be found which had a direct bearing upon the relation of the modulus of elasticity to various mixtures. Many tests have been made which show the relation of unit deformation to applied loads, from which the modulus of elasticity may be computed. Bulletins No.1 and No.4 of the University of Illinois Engineerinf Experiment Station, contain some data along this line, especially Bulletin No.4. In this bulletin the value of the modulus of elasticity of a 1-3-6 mixture is found. Curves are also plotted showing the relation of "n", which is the ratio of the modulus of elasticity of the steel to that of

the concrete, to the position of the neutral axis for different percents of reinforcement.

It is also interesting to observe results in Bulletin No.10 of the University of Illinois Engineering Experiment Station, which treats of concrete and reinforced concrete columns. Although the conditions in the tests on columns are entirely different from those on beams, the results obtained for the modulus of elasticity permit of comparison for the 1-2-4 mixture. Results are also given for tests performed at the Watertown Arsenal. These last tests were on columns averaging 100 days of age. Comparisons of these results and those obtained in this work will be made in Part IV.

II. MATERIALS, TEST PIECES AND TESTING.

Materials.

In order that the materials should not differ from materials used in general practice, the stone, sand and Chicago AA cement were purchased in the open market. The Universal cement and the reinforcing bars were furnished through the courtesy of the Illinois Steel Company. This cement was a part of a shipment to the Illinois Traction System, and it probably did not differ from that used in general practice. Table No. 6 shows which brand of cement was used for each beam.

Stone. - The stone used was Kankakee limestone ordered screened through a 1-in.screen and over a 1/4-in.screen. The percentage of voids in the stone was determined by filling a vessel with the stone and then determining the amount of water that could be put into the vessel with the stone. The percentage of voids in the stone was found to be 43.8 per cent.

Table No. 2.

Analysis of Stone - Fineness.

Mesh inches	Percent passing.	
	First trial	Second trial
2	100.0	100.0
1 1/4	95.8	92.6
1	82.5	65.0
1/2	9.3	2.6
1/4	1.9	0.9
1/8	1.0	0.7
All	0.0	0.1

Sand.- The sand came from near the Wabash River at Attica, Indiana, and while not very sharp , it was fairly clean and of good quality.

The percentage of voids in the sand as shown by the test described for the stone, was found to be 32.6 per cent.

Table No.3.

Analysis of Sand - Fineness.

Sieve No.	Percent passing.
5	98.5
10	74.3
16	65.5
18	56.1
20	43.0
30	27.3
40	17.9
50	12.5
74	5.6
150	1.4
200	1.1

Cement.- The cement used was Universal Portland cement, with the exception of that used in Beams No.-415.1 and No.-415.2, which were made of Chicago AA Portland cement. Table No.4 gives the tensile strength of neat Universal cement, and 1:3 mortar. The briquettes were stored in damp air for one day and under water for the remainder of the time.

Table No.4.

Tensile Strength of Universal Portland Cement.

Ref. No.	Ultimate strength - lb.per sq.in.			
	Age 7 days		Age 28 days	
	Neat	1:3 Mortar	Neat	1:3 Mortar
1	560	185	585	320
2	450	165	660	250
3	500	190	655	295
4	460	185	705	255
5	545	175	665	260
6	520	200	605	315
Average	506	183	646	282
Prob.Error	±12.3	± 3.3	±12.0	± 8.6

The results shown in Table No.5, as the results of the fineness test of Universal Portland cement, are the average of four tests.

Table No.5.

Analysis of Cement - Fineness.

Sieve No.	Percent passing.
74	98.65
100	96.20
200	80.95

Steel.- The bars used in reinforcing the beams were 1/2 in. in diameter, with the exception of those used in Beams No.-417.5 and No.-417.6, which were 5/8 in. in diameter. The material used was mild steel having an elastic limit of 38,500 lb.per sq.in. and a maximum breaking strength of 54,500 lb. per sq.in., and a percent elongation in 8 in. of 31.5

Concrete.- The concrete was of a good quality, being mixed by men who had had considerable experience in concrete work.

Test Beams.

The test beams used were all of standard size reccomended by the Joint Committee on Concrete and Reinforced Concrete, 8 in. wide, 11 in. deep, and 13 ft. long over all, with a test span of 12 ft. The beams were reinforced with four round rods, the center of the reinforcement being placed 10 in. below the top surface of the beam. 1/2-in. mild steel smooth round rods were used with the exception of the two beams noted before, the cross-section of the steel being .785 sq.in. The effective cross-section of the beam was 80 sq.in., making the proportion of the steel, in terms of the cross-section of the beam above the bars, .98 per cent. Bars 5/8 in. in diameter were used in beams No. 417.5 and No. 417.6, the proportion of steel being 1.53 per cent. In determining the proportion of steel, no reduction was made from the area of the beam, for the area taken by the metal. The bars were placed in a horizontal position 2 in. apart center to center, with their axes parallel to and 1 in. above the lower face of the beam.

The number of beams which were made from each different mixture, and the general data concerning the test beams is shown in table No. 6.

Table No. 6 .

Data on Test Beams.

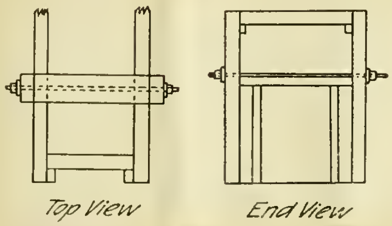
Beam No.	Kind of concrete	Age at test days	Kind of cement.	Amount of Reinforcement.	Percent of Reinforcement.
411.5	1-1-2	58	Universal	4-1/2in.round	.98
411.6	do	75	do	do	.98
411.1	do	77	do	do	.98
411.2	do	59	do	do	.98
412.5	1-1 1/2 -3	Broken	in handling.		.98
412.6	do	61	Universal	do	.98
413.5	1-2-4	57	do	do	.98
417.5	do	61	do	4-5/8in.round	1.53
417.6	do	63	do	do	1.53
414.5	1-3-6	59	do	4-1/2in.round	.98
414.6	do	61	do	do	,98
415.1	1-4-8	75	Chicago AA	do	.98
415.2	do	62	do	do	.98
415.5	do	57	Universal	do	.98
415.6	do	62	do	do	.98
416.5	do	65	do	do	.98
416.6	do	70	do	do	.98

Note:- Owing to an oversight, Beams No.-411.6, 411,1, 415.1 and 416.6 were not moved to the testing laboratory until they were considerably overdue.

Making of Beams.

The beams were made directly on the floor of the concrete laboratory, a strip of building paper being first spread on the floor. The forms were of 2-in. pine dressed on all surfaces, the ends being held in place by cleats on the sides. Braces were placed every 3 ft. to prevent bulging during the tamping of the concrete. The forms were made reversible to prevent warping, being reversed each time they were used. The details of the form are shown in Fig. 2 .

Fig. 2



Details of Forms.

All materials were measured by loose volume, and the mixing was done by hand with shovels. The cement and sand were mixed dry on a large sheet-steel plate, the stone was then added. After the stone had mixed with the cement and sand, water was added, and the mixture turned until a uniform consistency was obtained. Sometimes the sand and stone were much wetter than at other times, thus causing a wide variation in the amount of water used, as indicated in Table No. 7.

The concrete was deposited in the forms in layers of about 3 in., each layer being thoroughly tamped. After each layer had been tamped once a flat spade was forced between the face of the form and the concrete, thus flushing the mortar to the outside

of the beam, forming a smooth surface.

Table No. 7 .

Amount of Water used in mixing Concrete.

Beam No.	Mixture	Stone	Sand	Cement	Total	Weight of water lbs.	Percent of water to total
411.5	1-1-2	725	408	346	1479	125	8.5
411.6	do	736	395	287	1418	144	10.2
411.1	do	671	378	342	1391	131	9.4
411.2	do	697	484	330	1511	148	9.8
412.6	1-1 1/2 -3	689	408	228	1325	112	8.5
413.5	1-2-4	770	463	192	1425	92	6.5
417.5	do	719	436	180	1335	100	7.5
417.6	do	726	486	189	1401	100	7.1
414.5	1-3-6	724	434	130	1288	107	8.3
414.6	do	705	400	144	1249	100	8.0
415.1	1-4-8	771	410	97	1278	88	6.9
415.2	do	846	421	90	1357	90	6.6
415.5	do	751	430	95	1276	88	6.9
415.6	do	733	415	92	1240	87	7.0
416.5	1-5-10	830	458	82	1370	88	6.4
416.6	do	834	439	72	1343	60	4.5

Storage.

The temperature of the laboratory in which the beams were stored was 40 to 70 degrees Fahrenheit. The forms were removed when the beams were seven days old, and the beams left in place until they were removed for testing. The beams were sprinkled twice a day as long as they remained in the laboratory.

Beam No.-412.5 was broken in handling in the testing laboratory, before testing.

Details of Tests.

The tests were made on the 200,000-lb. Olson testing machine in the Laboratory of Applied Mechanics of the University of Illinois.

The beams weighed about 1200 lb. each, and were moved to and from the machine by means of a portable crane provided with an overhead track which supported two running trolleys and tackles. The beams were run along the table of the machine on a dolly and lifted into place by two running tackles supported by an overhead track suspended from the ceiling.

The beams were supported at each end by rocking supports. 12 ft. apart center to center and resting on the table of the machine. The tops of these supports were curves of small radius, while their bases were cylindrical surfaces of 12-in. radius. The supports permitted a rocking action with changes in length of the lower surface of the beam due to to its deflection. Bearing plates 1 in. X 4 in. X 8 in. were placed between the rocker and the beam, a layer of plaster of paris being placed between the plates and the beam to overcome any uneveness of the surface.

Similar bearing plates, resting on a cushion of plaster of paris,were placed at each 1/3 point. On top of each of these plates a 1 1/2-in. turned roller was placed with its axis at right angles to the axis of the beam and directly over the 1/3 point. These rollers were spanned by a 7-in. I-beam, wooden blocks with steel plates fastened to their bearing surface being placed between the beam and each roller. The load was transferred from the head of the machine to the I-beam by means of a knife-edge resting st the middle of the I-beam.

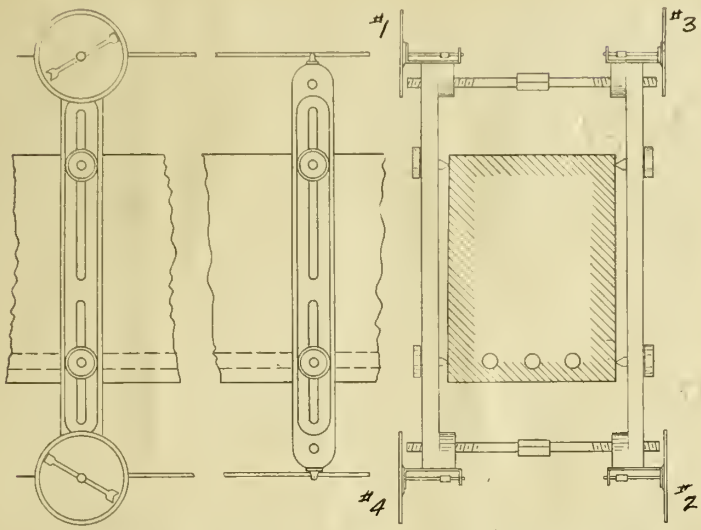
The deflections at the center of the beam were obtained by means of a fine thread stretched between two points over the supports at the middle of the depth of the beam, and passing in front of a mirror-scale placed at the middle of the beam. This scale was graduated to 1/50-in. and was read by lining up the thread and its reflection in the mirror. The thread was fastened at one end and kept taut by means of a suspended weight at the other end.

Deformations in the upper and lower fibers were obtained by means of the extensometer apparatus shown in the accompanying figure.(page 19). The two yokes were placed symmetrically with respect to the center of the span and 40 in. apart. The upper contact points were attached 1/2 in. below the surface of the beam, and the lower contact points at the level of the reinforcement, the two contact being 9 1/2 in. apart. The centers of the upper dials and rollers are in a straight line with the axis of the yoke, and 5 in. above the upper contact points. The lower dials are 5 1/4 in. below the lower contact points, the rollers being 20 1/4 in. apart vertically. The other yoke is provided with fixed pins in place of the rollers on the first yoke. Upon these pins rest 1/4-in. brass pipes with steel strips at the other end resting upon the rollers. The dials are 4 in. in diameter, and by means of a vernier scale may be read to .0001 in.

The loads were applied by the slowest speed of the machine, the downward movement of the head being at the rate of about .04 in. per minute. The load was applied in increments of 1000 lb., the machine being stopped after each increment, and readings of the extensometers and deflections taken.

As the test progressed the beam was carefully watched for cracks and a record kept of all cracks, showing their position and the load at which they appeared.

Fig. 3



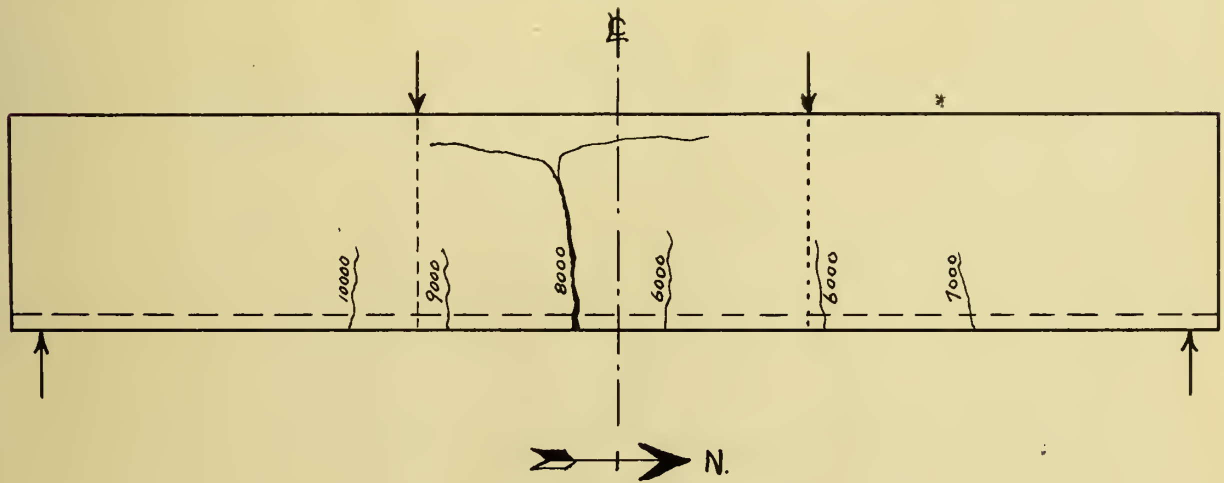
Extensometer Apparatus.

III. OBSERVED DATA.

In this division of the work a discussion will be made of the action of each beam, telling where and at what loads the first sign of failure and the ultimate failure occurred, and anything unusual in the action of the beam. Figures will be drawn showing the positions of the main cracks.

Beam No.-411.1 1-1-2 mixture .98 o/o reinforcement.

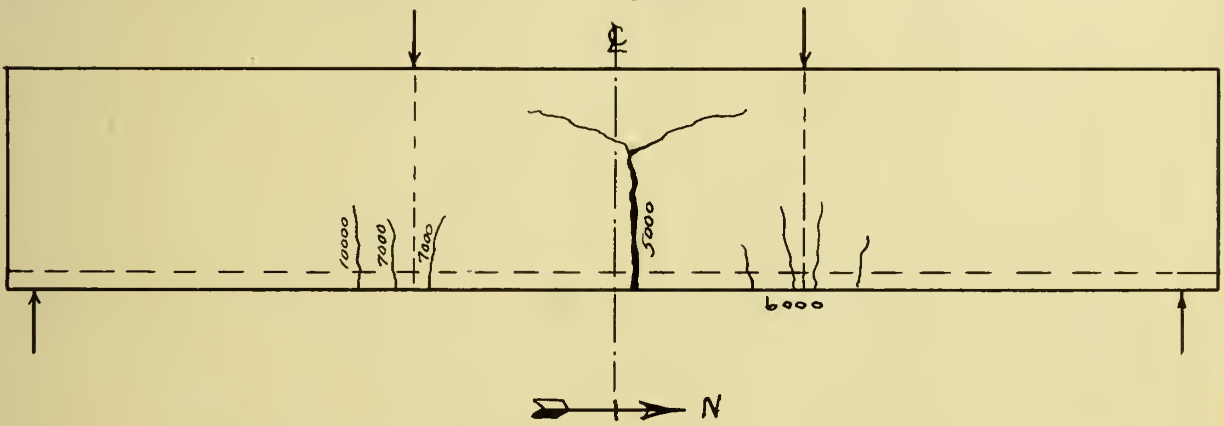
The first visible cracks appeared near the center of the span and near the north load-point at a load of 6000 lb. At a load of 11170 lb. the beam failed, a large crack forming 6 in. to the south of the center of the span. Upon applying the load further the crack became wider and not until the beam had deflected 2 in. did the concrete crush on top.



Sketch showing Beam No.-411.1 after failure.

Beam No.411.2 1-1-2 mixture .98 o/o reinforcement.

The first hair crack appeared at the middle of the span at a load of 5000 lb. At a load of 6000 lb. numerous hair cracks appeared near the north load-point, and at 7000 lb. other hair cracks appeared at the south load-point. These cracks extended about half-way up the beam. At a load of 11000 lb. the beam failed, the vertical crack at the center opening up. A faster speed was then put on the machine, and the beam crushed on top at a load of 10200 lb., the beam deflecting about 1 1/4 in. more than at the time of failure.

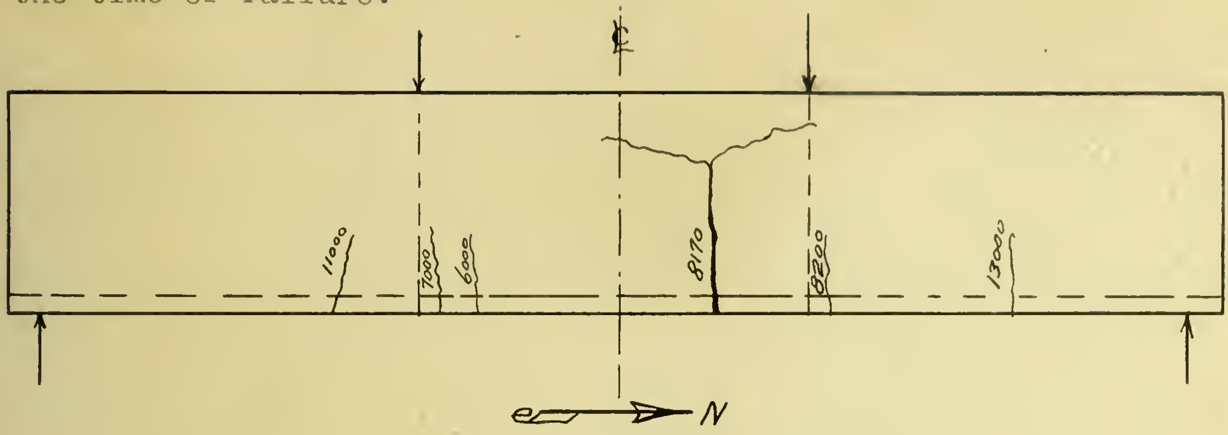


Sketch showing Beam No.-411.2 after failure.

Beam No.-411.5 1-1-2 mixture .98 o/o reinforcement.

At a load of 7150 lb. the first hair crack appeared near the south load-point. At a load of 8175 lb. a crack appeared near the north load-point and a vertical crack appeared at the middle of the span. Other hair cracks appeared near the south load-point at loads of 11000 lb. and 11500 lb., and the beam failed in tension at the vertical crack at the center at a maximum load of 13000 lb. The faster speed was then applied and the beam crushed on top at a load of 10300 lb., a horizontal crack appearing slightly below the upper surface of the beam. The deflection was about 3/4 in. more than at

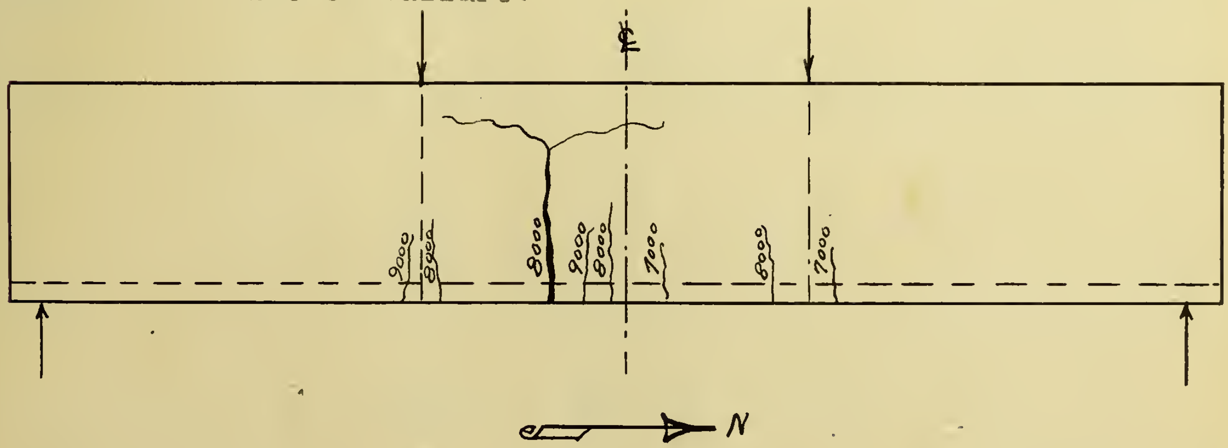
the time of failure.



Sketch showing Beam No.-411.5 after failure.

Beam No.-411.6 1-1-2 mixture .98 o/o reinforcement.

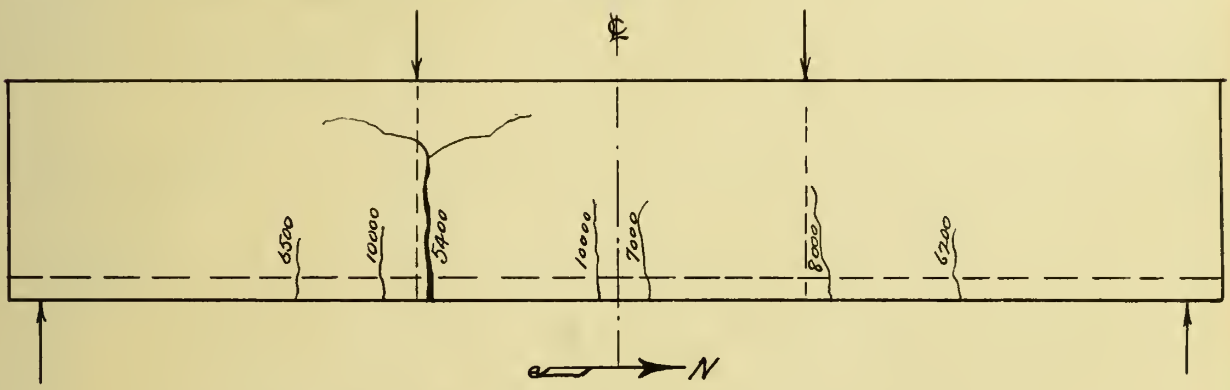
At a load of 7000 lb. a vertical hair crack appeared near the north load-point and one near the center of the span. At a load of 8000 lb. other hair cracks appeared at the center and near the south load-point and at a point half-way between the center of the span and the load-point. Other cracks appeared at a load of 9000 lb., and at a load of 11300 lb. the beam failed in tension, the vertical crack half-way between the center and the south load-point opening up. The concrete crushed at a load of 10000 lb. after deflecting about 1 in. further than at the time of failure.



Sketch showing Beam No.- 411.6 after failure.

Beam No.-412.6 1-1 1/2-3 mixture .98 o/o reinforcement.

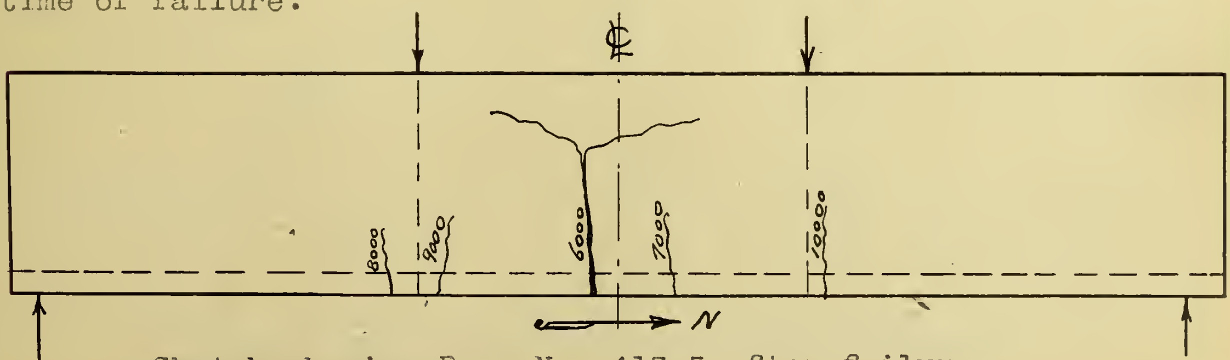
The first hair crack was vertical and appeared at a load of 5400 lb. at the south load-point. Other cracks appeared near the north load-point at a load of 6200 lb. and at the center and south load-point at loads of 8000 lb. and 10000 lb. The beam failed in tension, the crack which appeared at a load of 5400 lb. opening up at a load of 11885 lb. The beam was crushed on top at a load of 10300 lb. after deflecting about 3/4 in. further than at the time of failure.



Sketch showing Beam No.-412.6 after failure.

Beam No.-413.5 1-2-4 mixture .98 o/o reinforcement.

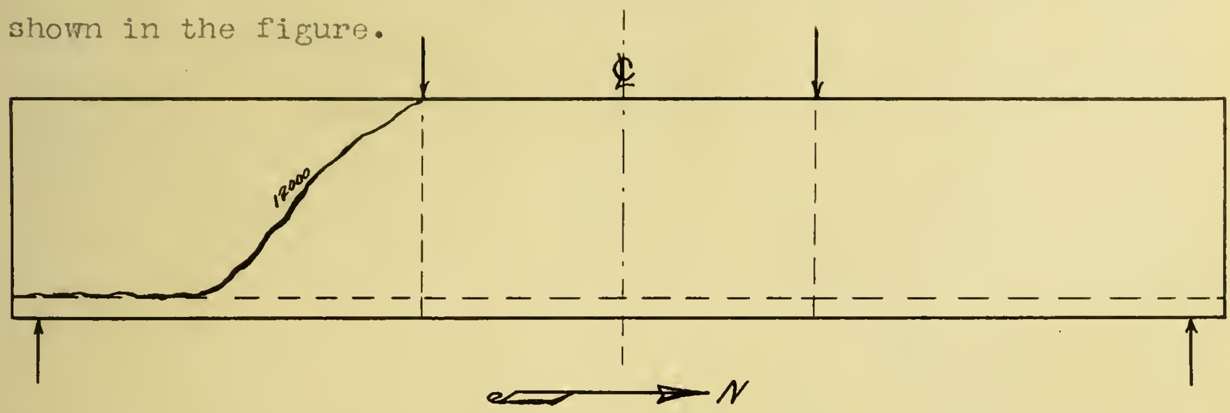
At a load of 11950 lb. the beam failed in tension, the vertical tension crack appearing about 6 in. south of the center of the span. The concrete crushed on top at a load of 10140 lb. after deflecting only slightly further than at the time of failure.



Sketch showing Beam No.-413.5 after failure.

Beam No.-417.5 1-2-4 mixture 1.53 o/o reinforcement.

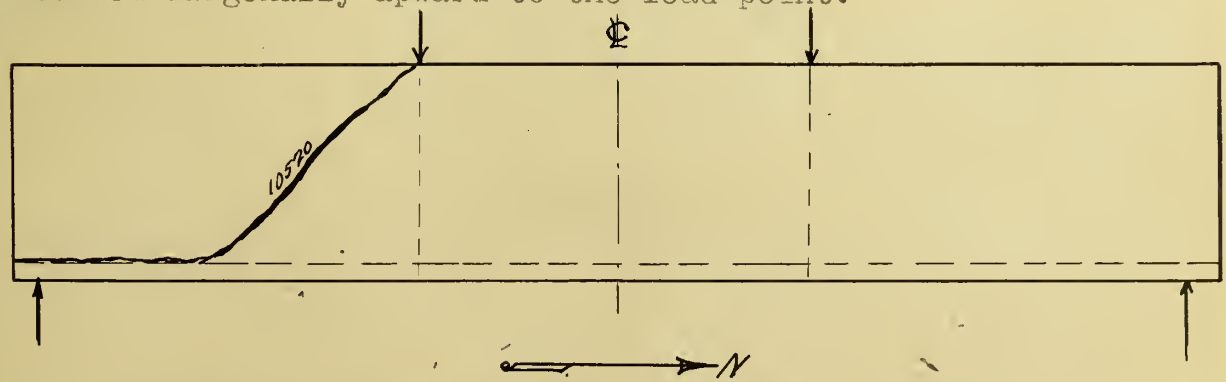
There was no sign of failure until at a load of 12000 lb. the beam failed suddenly in diagonal tension. The machine was stopped at a load of 12000 lb. to take readings. The load was again applied and before it increases any the beam failed, dropping on the machine without warning. The crack commenced at the south support and extended diagonally toward the load-point as shown in the figure.



Sketch showing Beam No.-417.5 after failure.

Beam No.-417.6 1-2-4 mixture 1.53 o/o reinforcement.

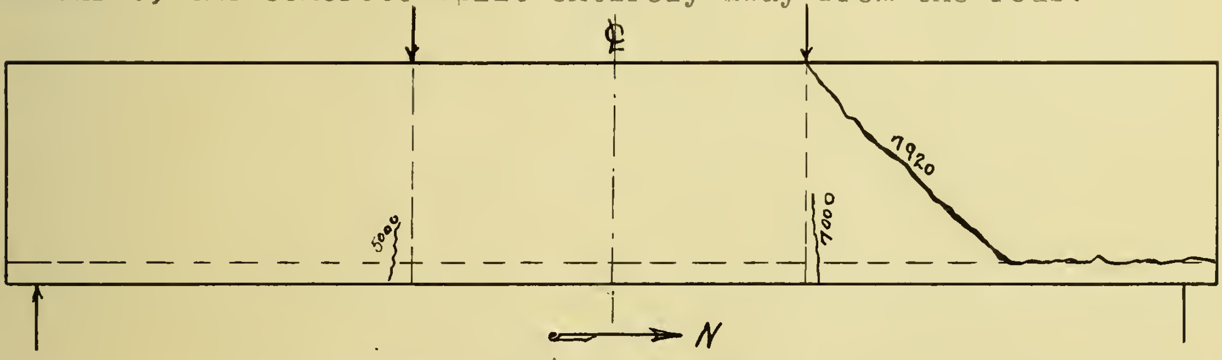
This beam failed in a manner similar to the failure of Beam No.-417.5, the failure coming without warning, at a load of 10520 lb. by diagonal tension. The crack commenced at the bottom of the beam about 2 ft. south of the south load-point, and extended diagonally upward to the load-point.



Sketch showing Beam No.-417.6 after failure.

Beam No.-414.5 1-3-6 mixture .98 o/o reinforcement.

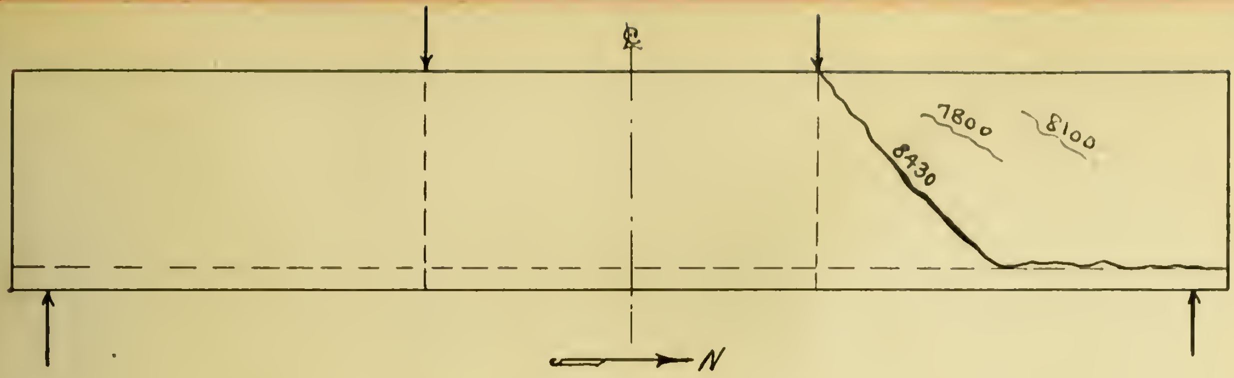
The first hair crack appeared at the south load-point at a load of 5000 lb. At a load of 7000 lb. another hair crack appeared at the north load-point, and the beam failed in diagonal tension at a load of 7920 lb., the crack commencing near the top of the beam at the north load-point and running diagonally downward until it struck the reinforcement at a point 2 ft. north of the load -point. From this point the crack followed the reinforcement to the north support. In taking the beam from the machine, the concrete split entirely away from the rods.



Sketch showing Beam No.-414.5 after failure.

Beam No.- 414.6 1-3-6 mixture .98 o/o reinforcement.

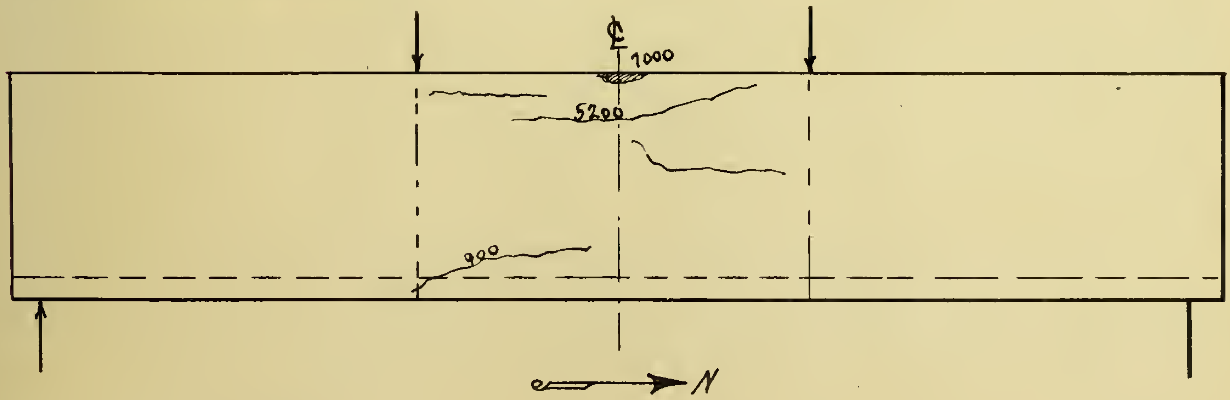
At a load of 7800 lb. a short diagonal hair crack appeared at a point 1 ft. to the north of the north load-point, and another one appeared a little to the north of the first one at a load of 8100 lb. At a load of 8430 lb. the beam failed in diagonal tension, the crack extending from a point at the bottom of the beam 2 ft. north of the north load-point, diagonally upward to the load-point.



Sketch showing Beam No.-414.6 after failure.

Beam No.-415.5 1-4-8 mixture .98 o/o reinforcement.

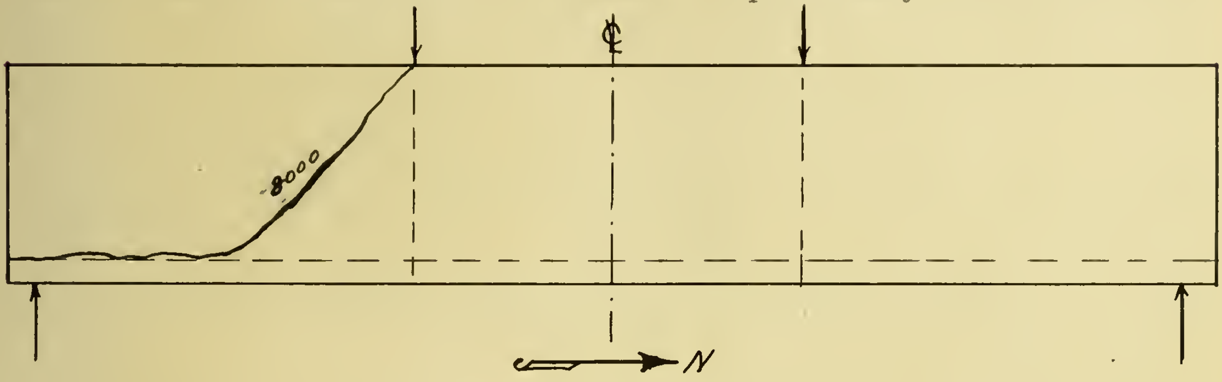
This beam failed in compression at a load of 7000 lb., the concrete crushing on top at the center of the span. The head was then run down at a faster speed, the load dropping off , until at a load of 5200 lb. a horizontal crack appeared, commencing at a point at the middle about 2 1/2 in. below the top surface of the beam, and extending about 1 ft. to the north. At a load of 900 lb. horizontal cracks appeared near the bottom of the beam at the center of the span and extending slightly past the north load-point.



Sketch showing Beam No.-415.5 after failure.

Beam No.-415.6 1-4-8 mixture .98 o/o reinforcement.

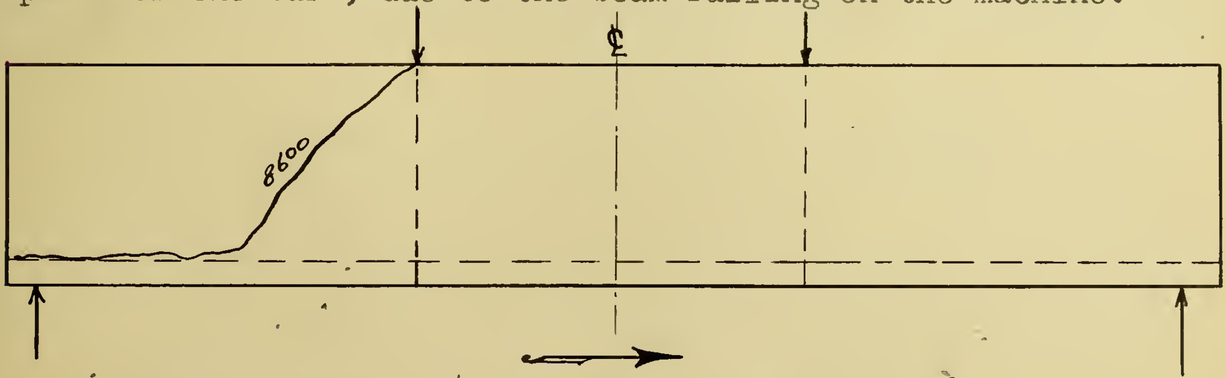
The beam failed in diagonal tension at a load of 8000 lb. without any preliminary sign of failure. The crack commenced at a point about 2 ft. south of the south load-point and extended diagonally upward to the load point. The beam dropping on the machine caused the concrete to pull away from the rods.



Sketch showing Beam No.-415.6 after failure.

Beam No.-415.1 1-4-8 mixture .98 o/o reinforcement.

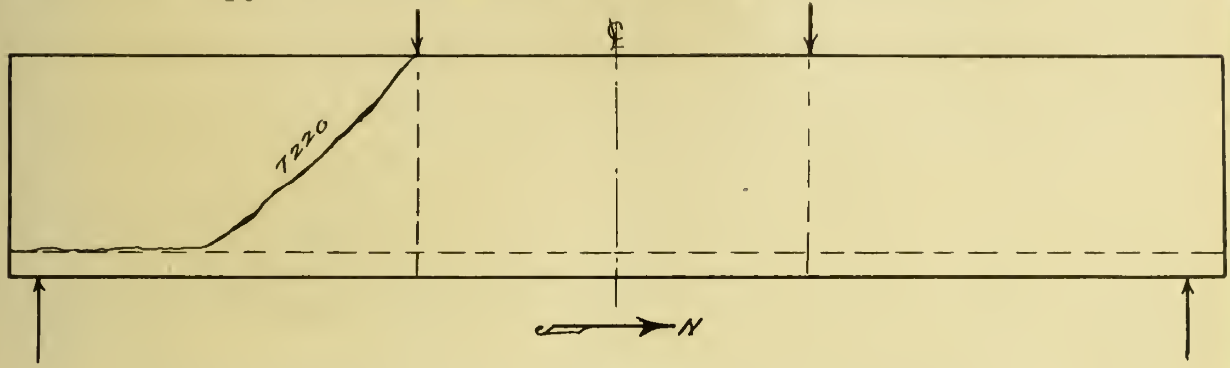
The first crack appeared about 2 ft. south of the south load point at a load of 8600 lb. At a load of 8680 lb. the beam failed in diagonal tension before the deflection for the load could be read. The crack extended diagonally upward toward the south load point from the first crack which appeared, and also toward the left from this crack, the concrete being stripped from the bars, due to the beam falling on the machine.



Sketch showing Beam No.-415.1 after failure.

Beam No.-415.2,1-4-8 mixture .98 o/o reinforcement.

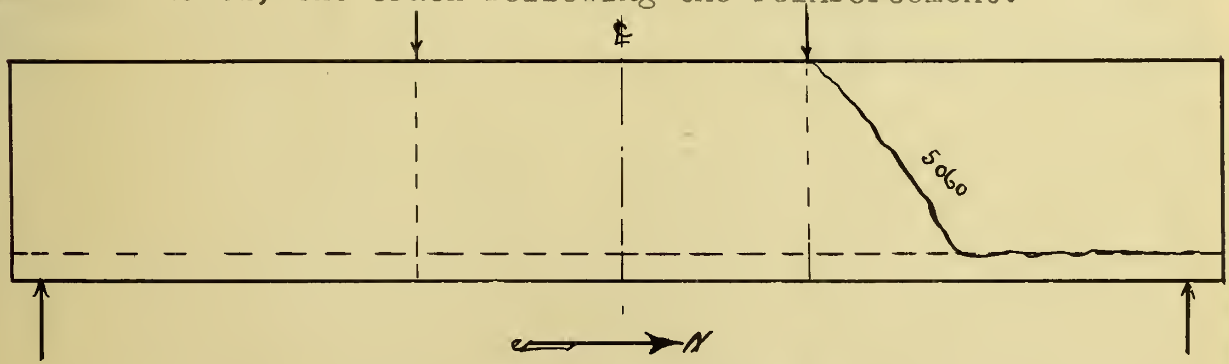
The beam failed in a manner similar to the failure of Beam No.-415.1, excepting that the beam showed no sign of failure until it dropped on the machine at a load of 7220 lb.



Sketch showing Beam No.-415.2 after failure.

Beam No.-416.5 1-5-10 mixture .98 o/o reinforcement.

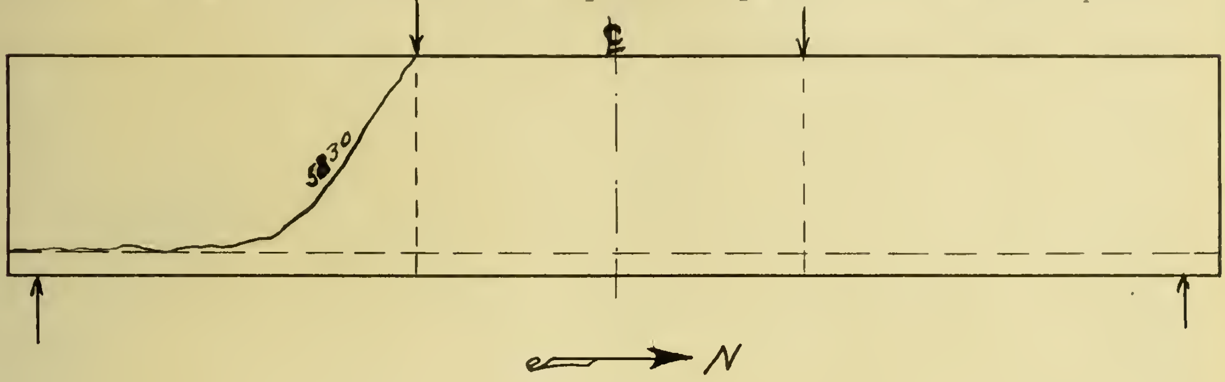
The beam failed in diagonal tension at a load of 5060 lb. without showing any preliminary sign of failure. The crack commenced at a point about 15 in. north of the north load point, extending diagonally upward to the load point, and also toward the north, the crack following the reinforcement.



Sketch showing Beam No.-416.5 after failure.

Beam No.-416.6 1-5-10 mixture .98 o/o reinforcement.

The beam failed in diagonal tension at a load of 5830 lb. The crack extended from a point at the bottom of the beam about 18 in. south of the south load point, upward to the load point.



Sketch showing Beam No.-416.6 after failure.

Discussion of Results.

Table No. 9 contains the modulus of elasticity for each beam as calculated from the tests. The average values of the modulus of elasticity for different mixtures are given. For the lean mixture, 1-5-10, a value of 1,100,000 lb. per sq.in. was obtained; for the 1-4-8 mixture, 1,400,000 lb. per sq.in.; for the 1-3-6 mixture, 1,600,000 lb.per sq.in.; for the 1-2-4 mixture, 1,800,000 lb.per sq.in.; for the 1-1 1/2-3 mixture, 2,460,000 lb.per sq.in., and for the 1-1-2 mixture, 2,700,000 lb.per sq.in. These results show clearly that the amount of cement in concrete has a direct bearing upon the modulus of elasticity. The curve plotted on page 38 shows the relation existing between the modulus of elasticity and the proportion by weight of the cement to the sand and stone. This curve is practically a straight line, indicating that the modulus of elasticity varies directly as the per-cent of cement in the concrete, for the range of mixtures used.

In Bulletin No.4 of the University of Illinois Engineering Experiment Station, the modulus of elasticity of a 1-3-6 mixture is calculated to be about 2,000,000 lb.per sq.in. The result obtained for this mixture, as shown above, is 1,600,000 lb.per sq.in. This result agrees favorably with given above. The values in the bulletin were given in round numbers.

Comparisons may also be made with the modulus of elasticity obtained in the column tests, as shown in Bulletin No.4, heretofore mentioned. For a 1-2-4 mixture the average modulus of elasticity obtained was 2,200,000 lb.per sq.in., as compared with 1,800,000 lb.per sq.in. obtained from the beam tests. From the column tests at the Watertown Arsenal, the value 2,500,000 lb. per Sq.in. was obtained. These last tests were made on pebble

concrete 105 days old. Obviously, the conditions were entirely different in the column tests and the beam tests, and the same value could not be anticipated. However, the variation is not large.

Many of the beams of a fairly rich mixture did not stress the steel to its yield point, but broke by diagonal tension. This naturally leads to an investigation of the vertical shear in these beams, as the shear developed may be taken as a measure of the diagonal-tensile stresses. The formula, $v = V/bd'$ was used in obtaining the shears. In Table No.8 are found the values of the vertical shear for all beams, and the manner of failure. The curve on page 37 shows the relation between the vertical shear and the percent by weight of cement to the sum of the weights of the sand and stone, in the beams that failed by diagonal tension. This curve also resembles a straight line, and it is of interest to note that if extended on down, it would nearly meet the zero point for both coordinates. From this curve it may be said that the maximum vertical shear is proportional to the ratio by weight of the cement to the sum of the weights of the sand and stone.

In the beams that broke by tension in the steel the maximum vertical shear was not developed. This is seen by comparing the vertical shear in the beams of the 1-1-2 mixture, which failed by tension, with the vertical shear in the Beams of the 1-2-4 mixture, which failed by diagonal tension. It is seen that in the richer mixture the vertical shear was about 85 lb.per sq.in, while for the leaner mixture about 80 lb.per sq.in. was developed. This clearly shows that in the beams which failed by tension in the steel the maximum vertical shear was not fully

developed.

In the same table are the calculated stresses in the steel. The two methods used gave results which checked favorably. In stresses calculated from the resisting moment, the weight of the beam itself and of the I-beam used in loading, is neglected. The addition of these loads would tend to increase the amount of the stress in the steel. The assumption is made that the concrete does not take tensile stress, but this is not true, except in the case of pure tension failures. This error would tend to compensate the one previously mentioned. The yield point of steel is exceeded only in the beams which failed by tension. Taking the value found, 31,500 lb.per sq.in., as the yield point, it is seen that in all other failures the steel could have taken more stress.

The results of the tests, taken as a whole, are satisfactory, and bring out clearly the variations of the modulus of elasticity for various mixtures. In the case of the 1-1 1/2-3 mixture, one of the two beams made was broken in handling. The value calculated from the other beam is high, and is not the average of two tests, as were the others.

An investigation of the average deflections for the different mixtures may also be made, as they show in a way the relative stiffness of the beams of different mixtures. It was found that the beams of the lean mixtures did not deflect as much as did those of the richer mixtures. For the 1-5-10 mixture the deflection at the center was .30 in. at the maximum load, while a deflection of .60 in. was noticed for the 1-1-2 mixture at its maximum load. Considering a certain load, as 5000 lb., and comparing the deflection for each beam, it is seen that while the

1-1-2 mixture deflected only .16 in., the 1-5-10 mixture deflected nearly .30 in. This shows that the rich mixtures have greater stiffness than the lean mixtures.

If further tests were to be made along this line, we would suggest that at least four beams of each mixture be tested, for the reason that many influences, such as storage, uniformity of mixture, handling, and testing, may cause wide variations in the results, and the greater the number of beams tested, the more trustworthy will be the results obtained. Also, to determine more accurately the modulus of elasticity, the ultimate unit deformation for each mixture should be determined. This could be done by testing beams reinforced with sufficient steel and in such a manner that the beams would fail by compression in the concrete. If this were done it would not be necessary to assume .002 as the ultimate unit deformation for every mixture, as was done in this work.

Explanation of Tables.

Table No. 8 .

In this table may be found the mixture and maximum applied load for each beam. For this maximum load the vertical shear was found for each beam. Under the heading, "Load considered" are the loads for which the stresses in the steel were calculated. For example, for Beam No. 411.5, the load considered was 10000 lb. The formula for calculating the stress in the steel from the resisting moment is, S = 24 W/d(1-.35k)A, in which the "W" was taken as 10000 lb., the "k" and "d" also depending upon this load. The stress calculated from the deformation was obtained by means of the formula, $S = E_s \varepsilon_s$, in which the unit deformation was taken for the load of 10000 lb.

Table No. 9 .

This table contains the calculated modulus of elasticity for each beam, for q = 1/4 and q = 1/2 .As before stated, the assumption was made that the ultimate unit deformation for each beam was .002 . Then for q = 1/4 the unit deformation .0005 was considered and the applied load and value of "k" found for this deformation. With this value of "k", the modulus of elasticity was calculated as described in Part I . For q = 1/2, the modulus was found in the same manner, using .001 as the unit deformation.

The values under the heading "Mean E" were an average of the results for each mixture, excluding those values which seemed to be in error.

Table No. 8 .

Data and Calculated Results for Reinforced Concrete Beams.

Beam No.	Mixture	Maximum applied load lbs.	Load considered lbs.	Neut. Axis k	Vert Shear V/bd'	Calculated stress in steel-lb.sq.in. From Resisting Moment	From Deformation.	Manner of Failure
411.5	1-1-2	13000	10000	.395	94	36800	31800	Tension
411.6	1-1-2	11130	10000	.395	81	35500	38550	do.
411.1	1-1-2	11170	11000	.380	81	38900	40800	do.
411.2	1-1-2	11000	10000	.415	81	35600	36600	do
412.6	1-1 1/2-3	11885	11000	.400	86	39100	43500	do
413.5	1-2-4	11950	10000	.475	90	36400	33900	do
417.5	1-2-4	12000	12000	.550	93	29000	27750	Diag.Ten,
417.6	1-2-4	10520	10000	.610	83	24600	26000	do
414.5	1-3-6	7920	7000	.470	59	25300	25000	do
414.6	1-3-6	8430	8000	.550	65	30000	31050	do
415.1	1-4-8	8680	8000	.540	67	29900	27500	do
415.2	1-4-8	7220	7000	.560	56	26600	24200	do
415.5	1-4-8	7000	6000	.560	54	22300	23550	Compress.
415.6	1-4-8	8000	8000	.497	61	29600	31350	Diag.Ten.
416.5	1-5-10	5060	4000	.560	39	15000	12500	do
416.6	1-5-10	5830	5000	.500	40	18500	18000	do
* 211.1	1-1 1/2-3	----	----	---	213			
* 211.2	do	----	----	---	136			
* 212.1	1-2-4	----	----	---	148			
* 212.2	1-2-4	----	----	---	154			
* 212.5	1-2-4	----	----	---	115			
* 212.6	1-2-4	----	----	---	126			

Note:- All of the beams had .98 o/o reinforcement, with the exception of Beams No.-417.5 and No.-417.6, which had 1.53 o/o reinforcement.

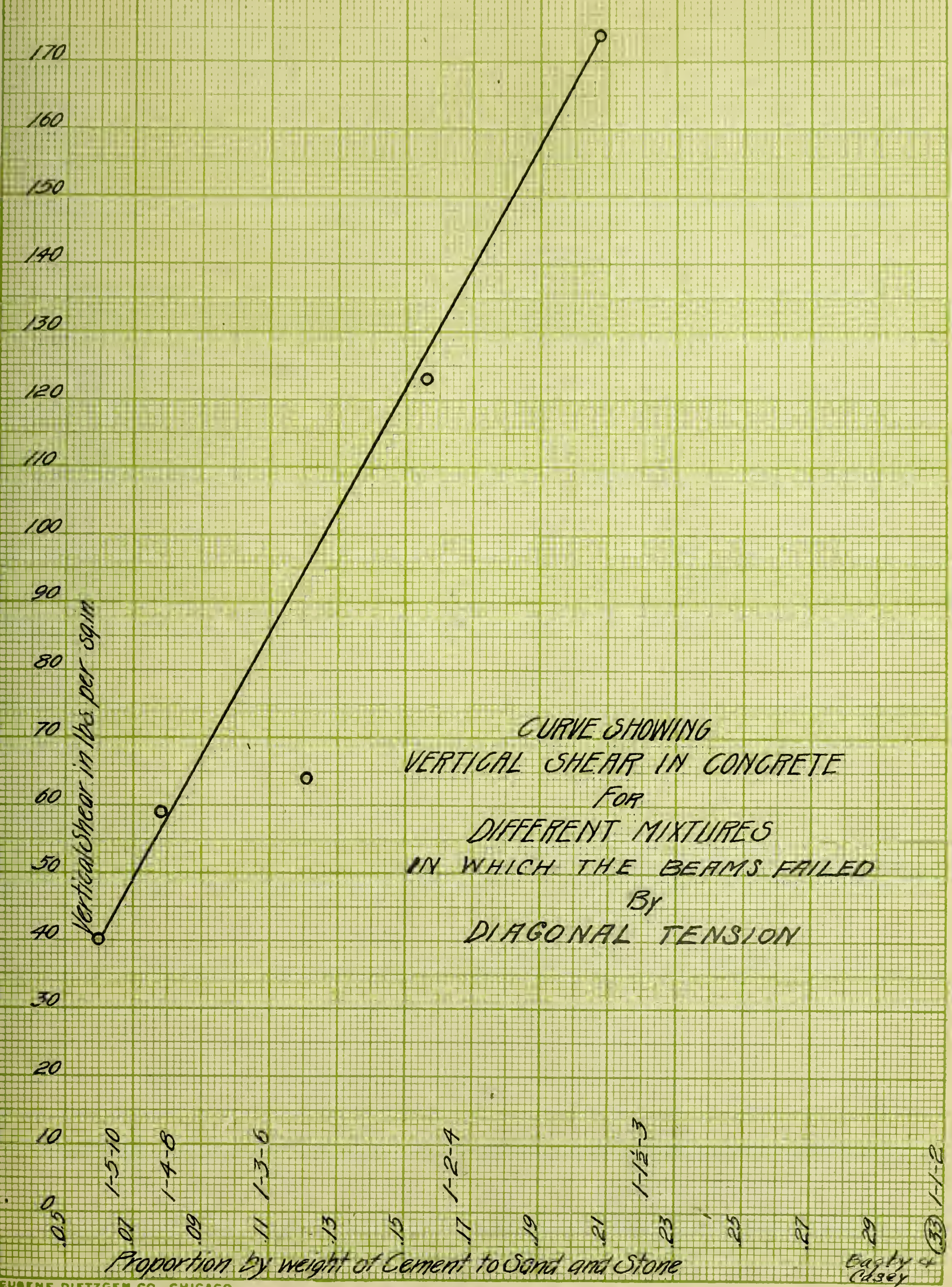
The results for the last six beams marked(*) were obtained from the thesis of Chester Alva Foreman, Carl James and Leon Browning Kinsey of the Class of 1907 of the University of Illinois. These beams were made of Chicago AA Portland cement.

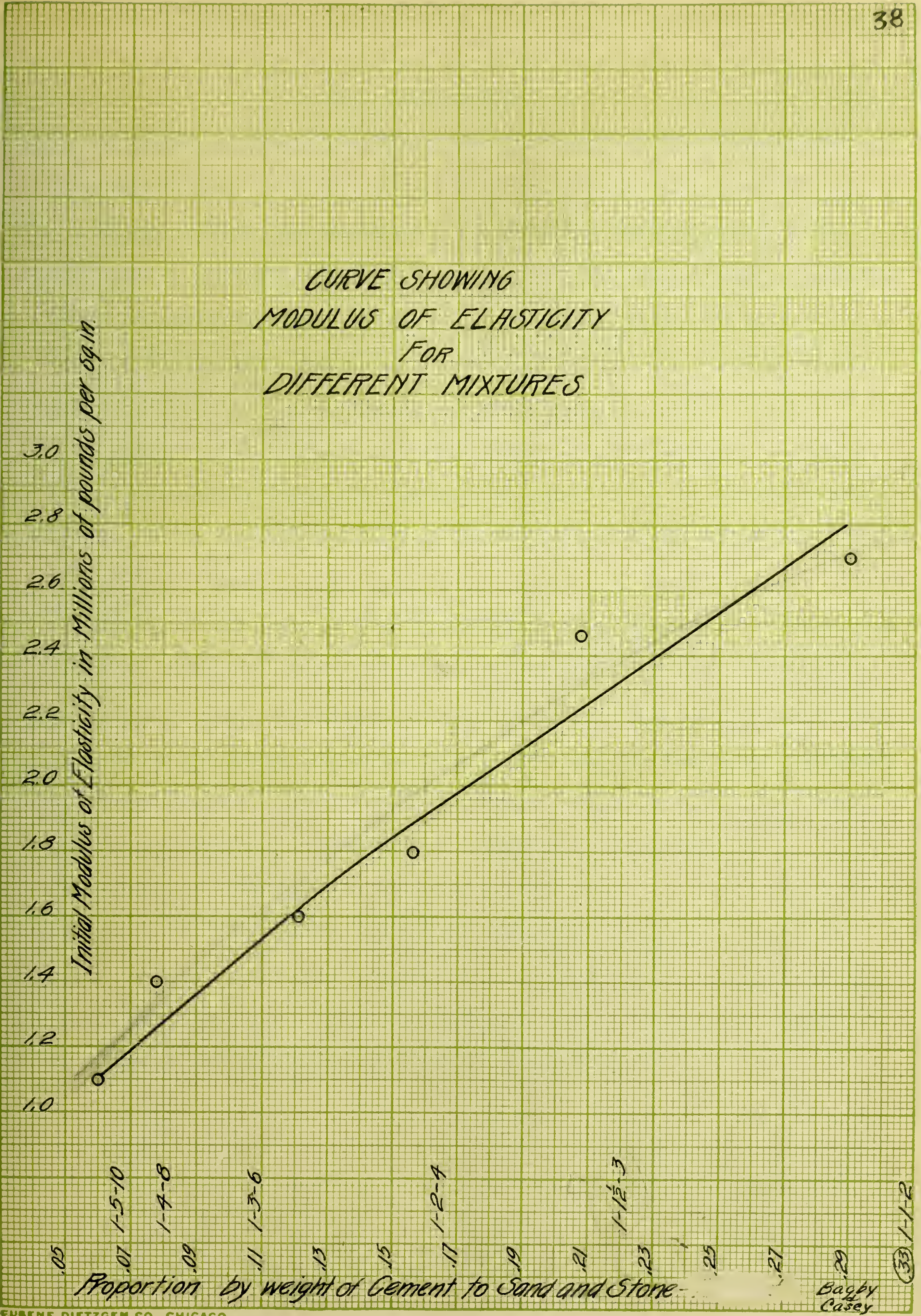
Table No.9.

Modulus of Elasticity for Different Mixtures.

Beam No.	Mixture	q = 1/4		q = 1/2		Mean E
		k	E	k	E	
411.5	1-1-2	.395	2,500,000	.395	2,730,000	
411.6	do	.415	2,175,000	.390	2,830,000	
411.1	do	.390	2,594,000	.390	2,830,000	
411.2	do	.405	2,270,000	.425	2,240,000	2,700,000
412.6	1-1 1/2 -3	.405	2,270,000	.400	2,650,000	2,460,000
413.5	1-2-4	.440	1,850,000	.460	1,820,000	
417.5	do	.507	1,960,000	.460	1,830,000	
417.6	do	.540	1,580,000	.570	1,460,000	
314.2*	do	.475	1,500,000	.472	1,700,000	
314.6*	do	.450	1,800,000	.450	1,930,000	
314.1*	do	.450	1,800,000	.430	2,180,000	1,900,000
414.5	1-3-6	.440	1,850,000	.460	1,820,000	
414.6	do	.475	1,500,000	.500	1,430,000	1,600,000
415.1	1-4-8	.485	1,410,000	.505	1,390,000	
415.2	do	.530	1,132,000	.540	1,120,000	
415.5	do	.480	1,445,000	.510	1,350,000	
415.6	do	.455	1,685,000	.480	1,600,000	1,400,000
416.5	1-5-10	.530	1,132,000			
416.6	do	.520	1,144,000			1,100,000

Note:- The data on the three beams marked(*), were secured from the thesis of Stanley Worcester Galhuly, Wilfred Lewis and Roy Austin Miller of the Class of 1907 of the University of Illinois.


CURVE SHOWING
VERTICAL SHEAR IN CONCRETE
FOR
DIFFERENT MIXTURES
IN WHICH THE BEAMS FAILED
BY
DIAGONAL TENSION
Vertical Shear in lbs. per sq. in.
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
1-5-10
1-4-8
1-3-6
1-2-4
1-1½-3
1-1-2
.05
.07
.09
.11
.13
.15
.17
.19
.21
.23
.25
.27
.29
.33
Proportion by weight of Cement to Sand and Stone


CURVE SHOWING
MODULUS OF ELASTICITY
FOR
DIFFERENT MIXTURES
Initial Modulus of Elasticity in Millions of pounds per sq.in.
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
1-5-10
1-4-8
1-3-6
1-2-4
1-1½-3
1-1-2
.05
.07
.09
.11
.13
.15
.17
.19
.21
.23
.25
.27
.29
Proportion by weight of Cement to Sand and Stone
Bagby
Casey
33

IV. CONCLUSIONS.

From the previous discussion the following conclusions may be derived:

First - From the curve on page 38 it is readily seen that there is a certain relation existing between the different mixtures of concrete, and the modulus of elasticity. Values of 1,000,000 lb. per sq.in. for a 1-5-10 mixture, to 2,700,000 lb. per sq.in. for a 1-1-2 mixture were obtained.

Second - The curve mentioned above is practically a straight line, indicating that the modulus of elasticity is directly proportional to the amount of cement in the mixture, for the range of mixtures used.

Third - As many of the beams failed by diagonal tension, the vertical shears in the beams were investigated. The values obtained were low, that for a 1-5-10 mixture being 40 lb.per sq.in., and for a 1-1 1/2-3 mixture, 170 lb.per sq.in. The curve on page 37 indicates that the average ultimate vertical shear developed is proportional to the amount of cement in the mixture.

Fourth - The tensile stress in the steel was calculated by two methods, the results obtained agreeing favorably. For mixtures 1-3-6, 1-4-8 and 1-5-10, the steelwas not stressed to the yield point, showing that the percent of reinforcement should be reduced for the most economical construction.

V. ORIGINAL READINGS AND CURVES.

Tables.

In the following part are given the original readings and a part of the derived values for the beams. The first column gives the applied load in pounds and does not include the weight of the beam or of the loading apparatus. The column headed "Deflection" does not contain the original readings from the scale but the deflections from the position of zero load. In the columns headed "Original Extensometer Readings" are given the original readings of the four extensometers. The sub-headings indicate the four extensometers. I and III are the upper extensometers and II and IV the lower. In the two columns headed "Deformation" are given the deformations per unit of length for the fiber indicated in the sub-heading, computed with reference to zero deformation at the first zero or applied load, by the methods explained on page 3 . The last column gives the position of the neutral axis for each applied load.

Beam No. 411.5 1-1-2 mixture.

Applied load lb.	Deflection in.	Extensometer Readings. I	II	III	IV	Deformation Upper fiber	Steel	Neutral Axis k
1000	.00	.0005	.0008	.0008	.0015	.000005	.00002	---
2000	.02	.0036	.0036	.0039	.0046	.0000475	.00005	.460
3000	.03	.0072	.0068	.0074	.0080	.000093	.00009	.495
4000	.05	.0112	.0108	.0118	.0125	.000145	.000165	.495
5000	.08	.0180	.0182	.0189	.0215	.0002275	.0002475	.477
6000	.14	.0266	.0305	.0284	.0341	.000318	.000418	.430
7000	.18	.0362	.0443	.0385	.0475	.000418	.000587	.400
8000	.27	.0459	.0570	.0483	.0603	.00052	.00079	.395
9000	.32	.----	----	----	----	-------	----	---
10000	.37	.0615	.0773	.0645	.0815	.000688	.00106	.390
11000	.40	.0700	.0878	.0729	.0823	.000813	.001115	.420
12000	.465	.0785	.0988	.0815	.1036	.000875	.001355	.390
13000	.610	.1057	.1603	.1093	.1679	------	------	---

Beam No. 411.6 1-1-2 mixture.

Applied load lb.	Deflection in.	I	II	III	IV	Upper fiber	Steel	k
1000	.015	.0039	.0034	.0039	.0032	.00005	.000035	.560
2000	.040	.0074	.0067	.0077	.0067	.0000975	.000075	.550
3000	.065	.0115	.0106	.0120	.0107	.000154	.000125	.540
4000	.095	.0173	.0165	.0178	.0164	.00023	.000193	.535
5000	.155	.0287	.0328	.0297	.0327	.0003475	.000417	.450
6000	.215	.0398	.0480	.0410	.0486	.0004675	.000632	.420
7000	.275	.0488	.0605	.0506	.0615	.000555	.000817	.405
8000	.335	.0579	.0738	.0600	.0739	.0006475	.000990	.395
9000	.390	.0669	.0844	.0690	.0855	.000750	.001137	.395
10000	.445	.0755	.0959	.0782	.0970	.000850	.001285	.395
11000	.505	.0844	.1078	.0879	.1099	.0009475	.001463	.390
11130	.550	----	----	----	----	-----	-----	--

Beam No. 411.1 1-1-2 mixture.

Applied load lb.	Deflection in.	I	II	III	IV	Upper fiber	Steel	k
1000	.030	.0028	.0022	.0032	.0029	.000043	.00003	.560
2000	.040	.0074	.0061	.0084	.0085	.000103	.000083	.540
3000	.075	.0120	.0122	.0138	.0145	.000163	.000168	.485
4000	.105	.0190	.0213	.0234	.0254	.000260	.000295	.465
5000	.150	.0268	.0315	.0309	.0375	.000335	.000453	.420
6000	.220	.0349	.0428	.0403	.0500	.000420	.000618	.405
7000	.265	.0422	.0532	.0484	.0614	.000498	.000775	.390
8000	.330	.0495	.0631	.0568	.0724	.000580	.009130	.390
9000	.370	.0570	.0730	.0648	.0830	.000660	.001045	.387
10000	.420	.0640	.0823	.0727	.0925	.000745	.001180	.390
11000	.480	.0721	.0944	.0826	.1067	.000825	.001360	.380
11170	.---	---	---	---	---	----	----	---

Beam No. 411.2 1-1-2 mixture.

Applied load lb.	Deflection in.	Extensometer Readings. in. I	II	III	IV	Deformation. Upper fiber	Steel	Neutral Axis k
1000	.025	.0032	.0020	.0013	.0030	.0000275	.000030	.460
2000	.040	.0071	.0052	.0047	.0067	.0000875	.000067	.490
3000	.065	.0129	.0107	.0082	.0124	.0001000	.000145	.460
4000	.110	.0202	.0283	.0153	.0209	.0002150	.000247	.460
5000	.155	.0305	.0312	.0250	.0336	.0003250	.000423	.430
6000	.230	.0401	.0455	.0352	.0467	.0004220	.000613	.405
7000	.275	.0502	.0579	.0448	.0592	.0005040	.000781	.405
8000	.330	.0591	.0691	.0534	.0702	.0006270	.000925	.405
9000	.385	.0680	.0799	.0620	.0811	.0007200	.001070	.400
10000	.440	.0760	.0906	.0708	.0918	.0008220	.001220	.400
11000	.510	.0862	.1037	.0808	.1047	.0009500	.001320	.415

Beam No. 412.6 1-1 1/2-3 mixture.

Applied load lb.	Deflection in.	I	II	III	IV	Upper fiber	Steel	Neutral Axis k
1000	.020	.0049	.0039	.0041	.0044	.0000575	.000045	.540
2000	.040	.0101	.0093	.0091	.0098	.000125	.000115	.505
3000	.075	.0157	.0159	.0148	.0162	.000193	.000197	.490
4000	.115	.0237	.0258	.0227	.0264	.000278	.000388	.445
5000	.160	.0319	.0367	.0312	.0380	.000365	.000487	.425
6000	.220	.0412	.0494	.0407	.0498	.000463	.000652	.410
7000	.280	.0500	.0613	.0494	.0613	.000555	.000818	.400
8000	.330	.0587	.0726	.0580	.0723	.000650	.000970	.400
9000	.395	.0689	.0850	.0679	.0845	.000758	.001130	.400
10000	.455	.0786	.0968	.0774	.0957	.000875	.001270	.405
11000	.520	.0889	.1093	.0876	.1075	.000985	.001450	.400
11885	.600	----	----	----	----	----	----	---

Beam No. 413.5 1-2-4 mixture.

Applied load lb.	Deflection in.	I	II	III	IV	Upper fiber	Steel	Neutral Axis k
1000	.015	.0035	.0036	.0032	.0034	.000043	.000043	.500
2000	.030	.0080	.0078	.0076	.0075	.000097	.000093	.510
3000	.070	.0139	.0143	.0130	.0135	.000167	.000175	.490
4000	.115	.0228	.0253	.0212	.0140	.000168	.000315	.455
5000	.170	.0328	.0372	.0308	.0358	.000375	.000470	.445
6000	.220	.0425	.0483	.0398	.0461	.000485	.000608	.440
7000	.275	.0525	.0593	.0487	.0564	.000590	.000750	.440
8000	.330	.0627	.0697	.0578	.0669	.000708	.000880	.445
9000	.390	.0729	.0803	.0670	.0754	.000880	.000990	~~.500~~ .460
10000	.455	.0838	.0913	.0768	.0859	.000963	.001130	.460
11000	.520	.0901	.1023	.0870	.0961	.001070	.001223	.465
11950	.655	---	---	---	---	----	----	---

Beam No. 417.5 1-2-4 mixture.

Applied load lb.	Deflection in.	Extensometer Readings in. I	II	III	IV	Deformation Upper fiber	Steel	Neutral Axis k
1000	.020	.0049	.0028	.0014	.0033	.000043	.000033	.540
2000	.045	.0091	.0077	.0042	.0079	.000073	.000105	.400
3000	.075:	.0149	.0127	.0097	.0132	.000153	.000163	.480
4000	.100	.0209	.0187	.0160	.0199	.000230	.000238	.490
5000	.135	.0271	.0248	.0223	.0259	.000310	.000310	.495
6000	.175	.0348	.0318	.0297	.0336	.000405	.000395	.505
7000	.195	.0422	.0385	.0370	.0410	.000498	.000485	.507
8000	.260	.0505	.0458	.0454	.0491	.000610	.000570	.515
9000	.300	.0591	.0534	.0541	.0570	.000723	.000663	.520
10000	.355	.0673	.0614	.0637	.0655	.000835	.000760	.520
11000	.410	.0761	.0702	.0750	.0745	.000965	.000858	.527
12000	.480	.0868	.0786	.0862	.0811	.001130	.000925	.550

Beam No. 417.6 1-2-4 mixture.

Applied load lb.	Deflection in.	Extensometer Readings in. I	II	III	IV	Deformation Upper fiber	Steel	Neutral Axis k
1000	.020	.0037	.0029	.0040	.0033	.000050	.000030	.600
2000	.040	.0084	.0070	.0088	.0076	.000115	.000080	.580
3000	.070	.0140	.0122	.0145	.0130	.000190	.000140	.565
4000	.100	.0212	.0194	.0218	.0199	.000285	.000225	.555
5000	.150	.0293	.0278	.0302	.0282	.000385	.000325	.540
6000	.190	.0389	.0368	.0395	.0368	.000510	.000428	.540
7000	.250	.0497	.0461	.0501	.0459	.000655	.000538	.547
8000	.310	.0626	.0565	.0626	.0560	.000825	.000643	.560
9000	.375	.0743	.0666	.0763	.0664	.001025	.000750	.570
10000	.460	.0936	.0800	.0954	.0792	.001280	.000868	.595
10520	.---	---	---	---	---	----	-----	---

Beam No. 414.5 1-3-6 mixture.

Applied load lb.	Deflection in.	Extensometer Readings in. I	II	III	IV	Deformation Upper fiber	Steel	Neutral Axis k
1000	.025	.0045	.0047	.0047	.0045	.0000475	.0000425	.520
2000	.065	.0119	.0132	.0108	.0124	.000138	.000165	.440
3000	.105	.0199	.0228	.0189	.0219	.000228	.000290	.440
4000	.155	.0289	.0333	.0278	.0325	.000333	.000425	.430
5000	.210	.0384	.0438	.0371	.0432	.000440	.000555	.440
6000	.270	.0486	.0545	.0468	.0541	.000555	.000705	.440
7000	.330	.0595	.0656	.0572	.0650	.000693	.000833	.450
7920	.420	----	----	---	---	-----	-----	--

Beam No. 414.6 1-3-6 mixture.

Applied load lb.	Deflection in.	Extensometer Readings. in. I	II	III	IV	Deformation Upper fiber	Steel	Neutral Axis k
1000	.030	.0062	.0053	.0055	.0057	.000075	.000065	.520
2000	.065	.0136	.0133	.0130	.0136	.000200	.000165	.500
3000	.130	.0233	.0245	.0227	.0248	.000288	.000308	.480
4000	.190	.0336	.0362	.0332	.0361	.000405	.000453	.470
5000	.250	.0455	.0485	.0451	.0475	.000553	.000595	.480
6000	.325	.0585	.0614	.0578	.0599	.000713	.000758	.485
7000	.395	.0728	.0740	.0720	.0723	.000910	.000893	.505
8000	.485	.0917	.0890	.0904	.0866	.001175	.001035	.530
8430	.565	----	----	---	---	----	----	---

Beam No. 415.1 1-4-8 mixture.

Applied load lb.	Deflection in.	I	II	III	IV	Upper fiber	Steel	k
1000	.030	.0034	.0046	.0041	.0054	.000040	.000068	.350
2000	.075	.0117	.0127	.0118	.0132	.000140	.000168	.450
3000	.120	.0216	.0227	.0214	.0233	.000265	.000288	.480
4000	.180	.0312	.0328	.0311	.0336	.000383	.000412	.480
5000	.245	.0418	.0431	.0417	.0441	.000515	.000543	.487
6000	.305	.0522	.0531	.0527	.0545	.000658	.000663	.495
7000	.375	.0639	.0638	.0647	.0655	.000810	.000788	.505
8000	.455	.0776	.0752	.0788	.0776	.000995	.000918	.520
8680	---	----	----	----	----	----	-----	---

Beam No. 415.2 1-4-8 mixture.

Applied load lb.	Deflection in.	I	II	III	IV	Upper fiber	Steel	k
1000	.030	.0064	.0053	.0060	.0045	.000085	.000050	.620
2000	.080	.0164	.0137	.0162	.0155	.000218	.000168	.550
3000	.130	.0266	.0239	.0267	.0265	.000348	.000295	.537
4000	.190	.0378	.0345	.0380	.0379	.000490	.000425	.530
5000	.260	.0495	.0454	.0502	.0494	.000648	.000560	.535
6000	.330	.0625	.0560	.0631	.0612	.000815	.000685	.540
7000	.410	.0775	.0674	.0779	.0731	.001023	.000808	.555
7220	---	----	---	----	---	----	----	---

Beam No. 415.5 1-4-8 mixture.

Applied load lb.	Deflection in.	I	II	III	IV	Upper fiber	Steel	k
1000	.035	.0047	.0048	.0051	.0053	.000060	.000050	.500
2000	.070	.0131	.0146	.0139	.0146	.000165	.000180	.470
3000	.120	.0230	.0243	.0242	.0262	.000290	.000315	.475
4000	.185	.0340	.0358	.0361	.0390	.000427	.000465	.480
5000	.250	.0452	.0473	.0491	.0517	.000580	.000613	.485
6000	.432	.0617	.0614	.0665	.0673	.000805	.000785	.507
7000	.470	.0920	.0810	.0998	.0918	.001260	.000983	.560

Beam No. 415.6 1-4-8 mixture.

Applied load lb.	Deflection in.	Extensometer Readings. in. I	II	III	IV	Deformation Upper fiber	Steel	Neutral Axis k
1000	.025	.0055	.0062	.0056	.0056	.000065	.000075	.460
2000	.080	.0149	.0172	.0144	.0155	.000178	.000210	.450
3000	.140	.0248	.0289	.0236	.0258	.000290	.000345	.450
4000	.190	.0350	.0405	.0323	.0360	.000408	.000488	.455
5000	.250	.0452	.0516	.0429	.0461	.000525	.000620	.457
6000	.315	.0569	.0638	.0543	.0572	.000628	.000768	.465
7000	.390	.0696	.0760	.0668	.0690	.000828	.000908	.480
8000	.480	.0850	.0898	.0824	.0820	.001065	.001045	.497

Beam No. 416.5 1-5-10 mixture.

1000	.030	.0061	.0049	.0062	.0059	.000080	.000060	.560
2000	.080	.0148	.0139	.0150	.0148	.000193	.000168	.520
3000	.145	.0252	.0246	.0258	.0250	.000353	.000295	.520
4000	.200	.0374	.0361	.0375	.0353	.000485	.000418	.530
5000	.285	.0520	.0476	.0513	.0461	.000688	.000504	.560
5060	.370	----	----	----	----	----	----	---

Beam No. 416.6 1-5-10 mixture.

1000	.040	.0061	.0067	.0072	.0077	.000093	.000080	.540
2000	.090	.0083	.0167	.0175	.0215	.000228	.000195	.530
3000	.145	.0120	.0268	.0284	.0395	.000373	.000313	.540
4000	.215	.0230	.0378	.0346	.0414	.000420	.000475	.470
5000	.290	.0235	.0493	.0483	.0536	.000600	.000605	.495
5830	.---	----	---	----	---	----	----	---

<u>Note</u>:- In Beam No. 416.6 the readings of Dials No.2 and No.3 only were used, as the others were evidently in error.

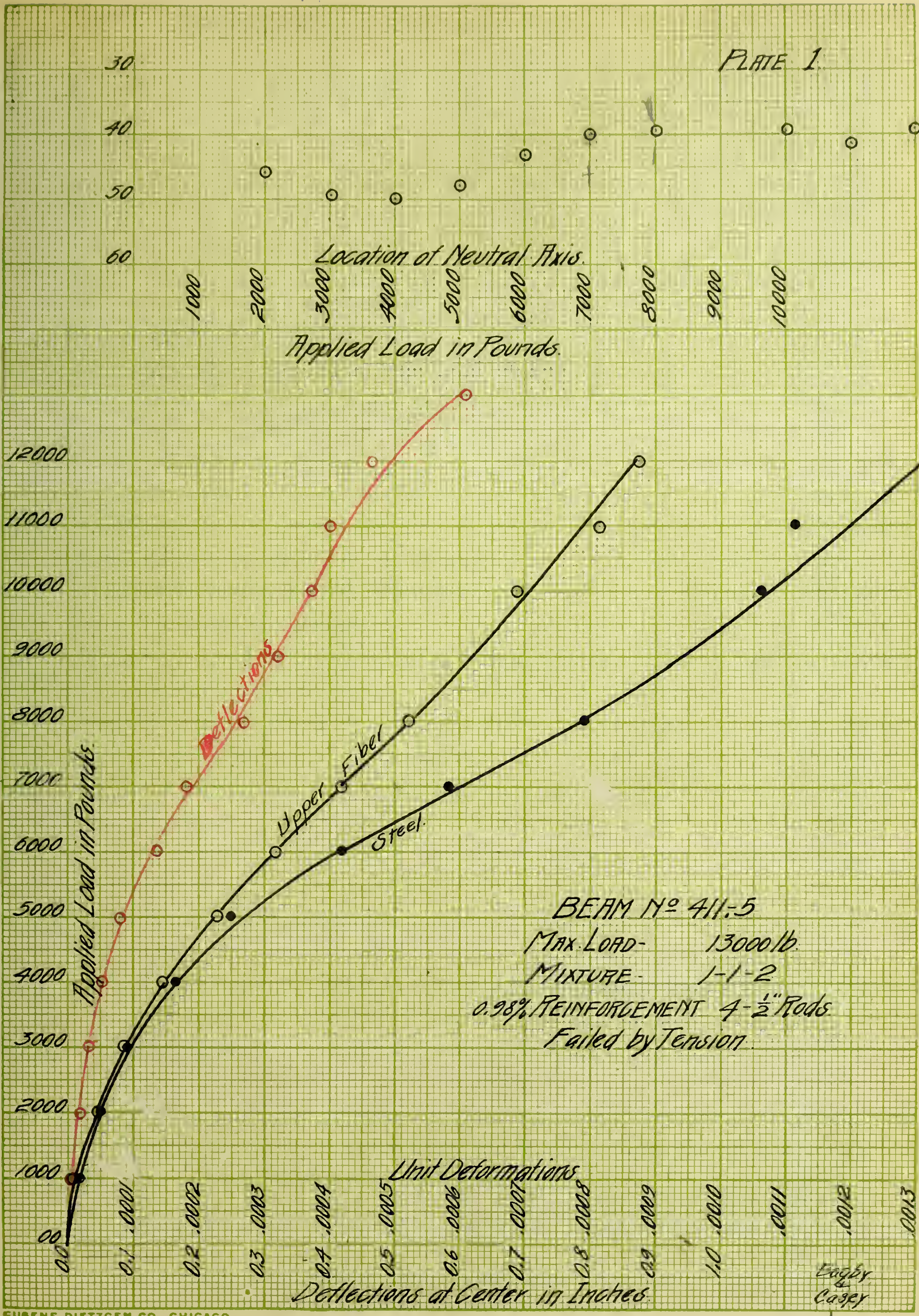


PLATE 1.
Location of Neutral Axis.
30
40
50
60
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Applied Load in Pounds.
Applied Load in Pounds.
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
00
Deflections
Upper Fiber
Steel
BEAM No. 411-5
Max. Load- 13000 lb.
Mixture- 1-1-2
0.98% Reinforcement 4-½" Rods
Failed by Tension.
Unit Deformations
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
.0011
.0012
.0013
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Deflections at Center in Inches.
Casey
EUGENE DIETZGEN CO., CHICAGO.

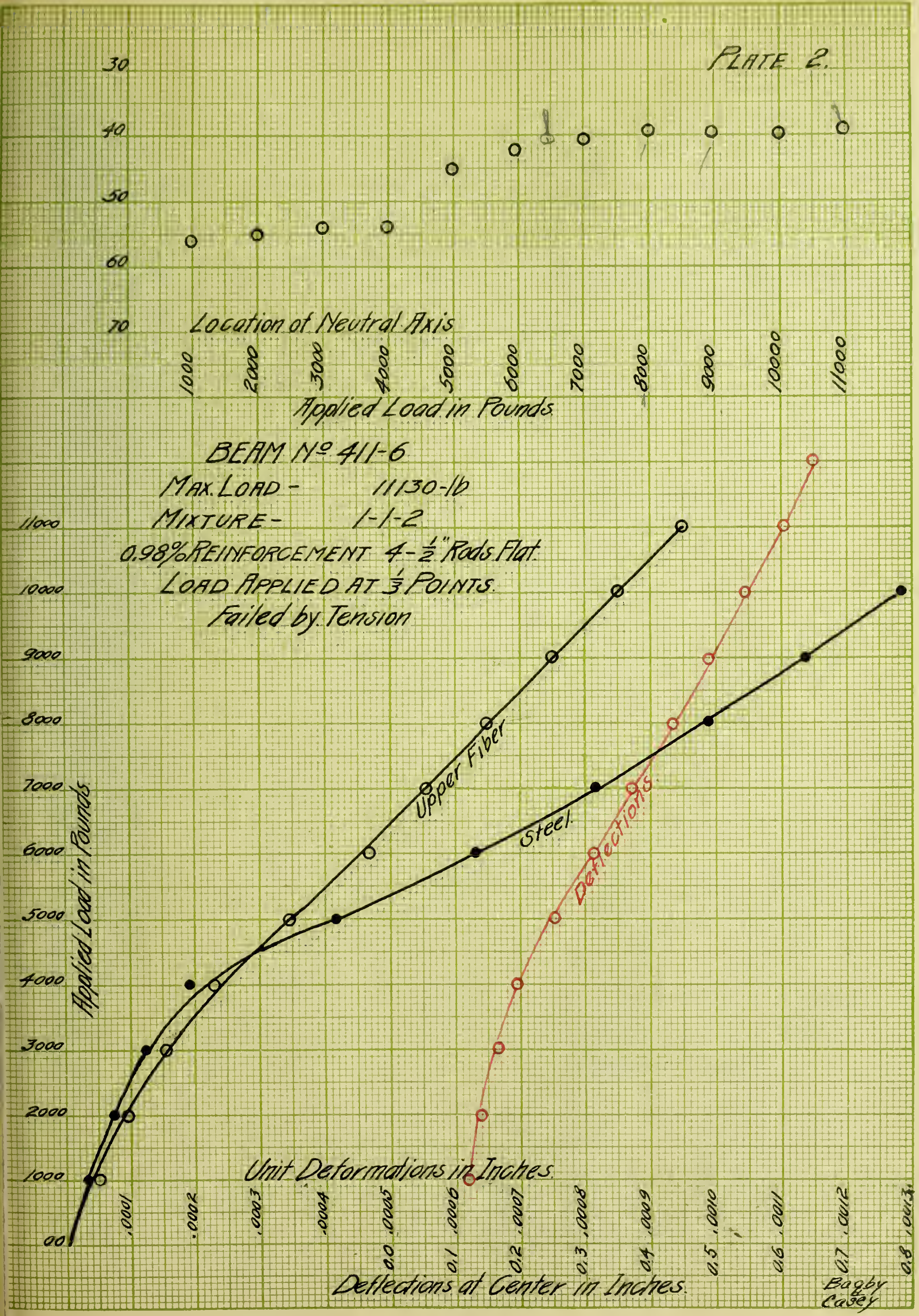


PLATE 2.
Location of Neutral Axis
30
40
50
60
70
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
Applied Load in Pounds.
BEAM No 411-6
MAX. LOAD - 11130-lb
MIXTURE - 1-1-2
0.98% REINFORCEMENT 4-1/2" Rods Flat.
LOAD APPLIED AT 1/3 POINTS.
Failed by Tension
Upper Fiber
Steel.
Deflections
Applied Load in Pounds
Unit Deformations in Inches.
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
.0011
.0012
.0013
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Deflections at Center in Inches.
Bagby
Casey

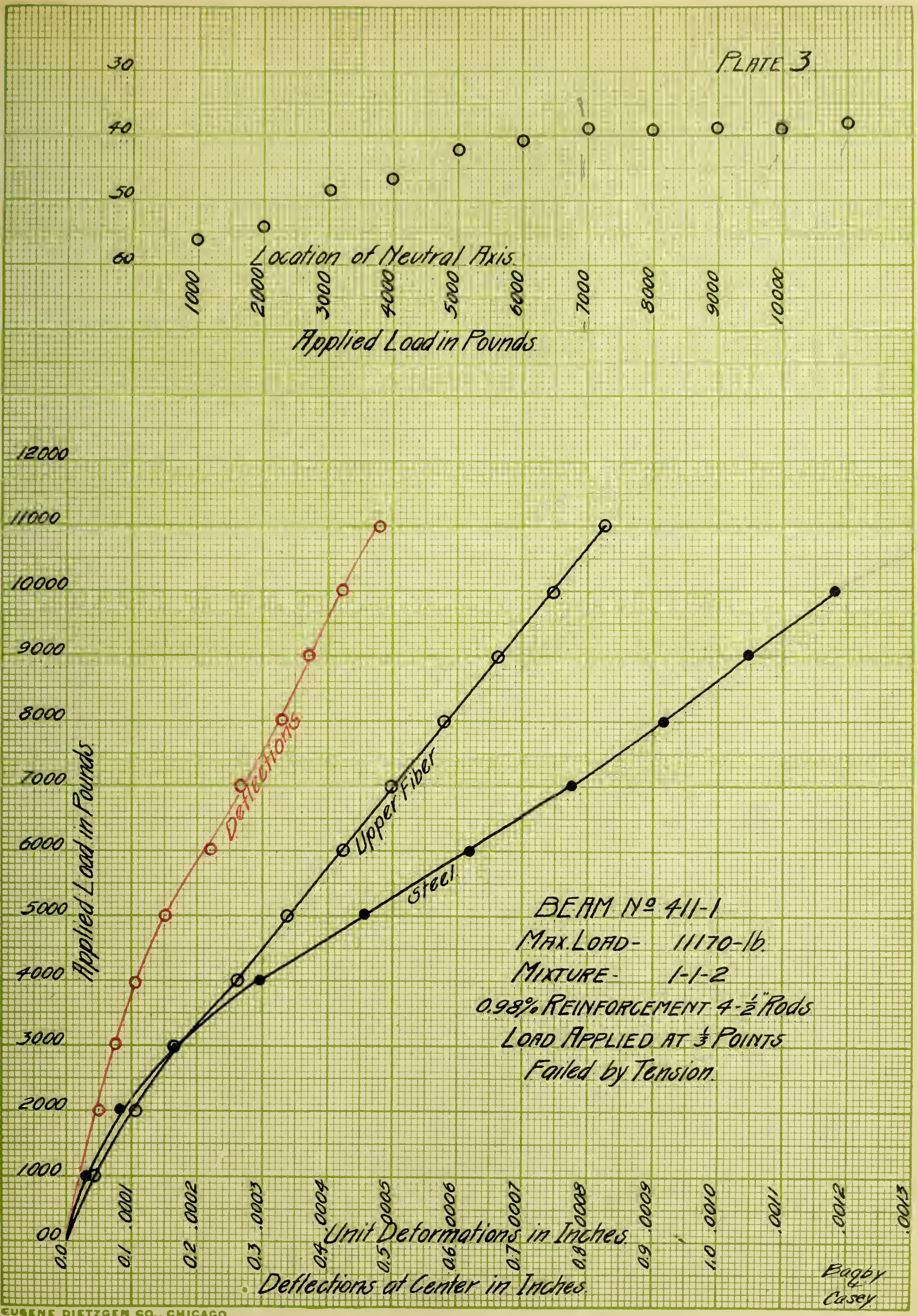


PLATE 3
30
40
50
60
Location of Neutral Axis
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Applied Load in Pounds
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
00
Applied Load in Pounds
Deflections
Upper Fiber
Steel
BEAM № 411-1
MAX. LOAD- 11170-lb.
MIXTURE- 1-1-2
0.98% REINFORCEMENT 4-½" Rods
LOAD APPLIED AT ⅓ POINTS
Failed by Tension.
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
.0011
.0012
.0013
Unit Deformations in Inches
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Deflections at Center in Inches
Bagby
Casey
EUGENE DIETZGEN CO., CHICAGO.

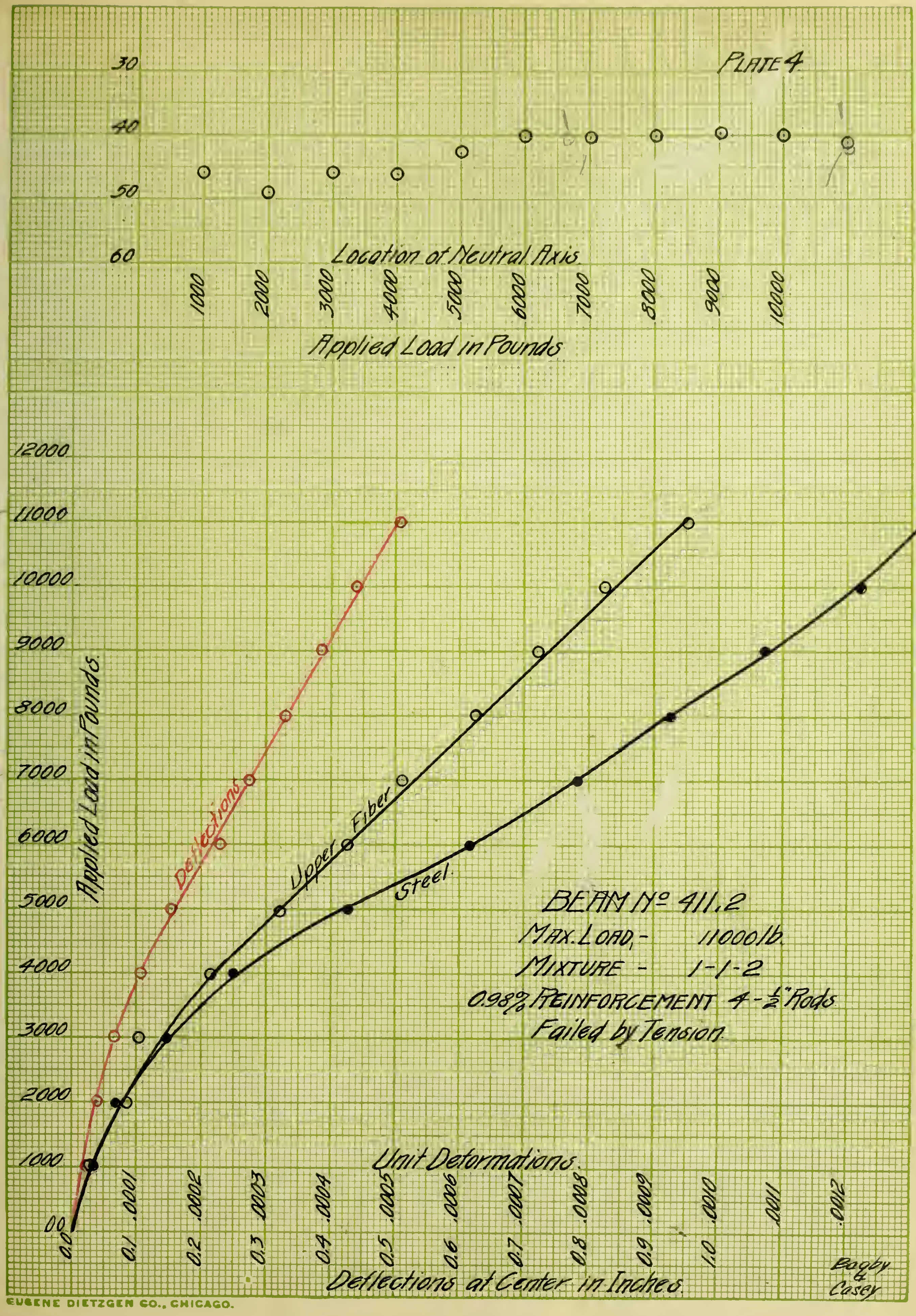


PLATE 4
30
40
50
60
Location of Neutral Axis
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Applied Load in Pounds
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0.0
Applied Load in Pounds
Deflections
Upper Fiber
Steel
BEAM Nº 411.2
MAX. LOAD, - 11000 lb.
MIXTURE - 1-1-2
0.98% REINFORCEMENT 4 - ½" Rods
Failed by Tension
Unit Deformations
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
.0011
.0012
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Deflections at Center in Inches
Bagby & Casey
EUGENE DIETZGEN CO., CHICAGO.

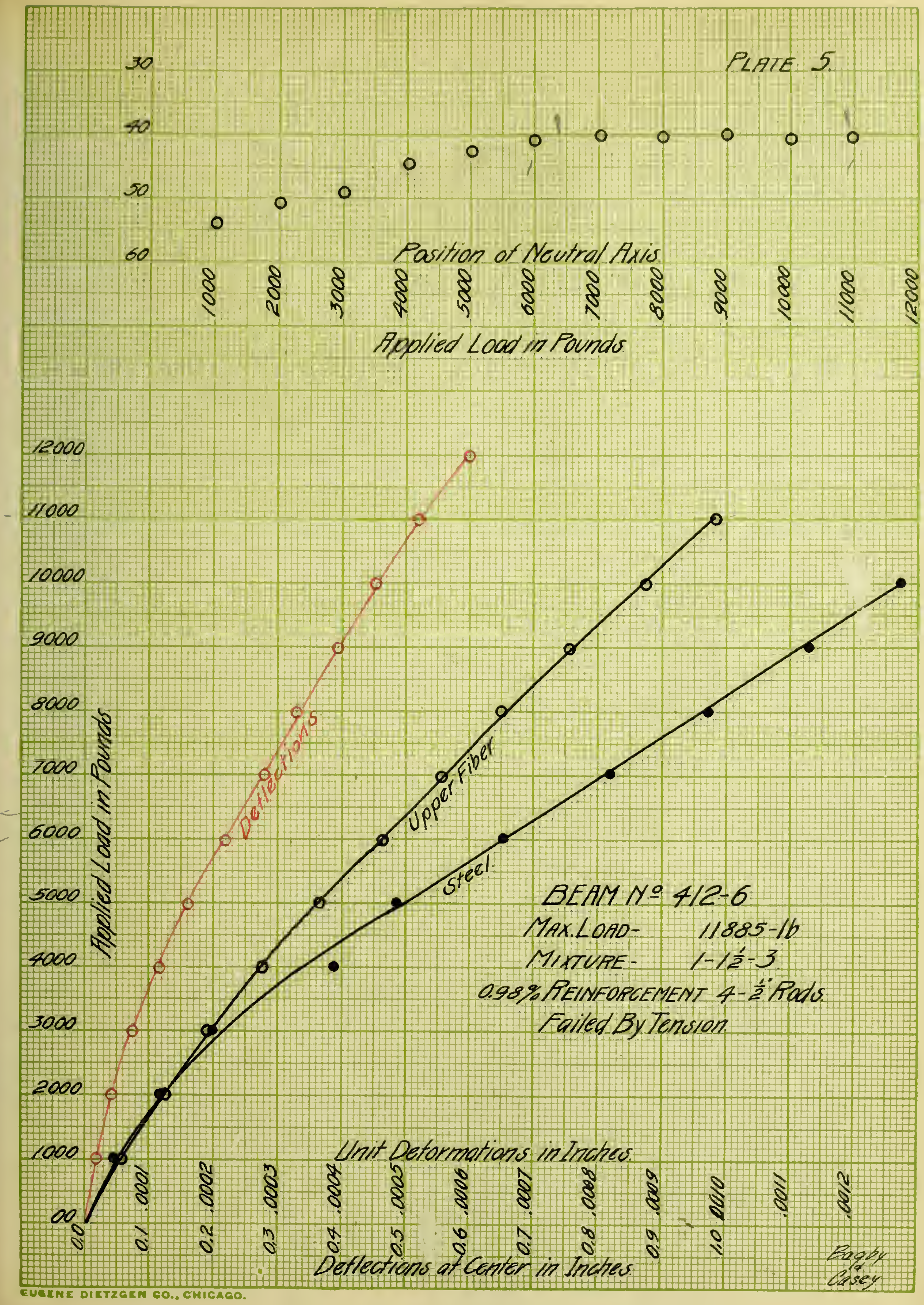


PLATE 5.
Position of Neutral Axis
Applied Load in Pounds
Applied Load in Pounds
Deflections
Upper Fiber
Steel
BEAM Nº 412-6
MAX. LOAD- 11885-lb
MIXTURE- 1-1½-3
0.98% REINFORCEMENT 4-½" Rods.
Failed By Tension.
Unit Deformations in Inches.
Deflections at Center in Inches
Bagby & Casey
EUGENE DIETZGEN CO., CHICAGO.

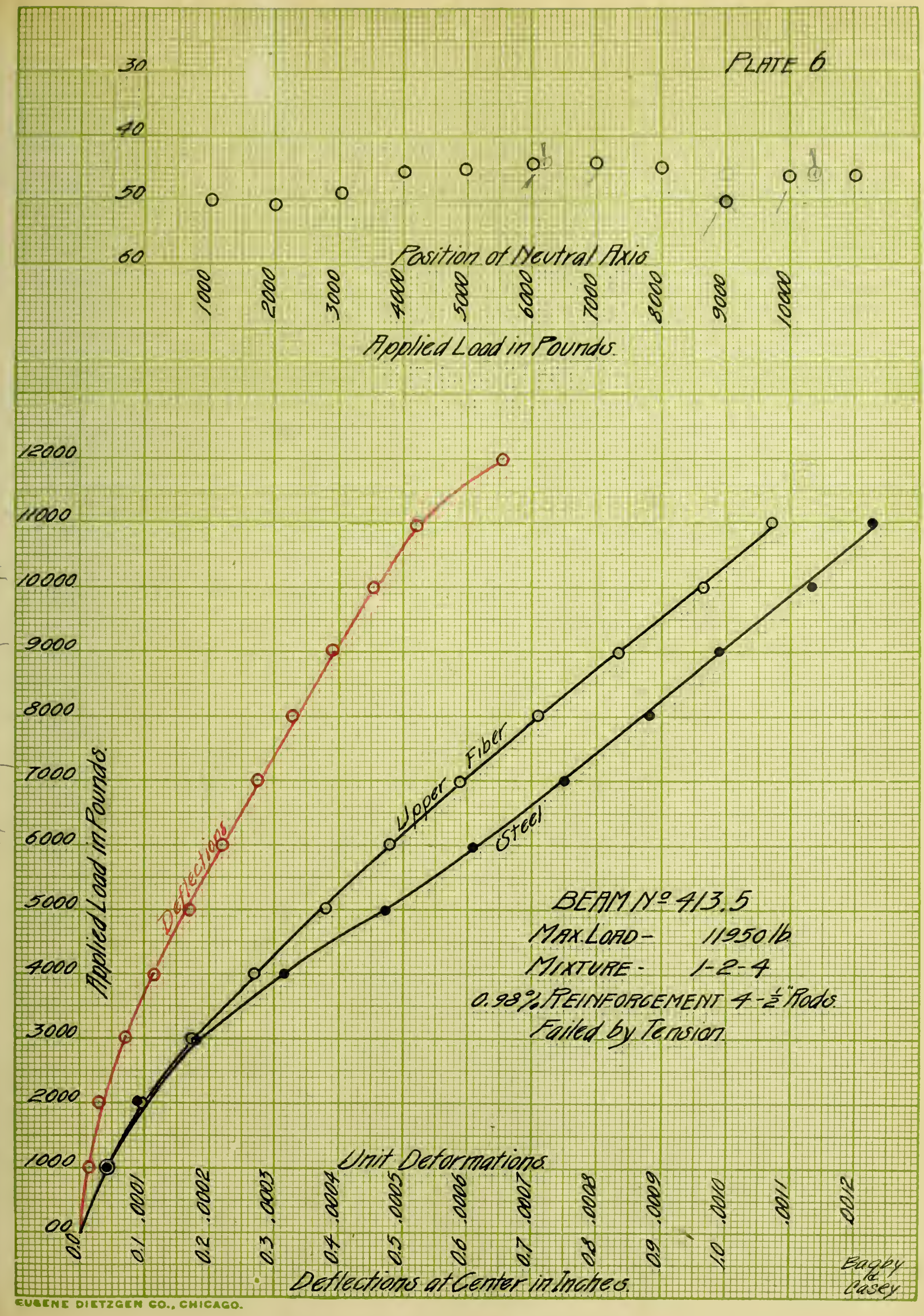


PLATE 6
Position of Neutral Axis
Applied Load in Pounds
Applied Load in Pounds
Deflections
Upper Fiber
Steel
Unit Deformations
Deflections at Center in Inches
BEAM Nº 413.5
MAX. LOAD - 11950 lb.
MIXTURE - 1-2-4
0.98% REINFORCEMENT 4-½" Rods
Failed by Tension
Bagby & Casey
EUGENE DIETZGEN CO., CHICAGO.

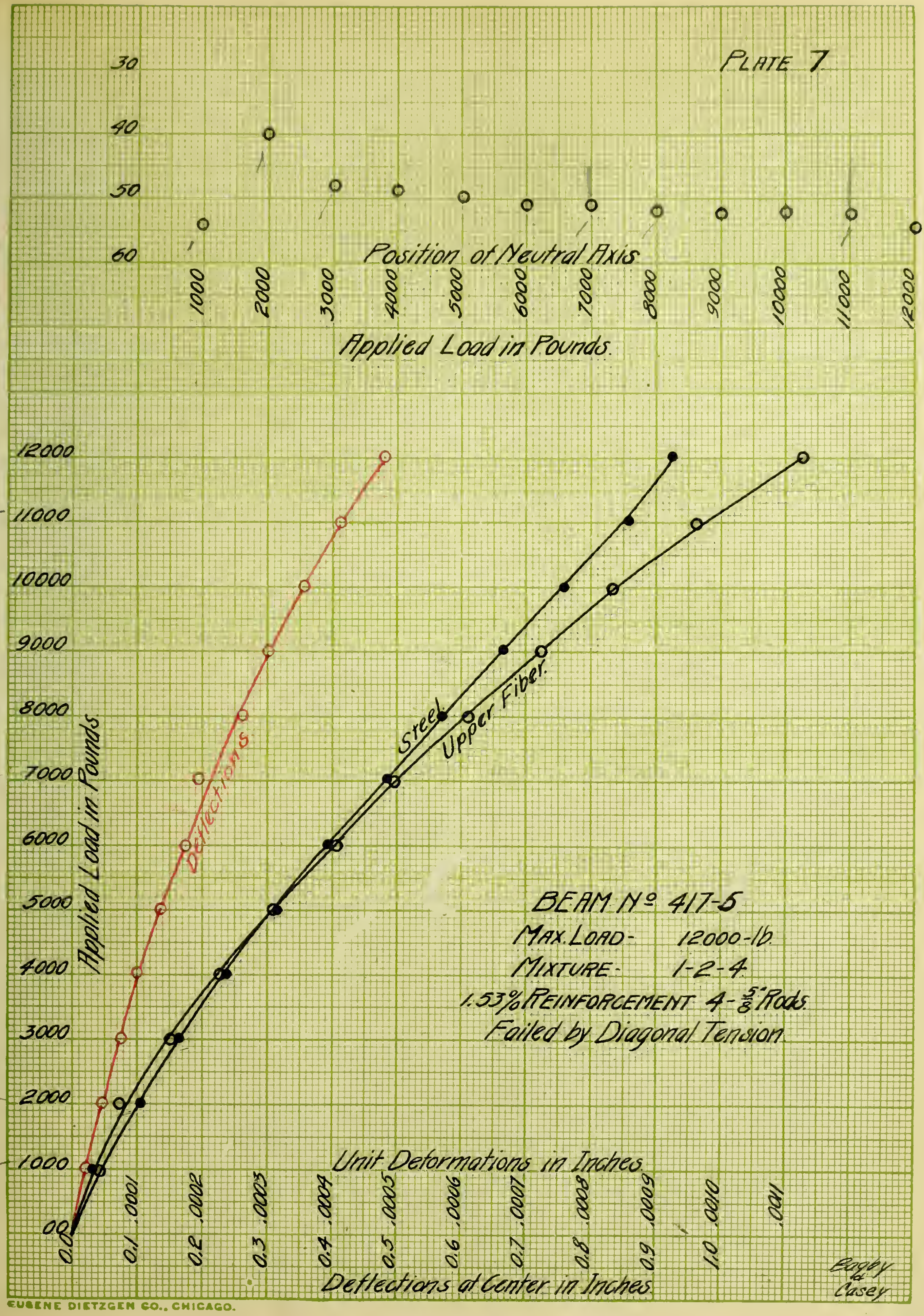


PLATE 7
30
40
50
60
Position of Neutral Axis
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
Applied Load in Pounds.
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0.0
Applied Load in Pounds
Deflections
Steel
Upper Fiber.
BEAM Nº 417-5
MAX. LOAD - 12000-lb.
MIXTURE - 1-2-4
1.53% REINFORCEMENT 4-5/8" Rods.
Failed by Diagonal Tension.
Unit Deformations in Inches.
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
.0011
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Deflections at Center in Inches.
Bagby & Casey
EUGENE DIETZGEN CO., CHICAGO.

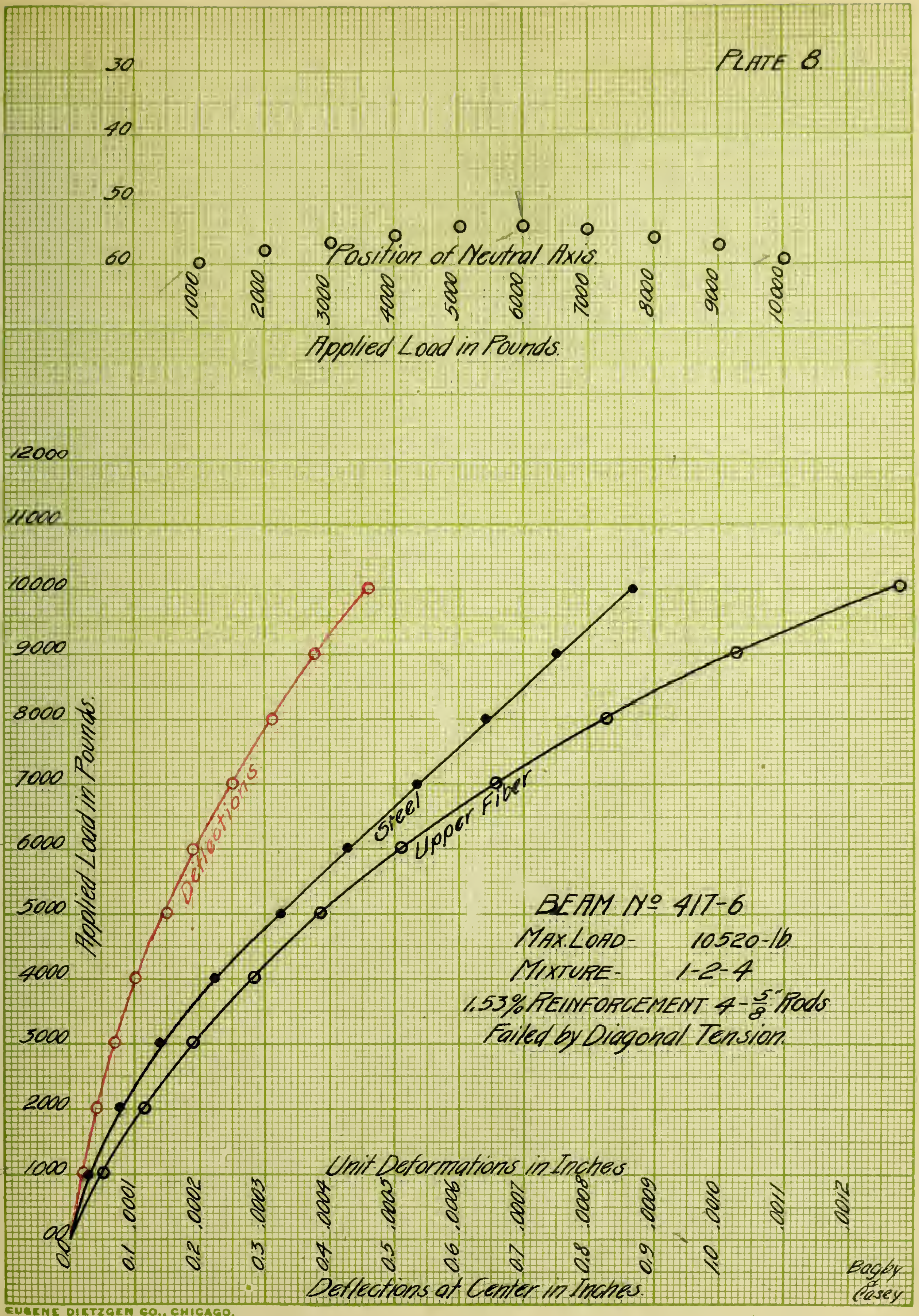


PLATE 8.
Position of Neutral Axis.
Applied Load in Pounds.
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
30
40
50
60
Applied Load in Pounds.
Deflections
Steel
Upper Fiber
BEAM No. 417-6
MAX. LOAD- 10520-lb.
MIXTURE- 1-2-4
1.53% REINFORCEMENT 4-5/8" Rods
Failed by Diagonal Tension.
Unit Deformations in Inches
Deflections at Center in Inches.
Bagby
Casey
EUGENE DIETZGEN CO., CHICAGO.

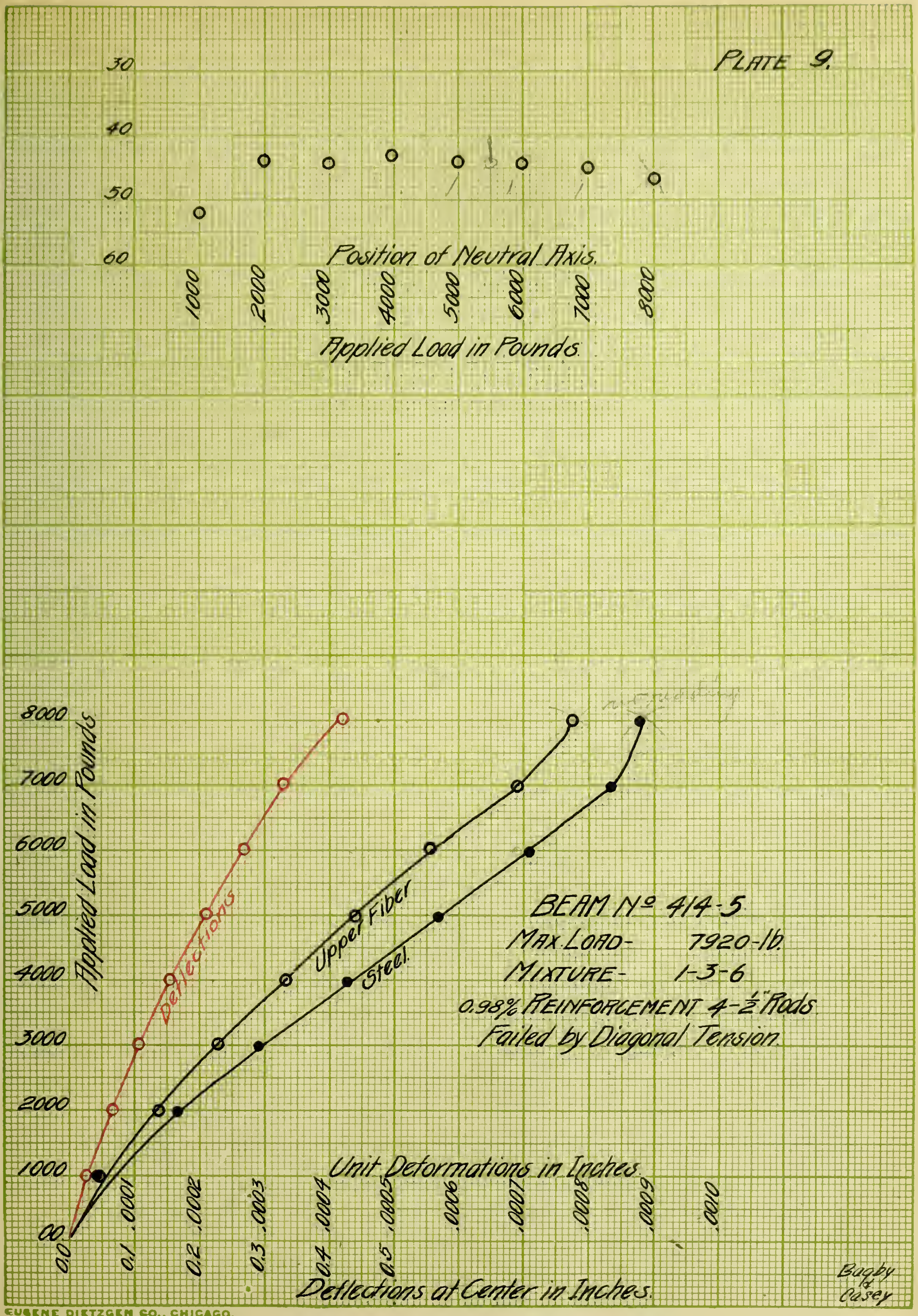
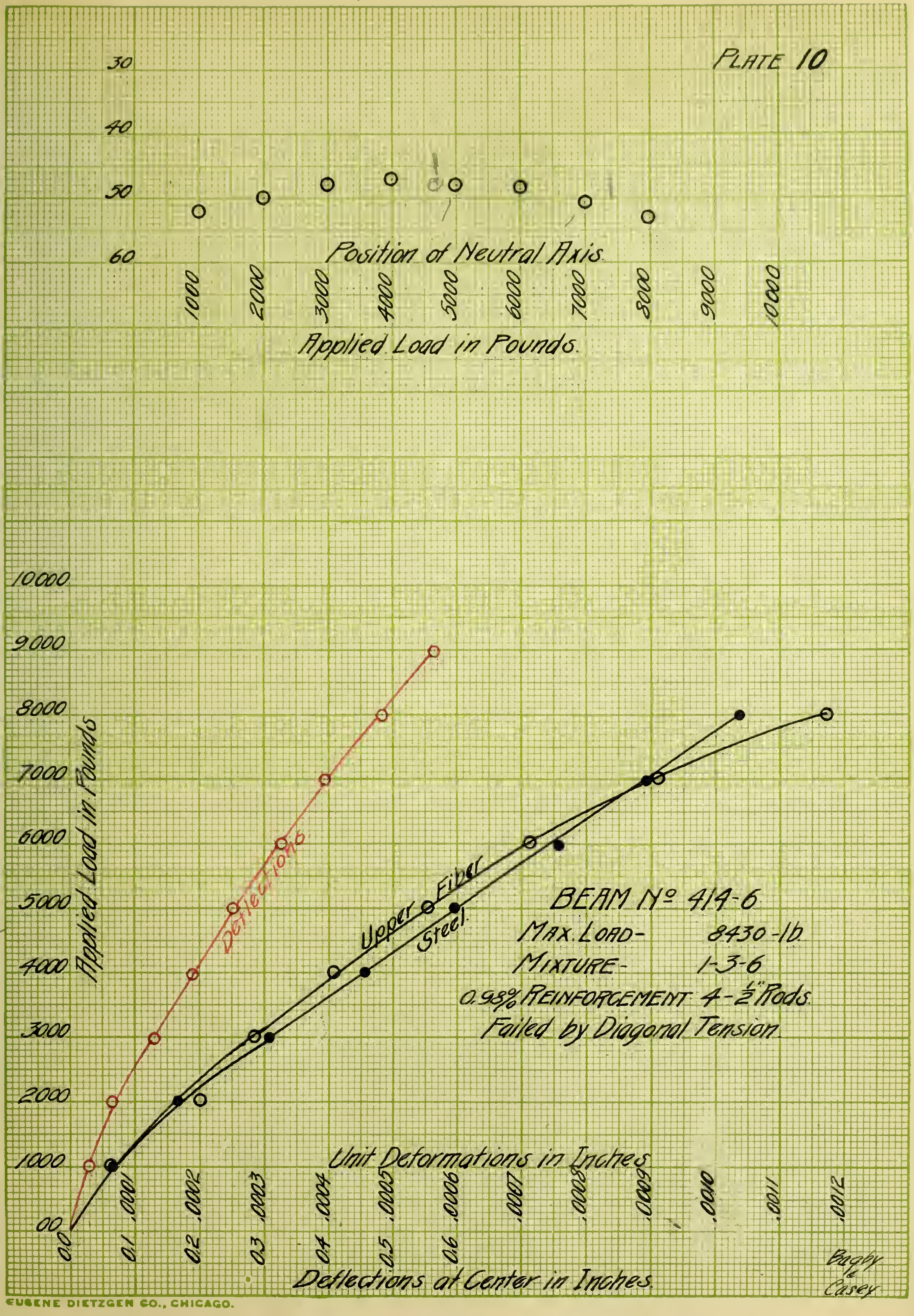


PLATE 9.
30
40
50
60
Position of Neutral Axis.
1000
2000
3000
4000
5000
6000
7000
8000
Applied Load in Pounds.
Applied Load in Pounds
8000
7000
6000
5000
4000
3000
2000
1000
0.0
Deflections
Upper Fiber
Steel.
BEAM No 414-5
MAX LOAD- 7920-lb.
MIXTURE- 1-3-6
0.98% REINFORCEMENT 4-½" Rods
Failed by Diagonal Tension.
Unit Deformations in Inches.
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
0.0
0.1
0.2
0.3
0.4
0.5
Deflections at Center in Inches.
Bagby & Casey



EUGENE DIETZGEN CO., CHICAGO.

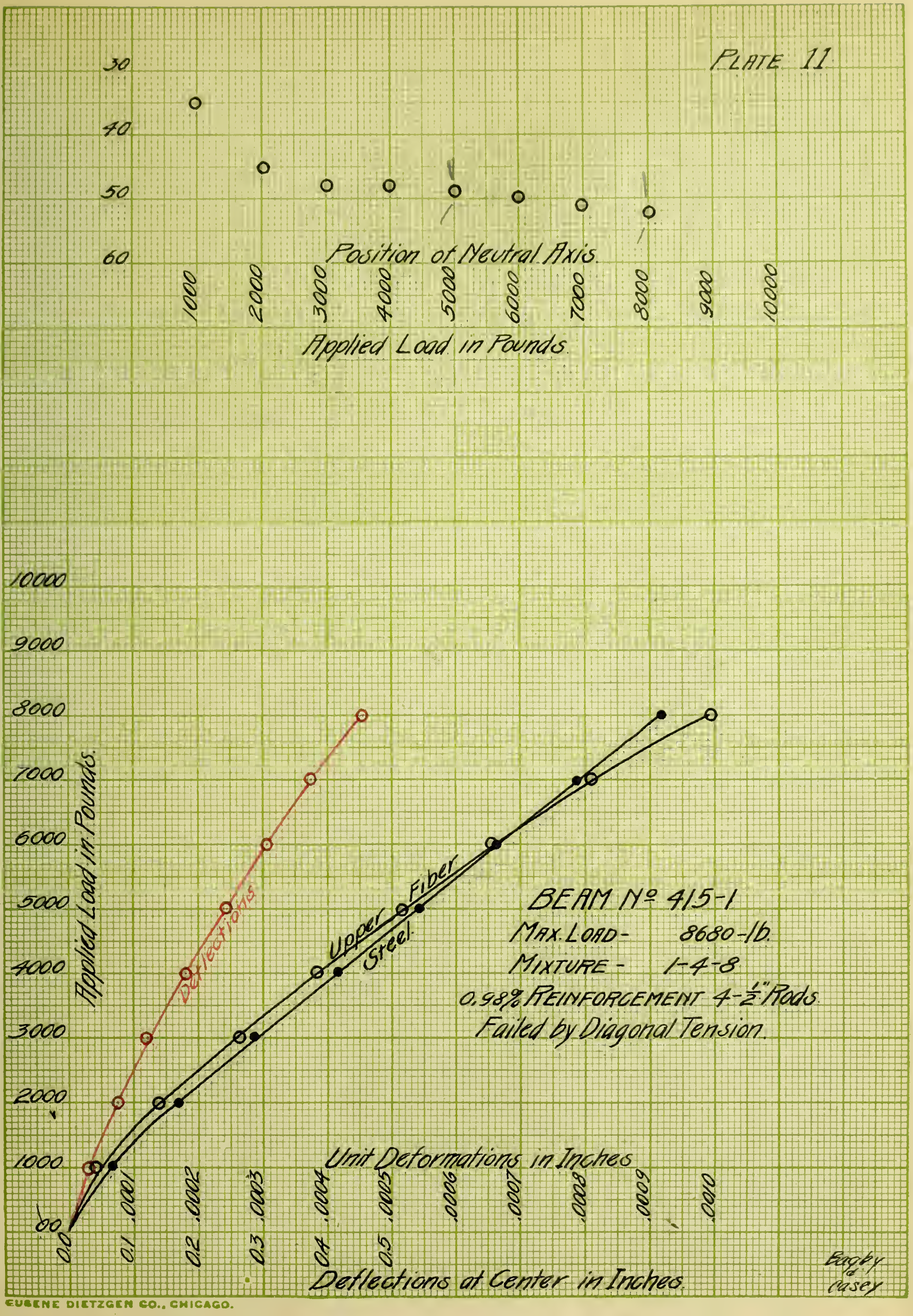


PLATE 11
30
40
50
60
Position of Neutral Axis.
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Applied Load in Pounds.
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
00
Applied Load in Pounds.
Deflections
Upper Fiber
Steel
BEAM № 415-1
MAX. LOAD - 8680-lb.
MIXTURE - 1-4-8
0.98% REINFORCEMENT 4-½" Rods.
Failed by Diagonal Tension.
Unit Deformations in Inches
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
0.0
0.1
0.2
0.3
0.4
0.5
Deflections at Center in Inches.
Bagby & Casey
EUGENE DIETZGEN CO., CHICAGO.

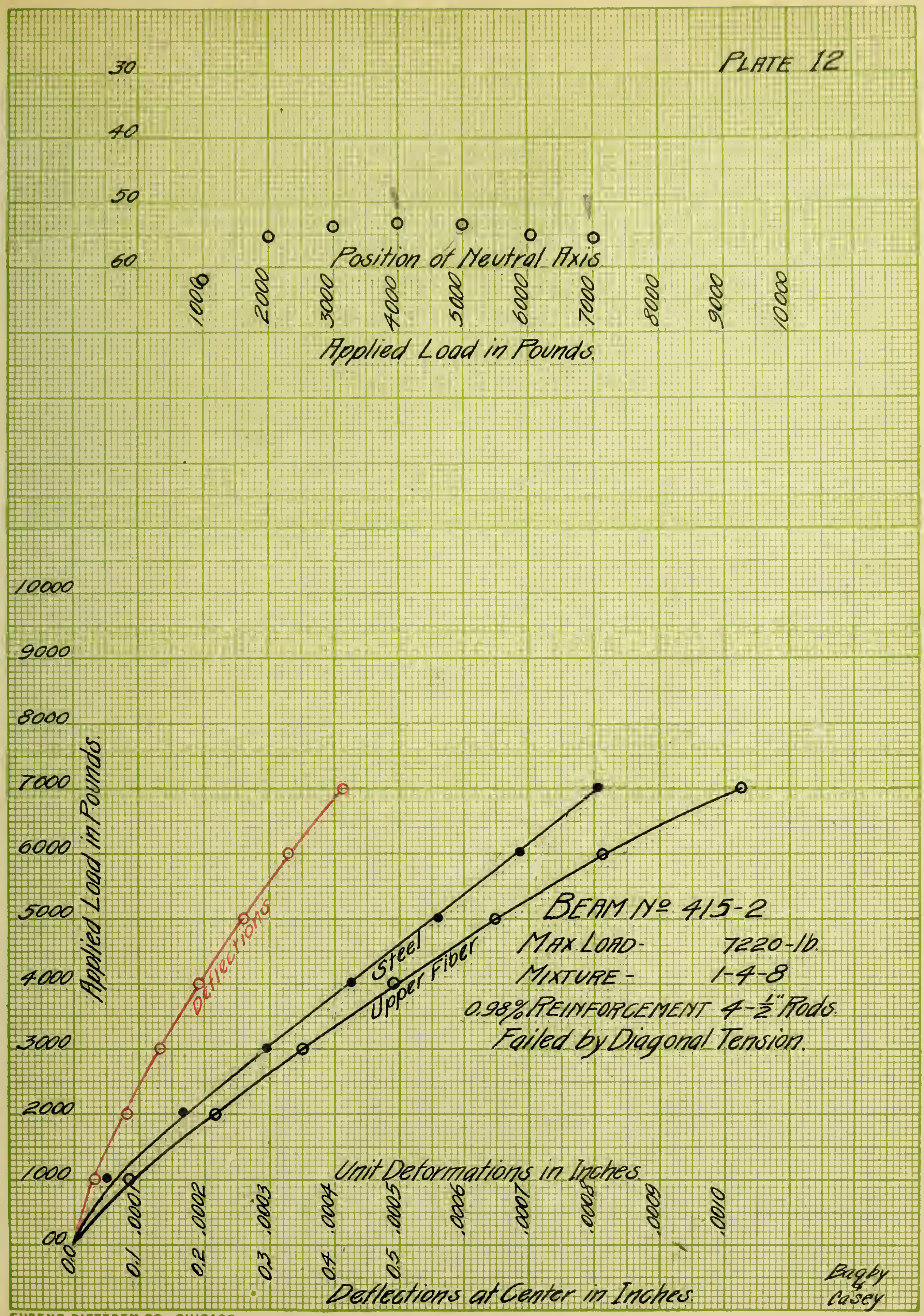


PLATE 12
30
40
50
60
Position of Neutral Axis
1000
2000
3000
4000
5000
6000
7000
8000
9000
10,000
Applied Load in Pounds.
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
00
Applied Load in Pounds.
Deflections
Steel
Upper Fiber
BEAM No 415-2
MAX LOAD - 7220-lb.
MIXTURE - 1-4-8
0.98% REINFORCEMENT 4-½" Rods.
Failed by Diagonal Tension.
Unit Deformations in Inches.
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
0.0
0.1
0.2
0.3
0.4
0.5
Deflections at Center in Inches.
Bagby & Casey
EUGENE DIETZGEN CO., CHICAGO.

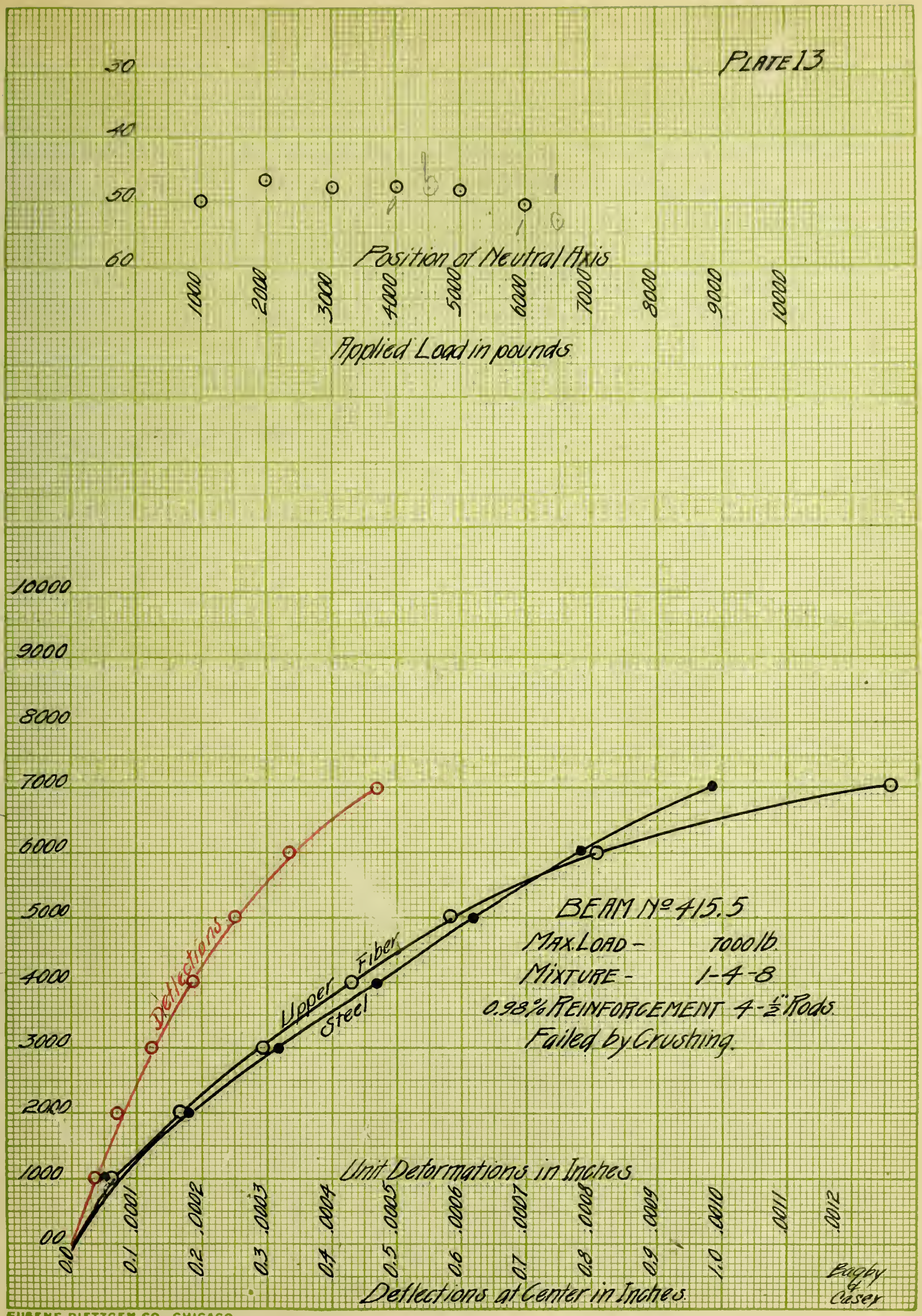
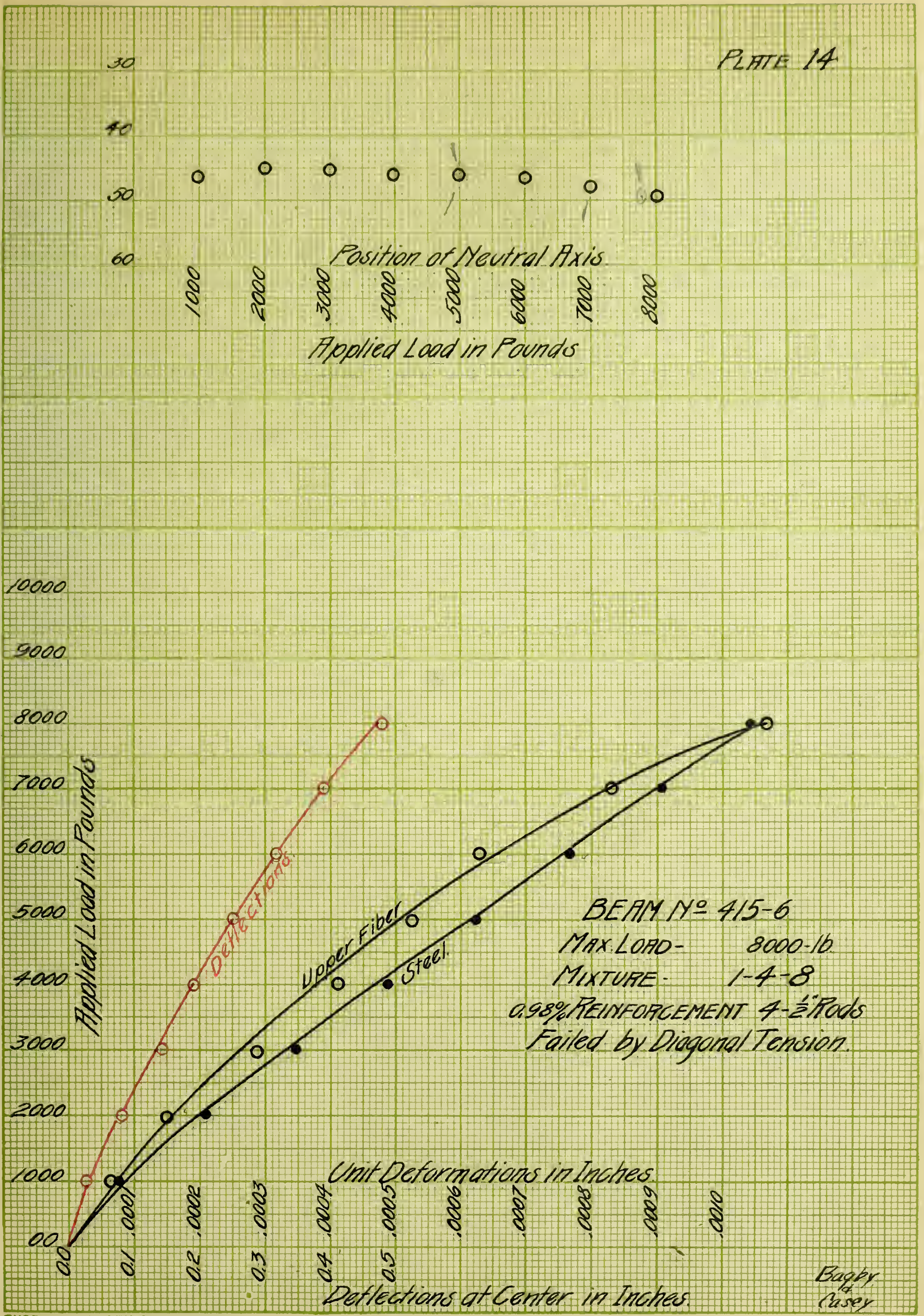


PLATE 13
Position of Neutral Axis
30
40
50
60
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Applied Load in pounds
Deflections
Upper Fiber
Steel
BEAM No 415.5
MAX. LOAD - 7000 lb.
MIXTURE - 1-4-8
0.98% REINFORCEMENT 4-½" Rods.
Failed by Crushing.
Unit Deformations in Inches
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
.0011
.0012
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Deflections at Center in Inches.
Bagby & Casey
EUGENE DIETZGEN CO., CHICAGO.

PLATE 14


30
40
50
60
Position of Neutral Axis
1000
2000
3000
4000
5000
6000
7000
8000
Applied Load in Pounds
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0.0
Applied Load in Pounds
Deflections
Upper Fiber
Steel
BEAM No 415-6
MAX LOAD- 8000-lb
MIXTURE - 1-4-8
0.98% REINFORCEMENT 4-½" Rods
Failed by Diagonal Tension.
Unit Deformations in Inches.
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
0.0
0.1
0.2
0.3
0.4
0.5
Deflections at Center in Inches.
Bagby & Casey

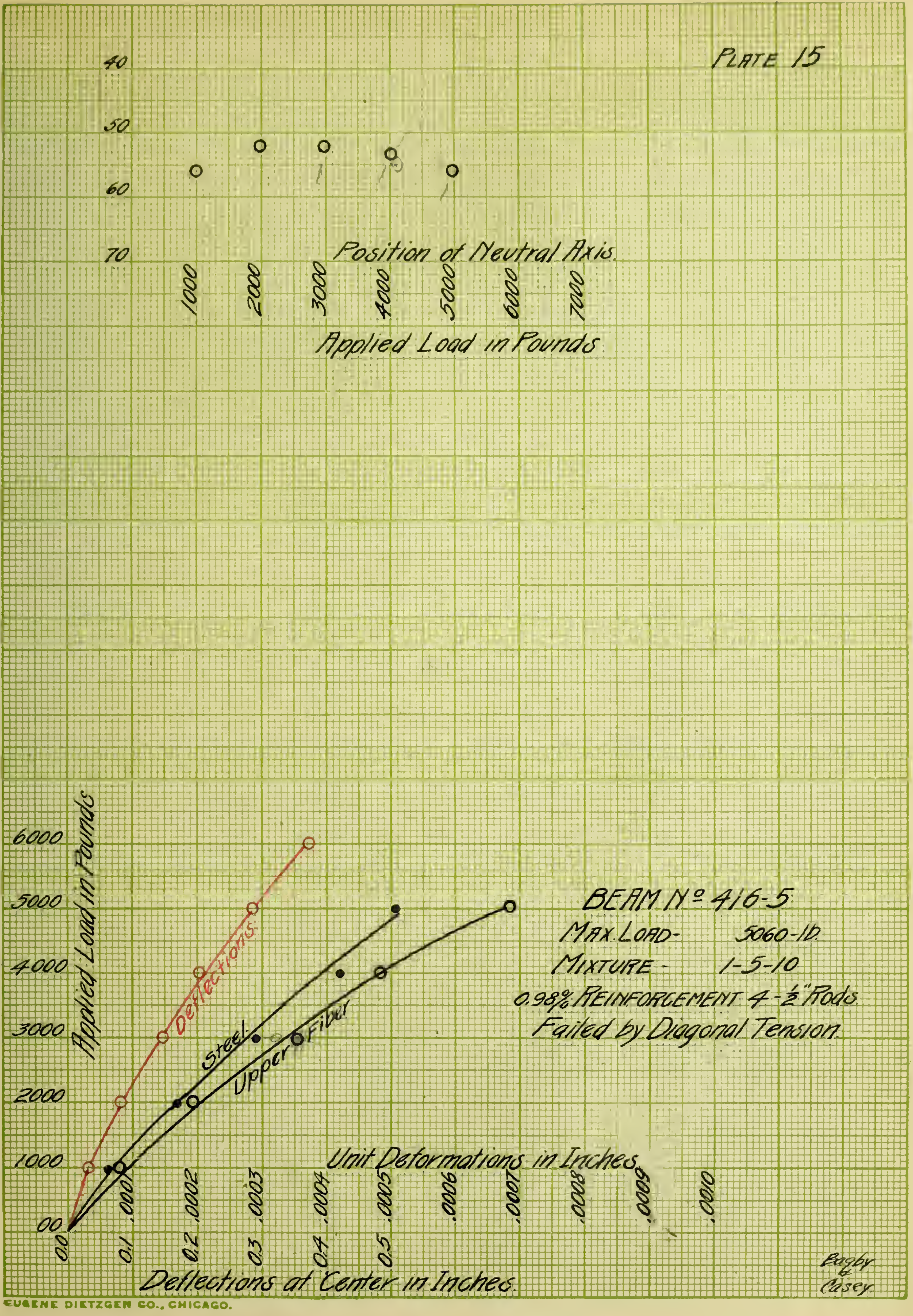


PLATE 15
40
50
60
70
Position of Neutral Axis.
1000
2000
3000
4000
5000
6000
7000
Applied Load in Pounds.
BEAM No 416-5.
MAX. LOAD- 5060-lb.
MIXTURE - 1-5-10
0.98% REINFORCEMENT 4-½" Rods.
Failed by Diagonal Tension.
Applied Load in Pounds
6000
5000
4000
3000
2000
1000
00
Deflections
Steel
Upper Fiber
Unit Deformations in Inches.
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
0.0
0.1
0.2
0.3
0.4
0.5
Deflections at Center in Inches.
Bagby & Casey.

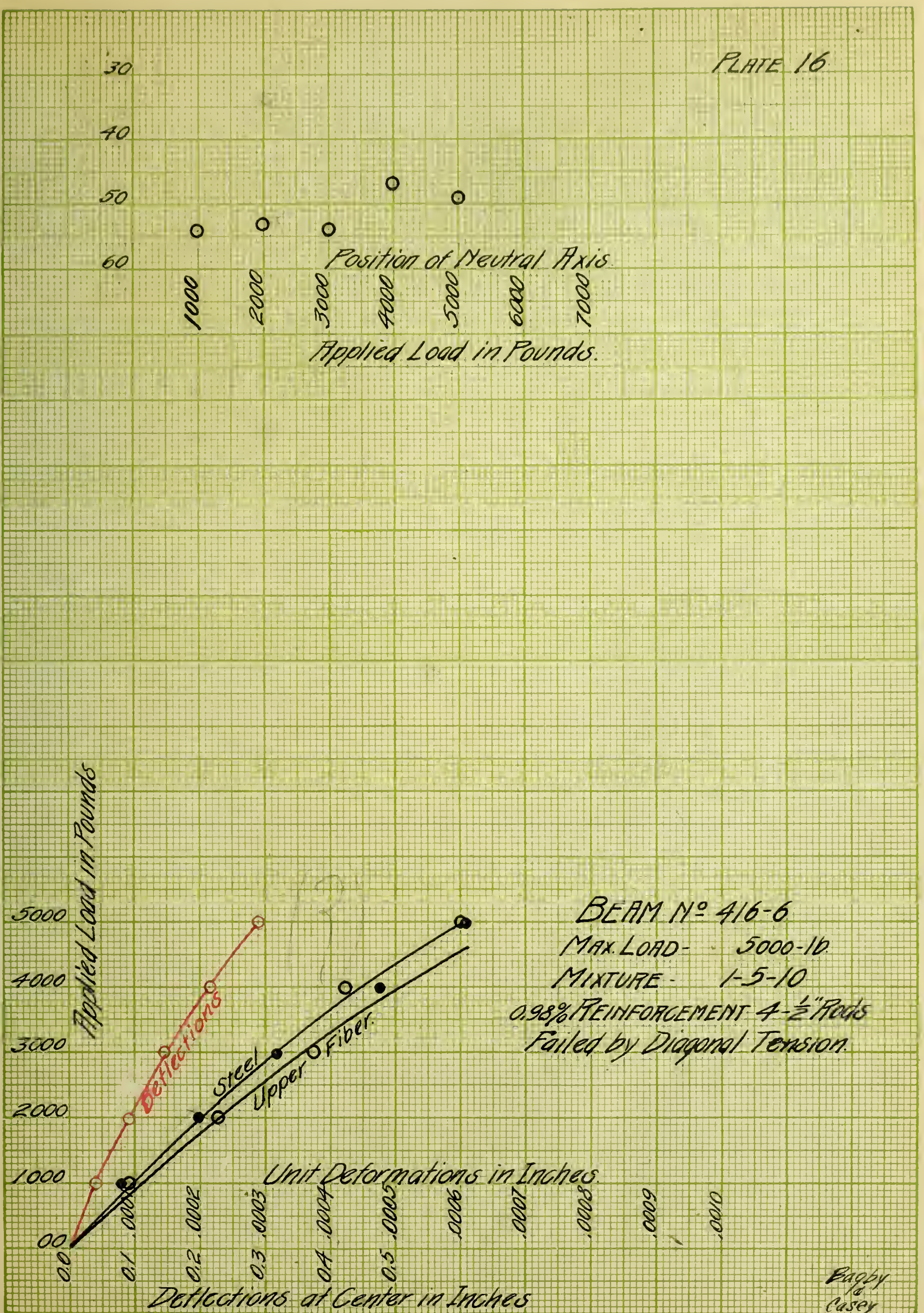


PLATE 16
30
40
50
60
Position of Neutral Axis
1000
2000
3000
4000
5000
6000
7000
Applied Load in Pounds.
Applied Load in Pounds
5000
4000
3000
2000
1000
00
Deflections
Steel
Upper Fiber
Unit Deformations in Inches
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
0.0
0.1
0.2
0.3
0.4
0.5
Deflections at Center in Inches
BEAM Nº 416-6
MAX. LOAD- 5000-lb.
MIXTURE- 1-5-10
0.98% REINFORCEMENT 4-½" Rods
Failed by Diagonal Tension.
Bagby & Casey