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Protocol for quantum-logical operations in an optical lattice using nonresonant absorption of photon pairs

Hema Ramachandran,[1] Anders Kastberg,[2] Andal Narayanan,[1] and R. Srinivasan[1,*]
[1]*Raman Research Institute, Sadashivnagar, Bangalore, India 560 080*
[2]*Department of Physics, Umeå University, SE-90187 Umeå, Sweden*
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Two nonidentical atoms may be entangled by inducing simultaneous cooperative absorption of a photon pair from a light field that is nonresonant to either atom in isolation. The atoms may be of two different elements, of different isotopes of the same element, or of a single isotope undergoing different transitions. Utilizing this cooperative process, we propose a protocol for a conditional operation with neutral atoms in an optical lattice. Current practical capabilities permit execution of the scheme.

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I. INTRODUCTION

The last few years have witnessed an enormous interest in the area of quantum computing, with many different schemes being proposed. Candidate systems in which quantum logic may be realized include ions in ion traps [1–3], bulk materials using nuclear magnetic resonance [4], quantum dots [5], trapped polar molecules [6], pairs of atoms in optical traps [7–9], and neutral atoms in an optical lattice [3,10,11]. Recently, single atoms in cavities [12], Bose Einstein condensates [13], Cooper pairs in superconductors [14], and atoms on "atom chips" [15] have also been considered. However, practical large scale quantum computers are yet to be realized. Effective implementation calls for seemingly contradictory requirements. The qubits are to be created in desired initial states and are to be kept isolated to prevent decoherence. They should also be made to interact in a controlled fashion to perform logical operations. Further, for practical application, qubit sites should be individually addressable for the initialization of qubits and their readout after interaction.

A. Optical lattices

Optical lattices are periodic arrays of irradiance maxima and minima formed by the interference of several laser beams [16]. With a judicious choice of frequency and polarization states of light, atoms in selected internal states may be confined. The conventional optical lattices, formed by standing-wave interference patterns, have lattice spacings of the order of λ, the wavelength of light used. Interaction between atoms in neighboring wells can be facilitated or inhibited by sliding the wells closer together or farther apart, by altering the polarization of the lattice beams. In a practical realization [17] of a "double lattice," two sets of lattice beams are used to form a pair of interpenetrating lattices that can be moved relative to one another by introducing unequal phase shifts in the two sets of beams.

As opposed to the "conventional" optical lattices, large period optical lattices can be formed either by employing long wavelengths [18], or by using macroscopic elements such as microlens arrays [19], diffraction optics [20], or Talbot grids [21]. The larger lattice spacing (10–100 μm) facilitates individual site addressing. The use of a superimposed spatially varying field has also been considered [9,22] for distinguishing atoms at different locations in conventional optical lattices.

B. Quantum logic with optical lattices

Several schemes have been proposed for performing quantum gate operations using atoms in an optical lattice. For example, Brennen *et al.* [10] consider alkali atoms trapped in an optical lattice formed by linearly polarized counterpropagating beams, with perpendicular polarizations that give rise to alternating potential wells of σ_+ and σ_- polarizations. Each well can preferentially trap atoms in two different hyperfine ground states labeled $|0\rangle_+, |1\rangle_+$ and $|0\rangle_-, |1\rangle_-$, respectively. A controlled-NOT (CNOT) operation is suggested by a gradual rotation of the beam polarizations, thereby merging the σ_+ and σ_- wells. Two neighboring atoms are thus brought together, and thereafter a "catalysis laser," resonant only for atoms in $|1\rangle_+$ and $|1\rangle_-$, is switched on. The effect is a conditional dipole-dipole interaction. A subsequent Raman π pulse, resonant to atoms with shifted energy levels, then causes complete transfer of population from $(|1\rangle_- * |1\rangle_+)$ to $(|1\rangle_- * |0\rangle_+)$ if and only if the two atoms had both initially been in their $|1\rangle$ states. This gives a controlled-PHASE (CPHASE) truth table, mappable to a CNOT operation [10].

In another proposal, Jaksch *et al.* [11] instead utilize phase shifts acquired by conditional collisions, while the atoms are brought together and apart in essentially the same way as above. This also provides the conditional dynamics for a CNOT operation.

In this paper, we show how even distinguishable atoms, say those of two different elements, may be entangled by a cooperative absorption, and using this, propose a mechanism for performing a conditional operation. We discuss how this scheme may be implemented using, e.g., conventional optical lattices, double optical lattices [17], and optical lattices based on microlens arrays [19]. We show that this may lead to quantum gates of high fidelity and also make feasible individual site addressing.

*Electronic address: hema@rri.res.in

II. QUANTUM LOGIC OPERATIONS USING NONRESONANT ABSORPTION OF PHOTON PAIRS

The proposed protocol utilizes the cooperative absorption of two photons, one each, by two distinguishable atoms in proximity. The catalysis light is not resonant for either atom, but the combined energy of two photons of the catalysis laser matches the sum of the transition energies of the two atoms.

When two atoms A and B, with transition frequencies ω_A and ω_B, are irradiated by light of frequency ω_L, the absorption peaks when $\omega_L=\omega_A$ or $\omega_L=\omega_B$ due to resonant one-photon absorption by one of the two atoms. An additional absorption peak is observed [23] at $\omega_L=(\omega_A+\omega_B)/2$ due to a two-photon process. The light is midway in frequency between the two resonant transitions, that is, it is as far to the red of the transition of one atom as it is to the blue of the other. If ω_A and ω_B differ considerably, the detuning of this laser is large and neither atom can, in isolation, make a transition. However, if the separation between the atoms is small, the pair can cooperatively make simultaneous transitions by a two-photon absorption, where one atom transfers its surplus energy to the other atom, to make good its shortfall [24]. Such cooperative two-photon transitions occur only in the presence of atom-atom interaction. We refer to this process as nonresonant absorption of photon pairs (NAPP). This process was predicted by Harris and Lidow [25], and first observed by White [23].

There are several theoretical investigations of the process in the literature [24,26]. Varada and Agarwal [24] solved numerically the density matrix of a pair of two-level atoms, treating it as a four-level system. With g_A, e_A, g_B, e_B denoting the ground state and excited states of A and B, respectively, transitions from $|g_A, g_B\rangle$ to $|e_A, e_B\rangle$ are possible via two paths: $|g_A, g_B\rangle \rightarrow |g_A, e_B\rangle \rightarrow |e_A, e_B\rangle$ and $|g_A, g_B\rangle \rightarrow |e_A, g_B\rangle \rightarrow |e_A, e_B\rangle$. For isolated atoms, the two paths interfere destructively. In the presence of a perturbative interaction, e.g., the dipole-dipole interaction, the intermediate levels $|g_A, e_B\rangle$ and $|e_A, g_B\rangle$ are mixed, resulting in an incomplete cancellation of the two paths, thereby making possible a cooperative two-photon absorption that cannot occur for free atoms. The rate of this two-photon transition increases inversely as the sixth power of the interatomic separation [24], becoming increasingly dominant as the atoms approach one another. Starting with the state $|g_A, g_B\rangle$, the probability of finding the system in the state $|e_A, e_B\rangle$ upon measurement will be periodic in time [27]. The rate of transition at the two-photon resonance is given by

$$R = \frac{2\pi V^2 (G_A^2 + G_B^2)}{\delta_A \delta_B}, \tag{1}$$

where V is the dipole-dipole interaction, $\delta_{A,B}$ denotes the detuning of the light from the respective single-photon transition, and G, the coupling constant of the atom to the field. For weak dipole-dipole coupling, using perturbation to the first order, the energy eigenvalues for $|g_A, g_B\rangle$ and $|e_A, e_B\rangle$ remain unchanged.

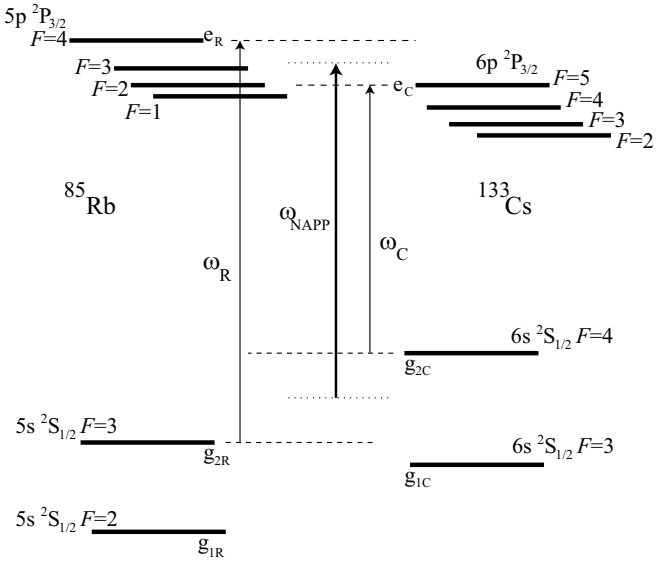

FIG. 1. The schematic energy level structure for ^{85}Rb and ^{133}Cs, showing the electronic energy levels relevant to our example. Energy separations are not to scale and have been offset. The thin vertical arrows indicate the individual resonances and the thick vertical arrow, the NAPP resonance.

Gate operation

As coherent oscillations occur between $|g_A, g_B\rangle$ and $|e_A, e_B\rangle$, NAPP provides a means of entangling two atoms. These may be a pair of identical atoms in different ground-state hyperfine levels, $|g_A\rangle$ and $|g_B\rangle$ [28]. Alternatively, they could be atoms of two isotopes of the same element, or two atoms belonging to different elements.

The NAPP process may be utilized for performing a quantum logical operation; a variety of possibilities exist, of which we outline two representative examples. In all cases, the qubits should be encoded in atomic states and atoms should be trapped in neighboring potential wells that may be manipulated, e.g., as in the schemes of [10,11]. In our examples, however, the entanglement is induced by the phenomenon of NAPP described above. Advantages of NAPP are that even though it scales as the inverse sixth power of the interatomic distance (as does the van der Waals coupling), the strength of the interaction can be tuned by varying the irradiance of the catalysis laser, and the coupling is internal-state dependent, making it a natural choice for a conditional operation.

1. Example 1: NAPP-based CNOT gate using different elements

We first discuss the case of quantum logic between trapped atoms of two different elements. We consider a two-species sample of cold atoms containing, e.g., ^{85}Rb and ^{133}Cs. The level structure is given in Fig. 1. Two interpenetrating lattices should be created that can be moved relative to one another [29]: one that preferentially traps Rb and the other Cs.

Initially, the atoms are kept in a noninteracting mode, with the Rb and Cs atoms at alternate lattice sites. The Rb lattice can trap Rb atoms in the ground-state hyperfine levels 5s

$^2S_{1/2}$, $F=2,3$ (which we denote $|g_{1R}\rangle$ and $|g_{2R}\rangle$) and the Cs lattice can trap Cs atoms in the corresponding levels 6s $^2S_{1/2}$, $F=3,4$ ($|g_{1C}\rangle$ and $|g_{2C}\rangle$). These constitute the qubits, which may be initialized, e.g., to the maximally entangled state, by optical pumping and a Raman pulse. The atoms are then brought close together where they may interact, in the presence of a catalysis laser. The catalysis laser is tuned halfway between the 5s $^2S_{1/2}$, $F=3\rightarrow 5p\ ^2P_{3/2}$, $F'=4$ transition in Rb (denoted $|g_{2R}\rangle\rightarrow|e_R\rangle$) and the 6s $^2S_{1/2}$, $F=4\rightarrow 6p\ ^2P_{3/2}$, $F'=5$ transition in Cs (denoted $|g_{2C}\rangle\rightarrow|e_C\rangle$).

When pairs of atoms, Rb and Cs, are brought together, four possibilities exist for the combined state of the Rb-Cs pair: $|g_{1R},g_{1C}\rangle$, $|g_{2R},g_{1C}\rangle$, $|g_{1R},g_{2C}\rangle$, and $|g_{2R},g_{2C}\rangle$. Of these only one, $|g_{2R},g_{2C}\rangle$, will result in a NAPP transition to $|e_R,e_C\rangle$. The intensity and duration of the catalysis laser is adjusted to obtain a 2π NAPP pulse, then $|g_{2R},g_{2C}\rangle$ evolves to $-|g_{2R},g_{2C}\rangle$. That is, the pair makes a joint, coherent transition to the corresponding excited level and then returns to the original state, having acquired a phaseshift of π. Thus, the following CPHASE truth table results

$$|g_{1R},g_{1C}\rangle\rightarrow|g_{1R},g_{1C}\rangle,$$

$$|g_{1R},g_{2C}\rangle\rightarrow|g_{1R},g_{2C}\rangle,$$

$$|g_{2R},g_{1C}\rangle\rightarrow|g_{2R},g_{1C}\rangle,$$

$$|g_{2R},g_{2C}\rangle\rightarrow-|g_{2R},g_{2C}\rangle. \quad (2)$$

After interaction, the atoms are separated again. The result of the operation may be determined as in other schemes [10,11].

This can be mapped to a CNOT operation by defining the basis $|\pm\rangle=|g_{1C}\pm g_{2C}\rangle$ for the Cs atoms, so that for the Rb-Cs pair, the usual CNOT truth table results

$$|g_{1R},\pm\rangle\rightarrow|g_{1R},\pm\rangle,$$

$$|g_{2R},\pm\rangle\rightarrow|g_{2R},\mp\rangle. \quad (3)$$

This NAPP-based scheme has several distinct advantages. It provides a situation where far detuned light is state selective. This gives the dual advantage of reducing spontaneous scattering while providing conditional dynamics. Provided that the atoms can be moved with great control, the catalysis laser may be on at all times, since this light can be incoherently scattered by the atoms only when pairs are in the appropriate states and at a close distance from one another, which happens for the very short duration of the conditional operation.

Using a scheme with two different elements implies some additional advantages. When using two dissimilar atoms, with well-separated resonant wavelengths, the segregation of the two types of qubits into two distinct lattices is better guaranteed. This also means that the process of transporting qubits selectively may be facilitated. On a more visionary note, one could imagine using three species, first entangling two, and then the third, in order to attempt to create GHZ states.

2. Example 2: NAPP-based CNOT with different isotopes of the same element

The NAPP-based CNOT operation can also be performed on atoms of the same element in different isotopic forms. For example, one could trap ^{85}Rb and ^{87}Rb in two interpenetrating lattices, using macroscopic lattices, or the double lattice scheme of [17], with ^{85}Rb in state 5s $^2S_{1/2}$, $F=2,3$, and ^{87}Rb in 5s $^2S_{1/2}$, $F=1,2$. Once again, when the atoms are brought together, four possibilities exist. A catalysis laser tuned midway to two distinct transitions causes NAPP for only one combination, resulting in a CPHASE operation, mappable to CNOT as earlier.

III. IMPLEMENTATION OF NAPP-BASED CNOT IN OPTICAL LATTICES

Having outlined the quantum logic protocol based on NAPP, we now present an example of how this may be practically implemented using cold atoms in optical lattices.

A microlens array appears to be an ideal tool for implementing the NAPP-based conditional logic operation for distinct elements. Dumke *et al.* [19] have experimentally demonstrated a pair of interpenetrating lattices of ^{85}Rb atoms, using a pair of laser beams that are incident on a microlens array at an angle to one another. The lattices were spatially manipulated by altering the angle between the beams. Individual lattice-site addressing had also been achieved.

To implement our scheme for a conditional operation in a cold cloud of, e.g., ^{85}Rb and ^{133}Cs atoms using a microlens array we utilize two lasers L1 and L2. One of the two lasers, L1, has to be red detuned of the ^{85}Rb 5s $^2S_{1/2}$, $F=3\rightarrow 5p\ ^2P_{3/2}$, $F'=4$ transition and the other, L2, red detuned of the ^{133}Cs 6s $^2S_{1/2}$, $F=4\rightarrow 6p\ ^2P_{3/2}$, $F'=5$ transition. Rb atoms are then trapped at the foci of L1 and Cs atoms at the foci of L2. The angle between the two lasers is such that the foci of L1 and L2 are well separated; this is the noninteracting situation, where the atoms may be initialized to desired states. To induce an interaction, the angle between L1 and L2 is reduced, so that a Rb atom is brought closer to one of its Cs neighbors. A catalysis laser, L3, tuned halfway between the ^{85}Rb 5s $^2S_{1/2}$, $F=3\rightarrow 5p\ ^2P_{3/2}$, $F'=4$ transition and the ^{133}Cs 6s $^2S_{1/2}$, $F=4\rightarrow 6p\ ^2P_{3/2}$, $F'=5$ transition is switched on for a duration corresponding to a 2π evolution of the NAPP transition. The atoms are once again separated by increasing the angle between the two lasers. The catalysis laser does not need to be sent through the microlens array—it suffices that this light illuminates the atoms.

IV. DISCUSSION

Conventional optical lattices have typical spacings of $\lambda/4$, and typical vibrational frequencies of hundreds of kilohertz. Using resolved side-band cooling [30–32], one can cool atoms to the lowest vibrational state.

In the case of microlens arrays, Dumke *et al.* [19] estimate radial vibrational frequencies of 10 kHz, and a

decoherence time of 50 ms. Displacement times of 1 μs are claimed, which is 20 times shorter than the oscillation period and 10^4 times shorter than the decoherence time. With a catalysis laser of power 1 W, focused to a spot of 1 mm diameter, we estimate the duration of the NAPP operation to be 0.02 μs, and thus the total gate time would be of the order of microseconds, suggesting that about a thousand gate operations would be possible before the atoms decohere. During a gate operation, the probability of incoherent scattering of a lattice photon is of the order of 10^{-4}.

For full-scale quantum computing, nonlocal operations must also be possible to perform. For this, the NAPP scheme is unsuitable. However, as long as these operations are strictly nonlocal and nonconditional, it is straightforward to complement the NAPP scheme with light fields resonant with the respective atoms, in order to perform, e.g., a Hadamard operation.

V. CONCLUSIONS

We have suggested means for performing quantum gate operations in optical lattices making use of NAPP. We have shown examples demonstrating the wide flexibility of the scheme; it can be used with atoms of different elements, with atoms in different isotopic forms of the same element, or with identical atoms undergoing different hyperfine transitions. Conventional optical lattices or the two-color double lattice may be used to implement this protocol. The use of a microlens array for the generation of the interpenetrating lattice, in addition, permits individual site addressing.

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