RUNDLES

Design of a Reinforced-Concrete Arch

Civil Engineering

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DESIGN OF
A REINFORCED-CONCRETE ARCH

BY

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THESIS

FOR

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IN

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I recommend that the thesis prepared under my supervision by EARL RUNDLES entitled Design of a Reinforced-Concrete Arch be approved as fulfilling this part of the requirements for the degree of Bachelor of Science in Civil Engineering.

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Recommendation approved

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Table of Contents.

DESIGN OF A REINFORCED CONCRETE ARCH.

INTRODUCTION.

Owing to the fact that the regular University curriculum for undergraduate work does not provide for instruction on the theory and design of arches, thesis work covering this field was chosen. The arch herein designed is intended to carry heavy highway traffic. It has a clear span of ninety-two feet and rise of eleven feet. The small rise was used so as to give ample room for the flow of flood waters near the haunches of the arch,

REINFORCEMENT.

The reinforcement consists of square twisted bars. Three quarter inch bars, placed six inches apart were used in the intrados and extrados. One half inch bars placed two feet apart were used in the transverse direction. This system was chosen rather than the Melan type in that tests show that a smaller unit gives greater strength, area for area.

METHOD.

Prof. Baker's graphical method according to the elastic theory was used in the design. The general dimensions were first assumed and then the design worked out to see whether the stresses obtained were within the allowable limit. The allowable stresses and the weights of the material are those which have been generally accepted.

DIMENSIONS.

Span (clear)	92 feet.
Rise	11 feet.
Earth fill over crown	6 inches.
Width of Roadway	20 feet.

REINFORCEMENT.

44 lines of 3/4"□ bars along intrados.

44 lines of 3/4"□ bars along extrados.

2 lines of 1/2"□ bars in each parapet wall.

LOADS.

Live load on one half of bridge, 200 lb. sq.ft.

Dead load of earth and concrete.

WEIGHTS OF MATERIALS.

Concrete and steel	150 lb. per cu. ft.
Earth filling	100 lb. per cu. ft.

DESIGN.

The neutral axis was divided so that each length divided by the moment of inertia of concrete and steel gave a constant. The length of the first division was taken so as to get a convenient size and number of divisions. See Table I.

The lengths were laid off in succession on the neutral axis from the crown to the springing lines. The centers of the divisions are marked a_1 a_2 - a_{22} as shown in Plate I.

Dead Load:- Vertical lines were drawn through a_1 a_2 - a_{22} and the areas included between the successive verticals, the intrados and the upper limit of the earth filling, found. These areas were multiplied by 100. Assuming one foot in width of arch, this gave the dead load for each division.

Live Load:- The live load over each division was found by scaling the distance between verticals and multiplying by 150.

Horizontal Pressure of Earth Filling:- The theory used does not take into consideration the horizontal pressure of the earth filling. On account of the flat arch this force could he safely neglected.

Plate I gives the dead (earth and concrete) and live load for each division. The loads were laid off at the center of gravity of the division between the verticals a_1 a_2 - a_{22} as shown in Plate I.

Construction of the trial equlibrum polygon,

1st. The load line 1 - 21 was laid off.

2nd. The trial pole was determined by an application of Navier's principle; T = pp (2.166 x 150.+ .50 x 100+200) 146. = 84,000 lb.

Therefore the trial pole distance was taken at 80,000 lb, and the trial equilibrium polygon drawn. It was necessary next to find the resultant of the positive forces and the closing line of the trial equilibrium polygon.

Table II gives the values of the co-ordinates x and y to the points of intersection of the lines of action and neutral line of the arch ring and also various intercepts and products employed in the work to follow. The resultant was found to be 129.75 and to act 0.944 ft. to the left of the center line of arch.

The trial closing line was assumed to be parallel to v_2, v_{22} and to be $\frac{R}{20 + 2}$ = 5.89 ft above it. This assumption simplified the subsequent work.

Taking monents about a point in T_l and T_r (Plate I) gives:-

$$\frac{\text{True } T_r}{\text{Trial } T} = \frac{\overline{x}_l - \overline{x}}{\overline{x}_l}$$

and

$$\frac{\text{True } T_e}{\text{Trial } T} = \frac{\overline{x}_r - \overline{x}}{\overline{x}_r}$$

Then if $m_1 m_{22}$ is the true closing line, the following proportion is true:

$$v_1 m_1 = \frac{\text{True } T_e}{\text{Trial}} v_1 n_1$$

or

$$v_1 m_1 = \frac{\overline{X}_r + \overline{X}}{\overline{X}_r} v_1 n_1 = 1.063 \times v_1 n_1 = 6.26 \text{ ft.}$$

and

$$V_{22}, m_{22} = \frac{x_e - x}{x_e} V_{22}, m_{22}$$

$= 0.935 \times 5.89$

$= 5.51$ ft.

The true closing line is obtained by drawing a line from m_1 to $m22$.

True Pole Distance; The true equilibrium polygon must give $\Sigma ck, y = a$ (Plate I) hence the trial pole must be moved accordingly.

True pole distance = Trial pole distance $\times \frac{\Sigma bmy}{\Sigma aky}$

$= 80,000 \times \frac{156.08}{133.28}$

$= 93,680$

True Equilibrium Polygon: The true pole is located by measuring the true pole distance from Q then beginning at K_r or K_l the true equilibrium polygon can be drawn.

Stresses Due to Dead and Live Load:

Let

a c = intercept between the neutral line and the true equilibrium polygon.

b = the breadth of the unit section of the arch.

c = the distance of the most remote fiber from the neutral line.

d = the depth of the arch ring

f = the unit fiber stress.

h = the true pole distance.

N = the component parallel to the radius at any point of the neutral line of all the forces to one side of the point.

T = the component parallel to the tangent at any point of the neutral line of all the forces to one side of the point.

v = the unit shearing stress.

$$f_b = \frac{b\ H\ a\ c}{d^2}$$

$$f_s = \frac{T}{d}$$

$$v = \frac{N}{d}$$

These stresses are recorded in Table II.

Effect of temperature Changes.

Let

l = span of the neutral line.

e = the expansion of concrete per unit of length per 1° Fahr.

t° = the difference in degrees between the mean and the actual temperature of the arch ring.

E = 1,500,000

l = 92 ft.

e = .000,005,4

Q = horizontal resisting force.

$$\frac{I}{\Delta S} = \frac{1}{1.165}$$

Then

$$Q = \frac{(1,500,000 \times 144)\ 92 \times .0000054 \times 20}{133,28 \times 1.17}$$

$= 1840''$

$$f_b = \frac{6 \times 1840 \times 5.}{3.35^2} = 4930^{\#}\Box' \text{ or } 34^{\#}\Box''$$

Conclusion:- Table III shows the stresses due to dead and live loads. **The stresses** used in checking the design of the arch ring are the maximum stresses which occur at the points shown in the table. The shearing stress at most points was almost negligable being too small to measure by graphical methods.

TABLE NO 1.

NO	DEPTH	I_c	I_s	$I+I$	ΔS	$\frac{\Delta S}{I}$
0	2.16	.839	2.08	2.92	3.38	1.160
1	2.18	.863	2.09	2.95	3.45	1.170
2	2.22	.912	2.18	3.09	3.58	1.160
3	2.25	.949	2.25	3.20	3.69	1.160
4	2.32	1.040	2.41	3.45	4.03	1.165
5	2.40	1.152	2.69	3.84	4.47	1.165
6	2.49	1.302	2.82	4.12	4.82	1.160
7	2.62	1.488	3.16	4.64	5.48	1.180
8	2.74	1714	3.49	5.20	6.32	1.210
9	2.89	1.945	3.92	5.93	6.90	1.161

TABLE NO. II

NO.	CO-ORDINATES		INTERCEPTS		PRODUCTS		INTERCEPTS		PRODUCTS		$\frac{bm\Sigma aKy}{\Sigma bmy}$
	x	y	bv	f	bv×x	f×x	bm	aK	bm×y	aK×y	cK
1	−48.2	0.00	0.00	5.89	+ 0.00	284.0	+ 6.26	+5.00	+ 0.00	−0.00	+5.35
2	−42.6	1.12	1.50	537	+63.90	228.5	+4.78	+3.80	+ 5.36	+6.00	+4.09
3	−35.8	3.00	4.00	4.50	+143.80	161.3	+ 2.16	+1.95	+ 6.48	+4.85	+1.85
4	−30.4	4.20	5.58	3.85	+169.63	117.0	+ 0.52	− .75	−2.18	+ 3.15	+ .44
5	−24.5	5.32	6.88	2.14	+168.80	52.4	−0.80	− .30	−4.15	− 1.60	−.68
6	− 20.0	600	7.55	2.55	+151.00	51.0	− 1.52	− .75	− 9.12	−4.50	−1.30
7	− 15.9	6.55	8.00	2.03	+127.20	32.3	−1.96	−1.50	−12.84	− 9.83	−1.67
8	−12.3	6.90	8.28	1.52	+102.00	18.7	−2.30	−1.95	−15.88	−13.47	−1.97
9	− 8.6	7.19	8.48	1.12	+73.80	9.6	−2.50	−2.22	−17.99	−15.99	−2.14
10	− 5.1	7.30	8.50	.62	+43.35	3.1	− 2.60	−2.35	−18.99	−17.15	−2.24
11	− 1.6	7.40	8.50	.20	+13.60	.3	− 2.60	−2.45	−19.25	−18.10	−2.24
12	+ 1.6	7.40	8.40		−13.44		− 2.50	−2.45	−18.50	−18.10	−2.13
13	+ 5.1	7.30	8.20		−41.80		− 2.40	−2.35	−17.53	−17.15	− 2.05
14	+ 8.6	7.19	8.00		−68.80		−2.21	−2.22	−15.97	−15.99	−1.89
15	+12.3	6.90	7.75		−95.40		−1.95	−1.95	−13.46	−13.47	−1.67
16	+15.9	6.55	7.38		−93.20		−1.56	−1.50	−10.45	−9.83	−1.33
17	+20.0	6.00	6.80		−136.00		−1.10	− .75	− 6.60	−4.50	− .93
18	+24.5	5.32	6.14		−150.50		− .46	+ .30	−2.44	+ 1.60	−.39
19	+30.4	4.20	5.05		−157.00		− .50	+ .75	+2.10	+3.15	−.42
20	+35.8	3.00	3.56		−127.50		+2.00	+1.95	+ 6.00	+4.85	+1.71
21	+42.6	1.12	1.20		−51.10		+4.35	+3.80	+4.87	+6.00	+3.72
22	+48.2	0.00	0.00		−00.00		+5.51	+5.00	+0.00	+0.00	+4.71
Σ		117.36	129.75		−122.34		−.33	− .02	−156.08	−133.28	−.18

TABLE NO. III

POINT	STRESS DUE TO				MAXIMUM STRESS		
	BENDING		THRUST		INTRADOS	EXTRADOS	SHEAR
	LB □'	LB □"	LB □'	LB □"	LB □"	LB □"	LB □"
a_1	16150	112	28600	200	312	86	16
a_2	16100	112	30500	212	324	100	11
a_3	6850	47	33300	230	277	183	4
a_4	9700	67	27400	190	123	257	—
a_{10}	13100	91	43000	300	209	391	—
a_{11}	25200	175	43200	300	125	475	—
a_{12}	38300	265	43200	300	35	565	—
a_{13}	35000	243	43200	300	57	544	—
a_{14}	37000	257	43200	300	57	557	—
a_{15}	31200	216	417000	290	82	506	—
a_{16}	17400	120	40000	277	161	397	—
a_{17}	16400	114	38000	264	132	252	—
a_{22}	13400	93	27500	192	285	92	6

350	200	1060	900	820	700	120	700	100	700									
5600	2930	2460	1940	1550	1370	1270	1180	1130	1120	1130	1180	1270	1370	1550	1940	2460	2930	5600
5500	2270	1400	900	600	400	290	200	180	170	180	200	290	400	600	900	1400	2270	550
8450	6400	4920	3740	2970	2470	2280	2080	2010	1990	1310	1380	1460	1770	2150	2840	3860	5100	[illegible]

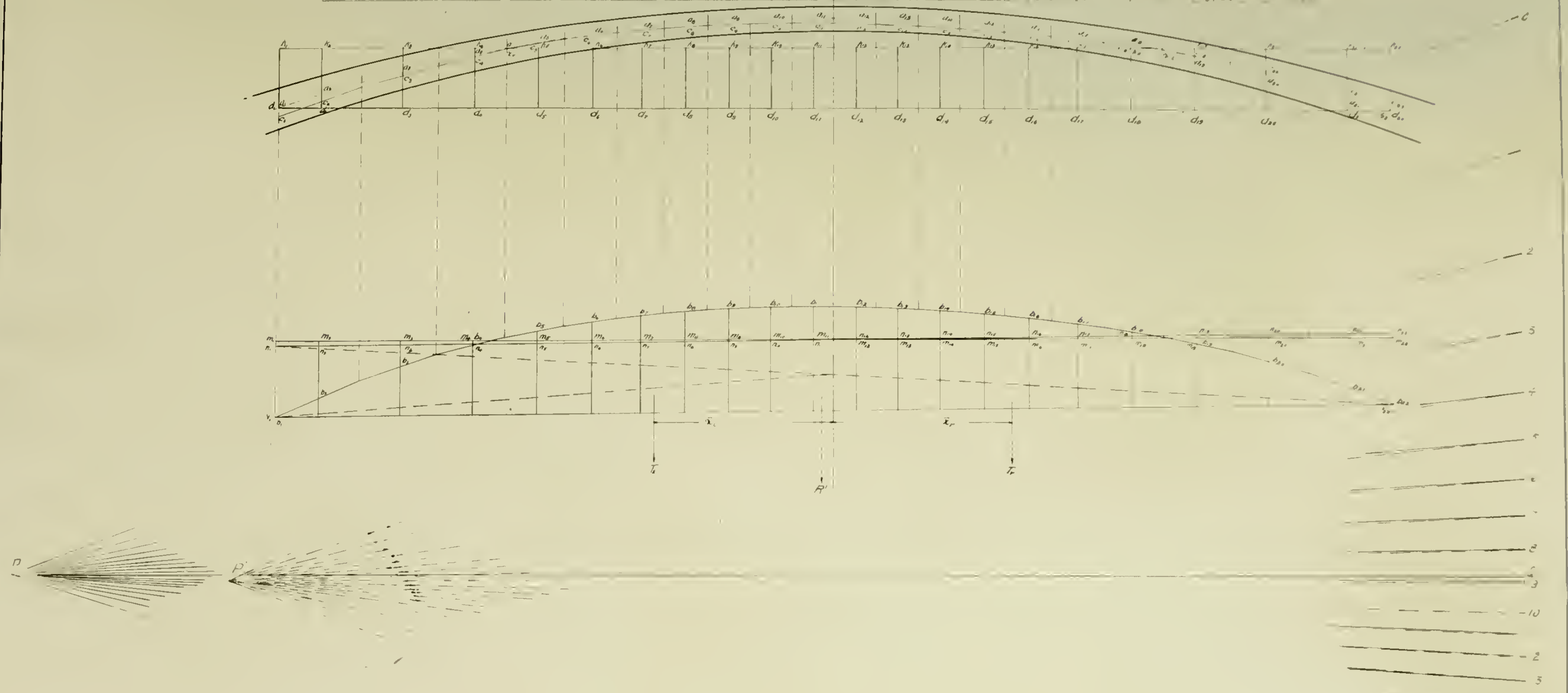

PLATE I

EQUILIBRIUM POLYGON FOR HINGELESS ARCH

SPAN 32 FT. RISE 11 FT.

SCALES

1 IN.= 3000 LB AND 1 IN.= 40 FT.

DRAWN BY, Earl Runnells

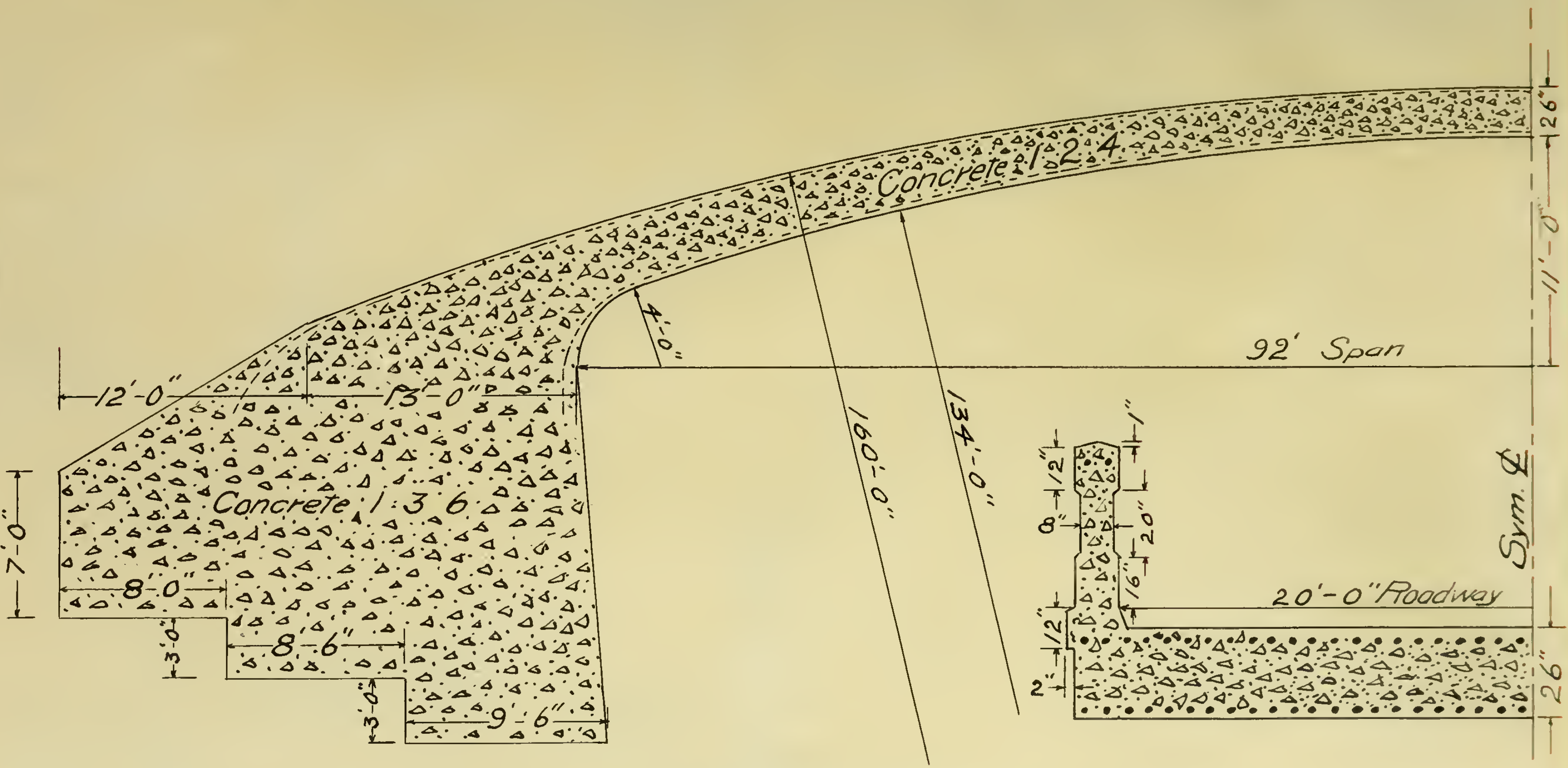

PLATE II
CROSS SECTIONS OF ARCH