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Chiral soliton model vs. pentaquark structure for Θ(1540)

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Abstract. The exotic baryon Θ^+(1540 MeV) is visualized as an expected (iso) rotational excitation in the chiral soliton model. It is also argued as a pentaquark baryon state in a constituent quark model with strong diquark correlations. I contrast these two points of view; observe the similarities and differences between the two pictures. Collective excitation, the characteristic of chiral soliton model, points toward small mixing of representations in the wake of $SU(3)$ breaking. In contrast, constituent quark models prefer near 'ideal' mixing, similar to $\omega - \phi$ mixing.

Keywords. Chiral soliton model; pentaquarks; QCD; large N.

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1. Introduction

An exotic baryon Θ^+(1540 MeV) with the quantum numbers of K^+n has been observed as a very narrow width state by several groups [1]. It has hypercharge $Y = 2$, the third component of isospin $I_3 = 0$. Since such a state has not been seen so far in the K^+p channel, $I = 1$ is ruled out and so Θ^+ is an isosinglet. While this is yet to be confirmed by some other experimental groups [2] that do not see evidence for a narrow state so far, there is a consensus that there is enough evidence to warrant its inclusion in the 2004 edition of *Particle Data Book* [3]. The minimal $SU(3)$ assignment for such a state is at the top $(Y = 2, I = 0)$ of $\{\overline{10}\}_F$ representation with $Y = 1$, $I = \frac{1}{2}$ (N^0, N^+), $Y = 0$, $I = 1$ $(\Sigma^-, \Sigma^0, \Sigma^+)$ and $Y = -1$, $I = \frac{3}{2}$ $(\Xi^{--}, \Xi^-, \Xi^0, \Xi^+)$ as other members of the family.

The term exotic refers to the fact that such a state is not realized as the usual three-quark composite, since positive strangeness for the baryon calls for an $\overline{s}$ quark in it. Minimal quark configuration is $udud\overline{s}$. Even though the spin and parity of Θ^+(1540) are yet to be determined experimentally, most of the theoretical analysis has carried a general prejudice that it is the $J^P = \frac{1}{2}^+$ state.

The static (low-energy domain) properties of the baryons are not easily derived from the underlying QCD on account of the non-perturbative features of the theory. It becomes necessary to look for other models that are inspired by QCD to throw further light on the structure and properties of hadrons (mesons and baryons) that are indeed color singlet composites of quarks and antiquarks. Even though the color degrees of freedom are hidden, it is useful to talk about the quark, anti-quark and gluon content of hadrons as revealed to an external electromagnetic or weak probe in a deep inelastic scattering by leptons ($e^{\pm}, \mu^{\pm}, \nu, \overline{\nu}$) or photons. These hadron structure functions (or more correctly their evolution as a function of resolution scale) are accessible to perturbative QCD. For other non-perturbative properties, one resorts to study QCD either on a lattice or use other effective theories, presumably derivable from QCD. Chiral Lagrangian dynamics is one such formalism in which QCD is seen to express its global (flavor) symmetries through the pseudoscalar meson degrees of freedom in the large $N_{\rm c}$ (where $N_{\rm c}$ is the number of colors) limit. Chiral Lagrangian with the octet of pseudoscalar mesons as primary fields admits a solitonic mode (skyrmion) [4], which provides the baryon sector. While the ground state in this sector is an $SU(3)$ octet of baryons, of which the nucleon is the $Y = 1$, $I = \frac{1}{2}$ member, other excited states are rotational (in ordinary and internal $SU(3)$ flavor space) excitations. Quantization in the collective coordinates associated with the skyrmion solution gives the spectrum of baryonic states.

It is possible to show that $\{\overline{10}\}_F$ baryons, of which $\Theta(1540)$ is a member as the next rotational excitation after the ground state $\{8\}$, which has the nucleon and the well-known $\{10\}$ representation of which Δ, the isospin quartet at 1232 MeV, is a prominent member. Indeed, Diakanov *et al* [5] predicted the mass and the narrow width of the observed Θ^+ on the basis of the chiral soliton model. There is now a vast literature accumulated on the subject treating Θ^+ in terms of the chiral soliton model on the one hand [6] and in terms of a constituent pentaquark model (with special correlations) on the other [7]. While both models can account for the presence of $\{\overline{10}\}_F$ states, they differ on what else is expected and what may be the consequence of $SU(3)_F$ symmetry breaking. We provide the similarities and contrasts of both points of view. There are also efforts to find pentaquark states in lattice QCD [8], which at the moment remains inconclusive.

2. Chiral soliton model

Effective Lagrangian embodies the chiral symmetry and is a function of $U(x)$, a unitary 3×3 matrix and $\partial_\mu U(x)$. The pseudoscalar octet of mesons are expressed through $U(x) \equiv \exp(\frac{i}{F_\pi}\lambda^a \phi^a(x))$, $a = 1, 2, ..., 8$; λ^a are Gell Mann matrices and x denotes $(\vec{x}, t)$. F_π is the pion decay constant and provides a scale for the masses in the theory. The theory admits finite energy static solutions (for the classical equations of motion), that has the form:

$$U(x) = U_0(\vec{x}) = \begin{pmatrix} \exp if(r)\vec{\tau}\cdot\hat{x} & 0 \\ 0 & 1 \end{pmatrix}.$$

The constraint that

$$f(r) \to \begin{matrix} \pi, & \text{when } r \to 0 \\ 0, & \text{when } r \to \infty \end{matrix},$$

ensures that $U(x)$ is both well-defined at the origin and can lead to finite energy configuration. Precise form of $f(r)$ can be obtained numerically from the radial equation of motion. Quantization is then carried out by identifying the collective coordinates, a set of parameters that label the transformations that will leave the classical solution invariant. Apart from the centre of mass of the skyrmion, the translation of which will indeed yield the overall momentum of the state and hence the kinetic energy, rotations in space as well as internal space are the other collective coordinates. To get their dynamics, it is convenient to identify the collective coordinates $\mathcal{A}$ by defining

$$U(\vec{x},t) = \mathcal{A}(t) U_0(\vec{x}) \mathcal{A}^{-1}(t); \quad \mathcal{A} \in SU(3). \tag{2.1}$$

Notice that the solution is left invariant under $\mathcal{A} \to L\mathcal{A}$; $L \in SU(3)$ denoting a $SU(3)$ flavor transformation and under $\mathcal{A} \to \mathcal{A}R$; $R \in SU(2)$, an element of rotation in space. $\mathcal{A}(t)$ constitute the relevant collective coordinates that embody the rotational and iso-rotational degrees of freedom for the skyrmion. The wave function for the baryons in various allowed irreducible representations of $SU(3)$ are given by the equivalent of Wigner D-functions $\mathcal{D}^{\{R\}}_{\alpha,\beta}(\mathcal{A})$, where $\{R\}$ stands for the $SU(3)_F$ representation of the state; $\alpha = (I, I_3; Y)$ and $\beta = (I' = J, I'_3 = J_3; Y' = 1)$ respectively denote the iso-rotational (isospin I, I_3 and hypercharge Y) and rotational (angular momentum J, J_3) state of the baryon [9,10]. The relevant Hamiltonian has the form:

$$H = M_0 + \frac{1}{2\mathcal{I}_1} \sum_{a=1}^{3} J_a^2 + \frac{1}{2\mathcal{I}_2} \sum_{a=4}^{7} J_a^2 + (1/N_\mathrm{c} \text{ corrections}), \tag{2.2}$$

where $\mathcal{I}_{1,2}$ are 'moments of inertia' for the rigid rotator model for the baryon. The interlocking of the spin and isospin is an essential feature of the soliton sector and generates a constraint on the states that is further made precise from the presence of the Wess–Zumino term in the effective Lagrangian for $SU(n_F)$, $n_F > 3$. For the $SU(2)$ skyrmion this is reflected by the fact that the allowed baryons have their $I = J$. For $SU(3)$, the constraint admits in the spectrum only those $SU(3)$ representations that have $Y = 1$ member and the spin J for the set will be the same as the isospin values of all the $Y = 1$ members of the representation. For $n_F > 3$, the angular momentum of the state assumes value(s) of isospin of the $Y = 1$, $SU(n_F - 2)$ singlet(s) [11].

Since every $SU(3)$ unitary irreducible representation is given by a pair of indices (p, q) (the wave function has p indices that transform like $\{3\}$ and q indices like $\{\overline{3}\}$) and the second Casimir operator $C_2(p,q) \equiv \sum_{a=1}^{8} J_a^2 = \frac{1}{3}(p^2 + q^2 + pq + 3(p+q))$ and $J_8 = -\frac{\sqrt{3}}{2} Y$, we can read off the mass spectrum (Wess–Zumino constraint implies $|p - q| = 0$ modulo 3) as:

$$E^J(p,q) = M_0 + \frac{1}{2\mathcal{I}_2} C_2(p,q) + \left(\frac{1}{2\mathcal{I}_1} - \frac{1}{2\mathcal{I}_2} \right) J(J+1) - \frac{3}{8\mathcal{I}_2}.$$

If $\mathcal{I}_1 > \mathcal{I}_2$, we will have the observed sequence of states; ground state $\{8\}_{1/2}$ followed by $\{10\}_{3/2}$ and the just discovered $\{\overline{10}\}_{1/2}$. From the central or average values of the masses in these multiplets, 1115 MeV for $\{8\}_{1/2}$, 1382 MeV for $\{10\}_{3/2}$ and 1755 MeV for $\{\overline{10}\}_{1/2}$, we may determine values of $\mathcal{I}_{1,2}$ and get the sequence of further excitations to be at (in MeV) 1784 $\{27\}_{3/2}$, 1967 $\{35\}_{5/2}$, 2155 $\{27\}_{1/2}$, 2570 $\{64\}_{5/2}$, 2588 $\{35\}_{3/2}$, 2707 $\{81\}_{7/2}$, 2959 $\{\overline{35}\}_{1/2}$ and so on.

An important observation made on the basis of the chiral soliton picture has to do with the narrow width of the baryons in $\{\overline{10}\}_{1/2}$. Diakanov *et al* [5] attribute this to the next-to-leading order (in $1/N_c$) correction for the meson baryon couplings. While the leading order for all transitions in the soliton sector is characterized by G_0, there are two further terms with strengths G_1 and G_2 in the next order. For example, $B_8B_8M_8$ has two distinct $SU(3)$ symmetric couplings D and F, usually given instead in terms of $g_{\pi NN}(= D+F)$ and $\alpha(= D/(D+F))$ that are expressed through

$$g_{\pi NN} = \frac{7}{10}\left(G_0 + \frac{1}{2}G_1 + \frac{1}{14}G_2\right), \tag{2.3}$$

$$\alpha = \frac{9}{14}\frac{G_0 + \frac{1}{2}G_1 - \frac{1}{6}G_2}{G_0 + \frac{1}{2}G_1 + G_2}. \tag{2.4}$$

In view of the fact that the experimental value of $\alpha = 0.65$ is very close to 9/14, we expect G_2 to be small and negligible.

The decouplet baryon decay couplings $B_{10}B_8M_8$ are characterized by the factor $G_0 + \frac{1}{2}G_1$ and the antidecouplet decays $B_{\overline{10}}B_8M_8$ are governed by the term $G_0 - G_1 - \frac{1}{2}G_2$. Diakanov estimates G_1/G_0 to be in the range 0.4–0.6 with the result $G_{\overline{10}}/G_{10} \sim 1/3$–$1/5$. Indeed, with a bit of adjustment in the value of G_1 and G_0, considerable suppression for the width of antidecouplet baryons can be realized.

When $SU(3)$ is explicitly broken there are two further consequences. There will be splitting within each $SU(3)$ representation resulting in the spectrum governed by Gell Mann Okubo mass relations. For octet baryons:

$$M_8(I,Y) = M_8^0 - bY + c(I(I+1) - Y^2/4) \tag{2.5}$$

and equal spacing of levels for both $\{10\}$ and $\{\overline{10}\}$ states:

$$M_{10}(Y) = M_{10}^0 - aY, \tag{2.6}$$

$$M_{\overline{10}}(Y) = M_{\overline{10}}^0 - a'Y. \tag{2.7}$$

A second related consequence of symmetry breaking is the mixing of various states with the same combination of (I, Y) states among different $SU(3)$ representations. In particular we expect that N_8 and $N_{\overline{10}}$ will mix to yield $N(939)$ and some N^* state. Similarly Σ_8 and $\Sigma_{\overline{10}}$ will mix. These mixings will induce a shift in masses from the above octet symmetry breaking relations for the mass eigenstates as well as cause suppression or enhancement of the decay amplitudes.

A. *Mixing angle from the spectrum*

If we identify N_8, $\Lambda(1115)$, Σ_8, $\Xi(1318)$ as octet states and $\Theta^+(1540)$, $N_{\overline{10}}$, $\Sigma_{\overline{10}}$ and $\Xi_{3/2}(1862)$ as antidecouplet states and assign $N(939)$ and $N(1710)$ as nucleon-like mass eigenstates and $\Sigma(1193)$ and $\Sigma(1880)$ as Σ-like states, we may find mixing angles θ as follows:

$$|N(939)\rangle = \cos\theta_N |N_8\rangle - \sin\theta_N |N_{\overline{10}}\rangle \tag{2.8}$$

$$|N(1710)\rangle = \sin\theta_N |N_8\rangle + \cos\theta_N |N_{\overline{10}}\rangle \tag{2.9}$$

and

$$|\Sigma(1193)\rangle = \cos\theta_\Sigma |\Sigma_8\rangle - \sin\theta_\Sigma |\Sigma_{\overline{10}}\rangle \tag{2.10}$$

$$|\Sigma(1880)\rangle = \sin\theta_\Sigma |\Sigma_8\rangle + \cos\theta_\Sigma |\Sigma_{\overline{10}}\rangle. \tag{2.11}$$

It is easily obtained that

$$939 + 1710 = \langle N_8|H|N_8\rangle + \langle N_{\overline{10}}|H|N_{\overline{10}}\rangle \tag{2.12}$$
$$= M_8^0 - b + \frac{c}{2} + M_{\overline{10}}^0 - a' \tag{2.13}$$
$$= 1115 + 1755 - a' - b + \frac{c}{2}. \tag{2.14}$$

From the masses of Θ^+ and $\Xi_{3/2}$, we get $a' = 107$ MeV, yielding $b - \frac{c}{2} = 114$ MeV. Using the masses of $\Lambda(1115)$ and $\Xi(1318)$ we find $b + \frac{c}{2} = 203$ MeV. We further have

$$\cos 2\theta_N (1710 - 939) = \langle N_{\overline{10}}|H|N_{\overline{10}}\rangle - \langle N_8|H|N_8\rangle \tag{2.15}$$
$$= M_{\overline{10}} - M_8 - a' + b - \frac{c}{2} \tag{2.16}$$
$$= 647 \text{ MeV}. \tag{2.17}$$

This implies that $\cos 2\theta_N = 0.84$, which translates into $\tan^2\theta_N = 0.087$.

If we denote $\langle N_8|H|N_{\overline{10}}\rangle = \langle \Sigma_8|H|\Sigma_{\overline{10}}\rangle = \delta$, we will have

$$2\delta = \left(M_{10} - M_8 - a + \left(b - \frac{c}{2}\right)\right)\tan\theta_N$$

yielding a value $\delta = 220$ MeV as signifying the extent of representation mixing.

We may carry out a similar analysis of the mass mixing of the Σ-sector to find

$$1880 + 1193 = 1755 + 1115 + 2c \tag{2.18}$$

$$\cos 2\theta_\Sigma (1880 - 1193) = 1755 - 1115 - 2c. \tag{2.19}$$

We obtain $\cos 2\theta_\Sigma = 0.63$ that corresponds to the representation mixing value of $\delta = 264$ MeV instead.

As a result of the mixing the expectation value of the N and Σ in the octet representation will be $M_{N_8} = M_8 - b + \frac{c}{2} = 1001$ MeV and $M_{\Sigma_8} = M_8 + 2c = 1293$ MeV. Gell Mann Okubo relation for the octet will imply

$$2M_{N_8} - M_{\Sigma_8} = 3M_\Lambda - 2M_\Xi = 709 \text{ MeV}.$$

Before mixing is applied the left-hand side of the expression will have $2 \times 939 - 1193 = 645$ MeV. When mixing is taken into account it is instead $2 \times 1001 - 1293 = 709$ MeV, much more precise. Indeed the nucleon (and Σ) needs $\{\overline{10}\}$ admixture to fit GMO formula better [11a]!

B. *Mixing angle from the decays*

The mixing angle can also be deduced by studying the decay width of the states. The couplings are assumed to be preserving $SU(3)$, but in the computation of the width, the phase space is computed with the actual [$SU(3)$ broken] mass values. We have already observed that while for $\{10\}_{3/2^+} \to \{8\}_{1/2^+} + \{8\}_{0^-}$, the rates are proportional to $(G_0 + \frac{1}{2}G_1)^2$, the transition rates for $\{\overline{10}\}_{1/2^+} \to \{8\}_{1/2^+} + \{8\}_{0^-}$ are governed by the factor $(G_0 - G_1 - \frac{1}{2}G_2)^2$. The partial decay widths for the antidecouplet baryons are given by

$$\Gamma(\Theta^+ \to KN) = \frac{3}{2\pi} \frac{(G_0 - G_1 - \frac{1}{2}G_2)^2}{(M_N + M_\Theta)^2} \frac{M_N}{M_\Theta} p^3_{\Theta\to KN} \times \left| \begin{pmatrix} 8 & 8 & \overline{10} \\ K & N & \Theta \end{pmatrix} \right|^2 \tag{2.20}$$

$$\Gamma(\Xi^{--} \to \pi^-\Xi^-) = \frac{3}{2\pi} \frac{(G_0 - G_1 - \frac{1}{2}G_2)^2}{(M_{\Xi^{--}} + M_{\Xi^-})^2} \frac{M_{\Xi^-}}{M_{\Xi^{--}}} p^3_{\Xi^{--}\to\pi^-\Xi^-} \times \left| \begin{pmatrix} 8 & 8 & \overline{10} \\ \pi^- & \Xi^- & \Xi^{--} \end{pmatrix} \right|^2 \tag{2.21}$$

with

$$p_{B_1\to B_2 M} = \sqrt{(M^2_{B_1} - (M_{B_2} + M_M)^2)(M^2_{B_1} - (M_{B_2} - M_M)^2)}/2M_{B_1}.$$

Notice that there is no $SU(3)$ symmetric coupling that will permit $N_{\overline{10}} \to \Delta + \pi$ (because $\{\overline{10}\}\{10\}\{8\}$ coupling does not exist). However, since $N(1710) \to \Delta(1232)\pi$ has a partial width of about 5 MeV, this is a clear indication that this state cannot be a pure antidecouplet. It is the octet part of the state that is responsible for the $\Delta\pi$ decay mode. Comparison of the partial width for $N(1710) \to \Delta\pi$ with that of $\Delta \to N\pi$ will give us the measure of the admixture. Since $|N(1710)\rangle = \cos\theta_N |N_{\overline{10}}\rangle + \sin\theta_N |N_8\rangle$, we get

$$\Gamma(N(1710) \to \Delta\pi) = \sin^2\theta_N \frac{3}{2\pi} \frac{(G_0 + \frac{1}{2}G_1)^2}{(1710 + 1232)^2} \frac{1710}{1232} p^3_{N^*\to\Delta\pi} \frac{4}{5} \sim 5 \text{ MeV}.$$

Comparing this with

$$\Gamma(\Delta(1232) \to N\pi) = \cos^2\theta_N \frac{3}{2\pi}\frac{(G_0 + \frac{1}{2}G_1)^2}{(939+1232)^2}\frac{1232}{939}p^3_{\Delta\to N\pi}\frac{1}{5}$$
$$= 120 \text{ MeV},$$

we find that $\tan^2\theta_N = 0.0035$. Note that a very small admixture is all that is needed to get a 5 MeV partial decay width for $N(1710) \to \Delta\pi$.

We conclude that mixing angle from both mass spectrum and decay data are reasonably small. The decay amplitudes appear to give a much smaller value of mixing parameter than the data on the mass spectrum of states.

It will be useful to contrast these results with other analysis. Pakvasa and Suzuki [12], who assumed the mixing octet to consist of $N(1440)$, the old Roper resonance and $\Sigma(1660)$ instead of $N(939)$ and $\Sigma(1193)$ along with appropriate excited states for both Λ and Ξ, find an even larger discrepancy in view of their choice for states. Since they use Roper resonance in place of the nucleon of the ground state baryon octet, they get a substantial mixing from the mass spectra and therefore cannot reconcile with a much smaller mixing angle that is indicated from the decay amplitudes. Weigel [13], who has made an extensive study of the spectrum of skyrmion states, identifies such an octet state with the radially excited skyrmion. It is not clear to us why $N_{\overline{10}}$ would prefer to mix with the radially excited state over the ground state. Perhaps we need to compute mixing with all the three levels. However in such an analysis, in the leading order, we expect the radially excited state to be orthogonal to the ground state and hence should not be giving any qualitatively different answers.

Mixing of states may arise also with higher iso-rotational levels in the skyrmion sector, that we have listed. Analysis have been carried out by including $\{27\}_{1/2}$ and $\{\overline{35}\}_{1/2}$ (which are expected to be much and very much heavier respectively) for $J = \frac{1}{2}$ states and $\{27\}_{3/2}$, $\{35\}_{3/2}$ states with $\{10\}_{3/2}$ baryons for $J = \frac{3}{2}$ levels.

We do not have much experimental support for higher states of baryons as of now and so such an analysis is of academic interest only. The basic premise that $\{\overline{10}\}$ states have a small admixture of the ground state octet baryons appears to be born out by experimental data so far available. While mixing angles in the most basic interpretation are indeed small, the decay data appear to suggest much smaller mixing. We will see that an alternative explanation in terms of constituent quark model that will keep track of the number of strange quarks in a state will imply substantial mixing of representations, when $SU(3)_F$ is broken.

3. Constituent quark model

In the constituent quark model framework, the hadrons are considered built using dressed (valence) quarks much the same way nuclei are built using nucleons and model effective interaction among dressed quarks. In a naive uncorrelated quark model for pentaquark baryons, we expect the ground state to be a $J^P = 1/2^-$ state with flavor quantum number to be both $\{\overline{10}\}_F$ and $\{8\}_F$. They expect to be accompanied by states with $J^P = 3/2^-$ with roughly the same difference in mass as between N and Δ. Jaffe and Wilczek [14] argue that there is a lower energy state with positive parity, exploiting a strong correlation among quarks that picks diquark pairs that are antisymmetric in color, spin and flavor. Call these condensates $\mathbf{Q}$ that are $\{\overline{3}\}_C$ and $\{\overline{3}\}_F$ scalars. Pentaquark baryons are then $\mathbf{QQ}\bar{q}$ composites. For two such diquarks, along with an antiquark, to form a color singlet they must be antisymmetric in color. In order to form a $\{\overline{10}\}_F$, the diquark pair have to be in a flavor symmetric $\{\overline{6}\}_F$, which in turn implies orbital excitation and $l=1$. When combined with an antiquark, we have the pentaquark states in $\{\overline{10}\}_F$ and $\{8\}_F$, expected to be degenerate, both having $J^P = 1/2^+$ and $3/2^+$, with the spin orbit coupling providing the splitting as found in the separation between N and Δ. The nucleon-like states and the Σ-like states of $\{\overline{10}\}$ and $\{8\}$, when subjected to $SU(3)$ breaking are expected to mix *ideally*, so that the lighter member has no strange quarks and the higher has a $s\overline{s}$ pair for the nucleon-like member. This is similar to the vector meson nonet with $\omega - \phi$ mixing, where $|\omega\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle)$ and $|\phi\rangle = |s\overline{s}\rangle$. Ideally mixed nucleon-like states then are:

$$|N_{ud}\rangle = \sqrt{2/3}\ |N_8\rangle + \sqrt{1/3}\ |N_{\overline{10}}\rangle, \tag{3.1}$$

$$|N_s\rangle = -\sqrt{1/3}\ |N_8\rangle + \sqrt{2/3}\ |N_{\overline{10}}\rangle, \tag{3.2}$$

with $|p_{ud}\rangle = |udud\bar{d}\rangle$, $|n_{ud}\rangle = |udud\overline{u}\rangle$, $|p_s\rangle = |udus\overline{s}\rangle$ and $|n_s\rangle = |udsd\overline{s}\rangle$. For ideal mixing $\tan\theta = \sqrt{2}$.

Similarly the Σ-like states are, indeed

$$|\Sigma_{ud}\rangle = \sqrt{2/3}\ |\Sigma_8\rangle + \sqrt{1/3}\ |\Sigma_{\overline{10}}\rangle, \tag{3.3}$$

$$|\Sigma_s\rangle = -\sqrt{1/3}\ |\Sigma_8\rangle + \sqrt{2/3}\ |\Sigma_{\overline{10}}\rangle, \tag{3.4}$$

with $|\Sigma^+_{ud}\rangle = |udus\bar{d}\rangle$, $|\Sigma^+_s\rangle = |usus\overline{s}\rangle$; $|\Sigma^0_{ud}\rangle = \frac{1}{\sqrt{2}}|udus\overline{u}\rangle + \frac{1}{\sqrt{2}}|udds\bar{d}\rangle$, $|\Sigma^0_s\rangle = |usds\overline{s}\rangle$; and $|\Sigma^-_{ud}\rangle = |udds\overline{u}\rangle$, $|\Sigma^-_s\rangle = |dsds\overline{s}\rangle$.

Jaffe and Wilczek expect the mass spectrum to be governed by the number of s quarks and $\overline{s}$ antiquark the baryon has. If $H = M_0 + (n_s + n_{\overline{s}})\alpha + n_s\beta$, it will imply ideal mixing of $\{\overline{10}\}$ and $\{8\}$. We will then have the sequence of pentaquark baryons with $N^*_{ud} < \Theta^+ < \Sigma^*_{ud}$, $\Lambda^* < N^*_s < \Xi^*_{1/2}$, $\Xi^*_{3/2} < \Sigma^*_s$. The states identified by them to fit this sequence, apart from $\Theta^+(1540)$, are $N^*_{ud}(1440)$, $N^*_s(1710)$, $\Lambda^*(1600)$, $\Sigma^*_s(1880)$. They agree with the above sequence if we accept their analysis of the systematics for exotic Ξ decays, where they argue that there are nearly degenerate $\Xi_{1/2}$ and $\Xi_{3/2}$ in the 1855–1860 mass region [15].

This large mixing angle is incompatible with the information from decay data. In particular, $\Gamma(N(1710) \to \Delta\pi)$ will turn out to be absurdly large, if $\tan\theta = \sqrt{2}$. Further the narrow width for Θ^+ and $\Xi^{--}_{3/2}$ will be in sharp conflict with the expected large width for N_{ud} and N_s. While the broad Roper resonance fits the bill for N_{ud}, it is not clear whether there is another broad nucleon-like state in that region instead, if $N(1710)$ is to be discounted as the candidate for N^*_s.

Equally dramatic is the prediction that both $\{8\}_F$ and $\{\overline{10}\}_F$ will be nearly degenerate before $SU(3)_F$ is broken. This is indeed the argument for interpreting that the Ξ resonances cover both $I = 1/2$ and $I = 3/2$ components in the same region.

Comparing with the chiral soliton model, the pentaquark model predicts in addition to $J^P = 1/2^+$ state, $J^P = 3/2^+$ states as well with an expected mass difference, which should be of the same order as $(M_\Delta - M_N)$. We expect many more baryonic states than what has been reported. We observe that there is no interlocking of spin and isospin that was characteristic in the soliton sector.

We need more definitive experimental evidence before we are able to rule in favor of either chiral soliton model or any of the specific constrained constituent quark models. There are other variants of Jaffe–Wilczek diquark correlations: for example the one by Karliner and Lipkin [16] postulates two separate clusters one made up of the same diquark and the other $(qq\bar{q})$ cluster in which the two quarks are in color symmetric $\{6\}_C$ state. The observed narrow width for Θ^+ is attributed to the fact that the clusters are kept apart due to the angular momentum barrier.

4. Comparisons and conclusion

Both in chiral model as well as in constituent pentaquark models with diquark correlations folded in, the spin parity assignment favored is $J^P = \frac{1}{2}^+$. However, while all rotational excitations in chiral soliton sector are necessarily of positive parity, there is no reason to exclude negative parity baryons in the pentaquark picture. The additional states in the CSM are attributed to radial excitations, all of which will have positive parity, and the same $SU(3)_F$ quantum numbers. In contrast, in CPQM we expect a more or less degenerate octet of states in addition to $\{\overline{10}\}$ baryons, both of spin 1/2 and 3/2 variety and further (perhaps a bit more massive) negative parity states. More detailed spectroscopy in this mass region can clarify whether such additional states are present.

In the CSM, nucleon-like states in the ground state octet and the exotic antidecouplet will indeed mix. These mixing angles remain small and generally found to be so with our assignment, which parallels Diakanov *et al*'s choice. Even so, the mixing angle as obtained from the decay widths appear further smaller than that obtained from mass spectrum. In contrast, when $SU(3)_F$ is broken in the CPQM we expect a large mixing of the nearly degenerate $\{8\}$ and $\{\overline{10}\}$ multiplets to give it the nature of ideal mixing. This will lead to a large strangeness content of one of the nucleon-like states, say $N(1710)$ and so it must have a significant branching ratio into ΛK and ΣK channels in order that Zweig rule is obeyed. Further, since it is made up of a substantial component of octet, the coupling to baryons and mesons such as $\Delta\pi$, ΛK etc. will be comparable to the strength of ΔNK coupling. This is in conflict with the observed branching ratio and small widths of this state. Admittedly more detailed analysis is called for before we can confirm $N(1710)$ as the candidate $uuds\bar{s}$ state.

Is there a deeper reason for the dramatic narrow width? We note that the degenerate octet and decouplet states arise from $(\bar{3}, \bar{6}) \oplus (\bar{6}, \bar{3})$ of the underlying $SU(3)_L \times SU(3)_R$ symmetry. In the same scheme baryon octet belongs to $(1, 8) \oplus (8, 1)$. In the limit of exact chiral symmetry (left-handed currents transforming as $(8, 1)$ and right-handed currents as $(1, 8)$) there will be no coupling between pentaquark baryons and the usual triquark octet. Thus $\Theta^+ \to NK$ decays are inhibited on account of the underlying chiral symmetry [17]. In exact chiral symmetry (limit in which pion masses vanish), we expect stable $\{\overline{10}\}$ baryon states [18].

Both viewpoints have room for many additional states; in CSM radial excitations will give further states with the same flavor quantum numbers and in CPQM many other permutations of correlated clusters are possible as well. Of course several of these features could be consequences of higher $(1/N_{\rm c})$ order and hence may not be very reliable predictions of CSM. Future experiments [19] should nail many of the predictions of both pictures.

It is satisfying to note, that the chiral soliton model, that starts from an underlying chiral symmetry has dynamical ingredients to account for its narrow width. In the CPQM, in contrast, small width is due to non-overlapping clusters on account of centrifugal barrier. Perhaps we need some ingredient that signals approximate chiral symmetry in the CPQM to reflect more similarity with the soliton picture. This calls for the possibility that we may be able to describe hybrid models that have features of both chiral soliton picture on the one hand, while being legitimately pentaquark constituent structures in terms of relevant variables. Main reason for such a prospect has to do with the feature that the narrow width is very likely a consequence of underlying approximate chiral symmetry.

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