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Adaptive control of spatially extended systems: Targeting spatiotemporal patterns and chaos

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We describe adaptive control algorithms whereby a spatially extended nonlinear system can be steered to a target state with desired spatiotemporal characteristics. Specifically we implement our control on a two-dimensional coupled map lattice, and successfully direct the system to desired targets ranging from spatiotemporal fixed points and regular spatial patterns to spatiotemporal chaos. The proposed methodology entails monitoring the local neighborhood of only one (arbitrary) site in order to regulate the entire lattice. Further, knowledge of the system's governing equations is not required. We also demonstrate the success of this method in controlling an unstable elastic array, a system of interest in engineering applications.
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The phenomenon of pattern formation can be observed in a variety of experimental situations such as lasers and chemical reactions [1]. Distributed control techniques capable of stabilizing complex patterns may shed some light on the mode of pattern formation in natural systems [2] and may also have many important practical applications in diverse contexts such as microelectromechanical systems, environmental monitors, drag reduction in fluid flows, compact data storage and improved material properties [3]. In all the above examples one requires a control mechanism that targets a regular spatiotemporal regime. On the other hand, the enhancement of spatiotemporal chaos has important practical applications in contexts as diverse as mixing flows, electronic systems, and chemical reactions, where the enhancement of chaos leads to improved performance [4], or in biological applications such as neural systems [5,6]. It is thus of considerable interest, and potential utility, to *devise control algorithms capable of achieving the desired type of spatiotemporal behavior* in such complex systems [2].

Here we describe simple and easily implementable *adaptive control* algorithms targeting desired complex spatiotemporal behavior. We explicitly show their success in targeting spatiotemporal fixed points, spatial patterns such as checkerboards and stripes, as well as in directing systems to enhanced spatiotemporal chaos [4,6]. Further, we demonstrate the success of such algorithms on an unstable elastic array studied in the context of ''smart matter'' [3].

Adaptive control algorithms have hitherto been implemented primarily on nonlinear systems with few degrees of freedom both for targeting periodic behavior [7,8] and for enhancing chaos [6]. The method applies a feedback loop in order to drive the system parameter (or parameters) to the values required so as to achieve a desired or target state. This is implemented by augmenting the evolution equation for the dynamical system,

$$\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X};\mu;t), \tag{1}$$

where $\mathbf{X} \equiv (X_1, X_2, \ldots X_N)$ are the state variables and μ is the parameter whose value determines the nature of the dynamics, by an additional equation for the evolution of the parameter itself

$$\dot{\mu} = \epsilon(\mathcal{P}^* - \mathcal{P}), \tag{2}$$

where $\mathcal{P}^*$ is the target value of some variable or property $\mathcal{P}$, and ϵ is the stiffness of control. Since the present implementation targets a desired spatiotemporal state the property $\mathcal{P}$ is chosen to reflect the spatiotemporal characteristics of the desired state. The feedback can be spatial or temporal depending on the nature of the target, a spatial feedback being effective for a spatial pattern (like squares), and either a spatial or temporal feedback proving to be effective for control to spatiotemporally periodic behavior. In addition, we choose a property $\mathcal{P}$ that can be simply defined, without the explicit knowledge of the system's equations of motion, and try to achieve control without monitoring a large number of sites. The above two features can be of considerable utility in the implementation of this control algorithm in an experimental situation.

We first demonstrate the success of our control algorithm in a two-dimensional lattice of coupled logistic maps, a system capable of exhibiting a rich variety of spatiotemporal patterns as well as spatiotemporal chaos [9]. The lattice evolves according to the equations:

$$x_{n+1}(i,j) = (1-\epsilon) f(\alpha, x_n(i,j)) + \frac{\epsilon}{4} \sum_{\mathrm{NN}} \{g[x_n(i,j)] - g[x_n(i_{nn}, j_{nn})]\}, \tag{3}$$

where $x_n(i,j)$ is the value of the variable defined at the site (i,j) at time step n, and NN denotes the four nearest neighbors of site (i,j). The local map is defined to be $f(x) = 1 - \alpha x^2$ with α indicating the strength of the nonlinearity, and parameter ϵ gives the strength of coupling among neighbors. Either parameter α or ϵ can be controlled [10]. Note that the method is quite general and can be directly applied to extended systems with continuous time evolution as well.

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The simplest spatiotemporal state one may wish to target is a synchronized frozen lattice, that is the spatiotemporal fixed point state. Targeting such fixed points could be desirable in a variety of situations ranging from the maintenance of steady states in biophysical processes under fluctuating environmental conditions, [11] to ''smart matter'' applications [3].

To reach and maintain a specific stable spatiotemporal fixed point x^* so that $\mathcal{P}\equiv x$, $\mathcal{P}^*\equiv x^*$, the control equation (for the control parameter α) is

$$\alpha_{n+1}=\alpha_n-\gamma[x_n(i_c,j_c)-x^*] \tag{4}$$

where (i_c,j_c) is the *single site chosen for monitoring feedback*, and the parameter is changed globally. If the desired state is any arbitrary synchronized fixed point, rather than a specific fixed point x^* as above, one can employ an alternate control strategy. The control parameter α now evolves via the equation

$$\alpha_{n=1}=\alpha_n-\gamma\mathcal{E}, \tag{5}$$

where the error signal $\mathcal{E}$ involves either spatial or temporal feedback. The error can be temporally defined as: $\mathcal{E}=|x_n(i_c,j_c)-x_{n-1}(i_c,j_c)|$. The demand that this error must be zero drives the lattice to the first spatiotemporal fixed point state (where $\mathcal{E}=0$) it encounters in parameter space. The same effect can be achieved using a spatial feedback, demanding that the local patch around the monitored site must be synchronized, i.e., the error signal is defined as $\mathcal{E}=|\Sigma_{\mathrm{NN}}[x_n(i_c,j_c)-x_n(i_{\mathrm{NN}},j_{\mathrm{NN}})]|$.

We now demonstrate the utility of our scheme in an interesting application involving continuous time evolution, viz. controlling an unstable elastic array, which has been used as a prototypical model for ''smart matter.'' It is clear that in such a context, where the system contains many elements, the effectiveness of control algorithms which rely on access to the full state of the system and detailed knowledge of its behavior is limited [3]. Hence the present approach can prove useful, as it needs local information from very few sites and no detailed knowledge of the dynamics in order to achieve the necessary control [12].

Consider a model of the buckling instability of beams [3]: an elastic array of N elements coupled to nearest neighbors by springs with spring constants α, and a destabilizing force coefficient f. The dynamics of the beam is given by

$$\frac{d^2\mathbf{u}}{dt^2}=A\mathbf{u}-G\dot{\mathbf{u}}, \tag{6}$$

where the N-dimensional vector $\mathbf{u}$ gives the displacements of the elements, the damping matrix G has the form gI, I being the identity matrix and the coupling matrix A has elements $A_{mn}=-2\alpha+f$ for $m=n$, $A_{mn}=\alpha$ for $m=n\pm1$, and $A_{mn}=0$ otherwise.

Our control principle can be implemented via an effective dynamics given by

$$\frac{d^2\mathbf{u}}{dt^2}=M\mathbf{u}-G\dot{\mathbf{u}}, \tag{7}$$

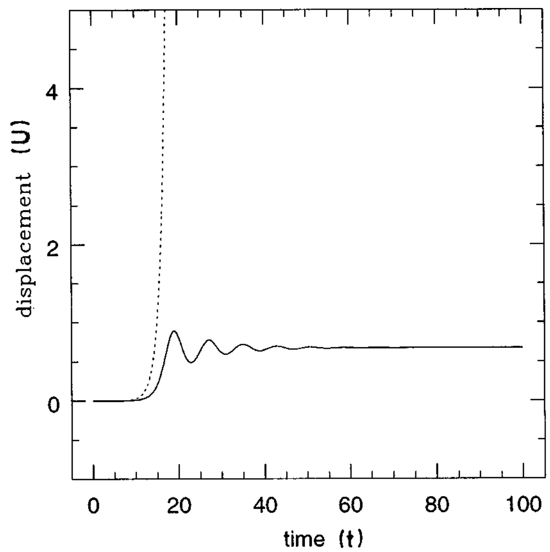

FIG. 1. Displacement u of a representative element in an elastic array with respect to time t in the uncontrolled (....) and controlled case (—) [i.e., Eqs. (6) and (7) in the text, respectively]. The number of elements N in the chain is 100. The chain is evolved via the fourth order Runge Kutta algorithm, with step size $\delta t=0.01$. Here we are controlling to a spatiotemporal fixed point by using spatial feedback (with stiffness $\gamma=1000\delta t=10$). In the absence of control the system moves exponentially away from the steady state, while under control it manages to maintain the steady state.

where the controlled matrix $M=A-C$, where C is the controlling part given by a diagonal matrix $\gamma\mathcal{E}I$ and I is the identity matrix [13]. The error signal $\mathcal{E}$, as described above, can be either spatial or temporal, and is computed using information from only one arbitrary site [14]. Figure 1 shows the displacements of a representative element of such an array in the controlled and uncontrolled case. In the absence of control, very weak environmental perturbations drive the system exponentially away from the desired configuration, as evident from the figure. Our control manages to achieve the goal, typical in smart matter applications, of maintaining the steady state where the beam is frozen in time.

The above method can also be used to target complex spatial patterns. To target spatial patterns we must use spatial feedback, which is obtained by measuring the *local neighborhood of the monitored site*. The feedback has to be specifically tailored according to the distinguishing characteristics of the desired targeted pattern. We demonstrate this for the case of two distinct patterns in the coupled map lattice (CML): the checkerboard (squares) and stripes.

In order to target checkerboard patterns, one can use its simplest characteristic, which is the requirement that $x(i,j)-x(i+1,j-1)=0$, $x(i,j)-x(i-1,j+1)=0$, $x(i,j)-x(i+1,j+1)=0$ and $x(i,j)-x(i-1,j-1)=0$, for all i,j. Utilizing the above to construct an error signal we have for the checkerboard: $\mathcal{E}=|\{x(i_c,j_c)-x(i_c+1,j_c-1)\}+\{x(i_c,j_c)-x(i_c-1,j_c+1)\}+\{x(i_c,j_c)-x(i_c+1,j_c+1)\}+\{x(i_c,j_c)-x(i_c-1,j_c-1)\}|$ where (i_c,j_c) is the site monitored for feedback. Hence the equation for controlling parameter α is $\alpha_{n+1}=\alpha_n-\gamma\mathcal{E}$ and for controlling parameter ϵ is [15] $\epsilon_{n+1}=\epsilon_n+\gamma\mathcal{E}$.

Now if one wanted to target a striped pattern the demand is $x(i,j)-x(i+1,j-1)=0$ and $x(i,j)-x(i-1,j+1)=0$.

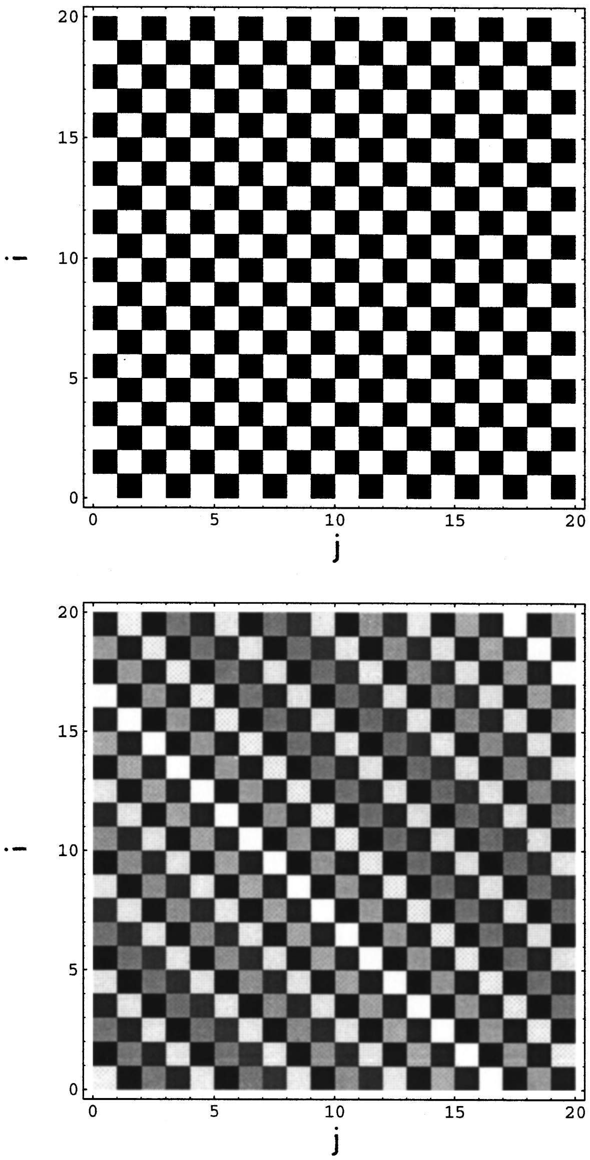

FIG. 2. Controlling to desired spatial patterns in a CML with $g(x)=x$ and $\alpha=2.0$, from random initial configurations. The parameter being controlled is ϵ, whose uncontrolled value is 0.2. Density plots of the targeted patterns in the 20×20 square lattice ($i=1,...,20$, $j=1,...,20$) are displayed: (a) a checkboard pattern (top), controlled by using spatial feedback, $\gamma=0.025$; (b) a striped pattern (bottom); controlled by using spatial feedback, $\gamma=0.001$. Note that for achieving striped patterns the control stiffness γ has to be small, or else the controlled system will miss the narrow regions of parameter space supporting the targeted patterns.

This gives the following error signal (to be used in the control equations): $\mathcal{E}=|\{x(i_c,j_c)-x(i_c+1,j_c-1)\}+\{x(i_c,j_c)-x(i_c-1,j_c+1)\}|$ where (i_c,j_c) is the site monitored for feedback.

This control method is found to drive the lattice to the targeted patterns, very effectively (see Fig. 2). The first (stable) configuration which satisfies the demand of error being 0 is obtained [16]. Note that the spatial periodicity achieved by targeting spatial patterns does not necessarily imply temporal periodicity (since the feedback does not have any temporal information here). In fact, in the cases above, the system, while being perfectly periodic in space, evolves as noisy cycles in time.

The adaptive method can also be used towards enhancing spatiotemporal chaos, an application of practical importance [4,6]. Now, if the desired state is chaotic rather than periodic, one needs to choose an appropriate property $\mathcal{P}$ which reflects the chaotic nature of the target state. An appropriate adaptive strategy is to take $\mathcal{P}$ to be the instantaneous local stretching rate Δx, in space or time [6]. The error signal is obtained by the difference between the current stretching rate and a prescribed target. The control equation for parameter α thus is $\alpha_{n+1}=\alpha_n+\gamma(\Delta x_{\text{target}}-\Delta x)$ and for parameter ϵ it is $\epsilon_{n+1}=\epsilon_n-\gamma(\Delta x_{\text{target}}-\Delta x)$. The local stretching Δx in time is given by $\Delta x=|x_n(i_c,j_c)-x_{n-1}(i_c,j_c)|$, where (i_c,j_c) is the site monitored for feedback.

Instead of a temporal feedback, like the one described above, one can also use a spatial feedback. For instance, one can demand that the local patch around the monitored site be very ''rough.'' A measure of this local ''spatial roughness'' (or local stretch in space) can be $\Delta x=|\Sigma_{\text{NN}}[x_n(i_c,j_c)-x_n(i_{\text{NN}},j_{\text{NN}})]|$. When the target $\Delta x_{\text{target}}=0$, a spatiotemporal fixed point is achieved, as noted before. When δx_{target} is large it leads the system to a more spatiotemporally chaotic state. The controlled parameter rapidly evolves in time to a suitable range and then fluctuates within a range of values, so as to keep the targeted stretch rate, on an average, satisfied.

Note, when targeting chaos by controlling parameter α, the control stiffness γ must not be too small. If γ is too small the system moves very slowly through parameter α space and if the initial value of α is small, the lattice elements tend to synchronize. Then the spatial error never manages to go to zero and the control is rendered unstable. For temporal feedback, very small stiffness leads the system to temporal chaos, but spatially again the system synchronizes. Thus one obtains synchronized chaos, which in some cases might be a desired target.

In summary, we have presented here several adaptive algorithms, utilizing both spatial and temporal feedbacks, which can be used to achieve desired spatiotemporal behavior in extended nonlinear systems. The techniques, which extend the adaptive control methods for low-dimensional systems [7,8,6] are rapid, powerful and robust. We have applied the scheme to achieve a wide range of spatiotemporal targets, from synchronization and spatial patterns to spatiotemporal chaos, and found the methodology to be very successful in extended systems, both in the case of discrete and continuous time evolution.

Two significant features of these methods are as follows.

(i) They can be implemented without explicit knowledge of the dynamics, which can be treated effectively as a black box. The only information necessary to implement adaptive control (or adaptive anticontrol) is either the difference between the current value of a variable and its previous value or the value of the monitored sites and a suitable set of neighbors.

(ii) Very few measurements are required to calculate feedback. Thus the scheme is not computationally intensive. In fact just one (arbitrary) site (and its local neighborhood) is monitored to obtain the required feedback and this is capable of regulating the entire lattice. We have demonstrated the efficacy of our method in an interesting practical context of relevance to smart matter where the above-mentioned features of our control method could be of considerable advantage. We hope that our techniques will find further applications in other realistic systems like coupled oscillator systems, chemical reactions, and Josephson junction arrays.

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[10] There may be situations where a certain parameter may not support any stable state corresponding to the target, at the given value of the other parameter. For instance, for $\alpha = 2.0$ there is no value of ϵ which supports a spatiotemporal fixed point, and so one cannot reach a fixed point by controlling ϵ alone.

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[12] The details of this application can be found in a longer version: S. Sinha and N. Gupte (unpublished).

[13] Alternately, the control can be effected through the additional dynamics $\dot{A}_{ii} = \gamma\mathcal{E}$ and $\dot{A}_{ij} = 0$, where A_{ii}, A_{ij} are the diagonal and off-diagonal elements of the matrix A in Eq. (6).

[14] The very successful smart matter controls based on markets, i.e., multiagent methods of control [3], depend on the information from many sites, with sensors located at various points. Here we need information from only one monitored site in order to compute the necessary control.

[15] While the sign of the control dynamics given here holds in generic cases, there may be some exceptions (arising from the nongeneric location of the initial state with respect to the target state in parameter space).

[16] Note that both parameters can be simultaneously controlled. But since single parameter controls are so effective one need not control both parameters. Further, stability of two parameter control is different, as one adds another nonlinear equation to the effective dynamics, and, in general, one parameter control has larger regions of stability in control stiffness γ space.