UCRHEP-T145
TIFR/TH/95-17
April 1995

Model of Four Light Neutrinos in the Light of All Present Data

Ernest Ma

Department of Physics, University of California, Riverside, California 92521

Probir Roy

Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India

Abstract

Motivated by existing and recent data on possible neutrino oscillations, we propose a model of four light neutrinos (ν_e, ν_μ, ν_τ, and a singlet ν_S) with a pattern of masses and mixing derivable from a discrete Z_5 symmetry and the seesaw mechanism. Atmospheric neutrino oscillations occur between ν_μ and ν_τ as pseudo-Dirac partners; whereas solar neutrino oscillations occur between ν_e and ν_S, a linear combination of which is massless. Additional oscillations may occur between ν_e and ν_μ to account for the recent observation of the LSND (Liquid Scintillator Neutrino Detector) experiment.

The physics of light neutrinos appears poised on the edge of major discoveries. Many hints have accumulated over the past years towards nonzero masses and flavor mixing of these special elementary fermions without charge. They include the solar neutrino deficit[1], the atmospheric neutrino anomaly[2], the need for a cosmological hot dark matter component[3], and finally the excess of $\bar{\nu}_\mu \to \bar{\nu}_e$ events observed recently by the LSND (Liquid Scintillator Neutrino Detector) experiment[4]. If all these strands are put together in a total picture, along with the nonobservations of neutrinoless double β-decay[5] and any depletion of reactor antineutrinos (such as from the Bugey experiment[6]), one is led naturally to a variation of a simple scenario already proposed[7]. We offer in this paper a theoretical understanding of this specific scenario in terms of a hierarchical seesaw model of the masses and flavor mixing of four light neutrinos with an assumed discrete Z_5 symmetry.

We recount the constraints from the above observations. In a two-oscillator picture involving ν_e, the solar neutrino deficit favors either of two matter-enhanced oscillation solutions with $\delta m^2_{e\alpha} \sim 10^{-6}$ to 10^{-5} eV^2, $\sin^2 2\theta_{e\alpha} \sim 5 \times 10^{-3}$ and with $\delta m^2_{e\alpha} \sim 10^{-6}$ to 10^{-4} eV^2, $\sin^2 2\theta_{e\alpha} > 0.4$ or a vacuum oscillation solution with $\delta m^2_{e\alpha} \sim 10^{-10}$ eV^2, $\sin^2 2\theta_{e\alpha} > 0.75$. Here $\delta m^2_{e\alpha}$ is the difference of the squares of the two neutrino masses and $\theta_{e\alpha}$ is the mixing angle between ν_e and another light neutrino ν_α. The atmospheric neutrino anomaly can be understood in a two-oscillator scenario involving ν_μ and another neutrino ν_β with $\delta m^2_{\mu\beta} \sim 10^{-2}$ eV^2 and $\sin^2 2\theta_{\mu\beta} = O(1)$. The LSND results, on the other hand, suggest $\delta m^2_{e\mu} \sim 0.5 - 10$ eV^2 and $\sin^2 2\theta_{e\mu} \sim (6 \pm 3) \times 10^{-3}$ once again in a two-oscillator picture. It is clear that with only the three known neutrino flavors $(\nu_e, \nu_\mu, \nu_\tau)$, we cannot explain all three δm^2's. The orders of magnitude of the latter are too disparate to be explained even by invoking three-flavor oscillations. We are thus prompted by the need to explain all of the above to add a singlet neutrino ν_S which is by itself noninteracting, but will be allowed to mix with the other neutrinos. In this way, we are also able to consider the cosmologically desirable requirement[3] that the masses of the usual three neutrinos sum up to about 5 eV.

With four neutrinos as suggested above, there is a simple hierarchical situation involving only two-oscillator scenarios[8]. Let ν_μ and ν_τ be nearly degenerate with masses of about 2.5 eV each and close to maximal mixing. The δm^2 here is about 10^{-2} eV^2. The singlet neutrino ν_S has a mass of order 10^{-3} eV whereas ν_e is very much lighter but mixes with ν_S as well as ν_μ by small amounts. In this scenario, because ν_e is lighter than ν_μ, there is a potential conflict with rapid neutron capture (r-process) in Type II supernovae[9]. However, if the hierarchy is inverted[10] to avoid this problem, then ν_e is a few eV in mass and the constraint of neutrinoless double β-decay that $m_{\nu_e} < 0.68$ eV[11] cannot be satisfied. Given the manifold uncertainties of the r-process calculation in a hot-bubble scenario, we choose to disregard it in favor of the double β-decay constraint.

We view the large mixing but small mass splitting of ν_μ and ν_τ as suggestive of their pseudo-Dirac origin. We also envisage a massless ν_e and a very light ν_S as two Majorana neutrinos with a small mixing. A unified seesaw model of these hierarchical masses would need a Dirac seesaw[12] for the former pair with a minimum 4×4 matrix and a Majorana seesaw[13] for the latter pair with a minimum 3×3 matrix. An additional small mixing between these two sectors is necessary for understanding the LSND results and can only come from both matrices being submatrices of one 7×7 neutrino mass matrix.

It may appear difficult at first sight to construct a model of the lepton sector generating naturally a mass matrix with the above requirements from a symmetry. However, as shown below, a reasonably simple model does emerge if one supplements the standard $SU(2)_L \times U(1)_Y$ electroweak gauge symmetry with a discrete Z_5 symmetry[14] which might be the product of a more fundamental underlying theory. This discrete symmetry will be broken spontaneously resulting in the appearance of domain walls. However, it is now known[15] that higher-dimensional operators, induced at the Planck scale, can make such domain walls collapse very quickly after formation so that we need not consider this as a potential problem.

We take the Z_5 elements to be ω^{-2}, ω^{-1}, 1, ω, and ω^2 with $\omega^5 = 1$. Let the three lepton families of left-handed electroweak doublets be denoted by $(\nu_\alpha, l_\alpha)_L$, with $\alpha = e, \mu, \tau$. Let the three right-handed charged lepton singlets $l_{\alpha R}$ be accompanied by four right-handed neutrino singlets $(\nu_{\alpha R}, \nu_{SR})$. The Z_5 transformations of the leptons bearing subscripts e, μ, τ, S are chosen to be 1, ω^{-2}, ω^2, ω^{-1} respectively. The scalar sector is assumed to consist of two doublets $\Phi_1 = (\phi_1^+, \phi_1^0)$, $\Phi_2 = (\phi_2^+, \phi_2^0)$ and a complex singlet χ^0 transforming as 1, ω^{-2}, and ω respectively. The charged lepton mass matrix linking $\bar{l}_{\alpha L}$ and $l_{\beta R}$ is now of the form

$$\mathcal{M}_l = \begin{bmatrix} a & 0 & d \\ e & b & 0 \\ 0 & 0 & c \end{bmatrix}, \tag{1}$$

where the diagonal entries a, b, c come from the nonzero vacuum expectation value of ϕ_1^0 and the off-diagonal entries d, e come from that of ϕ_2^0. The zeros of this matrix are protected at tree level by the assumed discrete Z_5 symmetry. As it stands, this mass matrix shows possible $e_L - \tau_L$ but very little $e_L - \mu_L$ mixing. There could be substantial $e_R - \mu_R$ mixing, but that is not observable as far as vector gauge interactions are concerned.

Turning to the neutrino sector, we find the mass matrix spanning $\bar{\nu}_{eL}$, $\bar{\nu}_{\mu L}$, $\bar{\nu}_{\tau L}$, ν_{eR}, $\nu_{\mu R}$, $\nu_{\tau R}$, and ν_{SR} to be given by

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & 0 & 0 & m_1 & m_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & m_7 & 0 & m_3 & 0 \\ m_1 & 0 & m_7 & M_1 & 0 & 0 & m_4 \\ m_6 & m_2 & 0 & 0 & m_8 & M_2 & 0 \\ 0 & 0 & m_3 & 0 & M_2 & m_9 & m_5 \\ 0 & 0 & 0 & m_4 & 0 & m_5 & 0 \end{bmatrix}, \tag{2}$$

where m_1, m_2, m_3 come from $\langle \bar{\phi}_1^0 \rangle$, m_6, m_7 from $\langle \bar{\phi}_2^0 \rangle$, m_4, m_9 from $\langle \chi^0 \rangle$, m_5, m_8 from $\langle \bar{\chi}^0 \rangle$, and M_1, M_2 are allowed mass terms even in the absence of symmetry breaking. The zeros are again protected at tree level by the assumed discrete Z_5 symmetry.

Note first that $\mathcal{M}_\nu$ has one zero mass eigenvalue, corresponding to an eigenstate which is mostly ν_e as we will show. In the absence of symmetry breaking, all the m's are zero and we have only M_1 and M_2, corresponding to one massive Majorana fermion and one massive Dirac fermion respectively. These allowed masses can be very heavy and will act as the anchors for the seesaw mechanisms which generate the requisite small neutrino masses of our model. Note that the 4×4 submatrix spanning $\bar{\nu}_{eL}$, $\bar{\nu}_{\mu L}$, $\bar{\nu}_{\tau L}$, and ν_{SR} is indeed identically zero, even in the presence of symmetry breaking.

Consider now the situation where only m_2, m_3 are made nonzero in addition to M_1, M_2. Then we have a Dirac seesaw mass $m_2 m_3/M_2$ for ν_μ and ν_τ, whereas ν_e and ν_S remain massless. Thus $\mathcal{M}_\nu$ has a global vector $U(1)$ symmetry as well as two chiral $U(1)$ symmetries in this case. If the other m's are also made nonzero, then these symmetries are broken except of course for that corresponding to the zero mass eigenvalue noted before. Hence it is natural[16] to assume that these other m's are much smaller in magnitude than m_2, m_3 which are in turn much smaller than M_1, M_2. As it turns out, we also need to make the ratio m_1/m_4 small because it corresponds to $\nu_e - \nu_S$ mixing for explaining solar neutrino data. To summarize, we assume

$$|m_1| << |m_{4,5,6,7,8,9}| << |m_{2,3}| << |M_{1,2}|. \tag{3}$$

For very large M_1, M_2, the seesaw reduction of $\mathcal{M}_\nu$ yields the following 4×4 mass matrix spanning ν_{SR}, $\bar{\nu}_{eL}$, $\bar{\nu}_{\mu L}$, and $\bar{\nu}_{\tau L}$:

$$\mathcal{M}'_\nu = -\frac{1}{M_1} \left[\begin{array}{cccc} m_4^2 & m_1 m_4 & 0 & m_4 m_7 \\ m_1 m_4 & m_1^2 & 0 & m_1 m_7 \\ 0 & 0 & 0 & 0 \\ m_4 m_7 & m_1 m_7 & 0 & m_7^2 \end{array} \right] - \frac{1}{M_2} \left[\begin{array}{cccc} 0 & m_5 m_6 & m_2 m_5 & 0 \\ m_5 m_6 & 0 & 0 & m_3 m_6 \\ m_2 m_5 & 0 & 0 & m_2 m_3 \\ 0 & m_3 m_6 & m_2 m_3 & 0 \end{array} \right]. \tag{4}$$

Taking the $m_2 m_3/M_2$ entries in the above mass matrix to be dominant, it is easily seen that a further seesaw reduction yields a 2×2 matrix spanning only ν_{SR} and $\bar{\nu}_{eL}$ with elements given by the upper-left-corner submatrix proportional to $1/M_1$. To find the eigenvalues λ' of $\mathcal{M}'_\nu$ which we denote by $-\lambda/M_1M_2$, we write down its characteristic equation:

$$
\begin{aligned}
0 &= \lambda\{\lambda^3 - \lambda^2 M_2(m_1^2 + m_4^2 + m_7^2) \\
&\quad - \lambda[M_1^2(m_2^2m_3^2 + m_2^2m_5^2 + m_3^2m_6^2 + m_5^2m_6^2) + 2M_1M_2m_1m_6(m_4m_5 + m_3m_7)] \\
&\quad + M_1^2M_2[m_1^2m_2^2(m_3^2 + m_5^2) + (m_3m_4 - m_5m_7)^2(m_2^2 + m_6^2)]\}. \qquad (5)
\end{aligned}
$$

Using the mass hierarchy assumed in Eq. (3), the four eigenvalues are easily obtained:

$$\lambda'_1 = 0, \qquad (6)$$

$$\lambda'_2 = -\frac{m_4^2}{M_1}, \qquad (7)$$

$$\lambda'_3 = \frac{m_2m_3}{M_2} - \frac{m_7^2}{2M_1} + \frac{1}{2M_2}\left(\frac{m_2m_5^2}{m_3} + \frac{m_3m_6^2}{m_2}\right), \qquad (8)$$

$$\lambda'_4 = -\frac{m_2m_3}{M_2} - \frac{m_7^2}{2M_1} - \frac{1}{2M_2}\left(\frac{m_2m_5^2}{m_3} + \frac{m_3m_6^2}{m_2}\right). \qquad (9)$$

The corresponding mass eigenstates are then related to the interaction eigenstates by

$$
\begin{bmatrix} \nu_S^c \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} -m_1/m_4 & 1 & m_5/m_3\sqrt{2} & m_5/m_3\sqrt{2} \\ 1 & m_1/m_4 & -m_6/m_2\sqrt{2} & m_6/m_2\sqrt{2} \\ -m_6/m_2 & 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -m_5/m_3 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{bmatrix}. \qquad (10)
$$

In Eqs. (6) to (10) we have consistently retained only the leading terms.

We now have our desired pattern of neutrino masses and mixing. We see that ν_μ and ν_τ are pseudo-Dirac partners with mass difference squared given by

$$\delta m_{34}^2 = \frac{2m_7^2m_2m_3}{M_1M_2}. \qquad (11)$$

The singlet neutrino ν_S is mostly ν_2 which has a small mass, whereas ν_e is mostly ν_1 which is massless. Their mixing is given by m_1/m_4. Furthermore, ν_μ oscillates to ν_e with mixing given by m_6/m_2 as shown by Eq. (10). For illustration, let $M_1 = M_2 = 100$ TeV, $m_2 = 10$ MeV, $m_3 = 25$ MeV, $m_4 = 0.5$ MeV, $m_6 = 0.4$ MeV, $m_7 = 0.6$ MeV, and $m_1 = 20$ keV; then the common mass of ν_μ and ν_τ is $m_2 m_3/M_2 = 2.5$ eV, the mass of ν_S is $m_4^2/M_1 = 2.5 \times 10^{-3}$ eV, $\delta m^2_{34} = 1.8 \times 10^{-2}$ eV^2, $\nu_e - \nu_S$ mixing is $m_1/m_4 = 0.04$, $\nu_e - \nu_\mu$ mixing is $m_6/m_2 = 0.04$, yielding an effective $\sin^2 2\theta = 6.4 \times 10^{-3}$ in either case as indicated by the solar-neutrino and LSND data.

With four light neutrinos, the nucleosynthesis bound[17] of $N_\nu < 3.3$ is an important constraint. Although ν_S is a singlet neutrino, it mixes with ν_e and may contribute significantly to N_ν through oscillations. However, for the matter-enhanced nonadiabatic $\nu_e - \nu_S$ oscillations which explain the solar data, this is not a problem[18]. Note that if we had made $m_1 >> m_4$ instead, then ν_S would be mostly massless and the resulting scenario would be excluded by nucleosynthesis. By the same token, if we had tried to let ν_μ oscillate into ν_S to explain the atmospheric neutrino data, it would also be in conflict with nucleosynthesis. These requirements are sufficient to pin down uniquely our model of four light neutrinos provided we restrict ourselves to only two-oscillator effects[8] and the viewpoint that two almost degenerate neutrinos should be pseudo-Dirac. Of course, this forces us to have maximal mixing in the $\nu_\mu - \nu_\tau$ sector and we must disregard part of the Frejus data[19], but if a smaller effective $\sin^2 2\theta$ (say 0.7) is desired, then the underlying symmetry as well as the mechanism for generating $\mathcal{M}'_\nu$ become much more complicated[7]. As it is, all we need is a discrete Z_5 symmetry and the seesaw mechanism.

Although $\nu_e - \nu_\tau$ mixing is very small as given by Eq. (10), it may instead come from the charged-lepton mass matrix of Eq. (1). Hence our model can also accommodate such an effect. Because two different Higgs doublets contribute to the lepton mass matrices, flavor-changing neutral-current processes do occur via the exchange of scalar bosons. For example, $\mu \to e\gamma$ is possible, but because all the Yukawa couplings are suppressed in this model, these are all negligible as far as present experimental bounds are concerned.

Regarding the Higgs sector which consists of two doublets and a singlet, it can be shown that the imposition of our discrete Z_5 symmetry actually results in a continuous U(1) symmetry which is of course broken in the Yukawa sector as χ^0 couples both to $\nu_{eR}\nu_{SR}$ and $\nu_{\tau R}\nu_{\tau R}$. This means that a pseudo-Goldstone boson appears with a mass of order $m_5/4\pi$ which is too small to be compatible with present data. To avoid this problem, a simple solution is to enlarge the scalar sector with a real singlet η_0 and a neutral complex singlet η_2 transforming under Z_5 as 1 and ω^2 respectively. In order that they do not couple directly to the leptons, they are also assumed to be odd under an extra discrete Z_2 symmetry. Because of the newly allowed terms $\eta_2\eta_2\chi$, $\eta_0\eta_2\bar{\chi}\bar{\chi}$, and $\eta_0\eta_0$, there is no longer any unwanted U(1) symmetry in this case and all the scalar bosons can be heavy.

The 4×4 neutrino mass matrix given by Eq. (4) is similar but not identical to those of Ref. [7]. We assume that ν_μ and ν_τ to be pseudo-Dirac partners whereas Peltoniemi and Valle took ν_μ and a linear combination of ν_τ and ν_e, while Caldwell and Mohapatra did not consider them as pseudo-Dirac partners at all. Our model also differs in that we have a massless eigenvalue in the neutrino mass matrix and they do not. However, the most important difference is in how we realize the desired form of the mass matrix. We use a simple discrete Z_5 symmetry and the seesaw mechanism whereas both papers of Ref. [7] require more complicated symmetries as well as radiative mechanisms for mass generation. Hence their scalar sectors are much more involved and contain many particles of exotic hypercharge whereas we have only the usual doublets and neutral singlets. Presumably, these are experimentally accessible at the electroweak energy scale and would serve as discriminants of one model against another.

In conclusion, we have shown that with all present desiderata, a simple model of four light neutrinos can be constructed with pairwise oscillations explaining the solar neutrino deficit ($\nu_e - \nu_S$), the atmospheric neutrino anomaly ($\nu_\mu - \nu_\tau$), and the LSND observation ($\nu_\mu - \nu_e$). If the δm^2 of the last experiment is indeed 6 eV^2, then there is also the cosmological connection of neutrinos as candidates for hot dark matter, though that would be in marginal conflict with another neutrino experiment[20]. If the experimental inputs chosen by us do indeed stand the test of time, the matrix of Eq. (2) with the hierarchy of Eq. (3) could well be the harbinger of a complete theory of neutrino masses and mixing.

ACKNOWLEDGEMENT

We thank Marc Sher for his helpful comments. One of us (P.R.) acknowledges the hospitality of the Physics Department of the University of California, Riverside for an extended visit. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-94ER40837.

References

[1] Reviews of this subject and many references are given in the following articles: R. S. Raghavan, Science **267**, 45 (1995); N. Hata, S. Bludman, and P. Langacker, Phys. Rev. D **49**, 3622 (1994); G. L. Fogli, E. Lisi, and D. Montanino, *ibid.* **49**, 3626 (1994); J. N. Bahcall and M. Pinsonneault, Rev. Mod. Phys. **64**, 885 (1992); K. V. L. Sarma, Tata Institute Report, hep-ph/9408277, to appear in Int. J. Mod. Phys. A (1995). Original discussions of matter-enhanced solar neutrino oscillations may be found in the papers by L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); *ibid.* **20**, 2364 (1979); S. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1985). Some additional references, among many others, are: V. Barger, R. J. N. Phillips, and K. Whisnant, Phys. Rev. D **43**, 1110 (1991); P. I. Krastev and S. T. Petcov, Phys. Rev. Lett. **72**, 1960 (1994).

[2] Y. Fukuda *et al.*, Phys. Lett. **B335**, 237 (1994) and earlier references therein; R. Becker-Szendy *et al.*, Phys. Rev. D **46**, 3720 (1992); D. Casper *et al.*, Phys. Rev. Lett. **66**, 2561 (1991).

[3] S. A. Bonometto and R. Valdarnini, Phys. Lett. **103A**, 369 (1984); L. Z. Fang, S. X. Li, and S. P. Xiang, Astron. Astrophys. **140**, 77 (1984); A. Deckel and S. J. Aarseth, Ap. J. **283**, 1 (1984); Q. Shafi and F. W. Stecker, Phys. Rev. Lett. **53**, 1292 (1984); A. Klypin, S. Borgani, J. Holtzman, and J. R. Primack, Ap. J. (1995) in press; J. R. Primack, J. Holtzman, A. Klypin, and D. O. Caldwell, Phys. Rev. Lett. **74**, 2160 (1995); D. Pogosyan and A. Starobinsky, Ap. J. (1995) in press.

[4] C. Athanassopoulos *et al.*, Los Alamos National Laboratory Report No. LA-UR-95-1238 (April 1995).

[5] M. K. Moe, Nucl. Phys. B (Proc. Suppl.) **38**, 36 (1995).

[6] Y. Declais *et al.*, Report No. LAPP-EXP-94-19, October 1994, to be published.

[7] D. O. Caldwell and R. N. Mohapatra, Phys. Rev. D **48**, 3259 (1993); J. T. Peltoniemi and J. W. F. Valle, Nucl. Phys. **B406**, 409 (1993).

[8] For a possible scenario involving three-oscillator effects, see E. Ma and J. Pantaleone, Univ. of California, Riverside Report No. UCRHEP-T140 (March 1995), to be published.

[9] Y.-Z. Qian, G. M. Fuller, R. W. Mayle, G. J. Mathews, J. R. Wilson, and S. E. Woosley, Phys. Rev. Lett. **71**, 1965 (1993).

[10] G. M. Fuller, J. R. Primack, and Y.-Z. Qian, Report No. ASTRO-PH-9502081 (February 1995), to be published.

[11] A. Balysh *et al.*, in Proc. of XXVII International Conference on High Energy Physics, Glasgow (July 1994), to be published.

[12] P. Roy and O. Shanker, Phys. Rev. Lett. **52**, 713 (1984); Phys. Rev. D **30**, 1949 (1984); M. Roncadelli and D. Wyler, Phys. Lett. **B133**, 325 (1983).

[13] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, ed. by P. van Nieuwenhuizen and D. Z. Freedman, (North Holland, Amsterdam, 1979), p. 315; T. Yanagida, in Proc. of *Workshop on the Unified Theory and the Baryon Number in the Universe*, ed. by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979).

[14] E. Ma, Phys. Rev. Lett. **68**, 1981 (1992).

[15] J. Preskill, S. Trivedi, F. Wilczek, and M. Wise, Nucl. Phys. **B363**, 207 (1991); B. Rai and G. Senjanovic, Phys. Rev. D **49**, 2729 (1994).

[16] G. 't Hooft, in *Recent Developments in Gauge Theories*, Proc. of the NATO Advanced Study Institutes, Ser. B, Vol. 59, edited by G. 't Hooft *et al.* (Plenum, New York, 1980).

[17] See for example T. Walker, G. Steigman, D. N. Schramm, K. Olive, and H.-S. Kang, Ap. J. **376**, 51 (1991).

[18] X. Shi, D. N. Schramm, and B. D. Fields, Phys. Rev. D **48**, 2563 (1993); see also K. Enqvist, K. Kainulainen, and M. Thomson, Nucl. Phys. **B373**, 498 (1992).

[19] Ch. Berger *et al.*, Phys. Lett. **B227**, 489 (1989); **B245**, 305 (1990).

[20] L. Borodovsky *et al.*, Phys. Rev. Lett. **68**, 274 (1992).