Bock & Fry

A Study of Steel Columns

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A STUDY OF STEEL COLUMNS

BY

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I recommend that the thesis prepared under my supervision by PAUL THEODORE BOCK and ALBERT STEVENS FRY entitled A Study of Steel Columns be approved as fulfilling this part of the requirements for the degree of Bachelor of Science in Civil Engineering.

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Assistant Professor of
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Recommendation approved

Ira O. Baker.

Head of Department of Civil Eng'g.

TO WHOM IT MAY CONCERN.

Please note that there are sundry memoranda in pencil on the face of the pages or on the margin. These memoranda were made by Professor Moore for his guidance in using the results of this thesis in the planning of future experiments and with the same subject. They are regarded by him as only temporary. It is hoped that no one will erase them until they have fully served their purpose.

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A STUDY OF STEEL COLUMNS.

I. INTRODUCTION.

The purpose of this thesis is to study the action of fixed ended columns and to determine the applicability of existing column formulae to their design. The results obtained are not expected to be conclusive owing to the limitations of the tests. The action of built up columns such as are in common use can not be definitely foretold from this series of tests since the columns used were of solid cross section.

Within the scope of this work, however, falls an interesting comparison of the authors' results with those obtained by the use of well known formulae. By this means, it is possible to place certain limitations upon the use of these formulae. In addition, it is intended, by a comparison with Huber and Shapland's "Tests of Steel Columns", a thesis prepared in 1912, to determine the relative merits of fixed and round ended columns and the relation of the quality of steel upon the strength of columns.

The latter portion of the thesis is devoted to a study of pin ended built I columns tested at the Watertown Arsenal in 1909. This was carried on along lines similar to the methods described by Professor A. N. Talbot and Mr. A. R. Lord in Bulletin No. 56, Engineering Experiement Station, University of Illinois, entitled "Tests of Columns". The intention was to find out the effect of length upon stress for low and high unit deformations.

II. THEORY AND AVAILABLE DATA.

The primary purpose of a column is to resist compression but whenever the ratio of the length to the least radius of gyration of the cross section becomes at all high, flexural stresses are set up which tend to cause failure by lateral bending before the compressive strength of the material is developed to the yield point. Altho initial eccentricity increases this tendency to failure, it does not alone bring it about. Even tho the bar be originally straight and the load be applied exactly along the longitudinal axis thru the center of gravity of the column, such a failure is almost certain to occur. The length and radius of gyration must evidently be considered in any formula which is intended to apply to long columns.

Many such formulae have been proposed. Among those best known are the Rankine, the Gordon, the Ritter, the Euler, and the straight line. Recently the American Bridge Company has adopted a new "three segment" formula. The first four of these are based upon a more or less imperfect theoretical basis, while the remaining two are purely empyrical. Neither class is entirely satisfactory for designing such columns as are ordinarily used, but owing to its greater simplicity with the same reliability of results, the straight line type takes precedence over all other formulae.

The general expression for Euler's formula is $P/A = uE(\frac{r}{l})^2$. In this equation, E is the modulus of elasticity of the material, $\frac{l}{r}$ is the slenderness ratio, u is a constant which varies with the condition of the ends, and A is the cross sectional area. P is the axial load which, when the column has delfected laterally, will just

hold it in equilibrium. Any load less than P will allow the column to return to its original position, while a slightly greater load will increase the bending until failure takes place. The derivation of this formula considers that lateral flexure alone causes the column to fail. This criterion is nearly true only for very long columns - so long in fact that they are but seldom used. For these columns, failure tends to occur by lateral bending before the material has reached the yield point. For all ordinary columns, however, the yield point is passed before failure takes place, the latter occurring when the deflection is not so great as Euler's formula would require. Since this is true, it is evident than Euler's formula can not safely be employed for such columns as are now used in buildings and bridges.

The Rankine formula is expressed by $\frac{P}{A} = \frac{S}{1 + d\left(\frac{l}{r}\right)^2}$.

In this case P is the ultimate load of the column, A the cross sectional area, $\left(\frac{l}{r}\right)$ the slenderness ratio, S the maximum unit strength of the material, and d an empirical constant varying with the material and end conditions. This formula has been widely used since it applies to columns for which the slenderness ratio ranges from 20 to 100 - cases which occur in engineering practice. The value S should not be taken as the ultimate of the material but as the yield point strength.

Ritter proposed the following formula $\frac{P}{A} = \frac{S}{1 + (Se/uE)\left(\frac{l}{r}\right)^2}$

This is really a modification of the Rankine formula in which the constant d has the value Se/uE. Se is the elastic limit and u is a constant depending on end conditions. This formula altho considering the important factor, the elastic limit, has not given

satisfactory results.

T. J. Johnson discovered that experimental values which he obtained could well be expressed by the use of a straight line formula. The general form for this is $\frac{P}{A} = S - C\frac{l}{r}$. C is a constant which changes with end conditions and materials. Due to the fact that unhomogeneity of material, initial stress, and other accidental conditions not dependent upon theory have lead to many column failures, and also to the fact that this type corresponds as closely to experimental results as Rankine's, the straight line formula has been adopted by many American railways and large cities. Various values of S and C are recommended by the different specifications and building ordinances.

In general, it may be stated that the design of a column is a more or less indefinite problem and that much remains to be completed in this field of investigation before column design will be placed upon a truly rational basis.

III. DESCRIPTION OF TESTS.

(a) Materials.

The columns used in the tests were all of the same material, soft steel.

(b) Test Pieces.

The tests were made upon small round columns which possessed cross-sectional areas slightly in excess of one half square inch. The values of $\frac{l}{r}$ for the different columns ranged from 40 to 168. Table 1 gives a summary of the dimensions of the columns together with such properties of the material as were determined.

(c) Apparatus.

The tests were made with a Philadelphia 100,000 pound machine and a Riehle 100,000 pound machine, the short columns being tested in the former and the long ones in the latter. Readings were taken over two-inch gage lengths with a Berry strain gage. In order to calculate the modulus of elasticity readings were taken over a length slightly less than that of the column by means of an extensometer devised by Professor H.F. Moore under whose supervision all the work was performed. This instrument consisted of a wooden bar, to one end of which was screwed a steel point at right angles to the bar. To the other extremity was attached an Ames dial also fitted with a steel point. It was found that by using sufficient care very satisfactory results could be obtained with this devise.

(d) Methods of Testing.

Before making the tests the bars were prepared as follows. Gage holes were drilled at the quarter points of the circumference and at intervals along the length of the columns. A gage

length of two inches was used.

Since the columns were to have fixed ends it was necessary to take precautions to make sure that the ends were rigidly fixed. This condition was established by first placing the specimen in the machine and then driving wedges between the jaws which held the column and the blocks of the machine. By this means, the experimenters were able to establish practically ideal fixed end conditions.

After the specimen had been carefully centered and adjusted in the machine, initial readings were taken over each gage length with the Berry gage and the extensometer. In order to eliminate temperature and personal equation errors, all readings were referred to a standard bar for correction. A reading on this bar was taken preceding each series, and at intervals of from five to ten readings thereafter. By the use of this bar, it was possible to eliminate, to a large extent, such errors as might otherwise enter into the test.

The use of these standard bar readings in relation to the reduction of data is shown in Plate A . Each division on the Berry instrument is equal to a deformation of .0002 inches. Since the gage length was two inches, the readings taken correspond to the actual unit elongation in terms of .0001 inches.

The manner of taking the data can be ascertained from the typical form of notes shown on pages 32 and 33. The reading recorded as the uncorrected average is the mean of five consecutive readings which agree closely with each other. After the first few columns were tested, the operator became proficient in making this reduction as he took the readings, so that only

the average was recorded on the data sheet. The same system of obtaining measurements was employed with the extensometer.

The application of the load was made in small increments in order to obtain an accurate record of the behaviour of the columns during various periods of the tests. The slowest speed to be obtained with the machine was used in all cases. After the load had been applied a short interval was allowed to intervene before reading the scale beam and taking gage measurements. In nearly every test, the machine was allowed to operate after the maximum load was passed in order to determine the effect of the further application of load and to accentuate the manner of failure of the column.

IV. DISCUSSION OF EXPERIMENTAL DATA.

The tests were made upon seven fixed ended columns, the distribution of stress at a number of points on the columns being ascertained for various increments of loads. The modulus of elasticity of the material was determined to be about 30,000,000 lbs. per square inch, this value being used to compute the stresses existing at the sections where deformations were measured. The yield point of the soft steel was considered as 37,000 lbs. per square inch, which value was taken from the tests made by Huber and Shapland, whose test columns were of the same material.

The main features of the tests have been embodied in the included tables and plates. Table I shows the dimensions of the columns together with such properties as were determined. Table II is a comparison of experimental ultimate unit loads with those calculated from well known formulas, the constants used in the latter being those given in Table III.

Table V gives the values of the stresses existing in different portions of the column for various average unit loads. These computations are based upon a value of E equal to 30,000,000. In the first column is given the location of the gage lines. In all cases the letter A designates the top series of holes, B the next lower set, etc. The letters N, E, S, and W represent the position of the gage holes with respect to their location around the circumference of the column. The above system was chosen arbitrarily for convenience of operation of the tests. In Table V, stresses were computed for three typical columns.

Table VI is a tabulation of average values of P/A for the

fixed ended soft steel columns tested by the authors, and for the rounded ended soft and cold rolled steel columns tested by Huber and Shapland in 1912.

On Plate I several curves are plotted showing the relation between the experimental values and those obtained from well known formulas. The curves shown by the full lines are based upon the constants given in Table III. The dotted curves represent values when S in the straight line and Rankine formulas is taken as 37,000, the yield point of the material. For values of $\frac{l}{r}$ less than 100 the formula values are too high. For $\frac{l}{r}$ greater than 100 the Rankine and Straight line formulas give slightly lower results than the tests indicate. It is to be noted that for these latter values of $\frac{l}{r}$, the observed and computed curves run very closely parallel. The curves given by considering the yield point give results which appear to be low. In the case of the straight line formula, this seems to show that the constant assumed was too great. The equation of the experimental curve, for values of $\frac{l}{r}$ less than 120, was found to be $P/A = 40\,000 - 50\,\frac{l}{r}$. The value of S obtained here indicates that the yield point of the material as determined was too low. From this plate is also seen the uselessness of Euler's formula. At $\frac{l}{r} = 200$, the Euler curve is just beginning to approach the experimental and other curves, a fact which leads one to believe that for fixed ended columns Euler's formula is of no value until an $\frac{l}{r}$ of nearly 300 is attained.

Plate II represents graphically the effect of quality and end conditions upon columns of varying lengths and radii of gyration. A discussion of these curves will be found in the next section of this work.

Plates III to XXV show the stress deformation curves for various portions of the columns. The significance of the designation "Point A", or "Point B" has been explained in connection with Table V. The curves on each plate illustrate the stress distribution around the periphery of a small portion of the column included between the four sets of two inch gage holes. It is seen from this series of curves that the stress is very uneven in its distribution.

Plates XXVI to XXIX show clearly the stress distribution along the sides of the column for various increments of unit loads. Columns 103 and 107 were plotted as representative columns.

An examination of Plates XXVIIIand XXIX shows that the ratio of the maximum to the average stress is greater at the top and bottom series of gages than in the center. This fact is further borne out by Column 101. According to the Rankine method of column analysis, we should have expected the stress to increase towards the middle. Under low loads the change in stress per unit length from top to bottom seems to be more uniform than under increased loads. This may be indicative of the fact that for short columns the fixedness of the ends does not effect the elastic curve as greatly for small as for large loads and that the tendency to bend is not so great. Column 107 also shows something of the latter tendency but the stress distribution in this column is in general too erratic to permit of conclusions being drawn from it. It is evident, however, that the stresses are more regular in their distribution than in Column 103. An inspection of the stress deformation curves for all the columns shows that in Column 103, the stresses were less evenly distributed than in any other case.

Plates VI and X show that for the short eight inch columns the ratio of maximum to average stress is at no place very great while the remaining plates show a tendency towards an increase in this ratio as the factor $\frac{l}{r}$ becomes larger.

In testing the columns, the machine was allowed to continue in operation in all cases but one after failure had occurred in order to observe the effects of fixed ends upon the curve assumed by the specimens. In the last column the load was removed immediately upon failure. The column returned very nearly to its original straight position thus bearing out the assumption upon which Euler's formula is based.

V. CONCLUSIONS.

In attempting to draw conclusions from the results obtained, it must be remembered that the test specimens were solid and not built up. In the latter type eccentrically applied loads, lack of straightness of the column itself, partial instead of complete fixity of ends, methods of lacing, and various other factors are likely to influence the strength of the column. In the tested columns almost ideal conditions were prevalent.

In the last section of this thesis, a formula was given which the authors have found fitted their experimental values up to $l/r = 120$. This was $P/A = 40,000 - 50\ l/r$. The tests were not sufficient in number to enable a claim for the validity of this formula to be set up. The value 40,000 is very close to that of the yield point of the material so that it appears to bear out the opinion that the strength of a column depends upon the yield point rather than the ultimate of the material.

The results of the tests substantiate the fact that the strength of a column decreases as the slenderness ratio increases. It appears that a straight line relation may be set up involving these factors for values of l/r less than about 120. For l/r greater than 120, the tests made by the authors were insufficient to determine whether the equation should be that of a curve or a second straight line. The former seems likely to be the case since it would be difficult to account for a sudden change in the slope of the straight line formula.

From the results obtained by Euler's formula, it is seen that this expression is of no value whatever for practical column

design owing to the fact that rational results for fixed ended columns are not obtained from its use until l/r equals about 300. Neither the Rankine, the Ritter, nor the Johnson straight line equation give results which coincide closely enough with the experimental strengths to warrant their usage for economical column design. For short columns, the Rankine equation using the yield point value for S gives fairly satisfactory results but the divergence of this curve from the test curve as l/r increases indicates that for longer columns the values obtained by its use are oversafe. Johnson's straight line curves appear to have too great a slope, thus overemphasizing the effect of the slenderness ratio.

The curves of Plate 2 for round and fixed ended soft steel columns afford a graphical illustration of the effect of end conditions upon strength. For values of l/r up to 50 the curves are roughly parallel, the fixed ended columns, however, showing the greatest strength. This fact would seem to indicate that end conditions play a small part in the strength of columns for which l/r does not exceed 50. From this point on the round ended columns show a rapid decrease in strength while the fixed ended columns do not begin to fall off until l/r equal to 120 is reached. For this value of $\frac{l}{r}$ the fixed ended ultimate is about ninety per cent greater than that for round ended columns.

A comparison of the round ended curves shows the effect of the quality of the steel upon the strength. For small values of l/r, the ratio of ultimate loads greatly favors the cold-rolled steel. As l/r increases this ratio decreases until for a length equal to 175 times the radius of gyration, the value of this ratio

is very close to unity. Reasoning by analogy with the fixed end soft steel specimens, the adoption of fixed ends for comparatively short columns would not greatly increase their strength. As l/r increases, however, the strength of the steel becomes less and less important so that we should expect the fixed end curve for cold-rolled steel to show a more rapid decrease in strength as l/r becomes greater until, in the neighborhood of l/r equal to 175, the values would be nearly the same as those for soft steel.

In general, it may be stated that for short columns where the strength of the material is more potent than the stiffness, a considerable benefit can be derived by the use of steel of high strength. On the other hand, for columns of even moderate lengths, the results obtained are not sufficient to justify its use in preference to steels of lesser strength. It must also be remembered that where the slenderness ratio becomes greater than about 50, end conditions play a very important part in the failure of a column. For such cases, it would probably be more economical to fix the column ends than it would to increase the strength of the steel itself.

The final conclusion arrived at by the authors is that the present-day design of columns is on a more or less irrational basis. Such unfortunate happenings as the Quebec bridge failure furnish costly examples of this truth. Among the various methods which have been proposed to place the design of columns upon firmer ground, there is one which at present shows promise of future development into a consistent and rational means of column design. That which is referred to is the method proposed by Professor H. F. Moore of the University of Illinois Experiment Station.

Inasmuch as the sidewise stiffness of a column varies directly with the moment of inertia of its cross section and since all formulae must, either directly or indirectly, recognize this fact, Professor Moore suggests that for a built up column, a factor of unity be established between the built up cross section and a similar solid cross section. This factor could then be used in the design of columns of like type and size. The determination of the factor of unity could be made by supporting the column as a beam. The deflection could be measured and the theoretical value for a one-piece section be computed. The ratio of these quantities would then represent the coefficient of unity.

The main disadvantages to this method are that a coefficient would have to be established for every different style and length of column. There is also the possibility that the coefficient would vary greatly for different beam loads. The feasibility of this mode of attack is largely a matter of conjecture, but it is worthy of consideration as a possible means for lessening the existing chaotic conditions.

VI. DISCUSSION OF WATERTOWN ARSENAL COLUMNS.

In Bulletin No. 56, Engineering Experiment Station, University of Illinois, Professor A. N. Talbot and Mr. A. R. Lord developed some interesting relations between the strength and length of plain steel columns of the Gray type. Working along lines similar to those used by the above named experimenters, the authors of this thesis investigated a series of pin-ended built I columns tested in 1909 at the Watertown Arsenal.

Table VI is a tabulation of the tests showing the general dimensions and properties of the twenty one columns. The reports of these tests state that the first three columns given in the table failed by buckling of the flanges. Triple flexure and buckling of the flanges caused failure in the next three specimens. Columns No. 2047, 2048, and 2049 failed by flexure, perpendicular to the plane of the web. The same is true for all the remaining columns with the additional phenomena of sudden sidewise springing when the maximum load was reached.

The stress-deformation curves were plotted from the data furnished in the report of the tests and are shown on Plates 30 to 34. The curves are practically straight lines under low loads with the exception of an irregularity at the zero point. This is due to the fact that the zero readings were taken with an applied load of 1,000 pounds per square inch.

In order to ascertain the relation of strength to length as the test progressed, the unit loads sustained by the columns for various unit deformations were taken from Plates 30 to 34, and plotted on Plates 35 and 36.

The equations of the lines thus determined were also found. These show that the effect of length upon stress increases with the deformation and that for small deformations the ratio $\frac{l}{r}$ is of little consequence. Plate 37 was drawn from the equations of the load-length diagrams. The column formula was considered to be $P/A = f - k \frac{l}{r}$. Values of f and k were then plotted for the various unit deformations. From this plate it appears that f increases along the path of a very flat curve. In Bulletin No. 56, previously referred to, the authors found that "up to a unit-deformation of .0004 the slenderness ratio has no effect and that beyond this deformation it increases in a constant ratio to the increase in deformation". The results of the Watertown tests in general bear out this statement. The values of k for low deformations are so small that they may readily be neglected. For l/r from .0006 to .001 the results obtained all fell along a straight line, which facts tend to substantiate the ratio of k to the unit-deformation is constant as both increase.

The chief difference existing between the built-I and the Gray columns is that higher values of f and lower values of k were found. This fact operates to produce greater unit loads for the Watertown columns. Practically however, the slope of the f curves is nearly the same for the two series of tests. The k curve of the Illinois University columns is much steeper than that of the Watertown columns. For the ultimate, Talbot and Lord used the formula $P/A = 36\,500 - 155 \frac{l}{r}$. For the columns at hand, the equation was found to be $\frac{P}{A} = 39\,000 - 65 \frac{l}{r}$.

The results of the study of the built I columns in comparison with the Gray columns shows that when the unit-deformation is low, the value of k, the coefficient of $\frac{l}{r}$ in the straight line formula, will also be small and may be considered zero. As the load increases and causes a greater deformation to occur, k increases in a constant ratio to the deformation and becomes an important factor in the column formula.

Both the Watertown and the University of Illinois series of tests give indications of a constant eccentricity thruout each test. Thus, we may consider the general equation,

$$\frac{P}{A} = f - k\frac{l}{r}$$

Then
$$f = \frac{P}{A} + k\frac{l}{r}$$

But from the curves, it appears that k is approximately directly proportional to $\frac{P}{A}$.

Whence
$$f = \frac{P}{A} + k'\frac{P}{A}\,\frac{l}{r}$$

or
$$f = \frac{P}{A}\left(1 + k'\frac{l}{r}\right) \qquad (1)$$

Considering the effect of eccentricity we have the formula

$$f = \frac{P}{A}\left(1 + \frac{e\,c}{r}\right)$$

But $c = k''r$ for any given section

Then
$$f = \frac{P}{A}\left(1 + \frac{k''e}{r}\right) \qquad (2)$$

It is seen that (1) and (2) have very similar form, which fact leads to the belief that the eccentricity must have been not far from constant thruout the tests.

VII.

TABLES.

TABLE I.

GENERAL COLUMN DATA.

Col. No.	Length in.	Diam. in.	Area sq.in.	$\frac{l}{r}$	Ultimate Load Total lbs.	Ultimate Load Unit lbs.per sq. in.	Modulus of Elasticity
104	8.1	.813	.519	39.9	20,610	39,700	- - - -
102	8.2	.812	.518	40.4	18,900	36,400	- - - -
101	14.3	.813	.519	70.4	20,000	38,600	- - - -
103	14.4	.811	.518	70.9	17,690	34,100	- - - -
105	24.1	.813	.519	118.7	18,100	34,900	30,000,000
106	34.2	.813	..519	168.3	12,800	24,600	26,000,000
107	34.1	.813	.519	168.0	13,480	26,600	31,000,000

TABLE II.

COMPARISON OF FORMULAS.

Col. No.	Values of l/r	Ultimate Unit Loads - Lbs. per sq. in.				
		P/A	Rankine	Ritter	Euler	St. Line
104	39.9	39 700	47 000	47 800	747 000	45 300
102	40.4	36 400	46 950	47 750	747 000	45 300
101	70.4	38 600	41 700	43 100	240 000	39 900
103	70.9	34 100	41 600	43 000	240 000	39 800
105	118.7	34 900	32 000	35 300	84 200	31 300
106	168.3	24 600	23 400	27 300	42 000	22 300
107	168.0	26 600	23 400	27 300	42 000	22 300

TABLE III.

TABLE OF CONSTANTS.

Quantity	Formula			
	Rankine	Ritter	Euler	St. Line
S	50 000	50 000	-------	52 500
D	.00004	.0000294	-------	------
C	------	-------	-------	179
E	------	30 000	30 000 000	
u	------		4	
Se		35 000		

TABLE IV.

DEDUCTION OF STRESSES.

Col. No. 4. Length 8.1 ins.

Location	Row	Unit Load lbs.per sq.in.	Corrected Difference	Computed Stress
A	N	5 720	-1.6	- 4 800
	E		+0.4	+ 1 200
	S		-3.3	- 9 900
	W		-3.0	- 9 000
"	N	11 400	-4.9	-14 700
	E		-1.6	- 4 800
	S		-5.6	-16 800
	W		-7.1	21 300
"	N	16 700	-8.4	-25 200
	E		-2.3	- 6 900
	S		-7.6	-22 800
	W		-7.5	
"	N	21 900	-8.3	-24 900
	E		-5.7	-17 100
	S		-8.2	-24 600
	W		-9.0	-27 000
"	N	26 500	-8.2	-24 600
	E		-7.5	-22 500
	S		-8.9	-26 700
	W		-9.0	-27 000
"	N	29 000	-9.0	-27 000
	E		-8.6	-25 800
	S		-10.4	-31 200
	W		-10.6	-31 800
"	N	31 800	-10.5	-31 500
	E		-10.3	-30 900
	S		-13.4	-40 200
	W		-13.7	-41 100
"	N	33 300	-11.5	-34 500
	E		-12.3	-36 500
	S		-13.5	-40 500
	W		-13.2	-39 600

Note. - In all cases A will designate the top series of gage holes, B the next lower, etc. In this case there was only one set.

TABLE IV. (Cont.)

DEDUCTION OF STRESSES.

Col. No. 7. Length 34.1 ins.

Location	Row	Unit Load lbs. per sq. in.	Corrected Difference	Computed Stress
A	N	6 400	-2.1	- 6 300
	E		-2.3	- 6 900
	S		-0.9	- 2 700
	W		-0.4	- 1 200
B	N	"	-4.4	-13 200
	E		-0.4	- 1 200
	S		-0.8	- 2 400
	W		-1.5	- 4 500
C	N	"	-5.5	-16 500
	E		+0.5	+ 1 500
	S		-2.3	- 6 900
	W		-3.3	- 9 900
D	N	"	-3.3	- 9 900
	E		-0.6	- 1 800
	S		+0.1	+ 300
	W		-3.9	-14 700
E	N	"	-4.0	-12 000
	E		+0.5	+ 1 500
	S		-1.1	- 3 300
	W		-5.3	-15 900
A	N	11 500	-4.1	-12 300
	E		-4.1	-12 300
	S		-1.6	- 4 800
	W		-0.7	- 2 100
B	N	"	-6.2	-18 600
	E		-4.8	-14 400
	S		-1.4	- 4 200
	W		+1.8	+ 5 400
C	N	"	-5.9	-17 700
	E		-1.6	- 4 800
	S		-1.4	- 4 200
	W		-1.4	- 4 200
D	N	"	-5.7	-17 100
	E		-1.8	- 5 400
	S		-2.0	- 6 000
	W		-3.0	- 9 000
E	N	"	-6.5	-19 500
	E		-1.7	- 5 100
	S		-2.6	- 7 800
	W		-3.0	- 9 000

TABLE IV. (Cont.)

DEDUCTION OF STRESSES.

Col. No. 7. Length 34.1 ins.

Location	Row	Unit Load lbs.per sq.in.	Corrected Difference	Computed Stress
A	N	16 300	-4.0	-12 000
	E		-6.3	-18 900
	S		-4.4	-13 200
	W		-4.9	-14 700
B	N	"	-7.6	-22 800
	E		-4.6	-13 800
	S		-2.4	- 7 200
	W		-2.1	- 6 300
C	N	"	-6.3	-18 900
	E		-2.8	- 8 400
	S		-1.4	- 4 200
	W		-4.7	-14 100
D	N	"	-5.1	-15 300
	E		-0.9	- 2 700
	S		-1.9	- 5 700
	W		-5.5	-16 500
E	N	"	-5.2	-15 600
	E		-2.1	- 6 300
	S		-5.9	-17 700
	W		-4.9	-14 700
A	N	19 300	-3.6	-10 800
	E		-7.9	-23 700
	S		-6.2	-18 600
	W		-5.3	-15 900
B	N	"	-8.8	-26 400
	E		-5.6	-16 800
	S		-6.9	-20 700
	W		-2.3	- 6 900
C	N	"	-10.0	-30 000
	E		- 3.6	-10 800
	S		- 6.1	-18 300
	W		- 6.1	-18 300
D	N	"	- 7.7	-23 100
	E		- 2.9	- 8 700
	S		- 3.8	-11 400
	W		- 6.6	-19 800
E	N	"	- 6.4	-19 200
	E		- 6.3	-18 900
	S		- 8.0	-24 000
	W		- 5.3	-15 900

TABLE IV. (Cont.)

DEDUCTION OF STRESSES.

Col. No. 3. Length 14.35 ins.

Location	Row	Unit Load lbs.per sq.in.	Corrected Difference	Computed Stress
A	N	6 090	-7.2	-21 600
	E		-5.3	-15 900
	S		+1.1	+ 3 300
	W		+0.8	+ 2 400
B	N	"	-4.3	-12 900
	E		-0.7	- 2 100
	S		-1.2	- 3 600
	W		-2.0	- 6 000
C	N	"	-2.0	- 6 000
	E		+2.0	+ 6 000
	S		-3.6	-10 800
	W		-6.1	-18 100
A	N	9 850	-9.0	-27 000
	E		-8.2	-24 600
	S		0.1	300
	W		-1.3	- 3 900
B	N	"	-6.7	-20 100
	E		-1.8	- 5 400
	S		-2.9	- 8 700
	W		-4.3	-12 900
C	N	"	-2.7	- 8 100
	E		+0.7	+ 2 100
	S		-7.3	-21 900
	W		-10.9	-32 700
A	N	13 900	-11.8	-35 400
	E		-10.3	-30 900
	S		- 0.9	- 2 700
	W		- 2.1	- 6 300
B	N	"	-9.7	-29 100
	E		-3.4	-10 200
	S		-2.6	- 7 800
	W		-4.7	-13 100
C	N	"	-3.6	-10 800
	E		0	0
	S		-8.6	-25 800
	W		-11.5	-34 500

TABLE IV. (Cont.)

DEDUCTION OF STRESSES.

Col. No. 3. Length 14.35 ins.

Location	Row	Unit Load lbs.per sq.in.	Corrected Difference	Computed Stress
A	N	17 700	-13.2	-39 600
	E		-11.6	-34 800
	S		- 1.6	- 4 800
	W		- 4.2	-12 600
B	N	"	-10.5	-31.500
	E		- 3.8	-11 400
	S		- 4.3	-12 900
	W		- 4.9	-14 700
C	N	"	-4.4	-13 200
	E		- 0.4	- 1 200
	S		-11.6	-34 800
	W		-13.4	-40 200
A	N	21 100	-14.1	-42 300
	E		-12.3	-36 900
	S		- 3.7	-11 100
	W		- 4.9	-14 700
B	N	"	-10.4	-31 200
	E		- 4.6	-13 800
	S		- 5.3	-15 900
	W		- 7.0	-21 000
C	N	"	- 5.7	-16 100
	E		- 0.8	- 2 400
	S		-12.3	-36 900
	W		-15.5	-46 500
A	N	24 600	-16.4	-49 200
	E		-14.8	-44 400
	S		- 5.8	-17 400
	W		- 9.8	-29 400
B	N	"	-12.3	-36 900
	E		- 6.2	-18 600
	S		- 7.9	-23 700
	W		- 8.9	-26 700
C	N	"	- 7.8	-23 400
	E		- 4.3	-12 900
	S		-14.9	-44 700
	W		-19.3	-57 900

TABLE IV. (Cont.)

DEDUCTION OF STRESSES.

Col. No. 3. Length 14.35 ins.

Location	Row	Unit Load lbs.per sq.in.	Corrected Difference	Computed Stress
A	N	27 000	-18.6	-55 800
	E		-14.8	-44 400
	S		- 5.0	-15 000
	W		- 8.1	-24 300
B	N	"	-12.4	-37 200
	E		- 8.4	-25 200
	S		- 7.4	-22 200
	W		- 8.9	-26 700
C	N	"	- 8.0	-24 000
	E		- 3.9	-11 700
	S		-18.4	-55 200
	W		-21.2	-63 600
A	N	29 800	-51.3	------
	E		-38.2	------
	S		-13.8	-41 400
	W		-25.4	------
B	N	"	-14.1	-42 300
	E		- 9.2	-27 600
	S		- 9.2	-27 600
	W		- 9.7	--29 100
C	N	"	-26.7	------
	E		-24.2	------
	S		-57.1	------
	W		-52.3	------

TABLE V.

VARIATION OF STRENGTH WITH END CONDITIONS AND QUALITY OF STEEL.

$\frac{l}{r}$	Values of P/A - Ultimate		
	Soft	Steel	Cold-Rolled Steel
	Fixed Ends	Round Ends	Round Ends
40.2	38 100	-------	-------
44.3	------	36 950	-------
49.3	------	------	56 500
70.7	36 300	------	------
87.0	------	29 700	------
87.2	------	------	39 700
108.2	------	22 000	------
118.7	34 900	------	------
135.4	------	------	18 900
168.2	25 600	------	------
173.0	------	------	11 100
173.0	------	10 580	------
175.0	------	10 380	-------

TABLE VI.

TABULATION OF COMPRESSION TESTS OF BUILT I STEEL COLUMNS, PIN ENDS.

Test made at Watertown Arsenal, 1909.

No. of Test	Length		$\frac{l}{r}$	Section-al Area	Ultimate Strength	
	Feet	Inches			Total	Per Sq.In
				sq.in.	Pounds	Pounds
2053	3	5 1/4	25	13.74	517 600	37 670
2054	3	5 1/4	25	13.74	520 300	37 870
2055	3	5 1/4	25	13.86	509 000	36 720
2050	6	10 1/2	50	13.89	467 200	33 640
2051	6	10 1/2	50	13.91	474 100	34 080
2052	6	10 1/2	50	13.93	470 900	33 800
2047	10	3 3/4	75	13.76	444 100	32 270
2048	10	3 3/4	75	13.61	435 500	32 000
2049	10	3 3/4	75	13.87	445 400	32 110
2044	13	9	100	13.54	432 500	31 940
2045	13	8 3/4	100	13.48	430 700	31 950
2046	13	8 3/4	100	13.53	447 500	33 070
2041	17	2	125	13.86	415 800	30 000
2042	17	2	125	13.92	401 600	28 850
2043	17	2	125	14.89	399 200	28 740
2038	20	7 1/4	150	14.17	388 300	27 400
2039	20	7 1/4	150	13.21	402 300	28 310
2040	20	7 1/4	150	13.84	404 000	29 190
2035	24	1/2	175	13.72	180 200	13 130
2036	24	1/2	175	13.74	371 100	27 010
2037	24	1/2	175	13.83	320 800	23 200

VIII.

PLATES.

FORM FOR REDUCTION OF STRAIN GAGE DATA

FOR SINGLE STANDARD BAR

Load	Interval	0	1	2	n-2	n-1	n
	Gage Line	Standard Bar	Arbitrary Numbering of Gage Lines				Standard Bar
Initial Load only	Uncorrected Average	S	R_1	R_2	R_{n-2}	R_{n-1}	S'
	Correction	0	$\frac{1}{n}C' = C_1$	$\frac{2}{n}C' = C_2$	$\frac{n-2}{n}C' = C_{n-2}$	$\frac{n-1}{n}C' = C_{n-1}$	$S - S' = C'$
	Corrected Zero Average	———	$R_1 + C_1 = A_1$	$R_2 + C_2 = A_2$	$R_{n-2} + C_{n-2} = A_{n-2}$	$R_{n-1} + C_{n-1} = A_{n-1}$	———
Any Subsequent Load P	Uncorrected Average	s	r_1	r_2	r_{n-2}	r_{n-1}	s'
	Uncorrected Difference	———	$A_1 - r_1 = d_1$	$A_2 - r_2 = d_2$	$A_{n-2} - r_{n-2} = d_{n-2}$	$A_{n-1} - r_{n-1} = d_{n-1}$	———
	Correction	$S - s = C$	$C + \frac{1}{n}(c' - C) = c_1$	$C + \frac{2}{n}(c' - C) = c_2$	$C + \frac{n-2}{n}(c' - C) = c_{n-2}$	$C + \frac{n-1}{n}(c' - C) = c_{n-1}$	$S - s' = c'$
	Corrected Difference	———	$d_1 - c_1 = e_1$	$d_2 - c_2 = e_2$	$d_{n-2} - c_{n-2} = e_{n-2}$	$d_{n-1} - c_{n-1} = e_{n-1}$	———

PLATE A

LABORATORY OF APPLIED MECHANICS
UNIVERSITY OF ILLINOIS

TEST DATA

of Column No. 1

Observer Bock
Date M.ch. 12.'13 Sheet 1

Load, Total	~~Series~~, Unit	Time	Temperature	Recorder	Gauge Line	St.Bar	N-A	N-B	N-C	E-A	E-B	E-C	St.Br	S-A	S-B	S-C	W-A	W-B	W-C	St.Br
					Corr. Zero Av.	38.0	56.1	68.7	14.7	11.1	74.4	31.2	38.0	80.0	56.8	22.1	9.6	28.3	66.9	38.0
250	500			Fry	Readings															
					Uncorrected Av.	38.0	56.1	68.7	14.6	11.0	74.3	31.0	37.8	79.8	56.7	22.0	9.5	28.3	66.9	38.0
					Uncorr. Difference															
					Correction	0	0	0	+0.1	+0.1	+0.1	+0.2	+0.2	+0.2	+0.1	+0.1	+0.1	0	0	+0.0
					Corr. Difference															
1290	2490				Readings															
					Uncorrected Av.	37.1	57.5	70	15.1	10.8	74.2	30	37.2	79.5	57.4	22.9	9.4	28.2	67.8	37.7
					Uncorr. Difference	—	-1.4	-1.3	-0.4	+0.3	+0.2	+1.2	—	+0.5	-0.6	-0.8	+0.2	+0.1	-0.9	—
					Correction	+0.9	+0.9	+0.9	+0.9	+0.9	+0.9	+0.9	+0.8	+0.7	+0.7	+0.6	+0.5	+0.5	+0.4	+0.3
					Corr. Difference		-2.3	-2.1	-1.3	-0.6	-0.7	+0.3		-0.2	-1.3	-1.4	-0.3	-0.4	-1.3	
2510	4850				Readings															
					Uncorrected Av.	37.8	59.9	70.3	17.4	10.4	72.8	29.6	36.3	79.7	57.4	21.6	9.4	28.4	68.7	36.8
					Uncorr. Difference		-3.8	-1.6	-2.7	+0.7	+1.6	+1.6		+0.3	-0.6	+0.5	+0.2	-0.1	-1.8	
					Correction	+0.2	+0.4	+0.6	+0.8	+1.0	+1.2	+1.4	+1.7	+1.6	+1.6	+1.5	+1.4	+1.4	+1.3	+1.2
					Corr. Difference		-4.2	-2.2	-3.5	-0.3	+0.4	+0.2		-1.3	-2.2	-1.0	-1.2	-1.3	-3.1	
5175	9980				Readings															
					Uncorrected Av.	36.7	57.3	71.2	18.8	11.2	74.1	30	36.1	81.3	57.7	22.3	11.2	29.4	70	36.3
					Uncorr. Difference		-1.2	-2.5	-3.6	-0.1	-0.3	-1.2		-1.3	-0.9	-0.2	-1.6	-1.1	-3.1	
					Correction	+1.3	+1.4	+1.5	+1.6	+1.7	+1.8	+1.9	+1.9	+1.9	+1.9	+1.9	+1.8	+1.7	+1.7	+1.7
					Corr. Difference		-2.6	-4.0	-5.2	-1.8	-2.1	-3.1		-3.2	-2.8	-2.1	-3.4	-2.8	-4.8	
10580	20400				Readings															
					Uncorrected Av.	35.1	60.8	73.4	22.7	12.8	75.9	31.8	35.5	82.3	59.4	23.6	14.9	30.8	71.9	34.9
					Uncorr. Difference		-4.7	-4.7	-8.0	-1.7	-1.5	-0.6		-2.3	-2.6	-1.5	-5.3	-2.5	-5.0	
					Correction	+2.9	+2.9	+2.8	+2.7	+2.7	+2.6	+2.6	+2.5	+2.6	+2.7	+2.8	+2.9	+3.0	+3.0	+3.1
					Corr. Difference		-7.6	-7.5	-10.7	-4.4	-4.1	-3.2		-4.9	-5.3	-4.3	-8.2	-5.5	-8.0	

LABORATORY OF APPLIED MECHANICS
UNIVERSITY OF ILLINOIS

TEST DATA

of Column No. 1

Observer Bock
Date Mch. 12. '13 Sheet 2.

Load Total	Unit	Time	Temperature	Recorder	Gauge Line	St.Br	N-A	N-B	N-C	E-A	E-B	EC	St.Br	S-A	S-B	S-C	W-A	W-B	W-C	St.Br
					Corr. Zero Av.	38	56.1	68.7	14.7	11.1	74.4	31.2	38	80	56.8	22.1	9.6	28.3	66.9	38
13830	26,700			Fry	Readings															
					Uncorrected Av.	33.9	60.0	75.8	23.8	12.1	76.9	34.3	33.2	83.6	62.0	25.5	17.7	33.4	74.5	33.3
					Uncorr. Difference		-3.9	-7.1	-9.1	-1.0	-2.6	-3.1		-3.6	-5.2	-3.4	-8.1	-5.1	-7.6	
					Correction	+4.1	+4.2	+4.3	+4.4	+4.5	+4.6	+4.7	+4.8	+4.8	+4.8	+4.8	+4.8	+4.7	+4.7	+4.7
					Corr. Difference		-8.1	-11.4	-13.5	-5.5	-7.1	-7.8		-8.4	-10.0	-8.2	-12.9	-9.8	-12.3	
15710	30,400				Readings															
					Uncorrected Av.	34.8	62.6	75	36.4	12.9	76.2	46.7	34.2	84.7	64.0	29.8	18.9	36.1	80.8	34.4
					Uncorr. Difference		-6.5	-6.3	-21.7	-1.8	-1.8	-15.5		-4.7	-7.2	-7.7	-9.3	-7.8	-13.7	
					Correction	+3.2	+3.3	+3.4	+3.5	+3.6	+3.6	+3.7	+3.8	+3.8	+3.7	+3.7	+3.7	+3.7	+3.6	+3.6
					Corr. Difference		-9.8	-9.7	-25.2	-5.4	-5.4	-19.2		-8.5	-10.9	-11.4	-13.0	-11.5	-17.3	
20000	Ultimate	of Column			Readings															
					Uncorrected Av.															
					Uncorr. Difference															
					Correction															
					Corr. Difference															
					Readings															
					Uncorrected Av.															
					Uncorr. Difference															
					Correction															
					Corr. Difference															
					Readings															
					Uncorrected Av.															
					Uncorr. Difference															
					Correction															
					Corr. Difference															

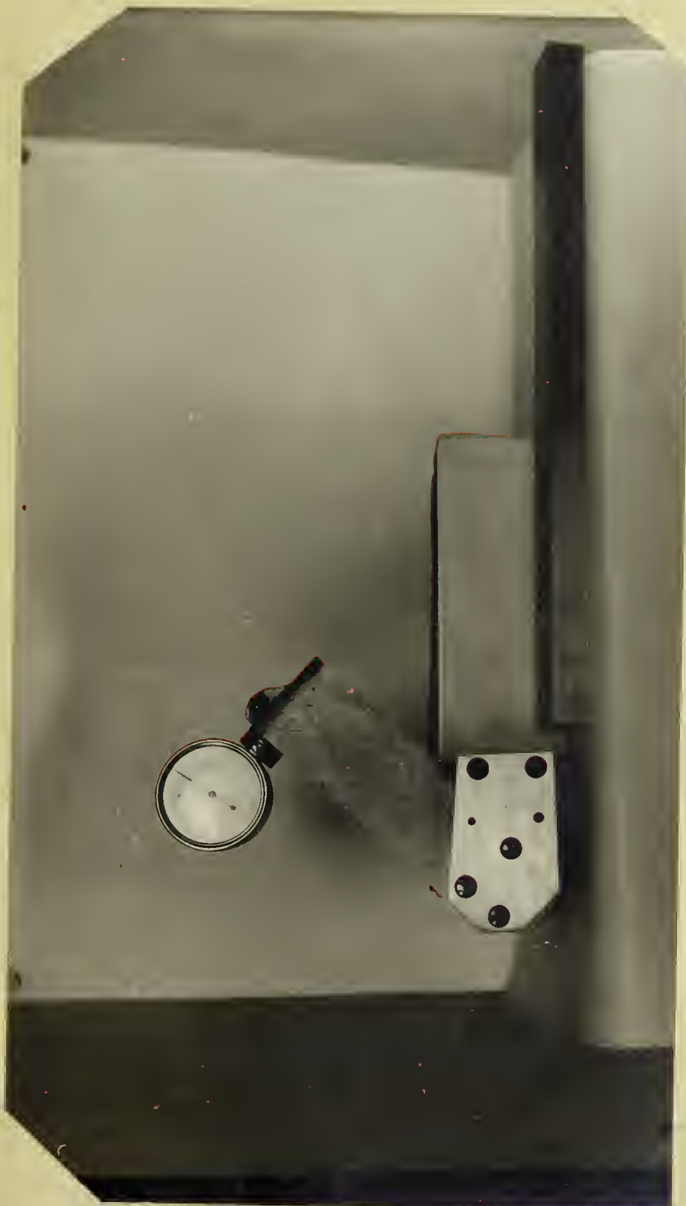


Plate B - Berry strain gage.

Plate C - Fixed Ended Columns after Failure.

Diagram Showing Method of Fixing the Ends of the Columns.

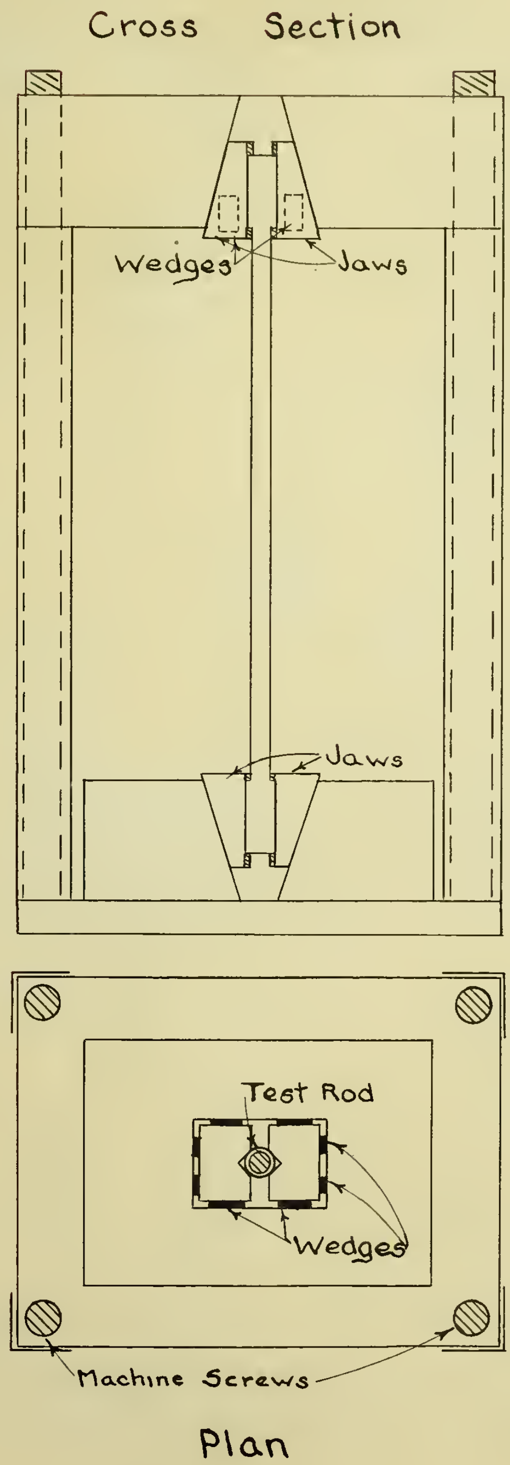


PLATE D

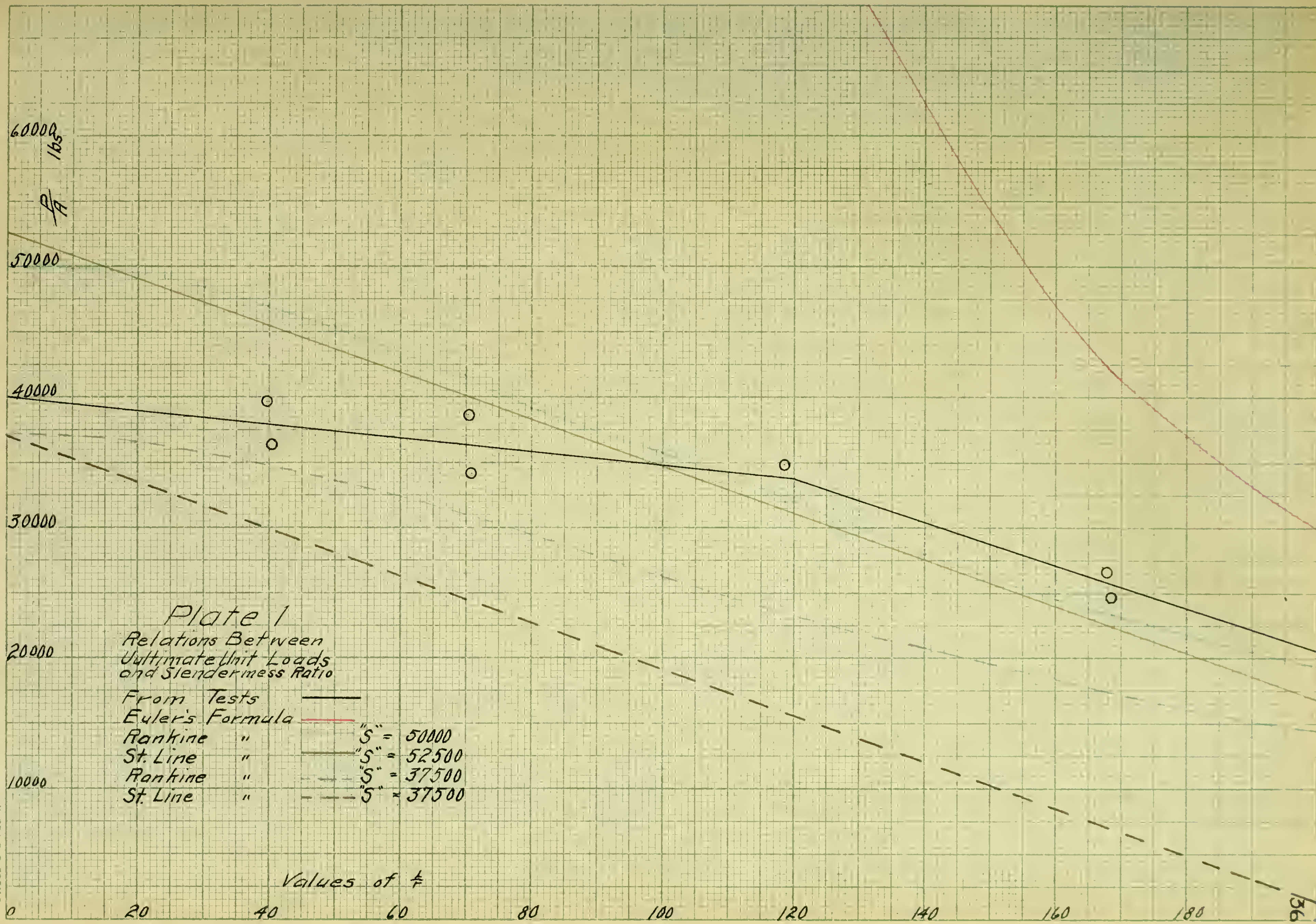
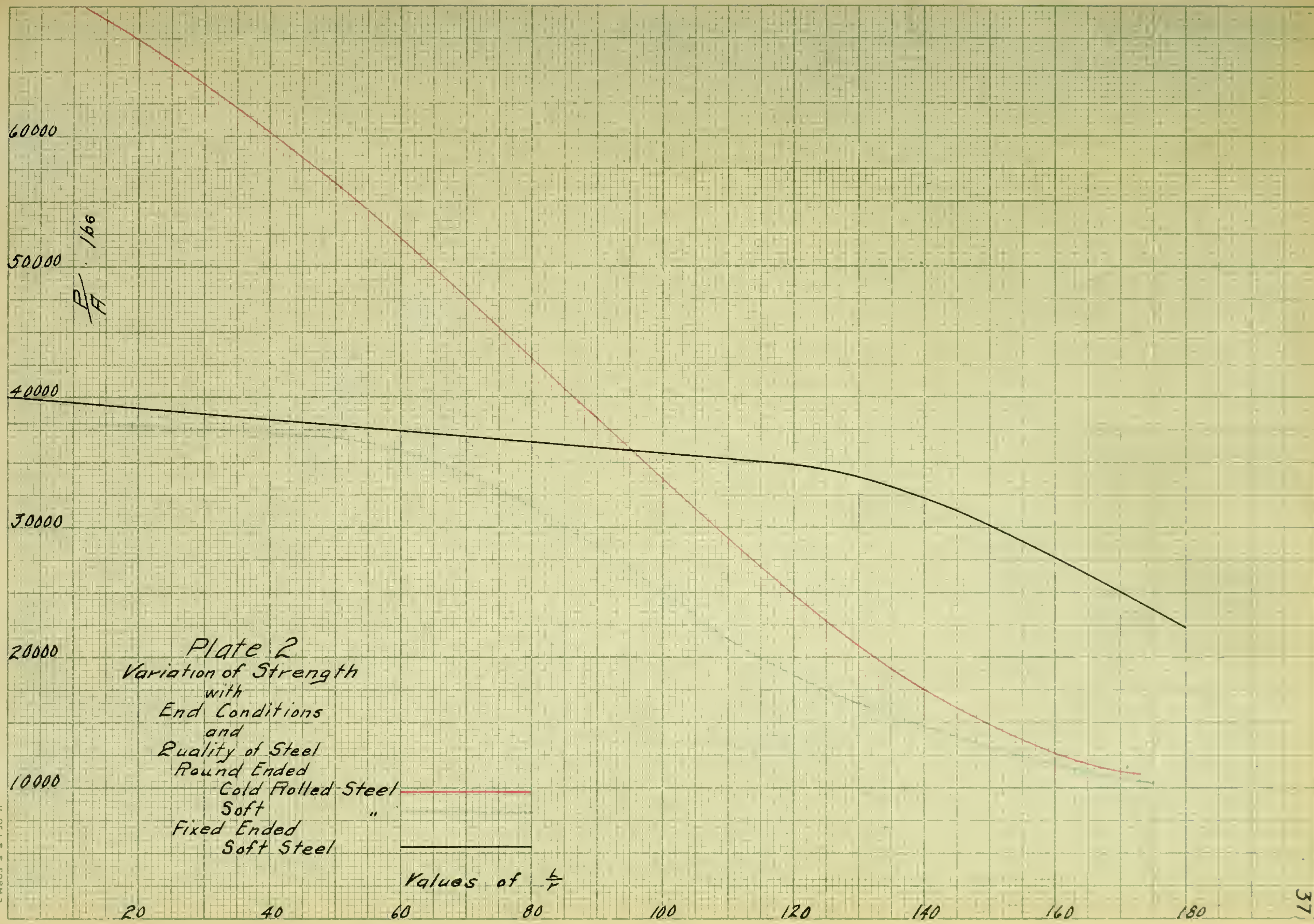


Plate 1
Relations Between
Ultimate Unit Loads
and Slenderness Ratio
From Tests
Euler's Formula
Rankine " "S" = 50000
St. Line " "S" = 52500
Rankine " "S" = 37500
St. Line " "S" = 37500
P/A lbs
60000
50000
40000
30000
20000
10000
Values of l/r
0
20
40
60
80
100
120
140
160
180
136
U. OF I. S. S. FORM 3


60000
50000
40000
30000
20000
10000
P/A lbs
20
40
60
80
100
120
140
160
180
Values of L/r
Plate 2
Variation of Strength
with
End Conditions
and
Quality of Steel
Round Ended
Cold Rolled Steel
Soft "
Fixed Ended
Soft Steel

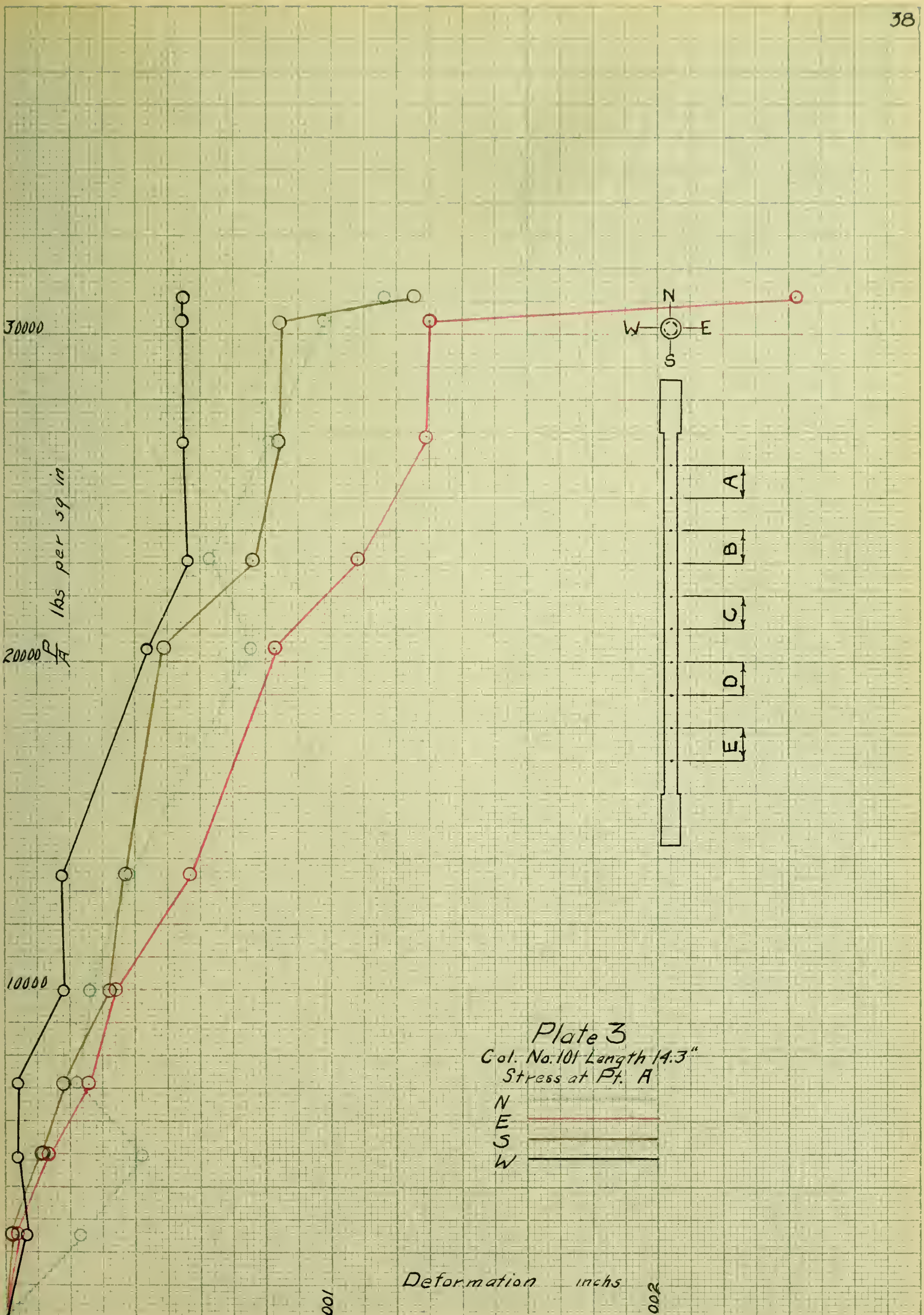
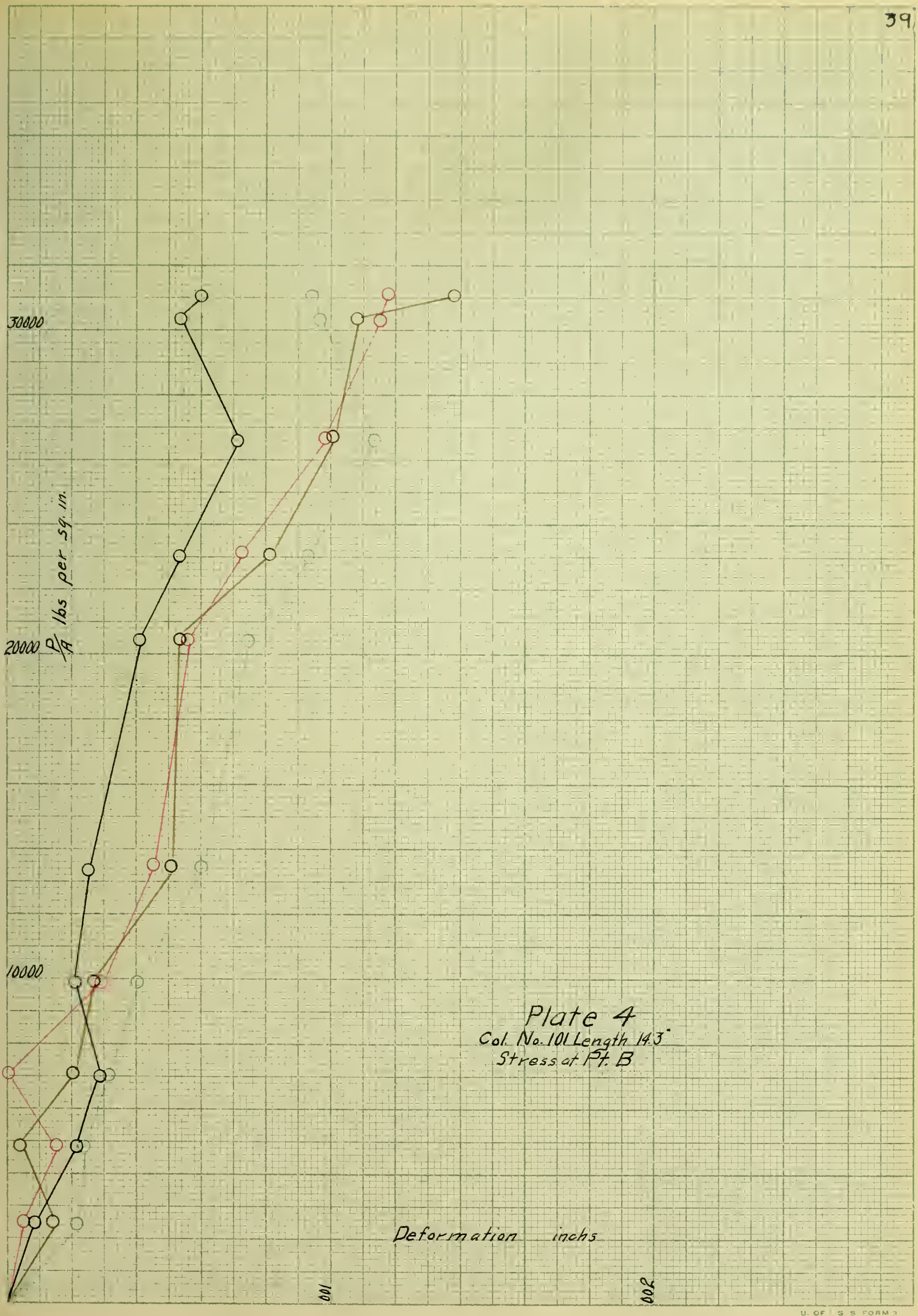
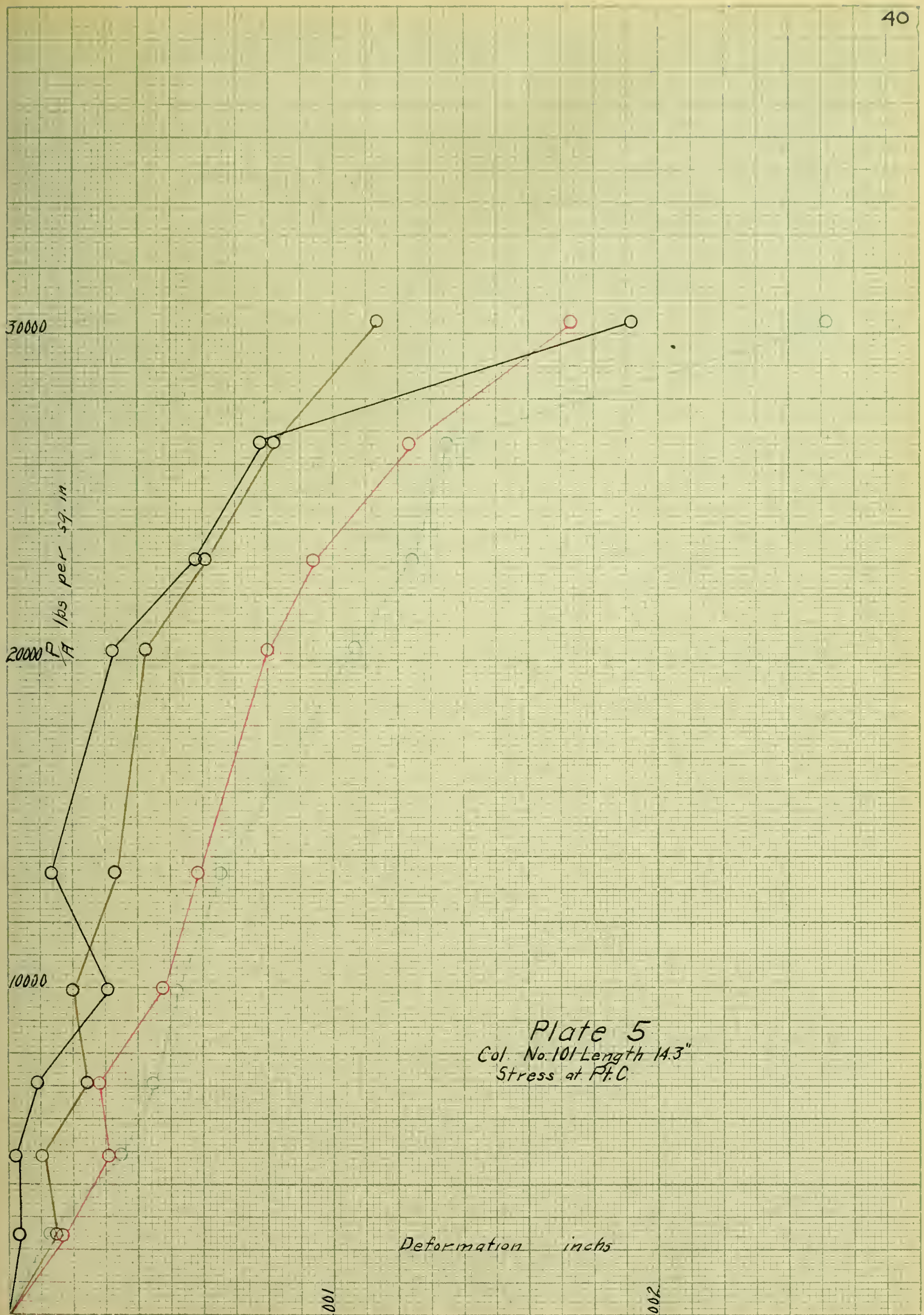
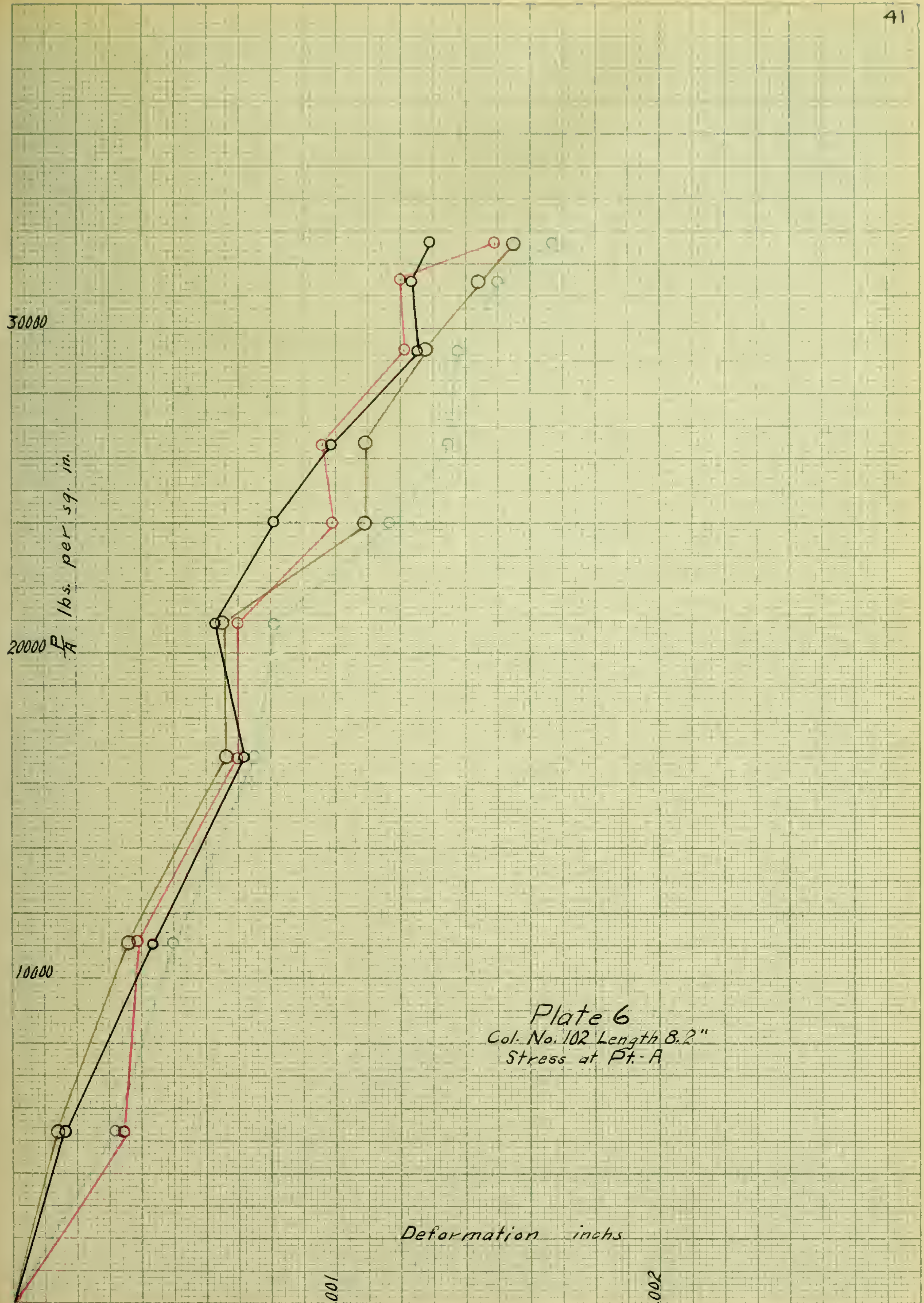
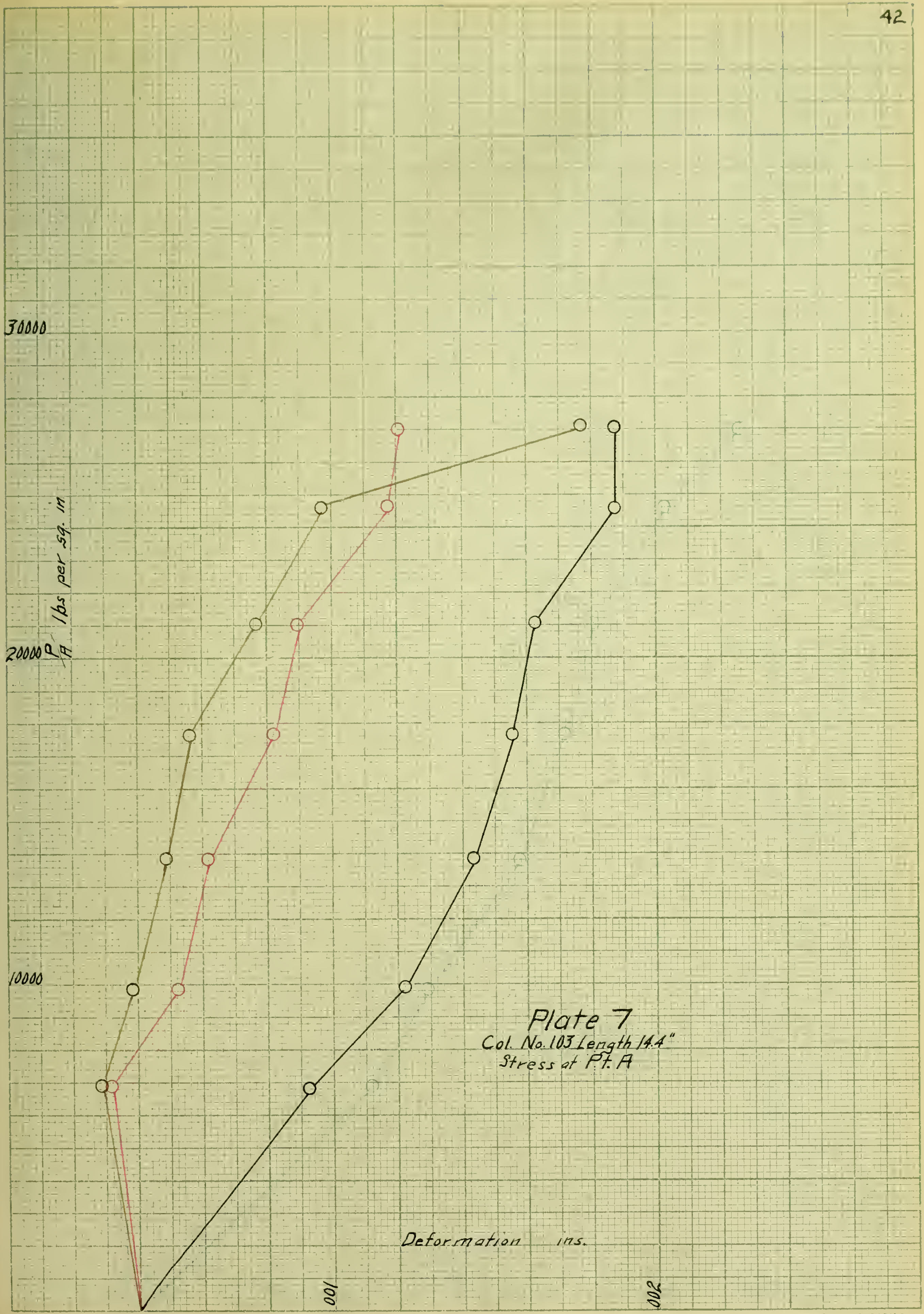


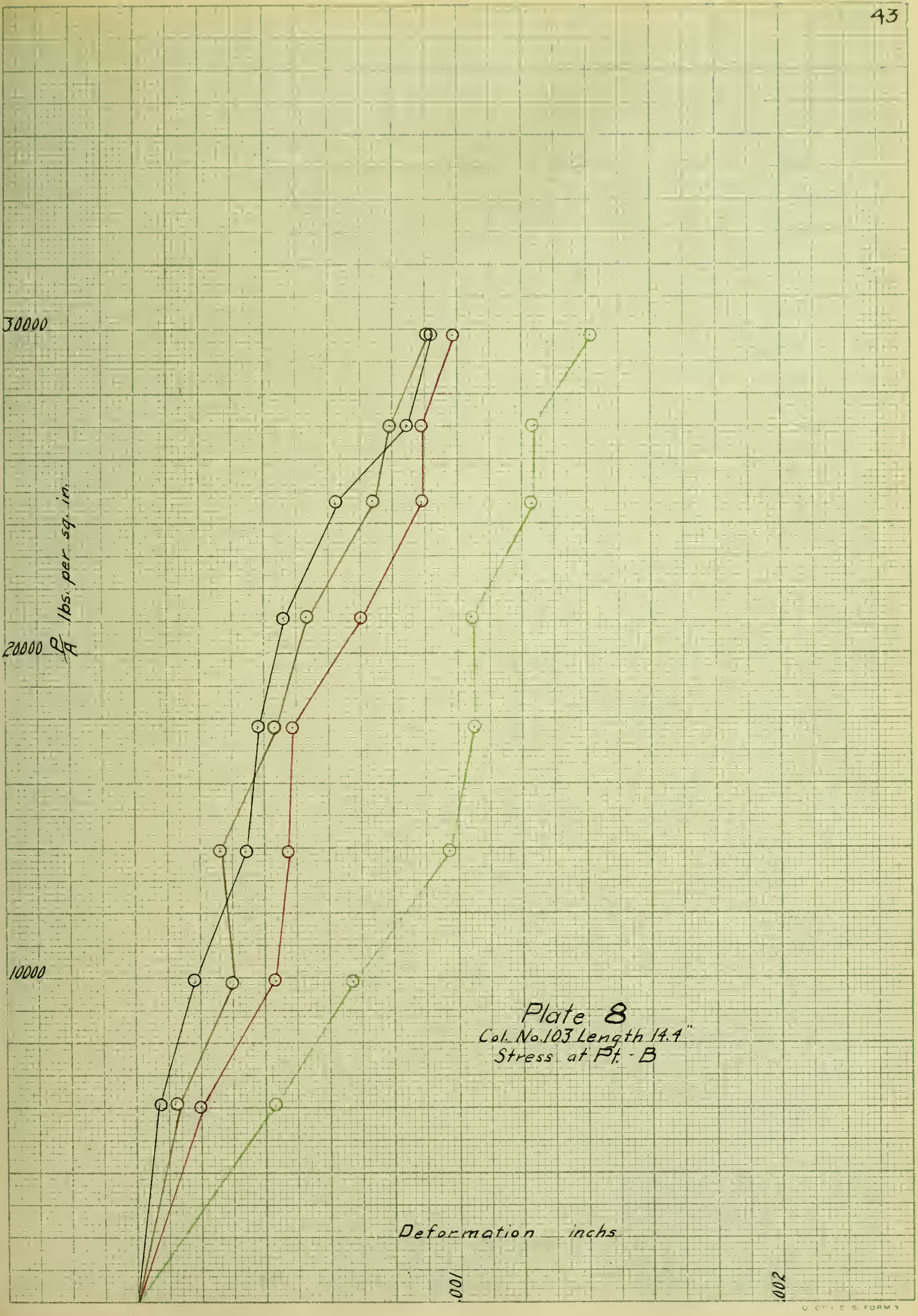
Plate 3
Col. No. 101 Length 14.3"
Stress at Pt. A

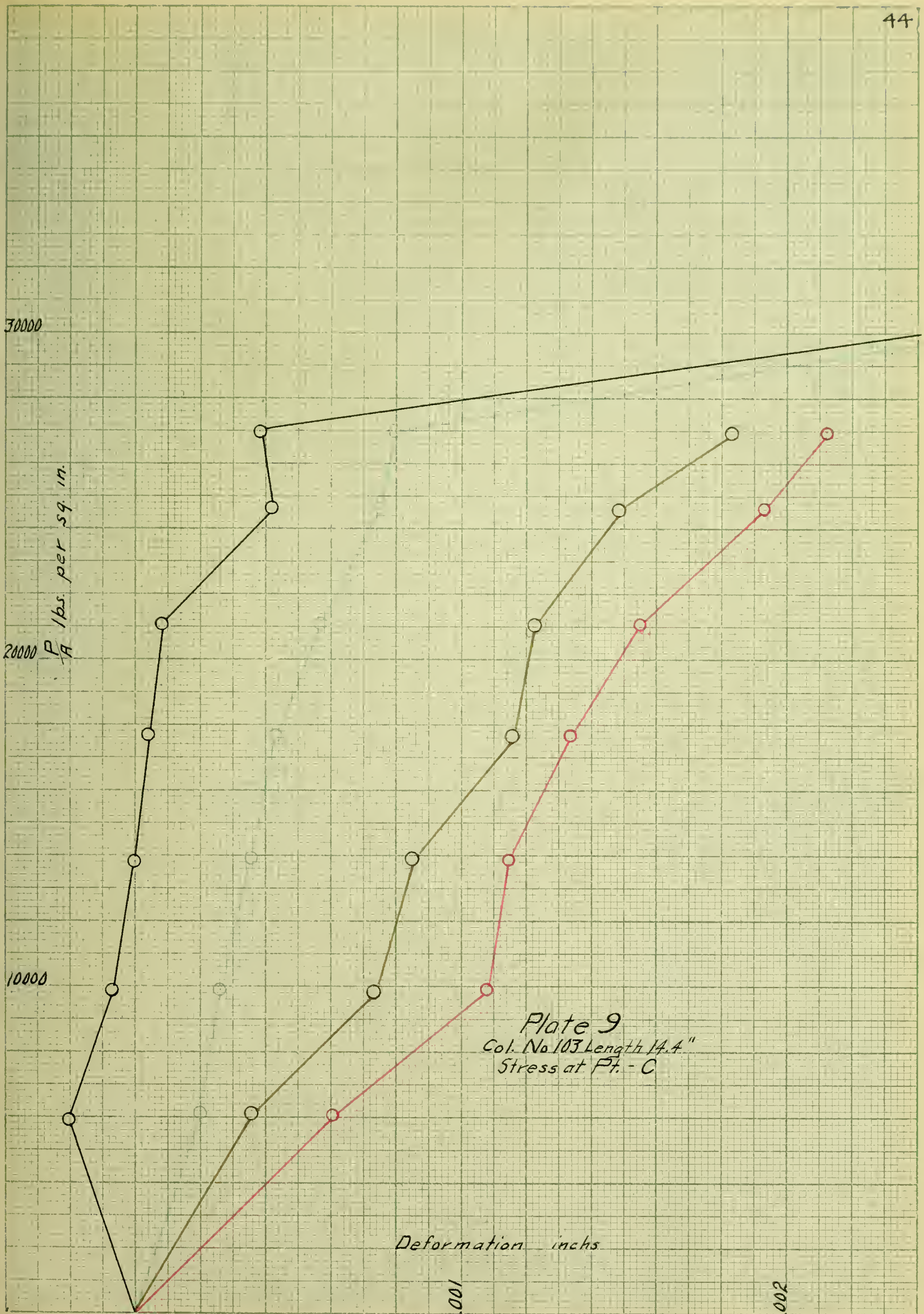

30000
20000
10000
P/A lbs per sq. in.
Plate 4
Col. No. 101 Length 14.3"
Stress at Pt. B
Deformation inchs
.001
.002
U. OF I. S. S. FORM 3

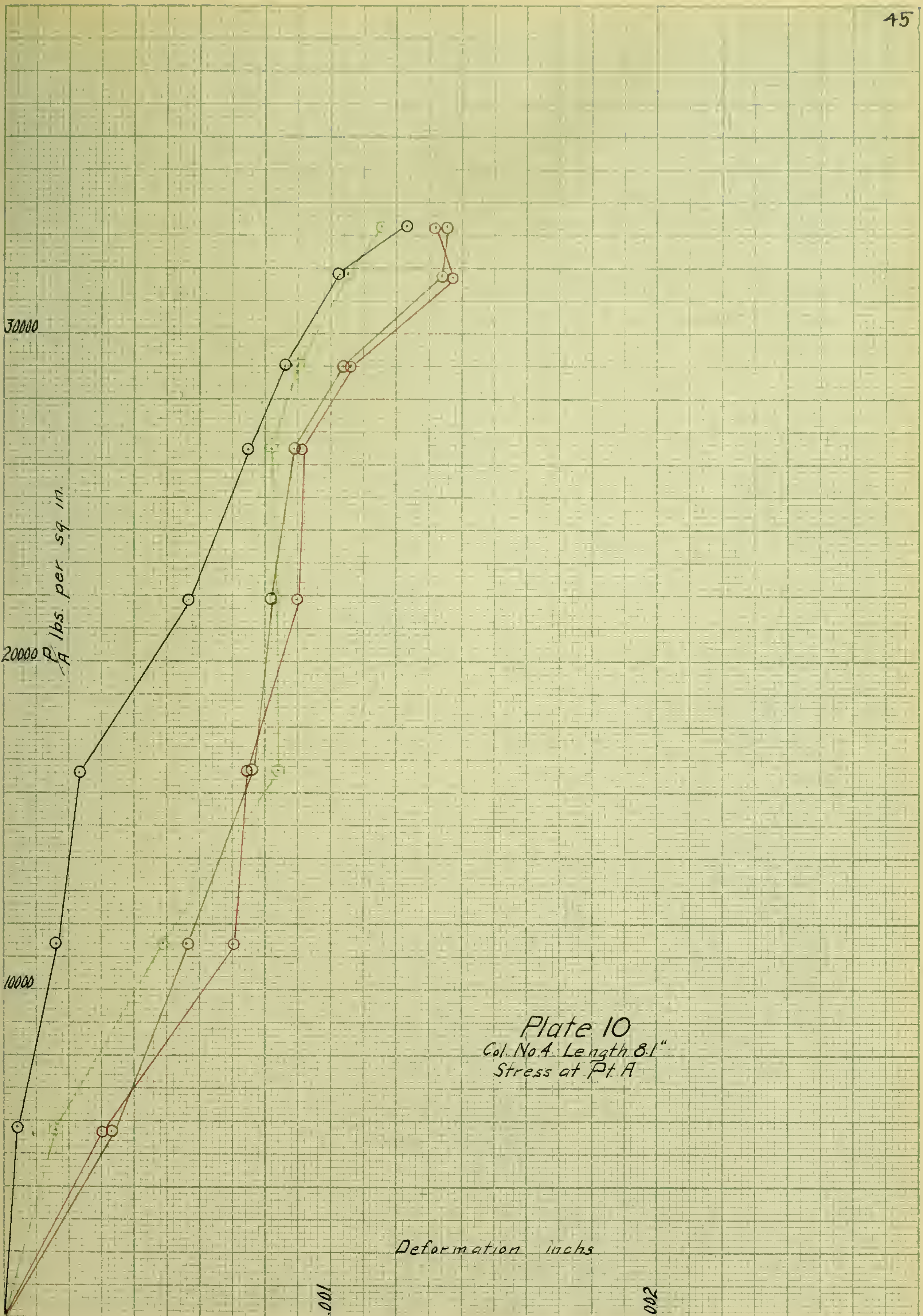

30000
20000 P/A lbs per sq. in
10000
Plate 5
Col. No. 101 Length 14.3"
Stress at Pt. C
Deformation inches
.001
.002

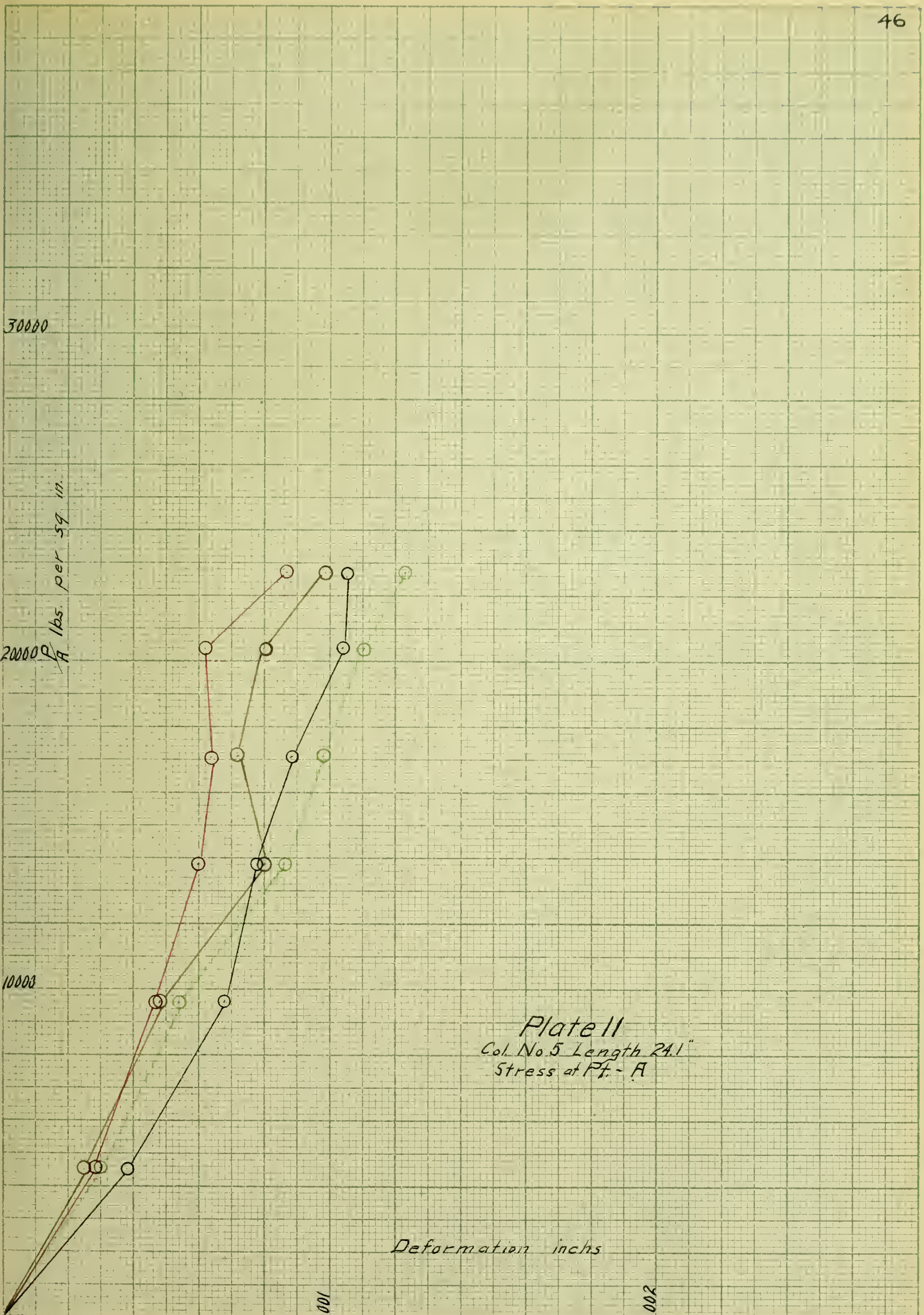

30000
20000 $\frac{P}{A}$ lbs. per sq. in.
10000
Plate 6
Col. No. 102 Length 8.2"
Stress at Pt.-A
Deformation inchs
.001
.002

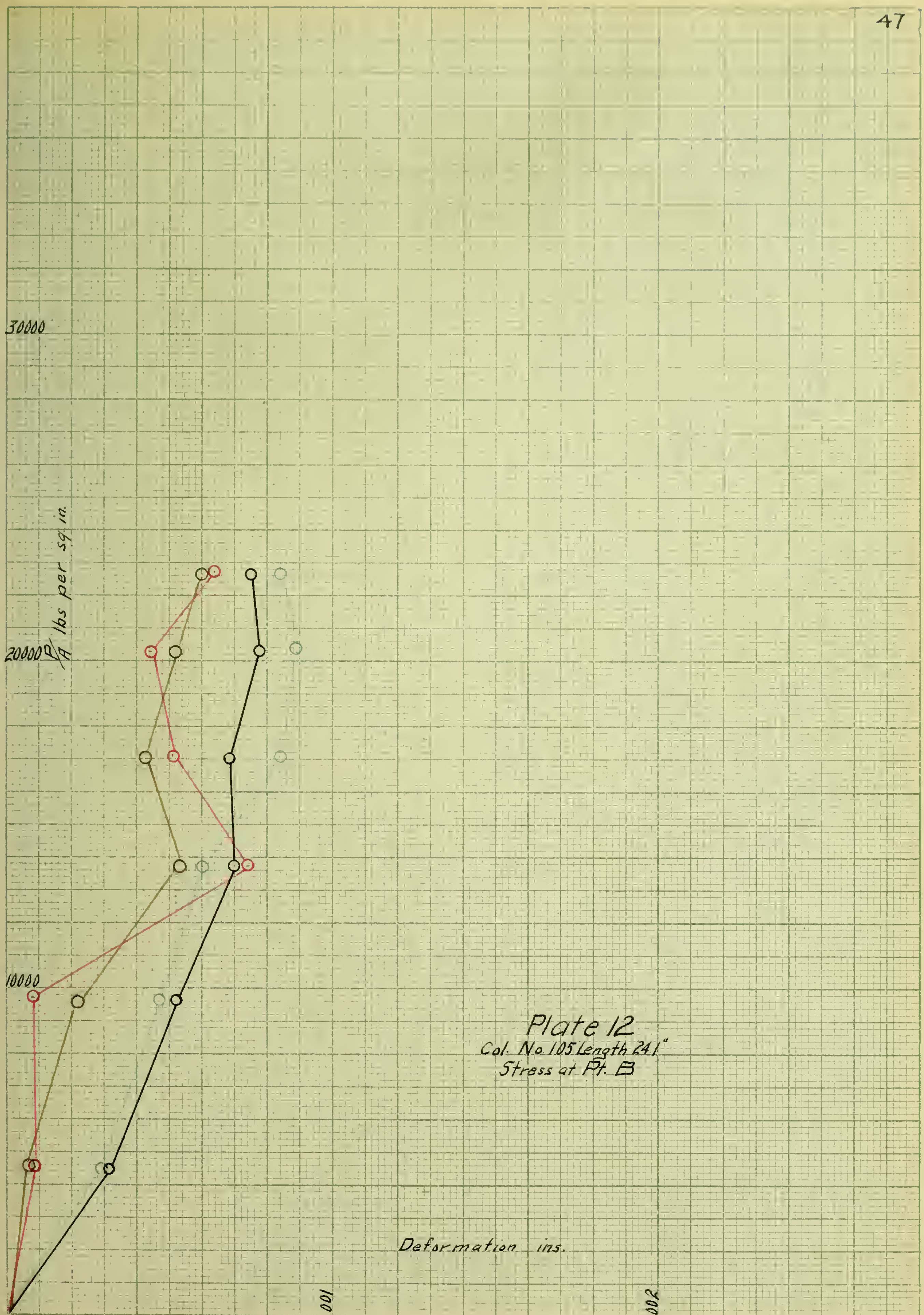

30000
20000
10000
P/A lbs per sq. in
Deformation ins.
.001
.002
Plate 7
Col. No. 103 Length 14.4"
Stress at Pt. A


30000
20000 P/A lbs. per sq. in.
10000
Plate 8
Col. No. 103 Length 14.4"
Stress at Pt. - B
Deformation inchs.
.001
.002


30000
20000
10000
P/A lbs. per sq. in.
Deformation inchs
.001
.002
Plate 9
Col. No 103 Length 14.4"
Stress at Pt. - C


30000
20000
10000
P/A lbs. per sq. in.
Plate 10
Col. No. 4 Length 8.1"
Stress at Pt. A
Deformation inchs
.001
.002


30000
20000
10000
P/A lbs per sq in.
.001
.002
Deformation inchs
Plate 11
Col. No. 5 Length 24.1"
Stress at Pt. - A


30000
20000
10000
P/A lbs per sq in.
Deformation ins.
.001
.002
Plate 12
Col. No. 105 Length 24.1"
Stress at Pt. B

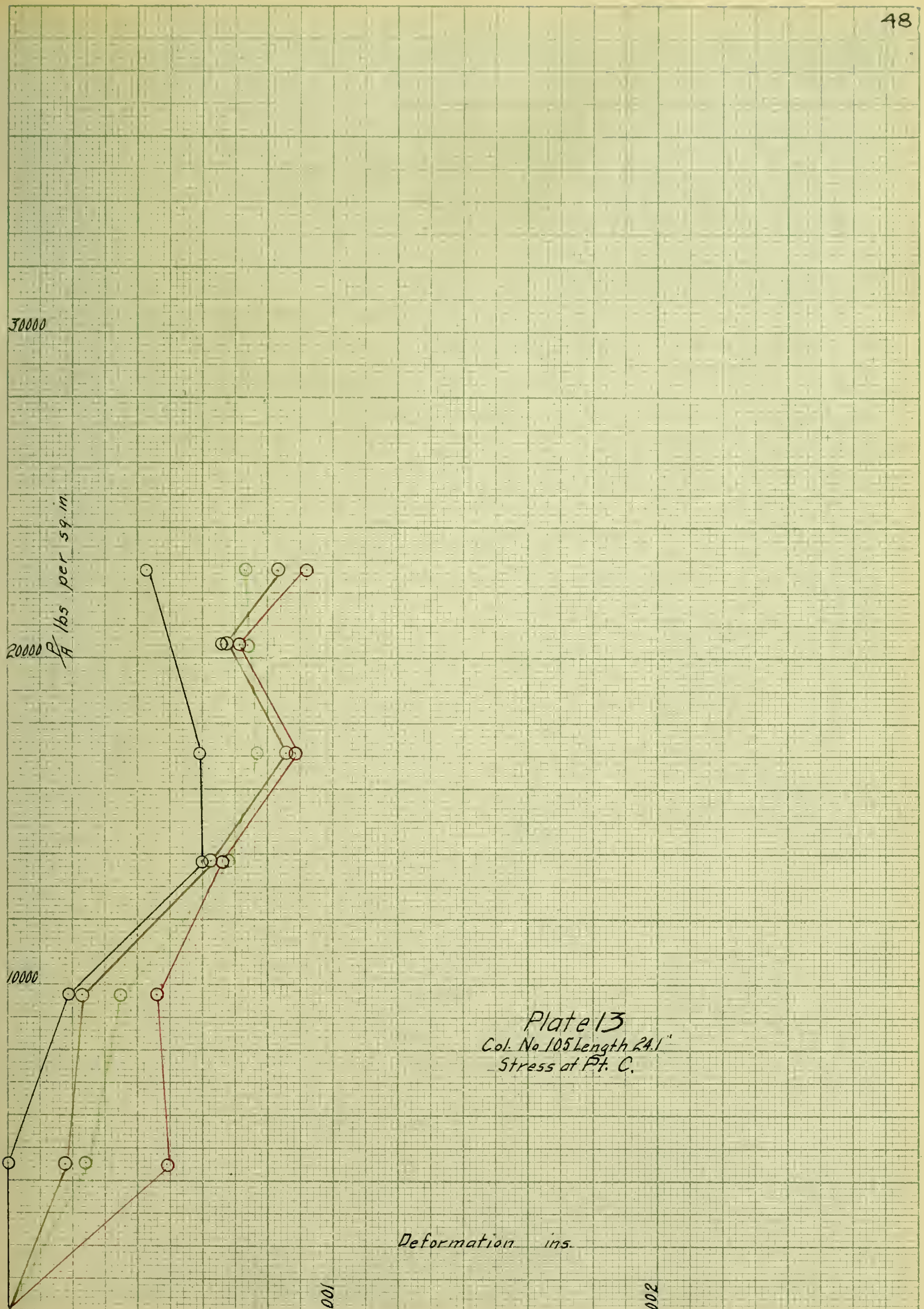
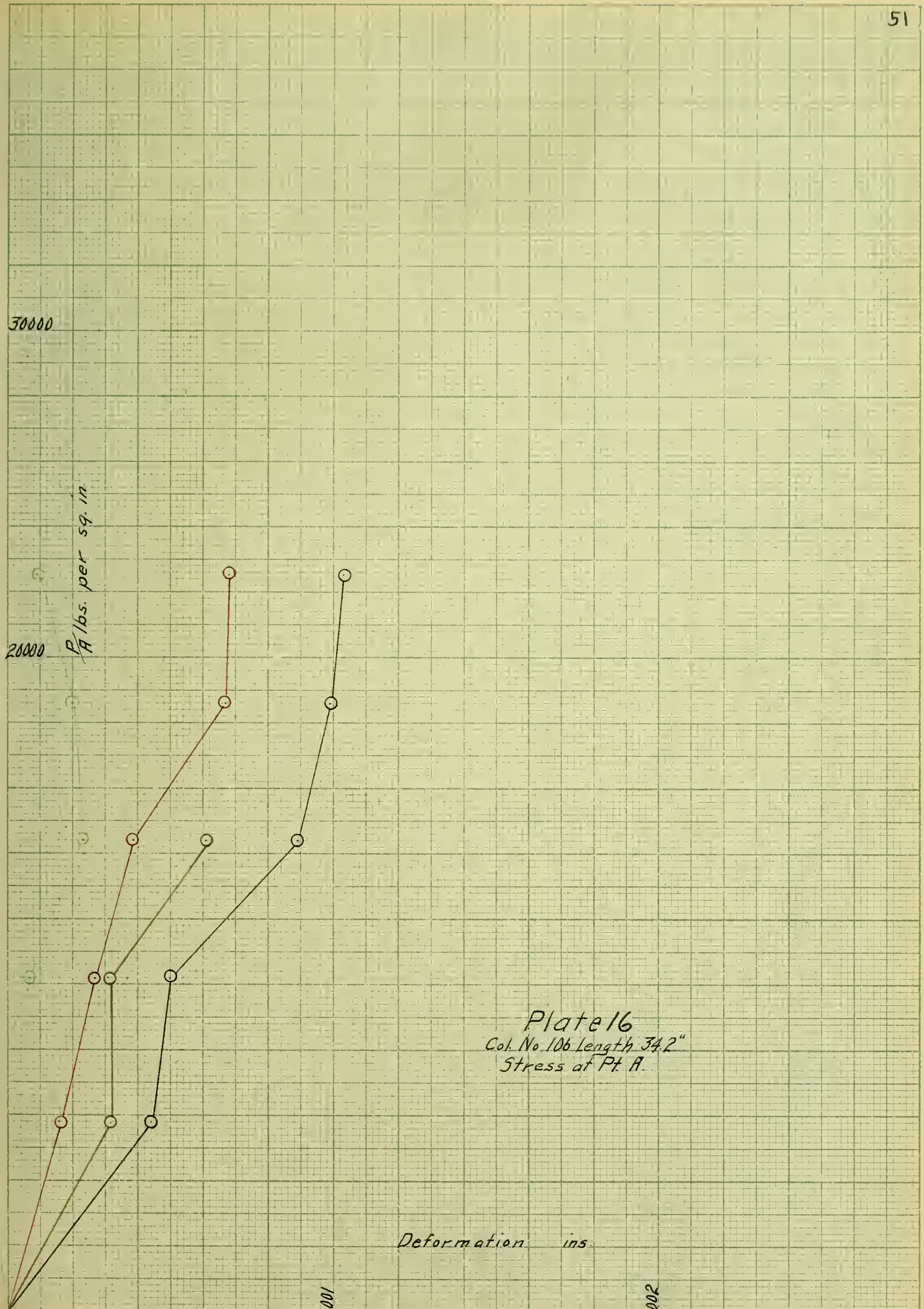


Plate 13
Col. No. 105 Length 24.1"
Stress at Pt. C.


30000
20000
10000
P/A lbs. per sq. in.
0
.001
Deformation ins
Plate 14
Col. No. 105 Length 24.1"
Stress at Pt.-D


30000
P/A lbs. per sq. in.
20000
10000
Plate 15
Col. No. 105 Length 24.1"
Stress at Pt. E
Deformation ins.
.001
.002


30000
20000
P/A lbs. per sq. in.
Plate 16
Col. No. 10b Length 34.2"
Stress at Pt. A.
Deformation ins.
.001
.002

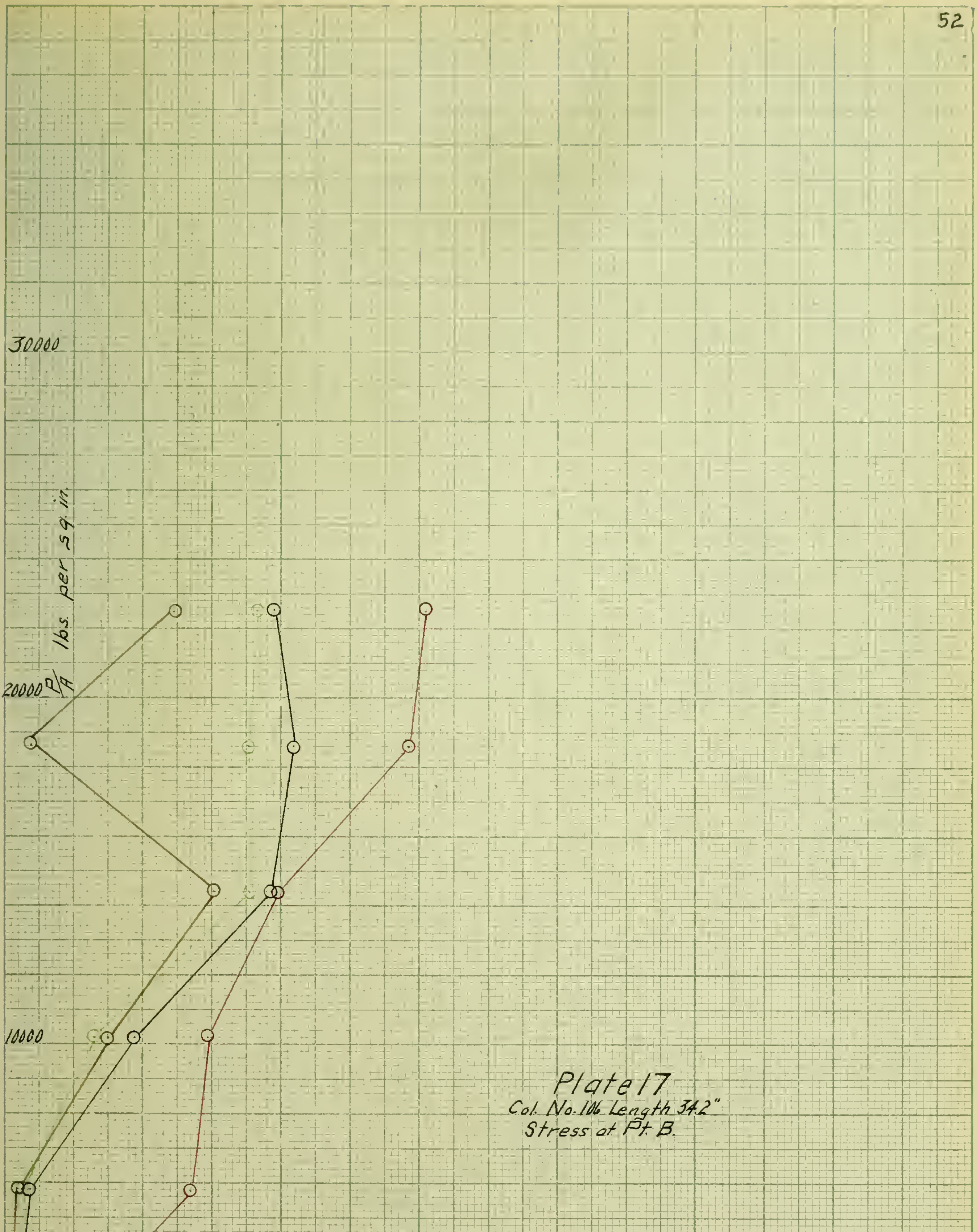
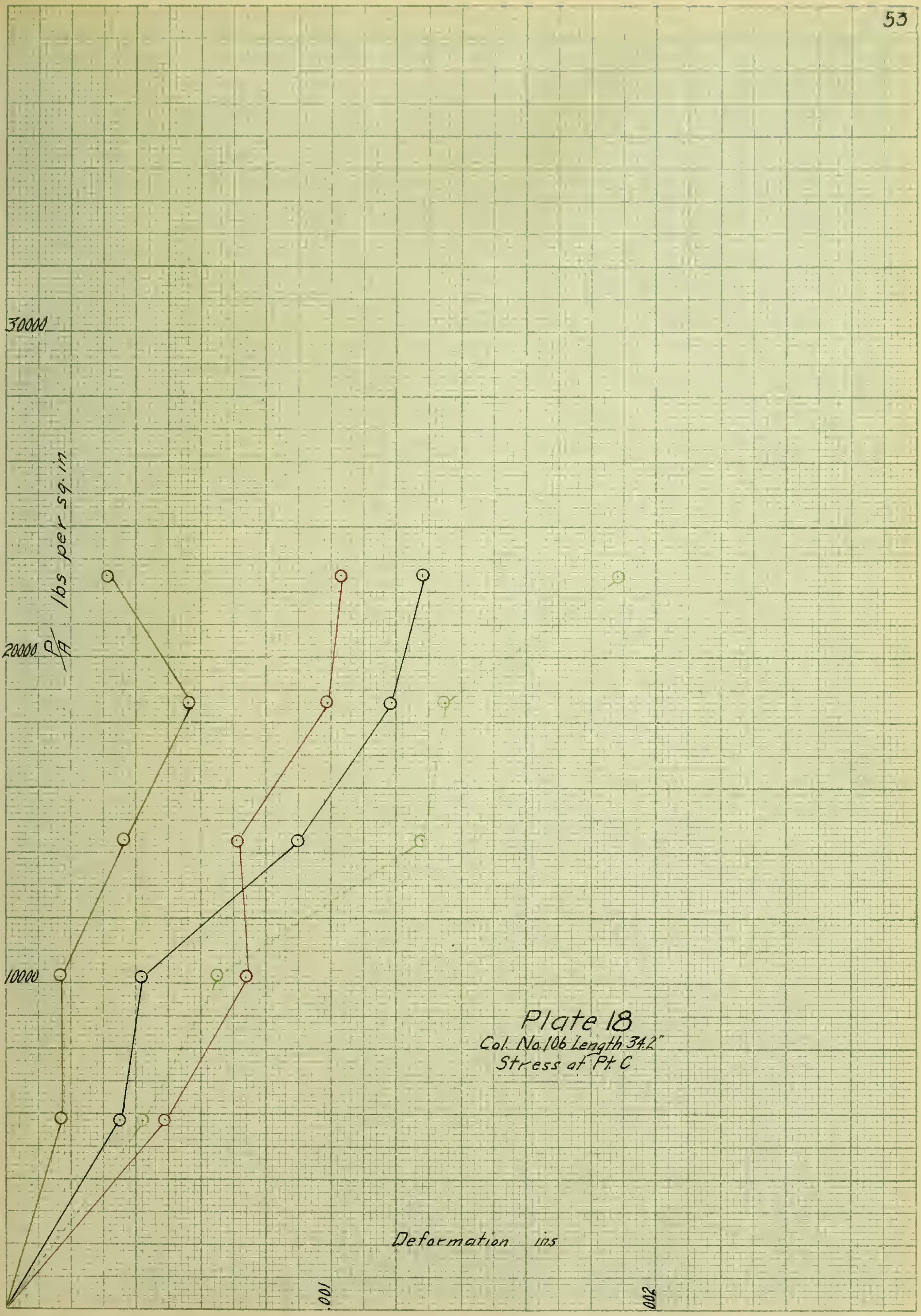
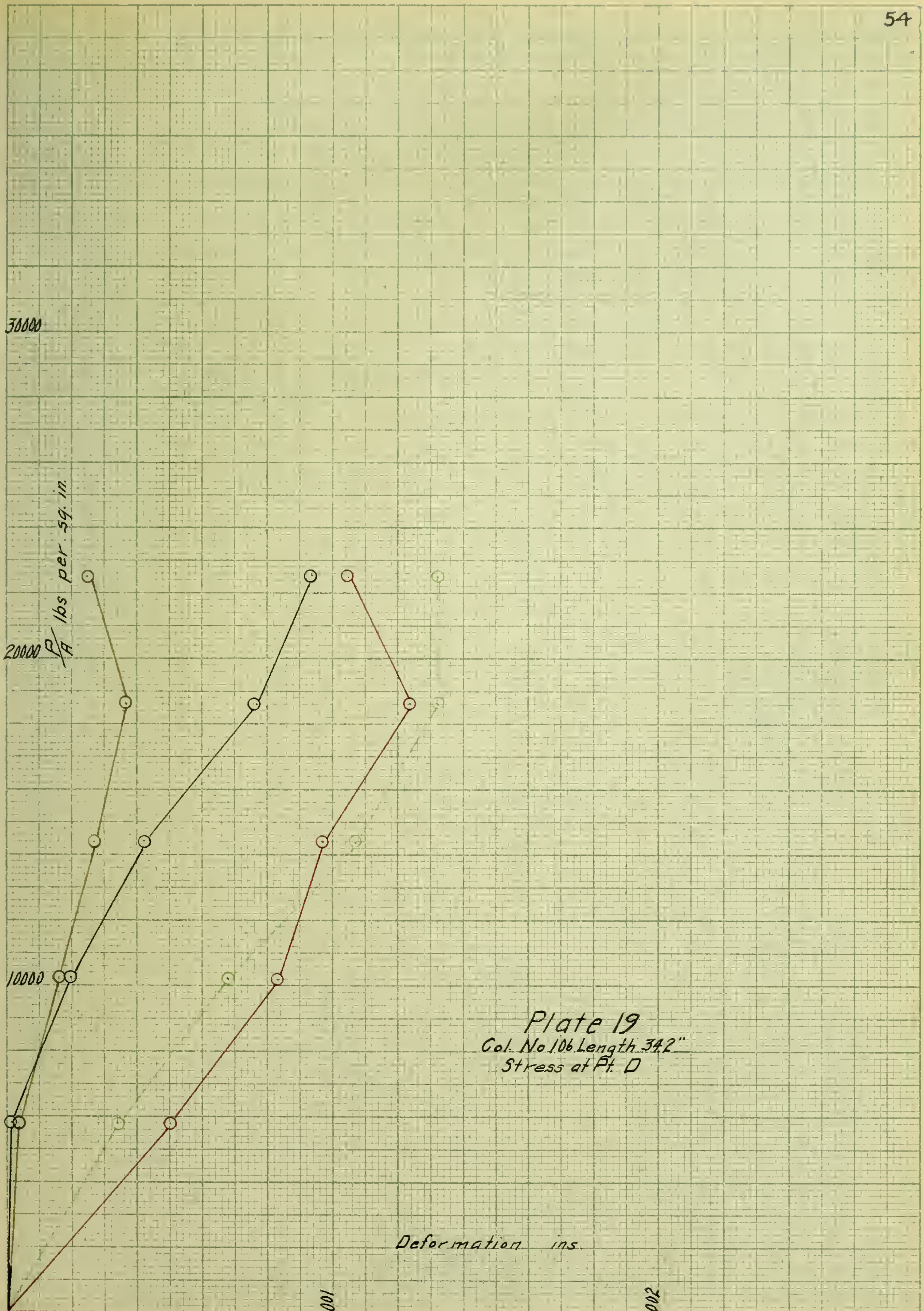
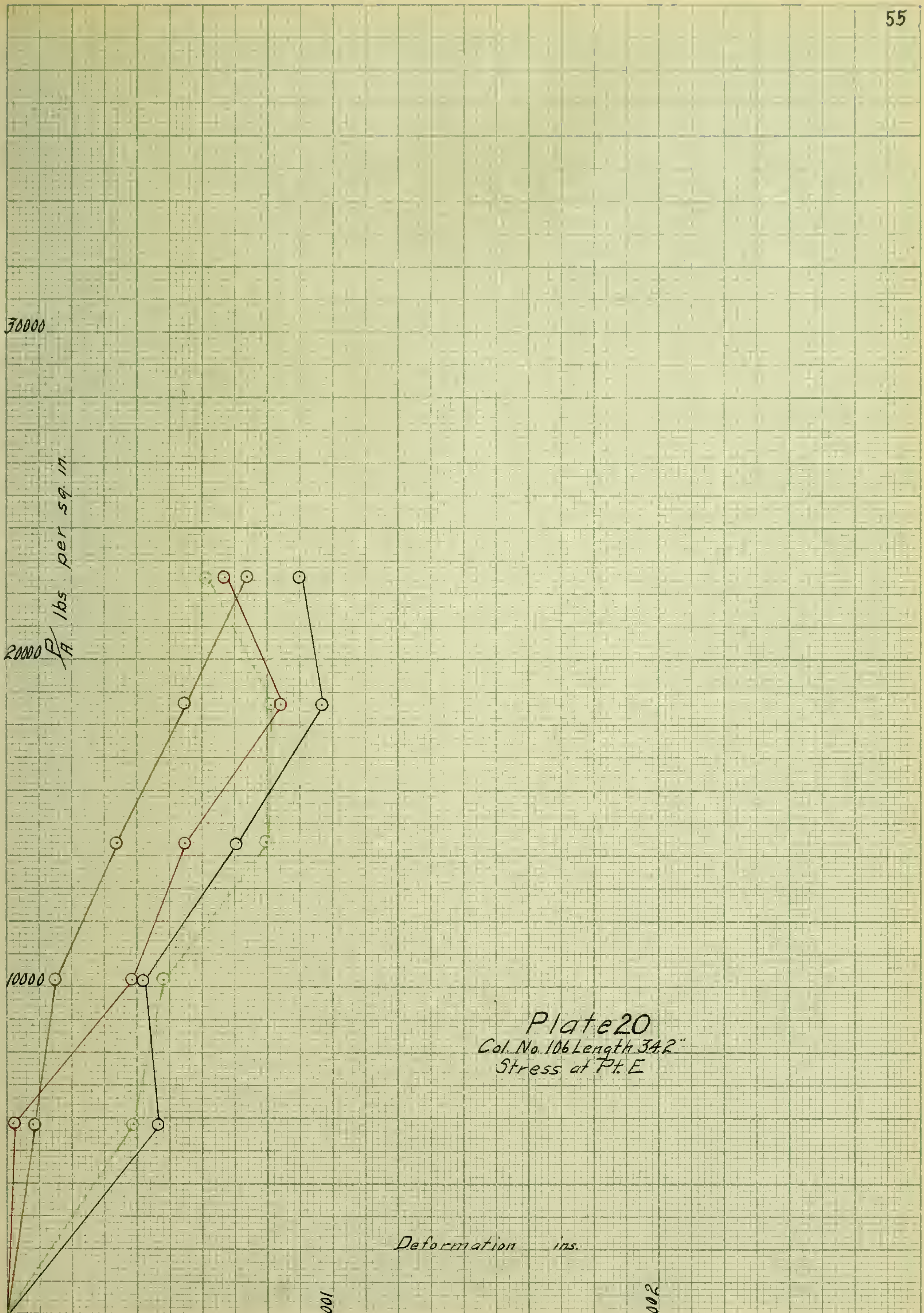


Plate 17
Col. No. 106 Length 34.2"
Stress at Pt. B.


30000
20000
10000
P/A lbs per sq. in.
Deformation ins
.001
.002
Plate 18
Col. No. 10b Length 34.2"
Stress at Pt. C


30000
20000
10000
P/A lbs per sq. in.
Deformation ins.
.001
.002
Plate 19
Col. No 106 Length 34.2"
Stress at Pt. D


30000
20000
10000
P/A lbs per sq. in.
Plate 20
Col. No. 106 Length 34.2"
Stress at Pt. E
Deformation ins.
.001
.002

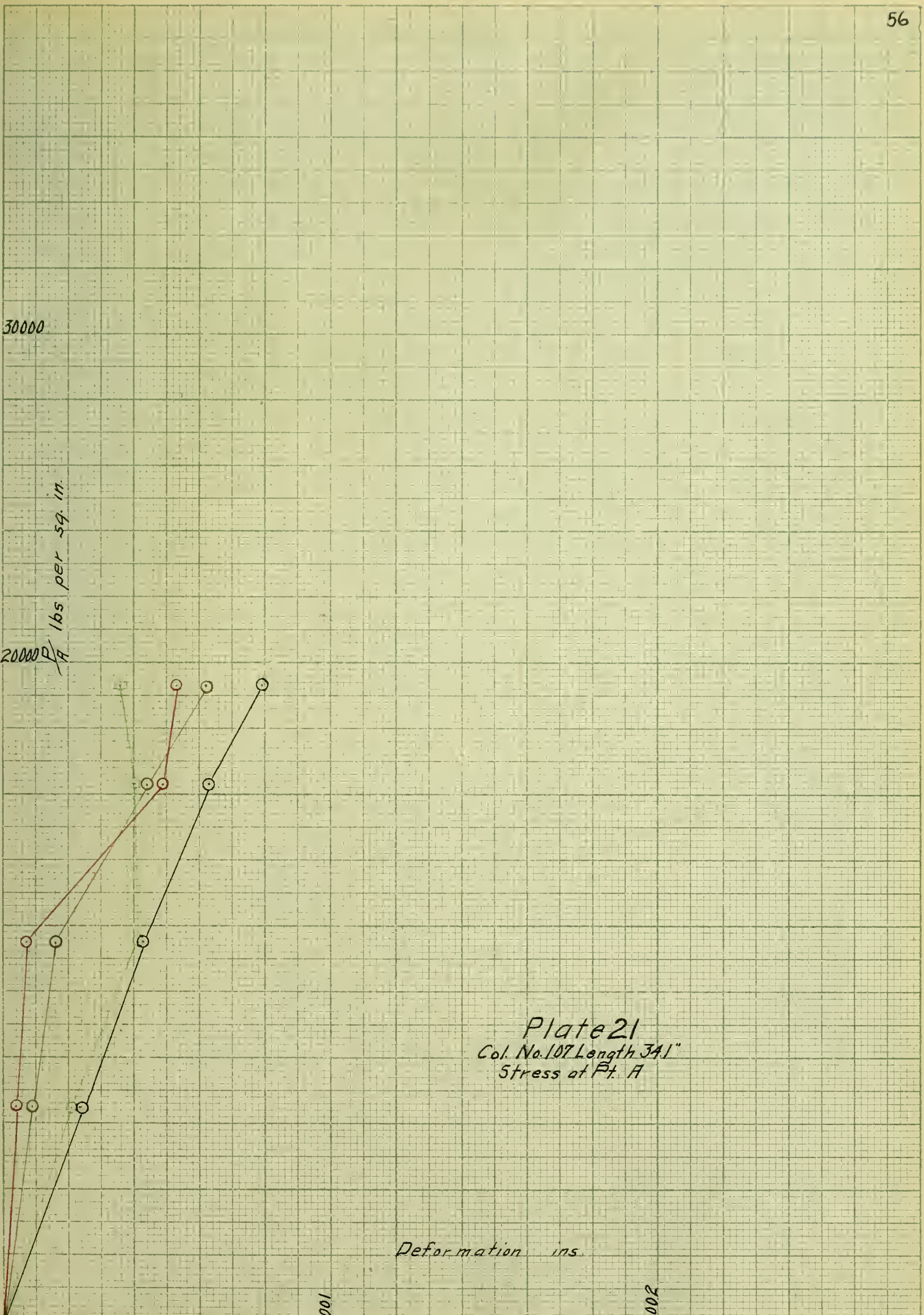
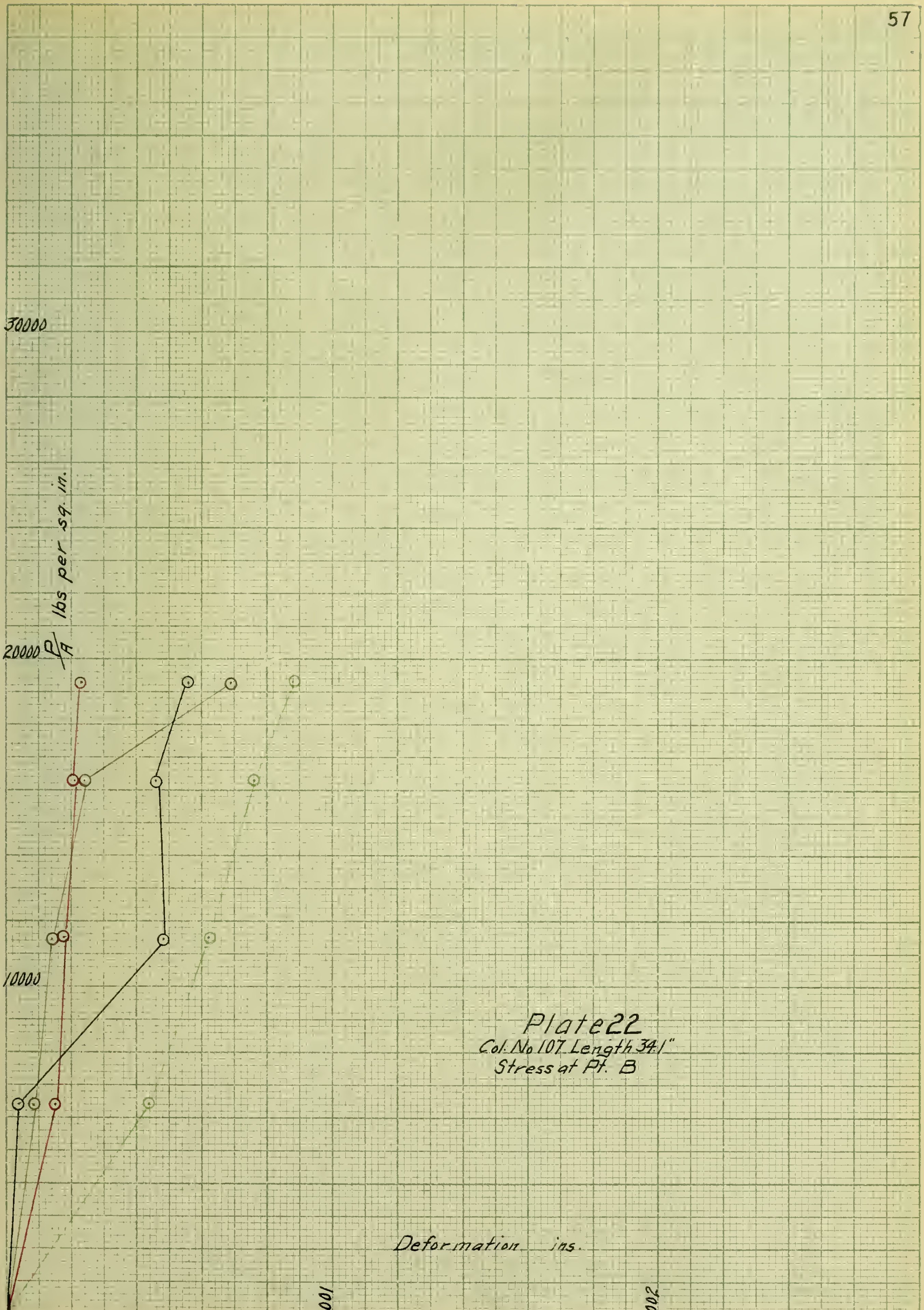


Plate 21
Col. No. 107 Length 34.1"
Stress at Pt. A


30000
20000
P/A lbs per sq. in.
10000
Plate 22
Col. No 107 Length 34.1"
Stress at Pt. B
Deformation ins.
.001
.002

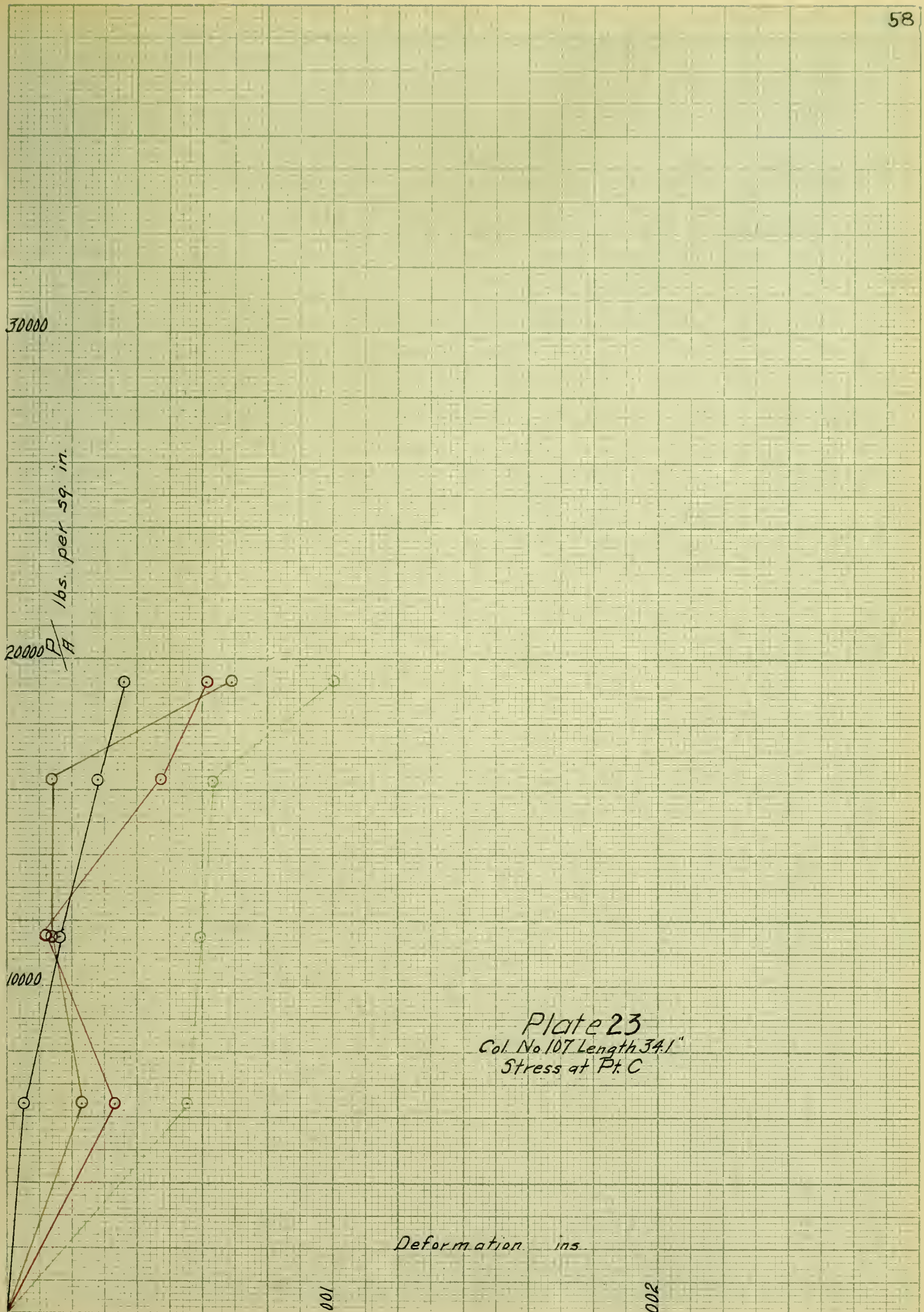


Plate 23
Col. No 107 Length 34.1"
Stress at Pt. C

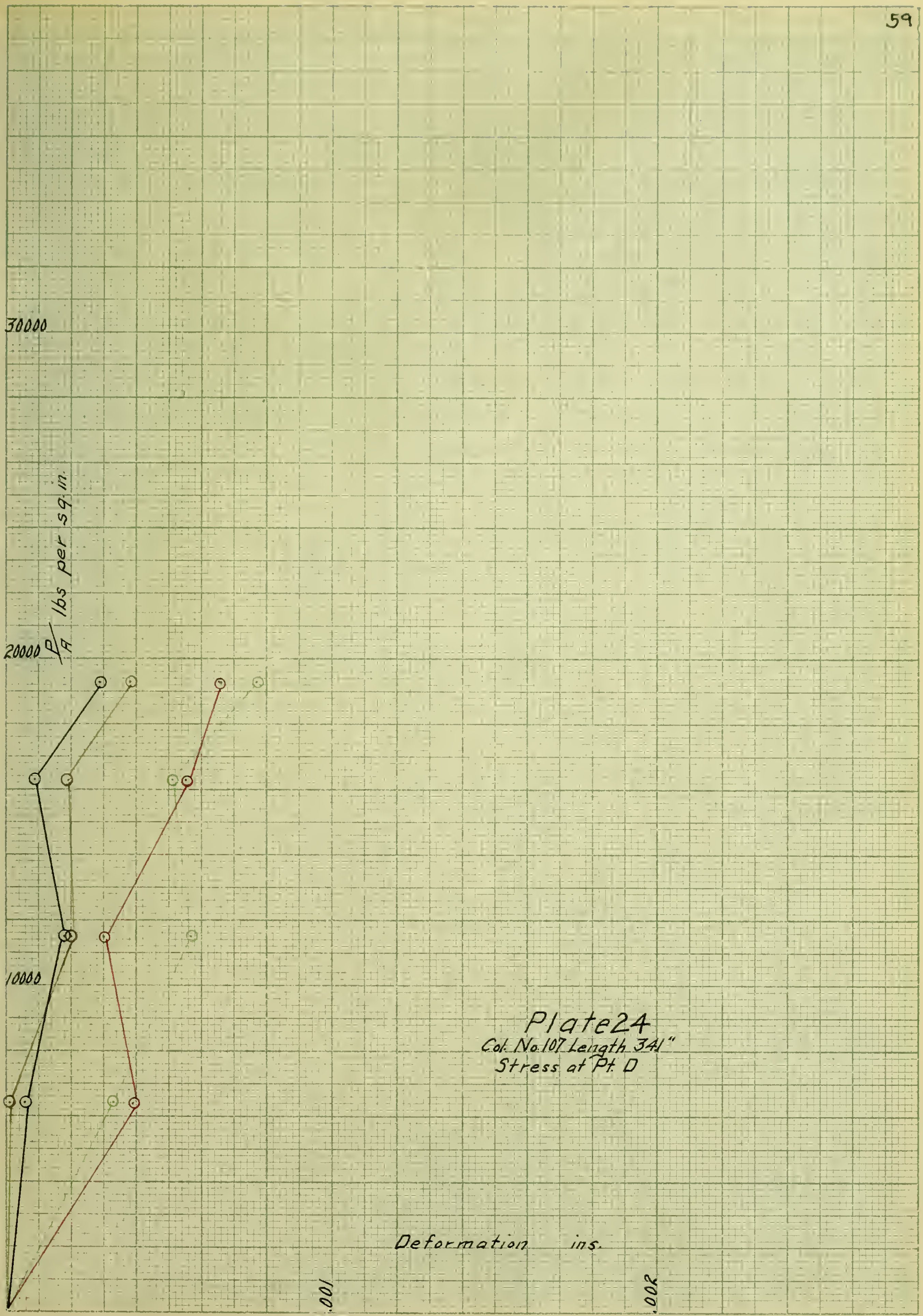


Plate 24
Col. No. 107 Length 341"
Stress at Pt. D

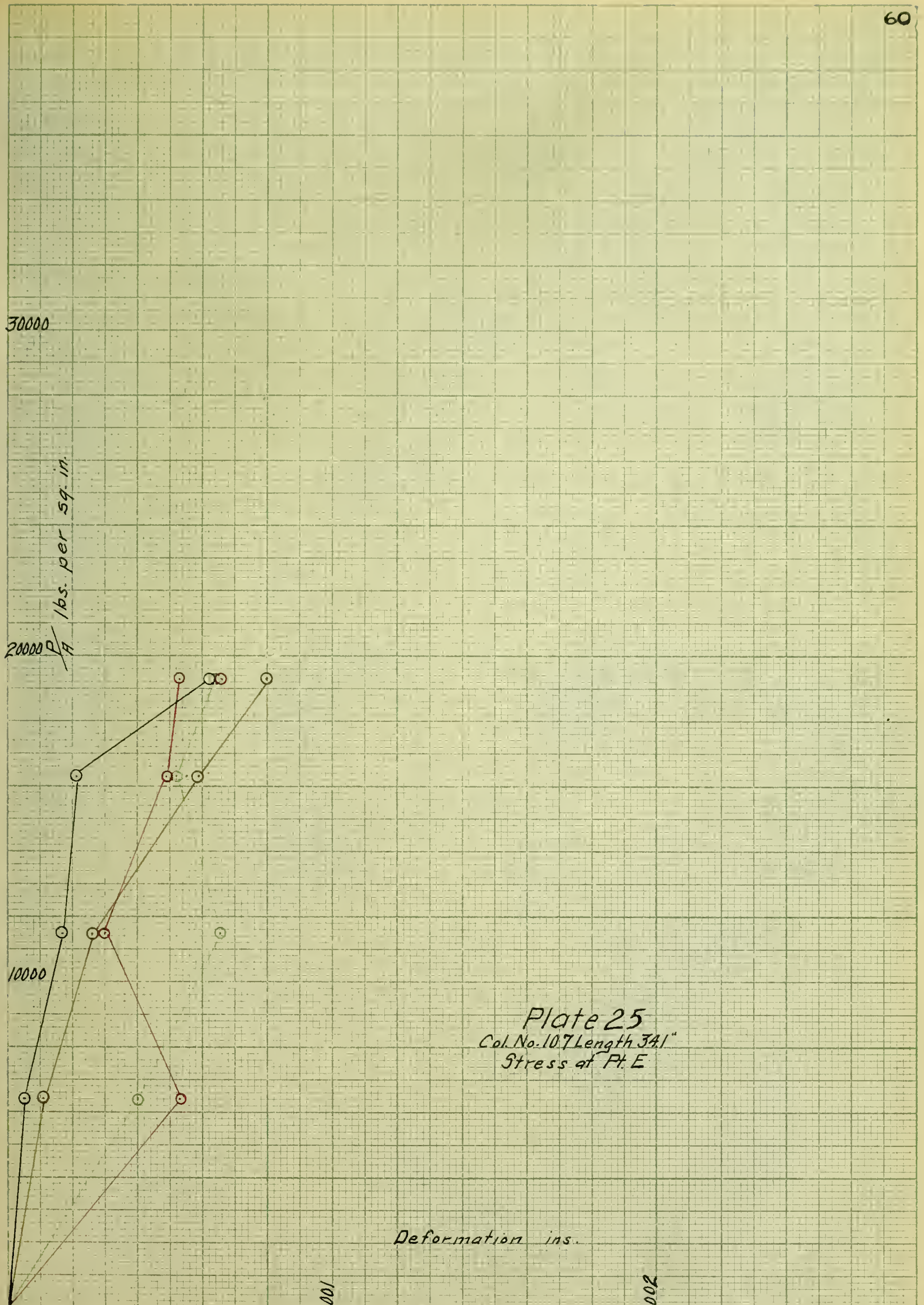


Plate 25
Col. No. 107 Length 34.1"
Stress at Pt. E

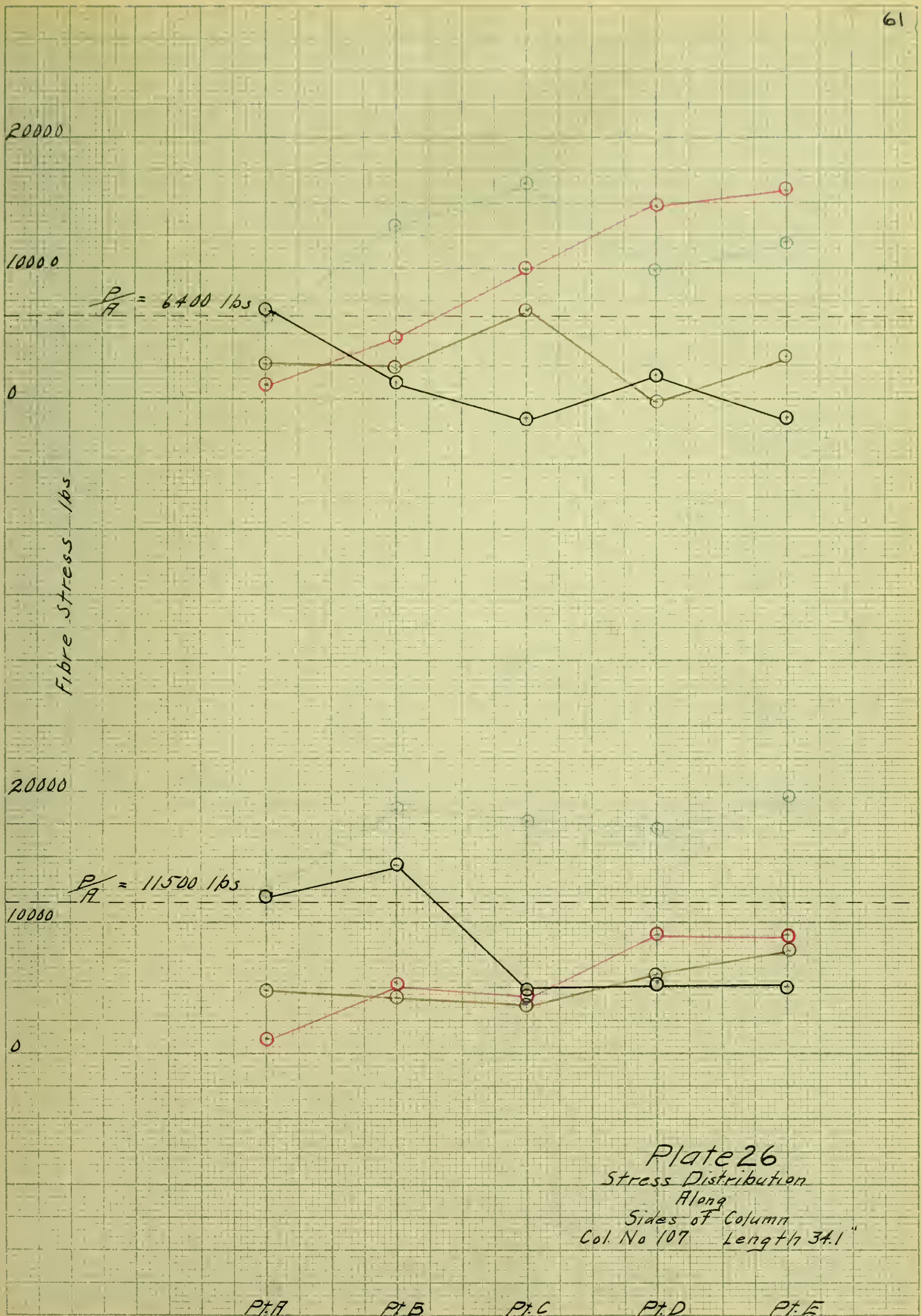


Plate 26
Stress Distribution
Along
Sides of Column
Col. No 107 Length 34.1"

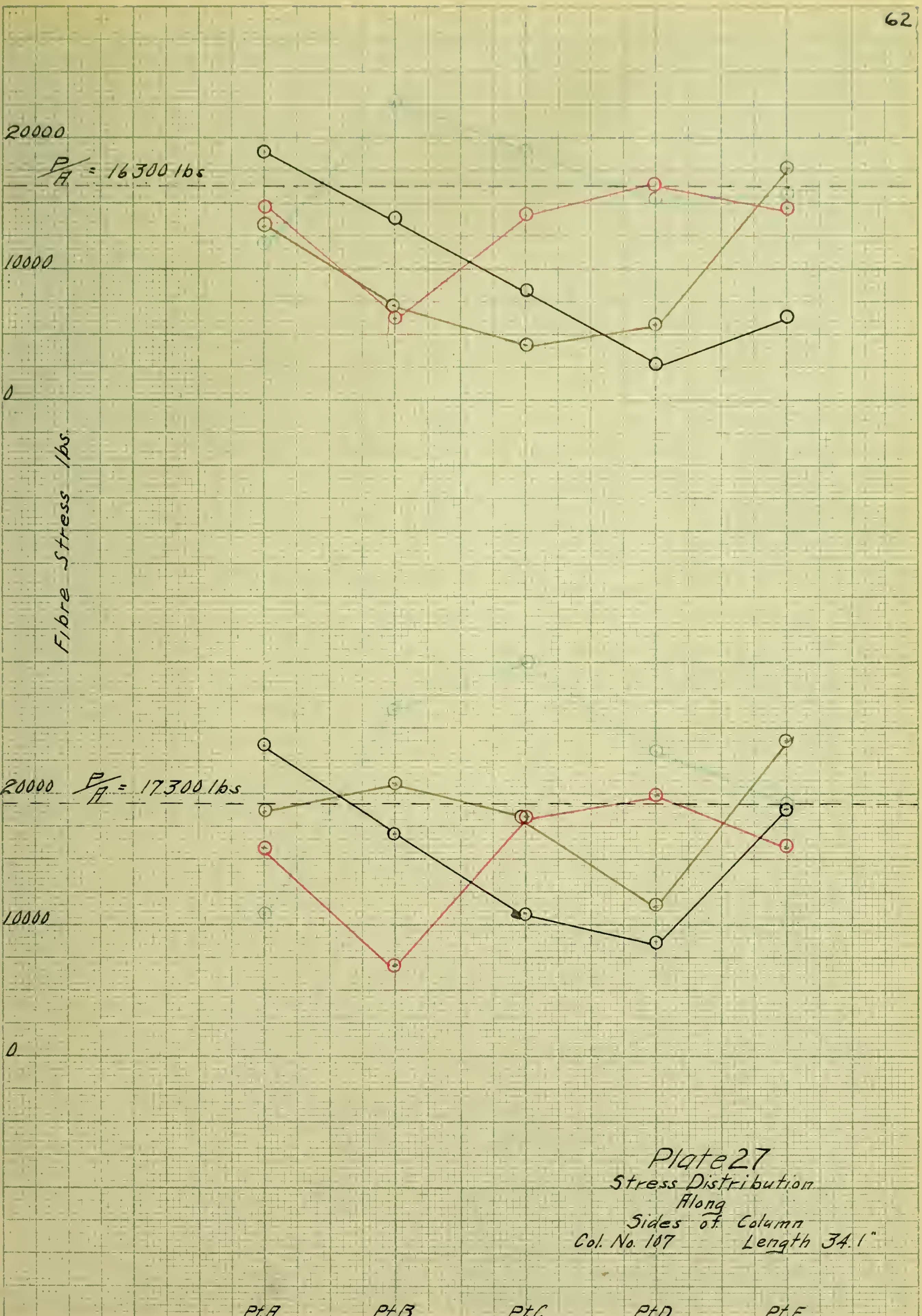


Plate 27
Stress Distribution
Along
Sides of Column
Col. No. 107 Length 34.1"

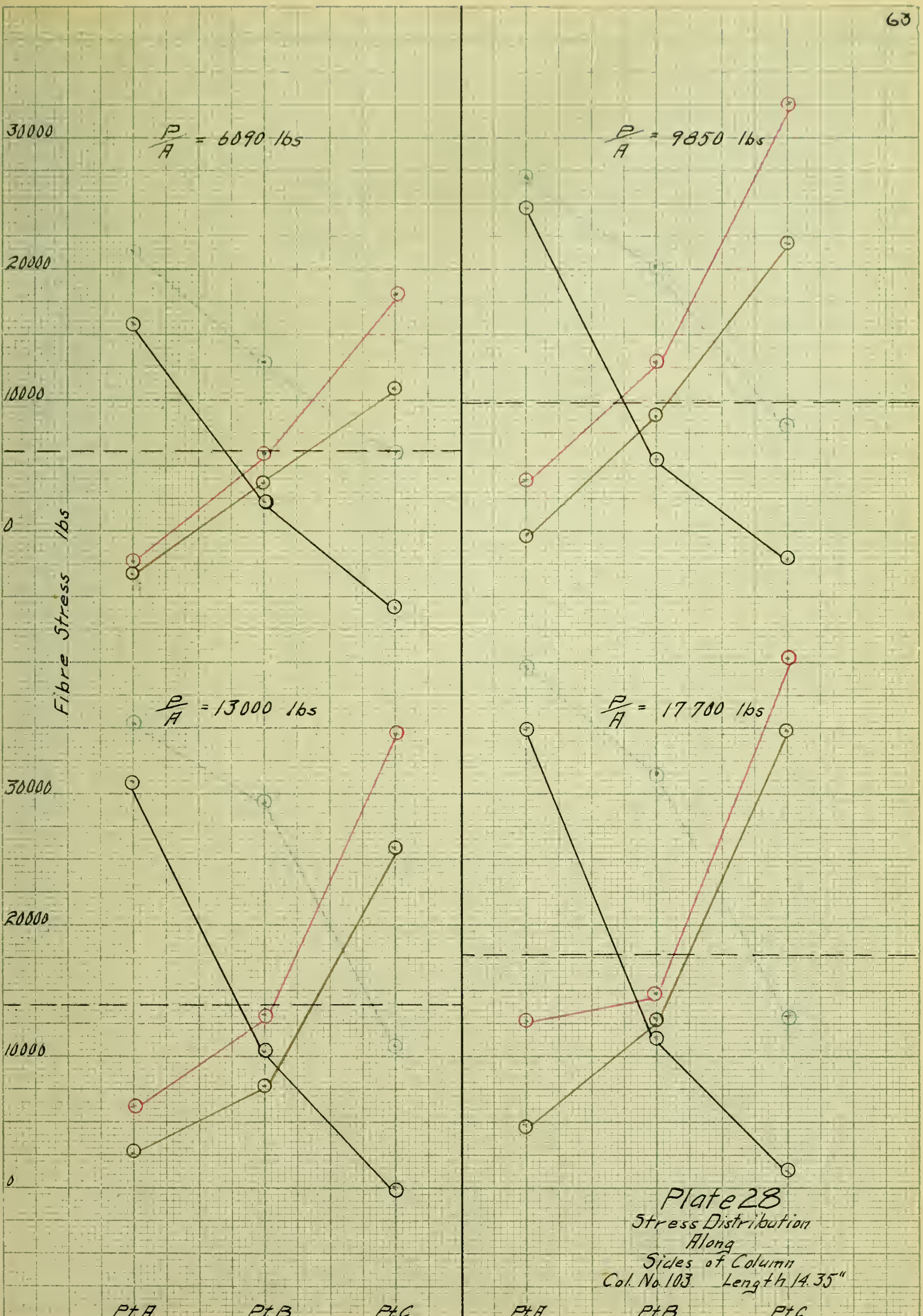


Plate 28
Stress Distribution
Along
Sides of Column
Col. No. 103 Length 14.35"

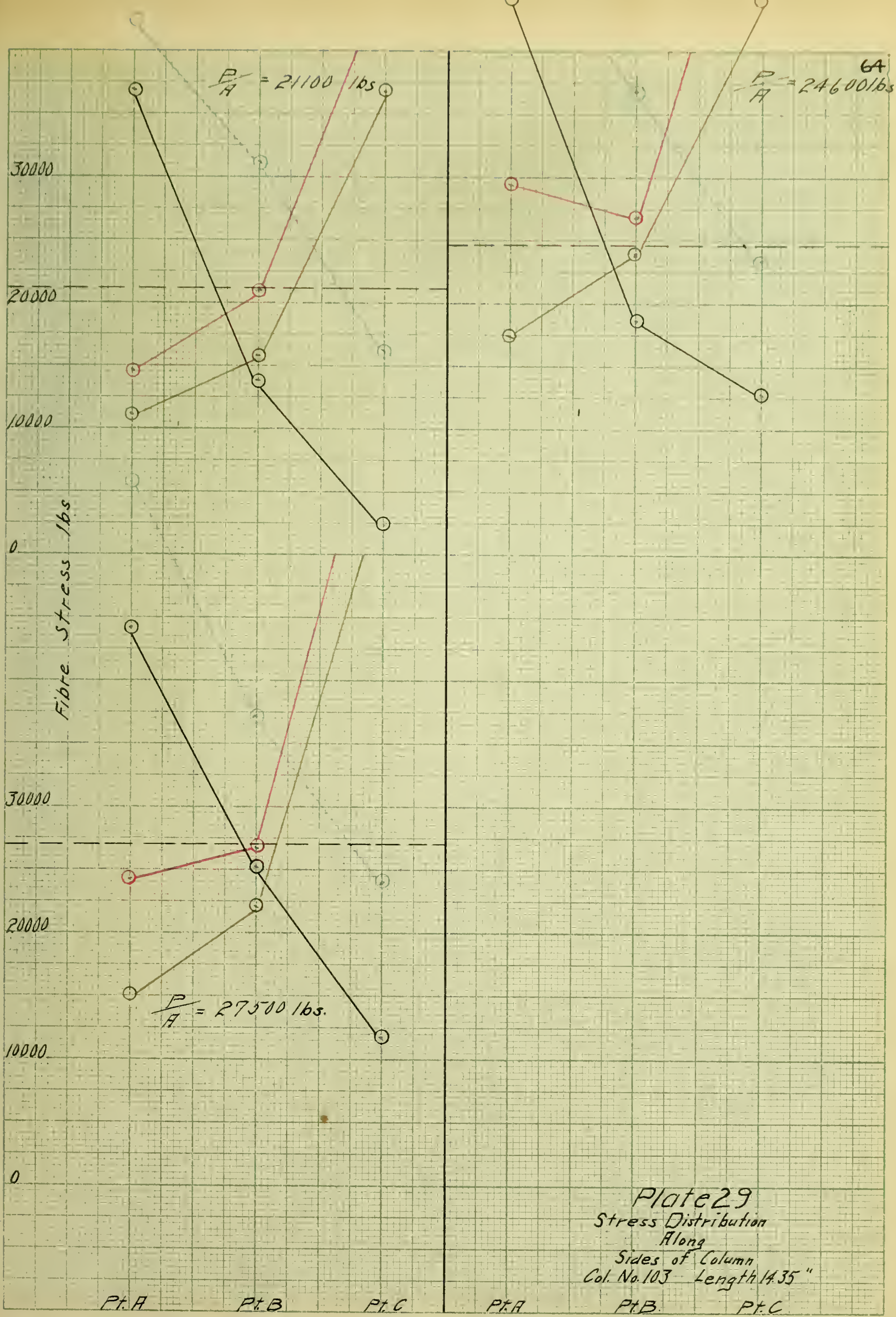


Plate 29
Stress Distribution
Along
Sides of Column
Col. No. 103 Length 14.35"

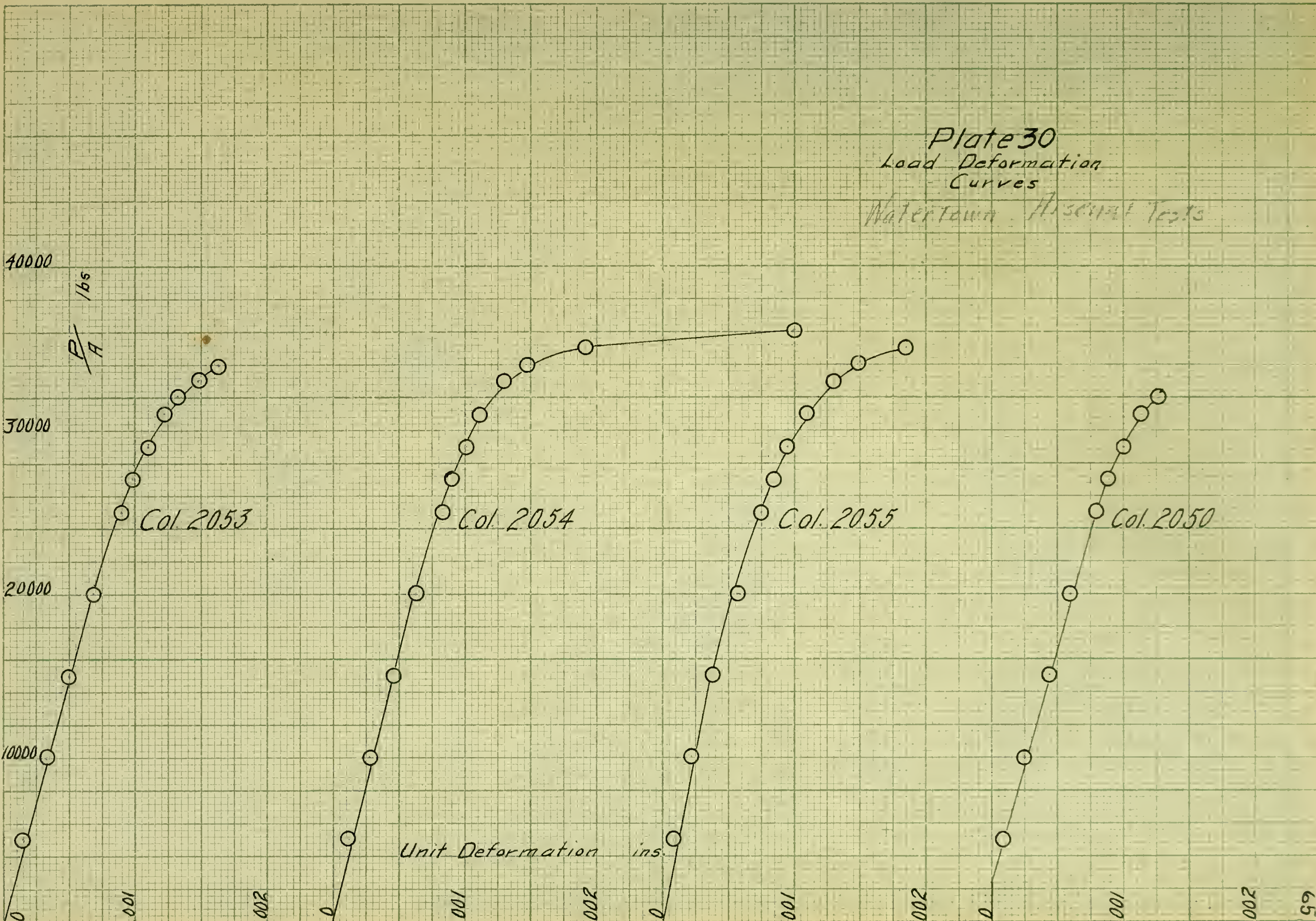


Plate 30
Load Deformation
Curves
Watertown Abscont Tests
40000
30000
20000
10000
P/A lbs
Col. 2053
Col. 2054
Col. 2055
Col. 2050
Unit Deformation ins.
0
100
200

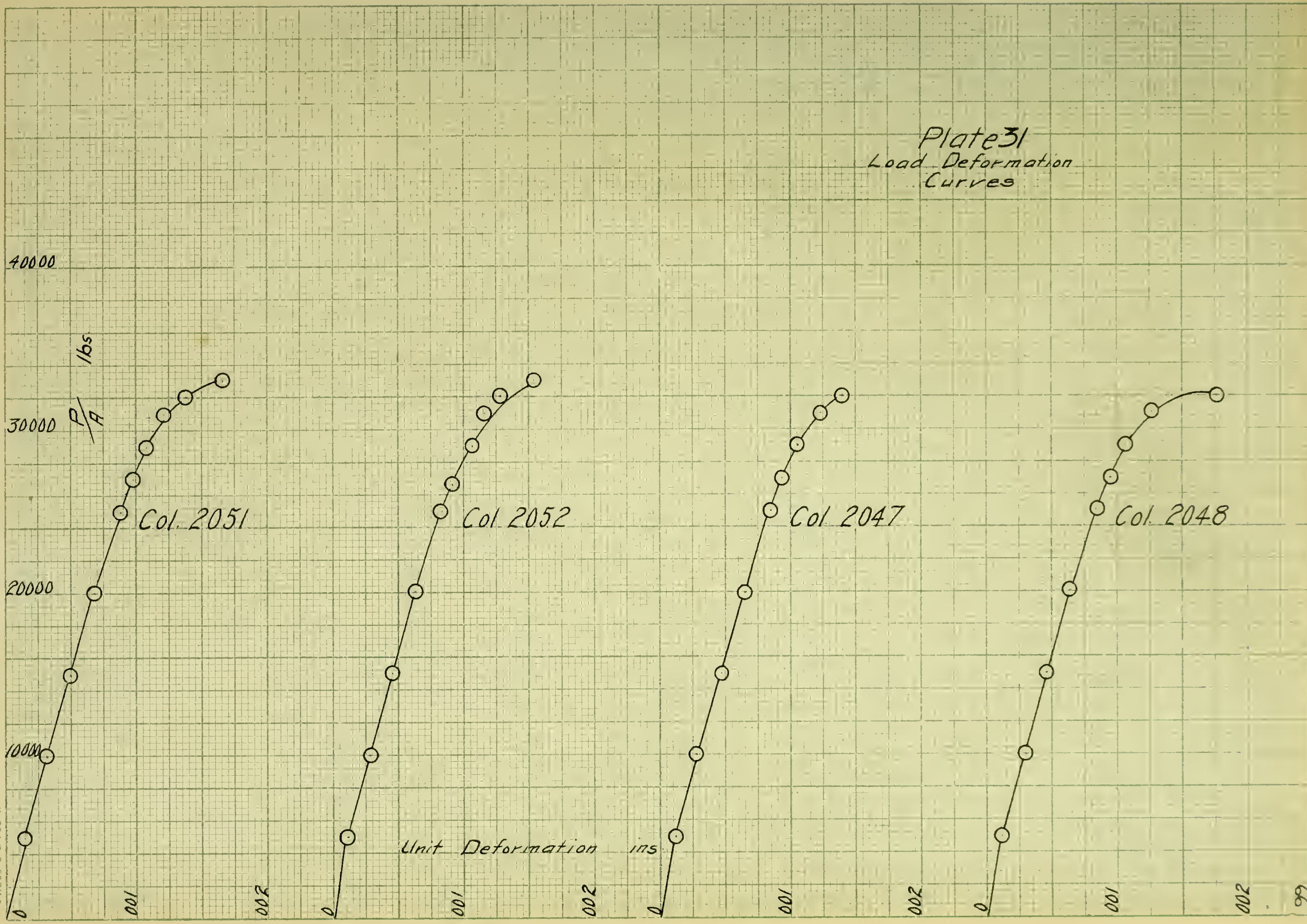
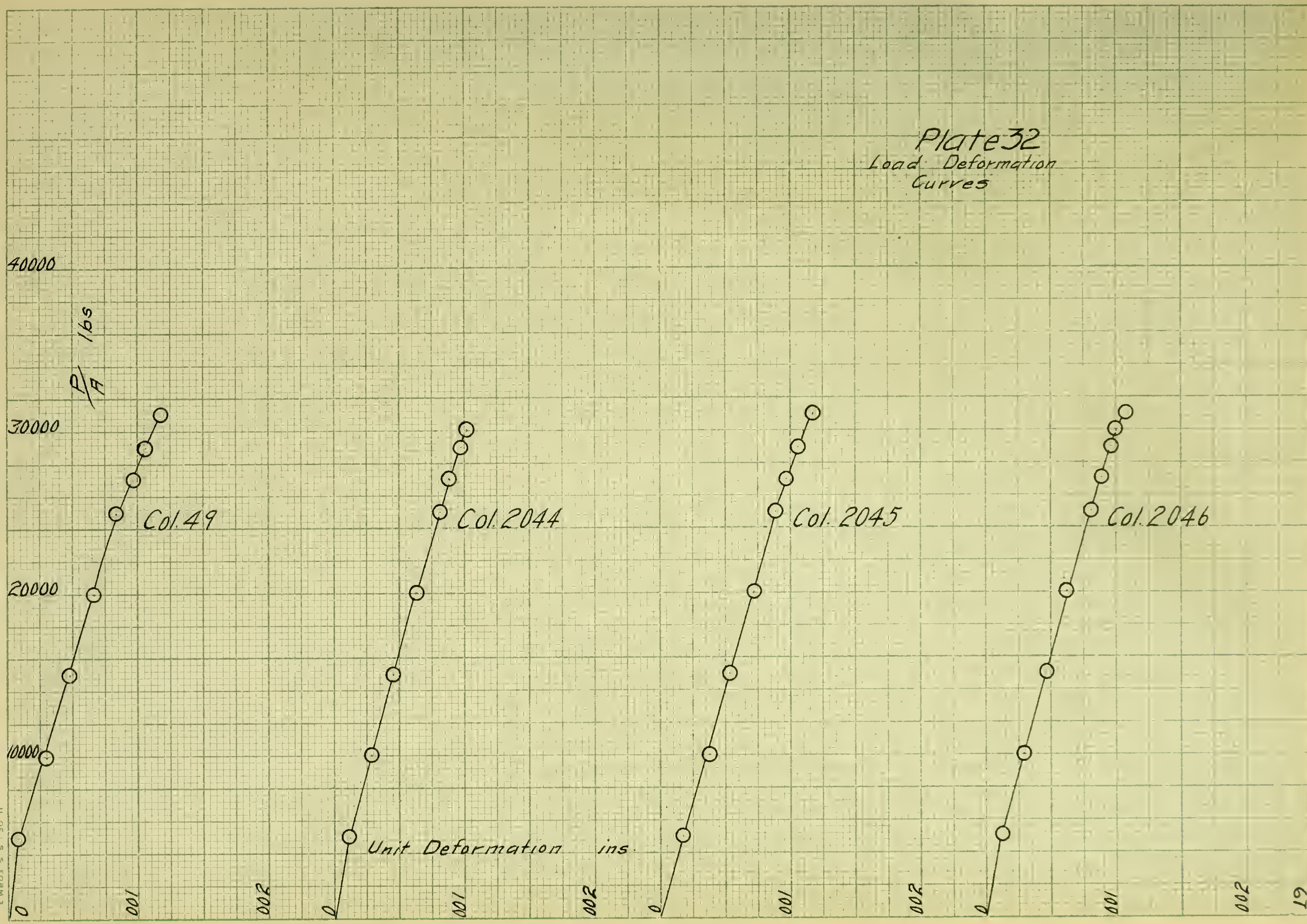


Plate 31
Load Deformation
Curves
40000
30000
20000
10000
P/A lbs
Col. 2051
Col. 2052
Col. 2047
Col. 2048
Unit Deformation ins
0
.001
.002



67
Plate 32
Load Deformation
Curves
40000
30000
20000
10000
P/A lbs
Col. 49
Col. 2044
Col. 2045
Col. 2046
Unit Deformation ins.
0
.001
.002
U. OF I. S. S. FORM 3

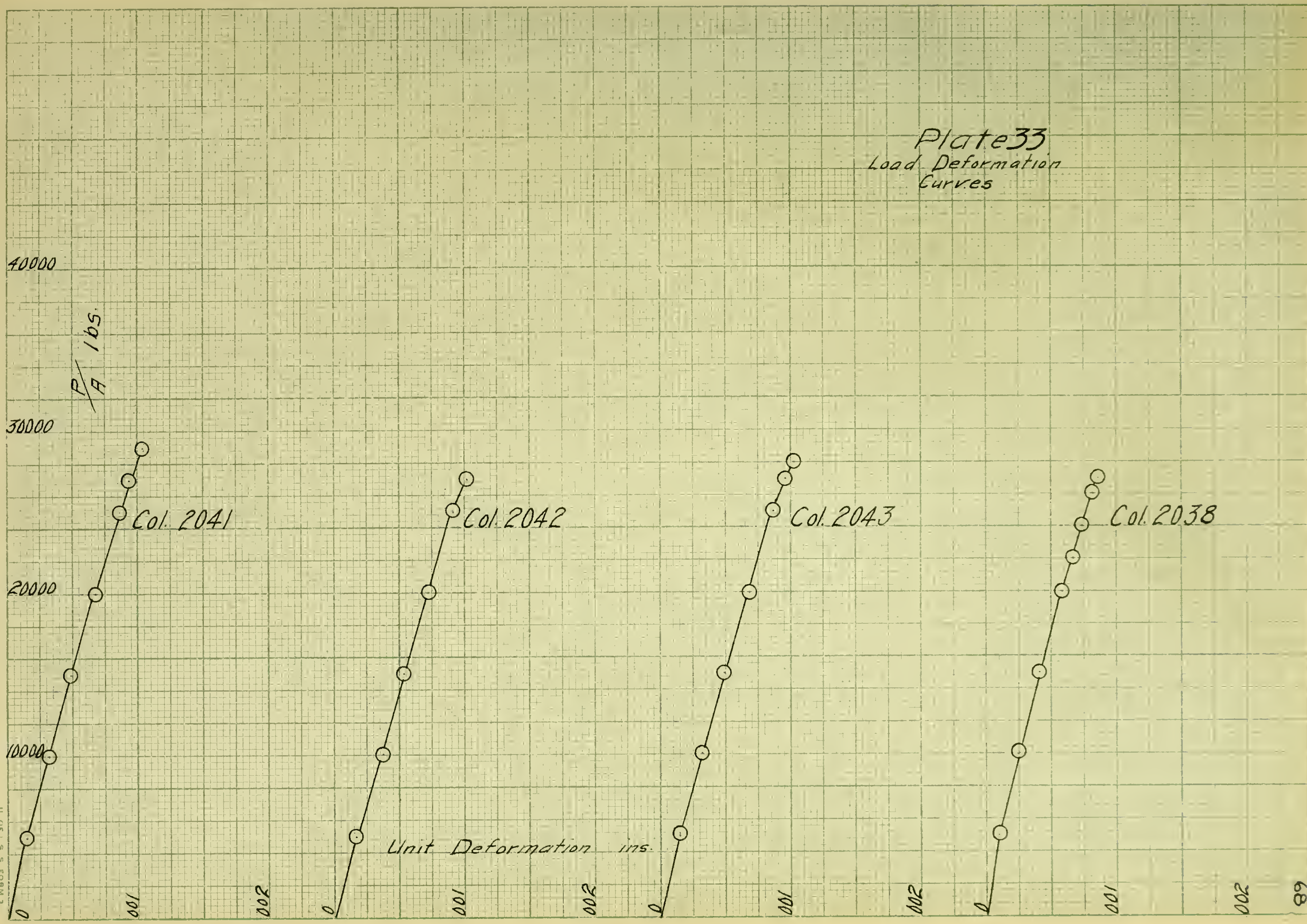


Plate 33
Load Deformation
Curves
P/A lbs.
40000
30000
20000
10000
0
001
002
Col. 2041
Col. 2042
Col. 2043
Col. 2038
Unit Deformation ins.

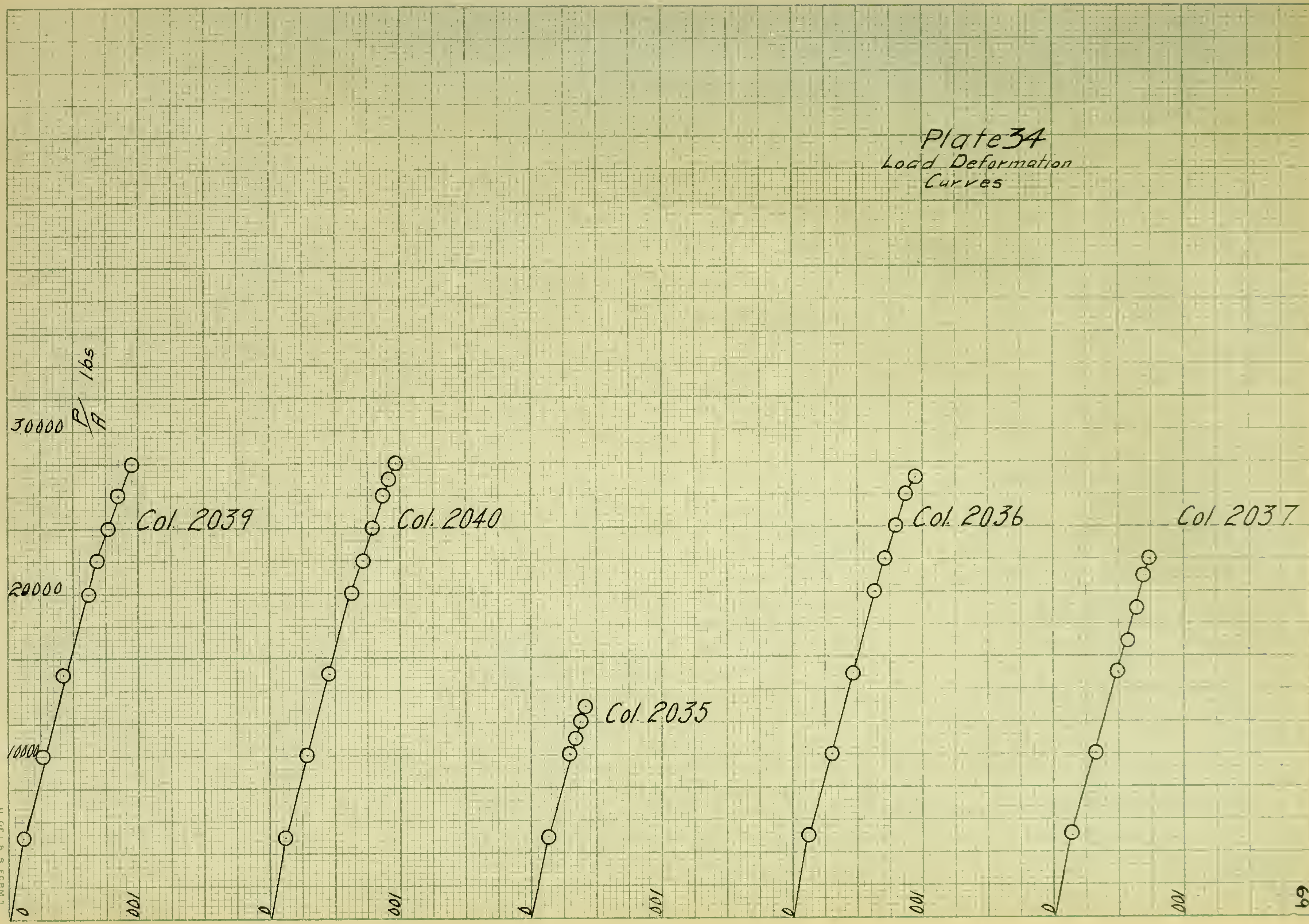


Plate 34
Load Deformation
Curves
30000 P/A lbs
20000
10000
0
100
Col. 2039
Col. 2040
Col. 2035
Col. 2036
Col. 2037
69

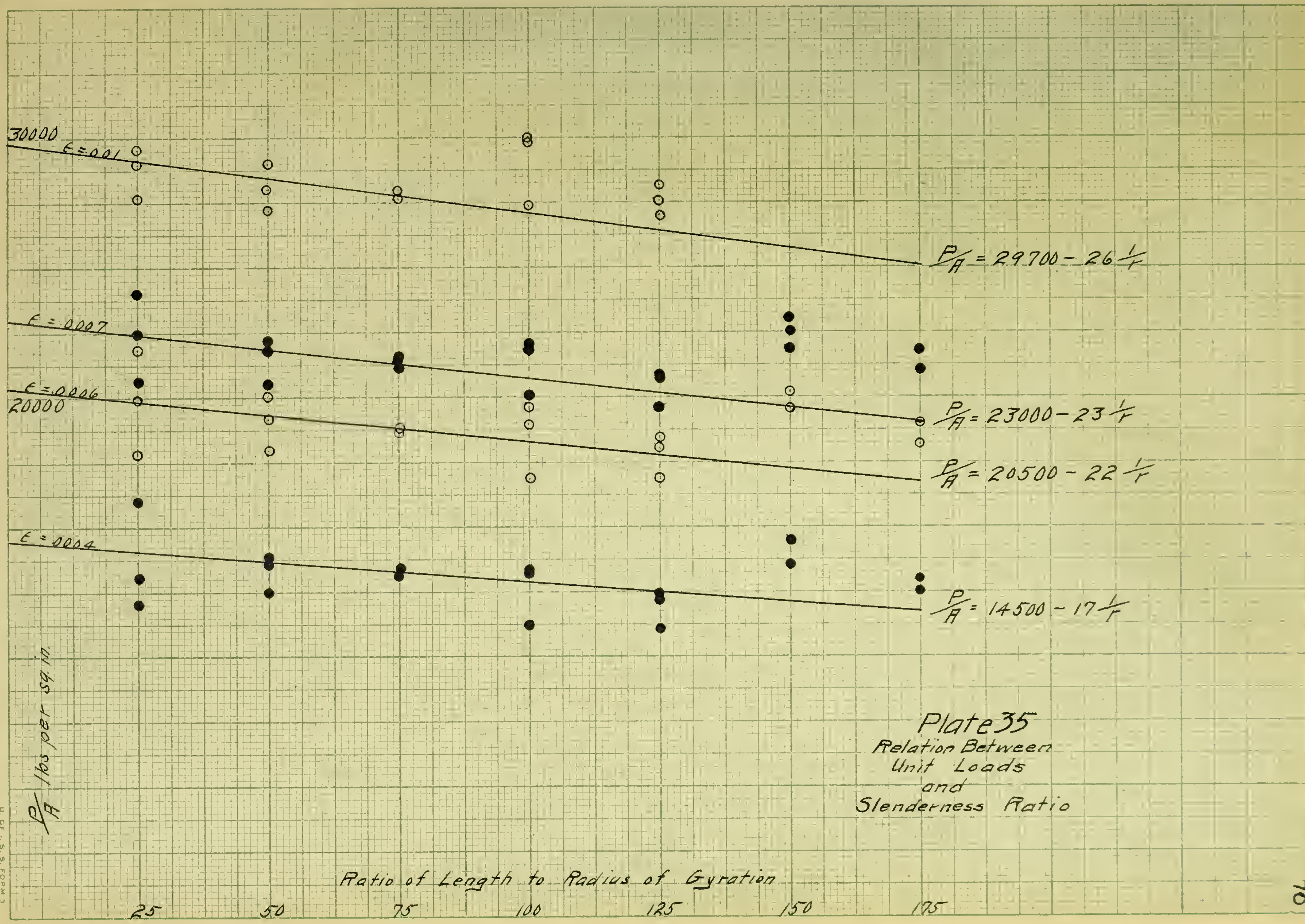
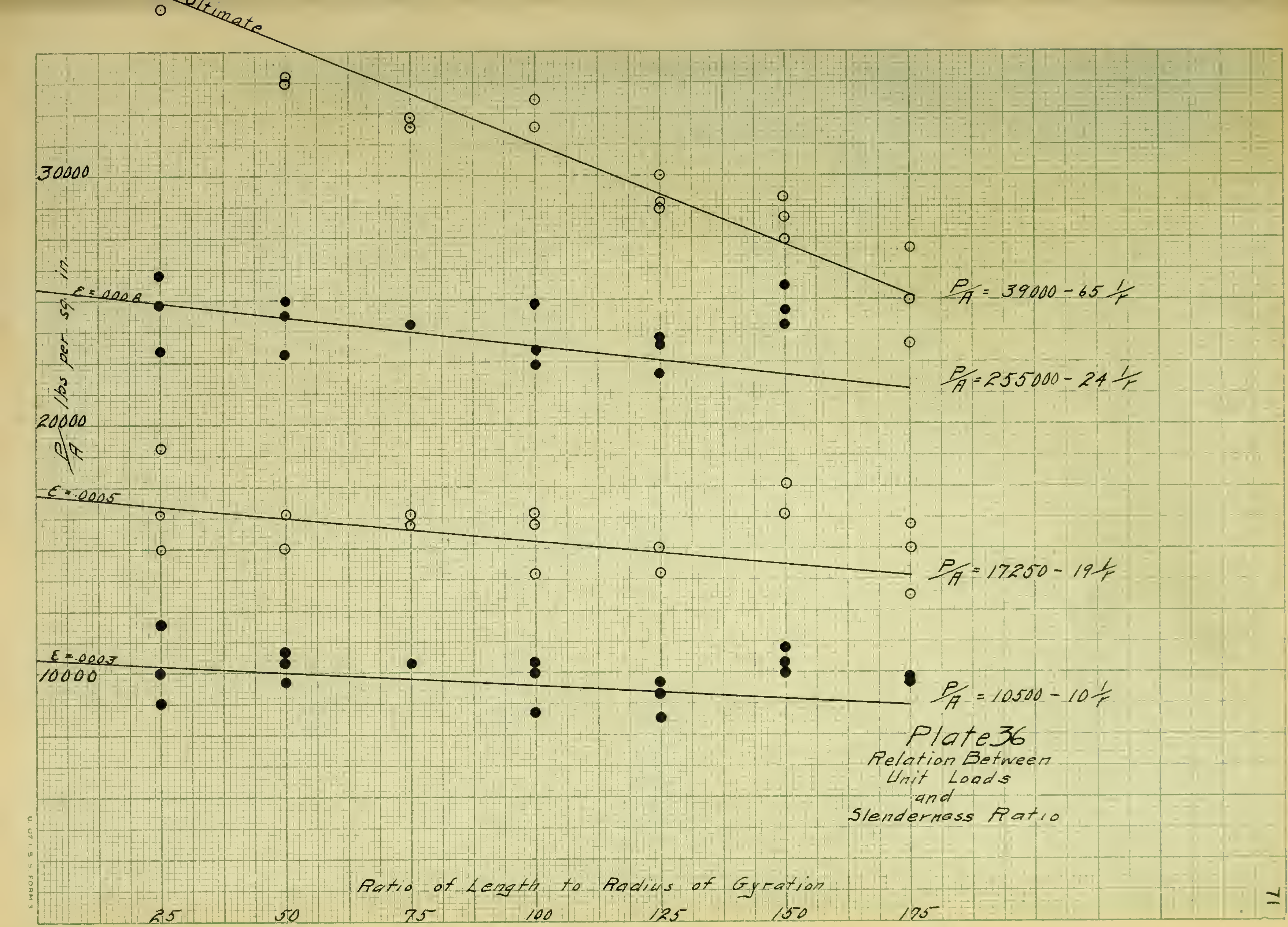
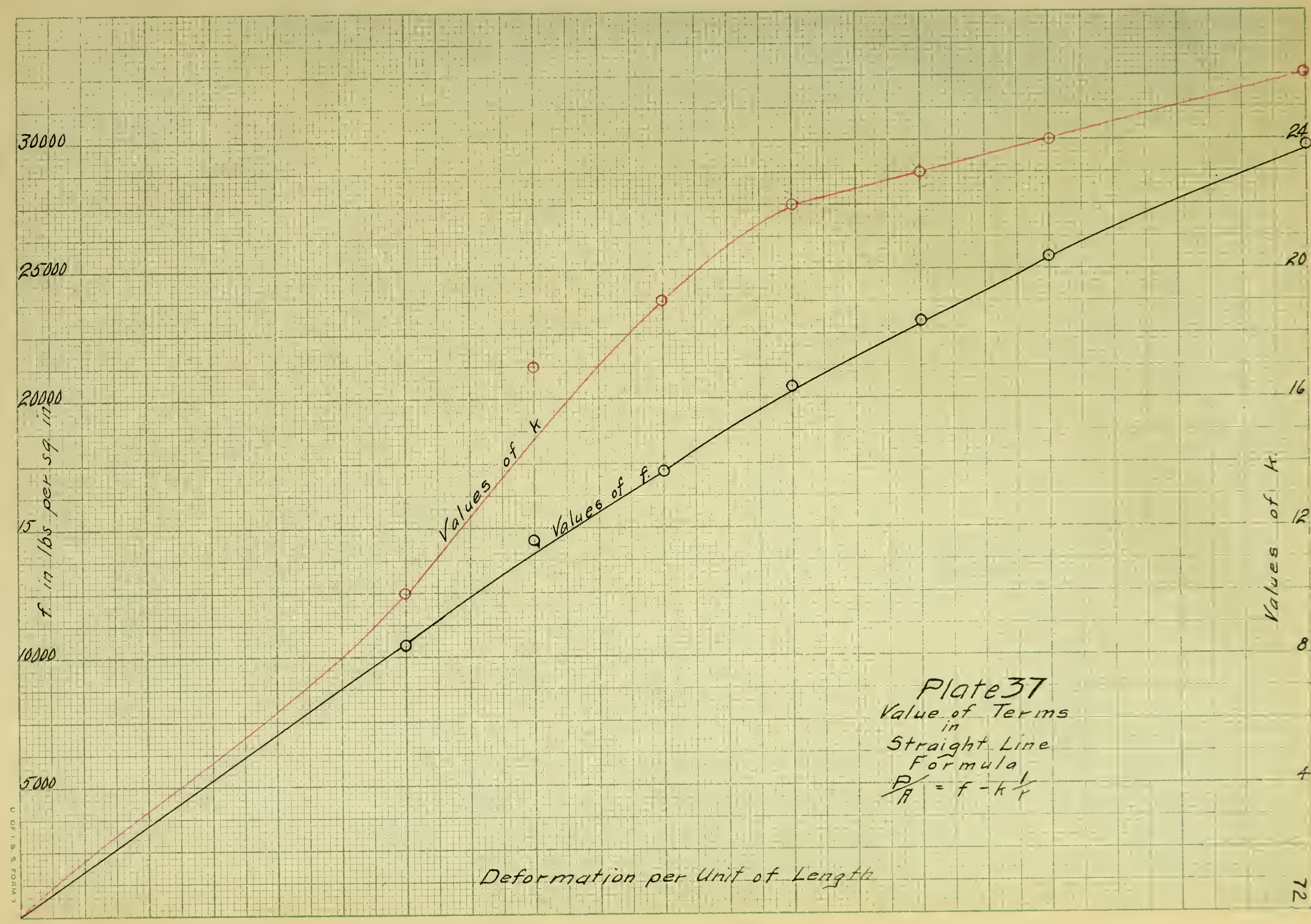


Plate 35
Relation Between Unit Loads and Slenderness Ratio



Ultimate
30000
20000
10000
P/A lbs per sq. in.
ε = .0008
ε = .0005
ε = .0003
P/A = 39000 - 65 l/r
P/A = 255000 - 24 l/r
P/A = 17250 - 19 l/r
P/A = 10500 - 10 l/r
Plate 36
Relation Between
Unit Loads
and
Slenderness Ratio
Ratio of Length to Radius of Gyration
25
50
75
100
125
150
175


30000
25000
20000
15
10000
5000
f in lbs per sq in
24
20
16
12
8
4
Values of k.
Values of k
Values of f.
Deformation per Unit of Length
Plate 37
Value of Terms
in
Straight Line
Formula
P/A = f - k l/r
72
U OF I. S. S. FORM 3