IMSc/2004/10/36
DESY 04-203
October 2004

An SO(10) GUT With See-Saw Masses For All Fermions

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Abstract

We propose an SO(10) grand unified theory which has the simplest Higgs structure discussed so far in the literature. We include only two Higgs scalars, a 210-plet and a 16-plet. In addition to the regular fermions we include one singlet, whose mass term breaks chiral symmetry, so that fermions can get masses. All fermions acquire see-saw masses, since there are no Higgs bi-doublets. Required neutrino masses with large mixing as well as leptogenesis are possible in this model.

The success of the standard model has motivated us to develop further our understanding of the basic interactions. Although the only knowledge we have for physics beyond the standard model comes from the neutrino mass, we have several theoretical motivations to think about the extensions ahead of any experimental inputs. The most natural extension of the standard model is grand unification, in which all the gauge coupling constants are unified at some high energy and we have a simple unified gauge group governing all gauge interactions of nature. The simplest grand unified theory is based on the gauge group $SU(5)$. But the minimal $SU(5)$ GUT has some problems that involve fermion masses, proton decay, gauge coupling unification, etc. These problems may be eliminated in some extensions of the model, but there is another compelling reason to consider larger grand unified group. In $SU(5)$ the left-handed and right-handed fermions are treated in different ways, so parity is never conserved.

A natural extension of the standard model is the left-right symmetric extension, in which parity is conserved at higher energies and could be broken spontaneously [1]. The simplest grand unified theory to accomodate the left-right symmetric extension of the standard model is based on the gauge group $SO(10)$. In recent times it has been noticed that in the $SO(10)$ GUTs several interesting features come out naturally [2, 3, 4, 5, 6]. It has been realized that the minimal $SO(10)$ GUT has considerable predictive power and thus attempts to construct GUTs with simplest Higgs choice have become a challenging question [2, 3, 4, 5].

In all the left-right symmetric models one requires a bi-doublet Higgs scalar (which is a doublet under both the left-handed and right-handed $SU(2)$ groups) to give masses to the fermions and break the electroweak symmetry. In a recent article it was pointed out that for symmetry breaking the most economic choice is to consider $SU(2)_R$ Higgs doublet to break left-right symmetry and a $SU(2)_L$ doublet to break the electroweak symmetry [7] and an explicit realization was then presented in a supersymmetric model [8]. Even without a bi-doublet of Higgs scalar it is possible to get fermion masses through see-saw contributions. Since a natural explanation of the smallness of the neutrino masses requires see-saw mechanism [9], it will appear more natural if all fermion masses have same see-saw origin. See-saw contributions of fermion masses were also studied extensively in the past with heavy fermions [10, 11] to understand the flavor structure of the fermions, but the present interest for see-saw fermion mass is solely from the point of view of minimality of the theory. In this article we intend to give a realization of this idea of left-right symmetric models without any bi-doublet Higgs scalars in an $SO(10)$ GUT with the simplest possible Higgs structure considered so far. Most of the $SO(10)$ GUTs considers Higgs triplets [12] to break the left-right symmetry for several reasons [2, 3, 4, 6]. However, Higgs doublets have also been considered for the left-right symmetry breaking [13]. In particular, in superstring inspired models and in recent times in orbifold GUTs Higgs doublets are the only choices [14].

All grand unified theories have the gauge hierarchy problem, which requires fine tuning of parameters to maintain the electroweak symmetry breaking scale light. This problem is solved by making the theory supersymmetric. In supersymmetric theories although we need to make the fine tuning at the tree level, there are no radiative quadratic divergences which need to be fine tuned at each order of perturbation theory. Supersymmetric theories are also advantageous because the cosmological constant vanishes in the limit of exact supersymmetry. However, in nature we have not seen any signals of supersymmetry upto the electroweak symmetry breaking scale and at the same time we have also found a non-vanishing cosmological constant. Supersymmetry cannot explain the smallness of the cosmological constant and we may need to invoke fine tuning to understand this.

Hence in recent times the question has arisen whether we really need supersymmetry [15]. If we need fine tuning to understand the cosmological constant, then perhaps we may need fine tuning to understand the gauge hierarchy problem. It is hoped that some new physics at very high energy will solve the fine tuning problems [16] without invoking low energy supersymmetry. The advantages of giving up supersymmetry at low energy are manifold since this gets rid of problems like the μ problem, CP problem and flavor problem, which are all associated with low energy supersymmetry. We shall thus not consider supersymmetry in our construction at any stage and do not bother about any fine tuning required to get any particular solution.

We first start with the different possibilities of constructing an $SO(10)$ GUT without any 10-plet of Higgs scalars, which contains a bi-doublet. Then we shall proceed to construct the simplest possible model in terms Higgs representations. The Higgs sector in our model is the smallest compared to all the existing models of $SO(10)$ GUT. We introduce only two Higgs scalars for all the symmetry breaking and for giving masses to all the fermions including neutrinos. For chiral symmetry breaking we have to introduce one heavy fermion singlet, but otherwise there are no new ingredients.

In $SO(10)$ GUTs the fermions of each generation (including a right-handed neutrino) belong to the 16-plet spinor representation, which transforms under the Pati-Salam subgroup ($G_{422} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$) as,

$$\psi_{iL} \equiv \mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}).$$

$i = 1, 2, 3$ is the generation index. The right-handed fermions (ψ_{iR}) then belong to the conjugate representation,

$$\psi_{iR} \equiv \bar{\mathbf{16}} = (\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}, \mathbf{1}, \mathbf{2}).$$

The generators of the left-right symemtric group $G_{3221} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are then related to the electric charge by

$$Q = T_{3L} + T_{3R} + \frac{(B-L)}{2} = T_{3L} + \frac{Y}{2}.$$

The quarks and leptons then transform as

$$\begin{aligned} (4,2,1) &= \left\{ \begin{array}{l} q_L = (\, u \quad d\,)_L \equiv (3,2,1,1/3) \\ \ell_L = (\, \nu \quad e\,)_L \equiv (1,2,1,-1) \end{array} \right. \\ (\bar{4},1,2) &= \left\{ \begin{array}{l} {q_R}^c = {q^c}_L = (\, d^c \quad u^c\,)_L \equiv (\bar{3},1,2,-1/3) \\ {\ell_R}^c = {\ell^c}_L = (\, e^c \quad \nu^c\,)_L \equiv (1,1,2,1) \end{array} \right. \end{aligned} \tag{1}$$

where (x,y,z) and (x,y,z,w) denote the transformation property under G_{422} and G_{3221} respectively. Similarly the right-handed fermions belong to $\overline{16}$.

The minimal Higgs representation required to break the groups $SU(2)_L$ and $SU(2)_R$ are two Higgs doublets, both of which belong to a **16**-plet Higgs representation Γ. There are two neutral components of Γ,

$$\begin{aligned} \chi_L &\equiv (1,2,1,-1) \subset (4,2,1) \subset 16 \equiv \Gamma \\ {\chi^c}_L &\equiv (1,1,2,1) \subset (\bar{4},1,2) \subset 16 \equiv \Gamma \end{aligned}$$

and the corresponding conjugates belonging to $\Gamma^\dagger$,

$$\begin{aligned} \chi_R &\equiv (1,1,2,-1) \subset (4,1,2) \subset \overline{16} \equiv \overline{\Gamma} \\ {\chi^c}_R &\equiv (1,2,1,1) \subset (\bar{4},2,1) \subset \overline{16} \equiv \overline{\Gamma}. \end{aligned}$$

The vacuum expectation values (*vev*s) of the Higgs fields χ_R and ${\chi^c}_L$ can break the left-right symmetry to the standard model ($G_{3221} \rightarrow G_{321} \equiv S(3)_c \times SU(2)_L \times U(1)_Y$) and the *vev*s of χ_L and ${\chi^c}_R$ can break the electroweak symmetry. However, these fields cannot give masses to the fermions since chiral symmetry is not broken by any of these Higgs scalars.

There is one more problem with these Higgs scalars, which has to be taken care of. Usually parity is conserved in most left-right symmetric models, so that spontaneous breaking of parity can give an explanation of the origin of parity violation. It is also possible to start with a theory, in which parity is explicitly broken so that the left-handed gauge couplings could be different from the right-handed gauge couplings. In theories with conserved left-right parity, it is difficult to give different and non-zero vevs to the fields χ_L and χ_R. The minimization of the potential leads to either equal values of these vevs or zero value for at least one of them. To solve this problem one may consider an extra $O(2)$ symmetry, which is most unnatural [17]. A natural solution to this problem is obtained by breaking parity. For any low energy theories, it is convenient to start with an explicit parity violation and hence with different left-handed and right-handed couplings. However, since we are starting from an $SO(10)$ GUT which includes parity as one of its generators, we shall consider a more appealing scenario in which parity is spontaneously broken before the left-right symmetry breaking by a parity odd singlet field [18].

In $SO(10)$ GUTs, the discrete left-right parity is a generator of the group, which is referred to as $D-$parity [18]. As a result, to start with, parity is always conserved. So starting from $SO(10)$ it is not possible to construct any model with explicit parity violation. However, it is possible to break parity at very high energy spontaneously, before the breaking of $SU(2)_R$, which will then allow us to have $\langle\chi_L\rangle \neq \langle\chi_R\rangle$. This is done by giving a *vev* to some $D-$parity odd field. Before we proceed further, let us discuss the other Higgs scalar in the model.

We also need one more Higgs scalar to break the $SO(10)$ group to its left-right symmetric subgroup. Although there are more than one possibilities, we choose the **210** dimensional representation, since it also serves another purpose which we shall discuss later. The **210** fields decompose under the Pati-Salam subgroup (G_{422}) as,

$$\begin{aligned}\Phi \equiv \mathbf{210} &= (\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{6},\mathbf{2},\mathbf{2}) + (\mathbf{15},\mathbf{3},\mathbf{1}) + (\mathbf{15},\mathbf{1},\mathbf{3}) \\ &\quad + (\mathbf{15},\mathbf{1},\mathbf{1}) + (\mathbf{10},\mathbf{2},\mathbf{2}) + (\overline{\mathbf{10}},\mathbf{2},\mathbf{2})\end{aligned}$$

The field Φ breaks the group $SO(10)$ to G_{3221} when the components $(1,1,1)$ and $(15,1,1)$ acquire *vev*s at the GUT scale M_U. The component $(1,1,1)$

is odd under D-parity of the group $SO(10)$. As a result, this allows the left-handed neutral components of Γ to become light while keeping the right-handed components heavy. This parity odd singlet plays a crucial role in giving neutrino masses. In the absence of this parity odd field the lightest neutrino always remain massless.

Since this is crucial for our discussion, we shall ellaborate this point. Consider the $SO(6)\times SO(4)$ subgroup of $SO(10)$, which is also the Pati-Salam subgroup since $SO(6)$ is isomorphic to $SU(4)$ and $SO(4)$ is isomorphic to $SU(2)\times SU(2)$. The **210** representation is a totally antisymmetric tensor of rank four Φ_{abcd} and the singlet $(1,1,1)$ is the component Φ_{6789} in the notation in which $a,b,c,d = 0,1,..,5$ are $SO(6)$ indices and $a,b,c,d = 6,7,8,9$ are $SO(4)$ indices. It can be shown that the action of the $D-$parity operator on this field gives -1 and hence it is odd. However, for an easy understanding we shall give a simple argument showing why this is odd under $D-$parity. There are two singlets in $\mathbf{16}\times\overline{\mathbf{16}} = \mathbf{1}+\mathbf{45}+\mathbf{210}$ which are, $A = (4,2,1)\times(\bar{4},2,1)$ and $B = (\bar{4},1,2)\times(4,1,2)$. Under $D-$parity $A \leftrightarrow B$, and hence any of these singlets A or B cannot be the singlet of $SO(10)$, since $D-$parity is a generator of $SO(10)$. So, the combination $A+B = 1$ of $SO(10)$ and the $A-B = (1,1,1) \subset 210$ and hence under $D-$parity, $\Phi_{6789} \leftrightarrow -\Phi_{6789}$.

We shall now come back to the Higgs doublets and show how the D-parity breaking will make the fields χ_L and χ_R^c as light as 100 GeV, so that *vevs* of these fields can break the electroweak symmetry $\langle\chi_L\rangle = u_L \sim 100$ GeV. The right-handed components χ_R and χ_L^c will remain almost as heavy as the GUT scale $\langle\chi_R\rangle = u_R \sim M_R \sim 10^{14}$ GeV. This requires fine tuning, but as we argued at the begining we allow it. The D-parity breaking also ensures the inequality of the *vevs* u_L and u_R.

We shall first write down the potential for the scalar fields in this model,

$$\begin{aligned}\mathcal{L}_s &= m_\Phi^2\Phi^2 + \eta\Phi^3 + \frac{\lambda_\Phi}{4!}\Phi^4 + m_\Gamma^2\Gamma^\dagger\Gamma + \frac{\lambda_\Gamma}{4}(\Gamma^\dagger\Gamma)^2 \\ &+ \frac{\lambda_\Gamma'}{4}[\Gamma^4 + (\Gamma^\dagger)^4] + M_D\Phi(\Gamma^\dagger\Gamma) + \lambda_{\Phi\Gamma}\Phi^2(\Gamma^\dagger\Gamma) \qquad (2)\end{aligned}$$

The coupling $\Phi\Gamma\Gamma^\dagger$ is the crucial term, which breaks the D-parity when the singlet component (1,1,1) of Phi acquires nonvanishing vev and gives the mass splitting between χ_L and χ_R. The *vevs* u_L and u_R also split and the lightest neutrino gets mass due to this term.

We shall now discuss the masses of the components of Γ and the *vevs*. The singlet component of Φ (1,1,1) is odd under the D-parity, which is the parity operator acting on the $SO(10)$ group space. If we denote this component as $\eta = \Phi(1,1,1)$, then the scalar potential responsible for the masses of the fields χ_L and χ_R is given by,

$$\begin{aligned} V &= m_\Gamma^2(\chi_L^c\chi_R + \chi_R^c\chi_L) + M_D\ \eta(\chi_L^c\chi_R - \chi_R^c\chi_L) \\ &+ \lambda_{\Phi\Gamma}\ \eta^2(\chi_L^c\chi_R + \chi_R^c\chi_L). \end{aligned} \tag{3}$$

The masses of these fields are then given by,

$$\begin{aligned} \mu_L^2 &= m_\Gamma^2 - M_D\langle\eta\rangle + \lambda_{\Phi\Gamma}\langle\eta\rangle^2, \\ \mu_R^2 &= m_\Gamma^2 + M_D\langle\eta\rangle + \lambda_{\Phi\Gamma}\langle\eta\rangle^2. \end{aligned} \tag{4}$$

With proper fine tuning we now get,

$$\begin{aligned} \langle\chi_L\rangle &= \langle\chi_R^c\rangle = u_L \sim \mu_L \sim 100 \text{ GeV} \\ \langle\chi_R\rangle &= \langle\chi_L^c\rangle = u_R \sim \mu_R \sim M_U \gg u_L. \end{aligned} \tag{5}$$

Thus the two components of the field Γ acquire widely different masses and *vev*s. u_L breaks the electroweak symmetry, while u_R breaks the left-right symmetry at a very high scale, close to the GUT scale.

This demonstrates how the fields Φ and Γ are sufficient to break the group $SO(10)$ to the left-right subgroup at the GUT scale and then at some intermediate scale break the left-right symmetric model to the standard model and subsequently break the electroweak symmetry. We shall now discuss how one can give masses to the fermions without introducing any new Higgs scalars, although we may introduce new fermions, which may acquire masses at very high scale breaking the chiral symmetry.

We shall first consider only tree level mass generation. Let us start with a list of dimension-5 operators, which can give Dirac masses to the quarks and leptons and Majorana masses to the left-handed and right-handed neutrinos. They are,

$$\begin{aligned} \mathcal{O}_1 &= (\overline{q_L}\chi_L)(q_R{\chi_R}^c) & \qquad \mathcal{O}_2 &= (\overline{q_L}{\chi_L}^c)(q_R\chi_R) \\ \mathcal{O}_3 &= (\overline{\ell_L}\chi_L)(\ell_R{\chi_R}^c) & \qquad \mathcal{O}_4 &= (\overline{\ell_L}{\chi_L}^c)(\ell_R\chi_R) \\ \mathcal{O}_5 &= (\ell_L{\chi_L}^c)(\ell_L{\chi_L}^c) & \qquad \mathcal{O}_6 &= (\ell_R{\chi_R}^c)(\ell_R{\chi_R}^c) \end{aligned}$$

$\mathcal{O}_1$ and $\mathcal{O}_2$ contributes to up and down quark masses, $\mathcal{O}_3$ gives Dirac masses to the neutrinos, $\mathcal{O}_4$ contributes to charged lepton masses and $\mathcal{O}_5$ and $\mathcal{O}_6$ are the Majorana masses for the left-handed and right-handed neutrinos.

These operators may be realized in three ways. A scalar field may mediate these terms, in which case we are back to the conventional models since the intermediate scalars have to be either **10**, **120** or **126**. The next possibilities are when these effective terms are generated by intermediate fermions. In this case we need two fermions for each of the first four operators, one left-handed fermion and one right-handed fermions with same quantum numbers, while for the last two operators we may not require any new fermions. With only one fermion it is not possible to generate any of the terms, since that will not break the chiral symmetry. So, we have to introduce two fields with same quantum numbers but opposite chirality and allow the mass terms of these fields in the Lagrangian which will break the chiral symmetry.

For $\mathcal{O}_1$ we need one left-handed and one right-handed fields both transforming as $U_{L,R} \equiv (3,1,1,4/3) \subset (15,1,1) \subset 45$ or 210. Then the essential terms in the Lagrangian which can allow $\mathcal{O}_1$ are given by,

$$\mathcal{L}_1 = a_1 \overline{U_L} q_R {\chi_R}^c + b_1 \overline{q_L} U_R \chi_L + m_U \overline{U_L} U_R + H.c. \tag{6}$$

Similarly, for the operator $\mathcal{O}_2$ we need the fields $D_{L,R} \equiv (3,1,1,-2/3) \subset (6,1,1) \subset 10$ or 126 or $D_{L,R} \equiv (3,1,1,-2/3) \subset (10,1,1) \subset 120$. For the operator $\mathcal{O}_4$ we need $E_{L,R} \equiv (1,1,1,-2) \subset (10,1,1) \subset 120$. The neutrino sector is somewhat simpler and we can manage with $S_{L,R} \equiv (1,1,1,0) \subset (1,1,1) \subset 1$ or 45. Thus the simplest possibility will be to have one 120 to give masses to the down quarks and charged leptons and one 45 to give masses to the up quarks and the neutrinos.

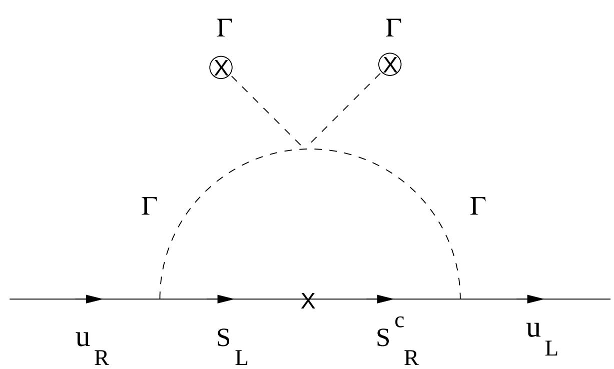

Figure 1: One loop diagram contributing to the fermion masses.

We shall now discuss the third and most interesting possibility with radiative generation of the fermion masses. We now include one heavy $SO(10)$ singlet fermions $S_{aL} \equiv \mathbf{1} = (\mathbf{1}, \mathbf{1}, \mathbf{1})$ per generation ($a = 1, 2, 3$). The most general Yukawa couplings are then given by,

$$\mathcal{L}_Y = f\overline{S^c_R}\psi_L\Gamma^\dagger + f\overline{S_L}\psi_R\Gamma + M_S S_L S_L + M_S S^c_R S^c_R. \tag{7}$$

We have written the Hermitian conjugate term separately for clarity. Since we are not discussing the question of CP violation, we assume all couplings are real. Chiral symmetry is broken by the mass term of the singlet M_S. We also assume that the scale of chiral symmetry breaking, M_S is close to the GUT scale.

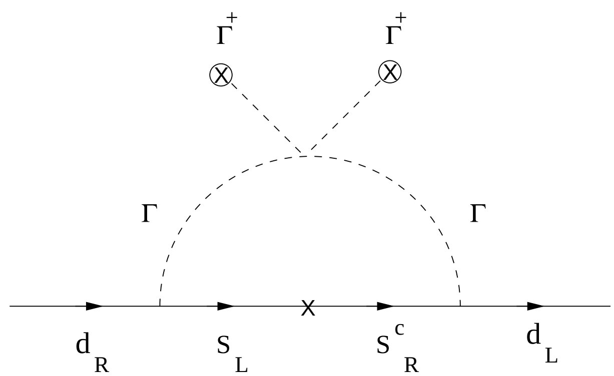

Figure 2: One loop diagram contributing to the fermion masses.

Once chiral symmetry is broken, fermions can get masses through one loop diagrams of figure 1 and figure 2. The diagram in figure 1 generates the effective operators $\mathcal{O}_1$ and $\mathcal{O}_3$, which are of the form

$$\overline{\psi_L}\ \psi_R\ \chi_L\ {\chi^c}_L \subset \overline{\psi_L}\ \psi_R\ \Gamma\ \Gamma. \tag{8}$$

On the other hand the diagram in figure 2 generates the effective operators $\mathcal{O}_2$ and $\mathcal{O}_4$, which are of the form

$$\overline{\psi_L}\ \psi_R\ \chi_R\ {\chi^c}_R \subset \overline{\psi_L}\ \psi_R\ \Gamma^\dagger\ \Gamma^\dagger. \tag{9}$$

Since the term $(\Gamma^\dagger\Gamma)^2$ enters in figure 1, while $(\Gamma^4 + {\Gamma^\dagger}^4)$ enters in figure 2, the up and down quark masses are not the same. The question of neutrino masses is somewhat complicated and we shall discuss it later.

We can now write down the fermion masses in this model. For convenience we define,

$$m_L = f u_L, \qquad \text{and} \qquad m_R = f u_R.$$

The diagram of figure 1 generates the up quark mass,

$$M_u = \frac{\lambda_\Gamma}{8\pi}\frac{m_R m_L}{M_X} \tag{10}$$

and figure 2 generates the down quark and charged lepton masses,

$$M_{d,\ell} = \frac{\lambda'_\Gamma}{8\pi}\frac{m_R m_L}{M_X}. \tag{11}$$

Here $M_X = M_\Gamma^2/M_S$ or M_S, depending on whether M_Γ or M_S is larger.

The up and down quark mass differences are explained by the different coupling constants λ_Γ and λ'_Γ. Although this also gives the $b-\tau$ unification, it does not give us the right fermion mass relations for the first and second generations. We hope that some new physics near the GUT scale can solve this problem. For example, if this $SO(10)$ GUT descends from a E_6 GUT, then the fundamental representation of E_6 will contain a $10-$plet and a singlet of fermion. The $10-$plet fermion can now be very heavy and can contribute to only the down quark sector, solving this fermion mass problem. In fact, if there are heavy fermions $D_{L,R}$ or $E_{L,R}$ in the representations 120 or 126, they can also solve this fermion mass problem. There could be other particles in the loop, which can contribute differently to the fermions solving this problem which was discussed in some of the earlier references [10, 11].

We shall now come to the question of neutrino masses. Neutrino masses with doublets and singlets have already been studied in the literature [5, 13, 19]. However, in the earlier papers Higgs bi-doublet was present and our scenario without any Higgs bi-doublet has a special feature due to $\mathcal{D}$ parity which we shall explain. Although there are radiative corrections to the neutrino masses, we may neglect them in comparison to the tree level contributions. For completeness of our discussions we shall consider them. The neutrinos will now mix with the singlet fermion S_L. Although there are no mass terms for the neutrinos, due to this mixing neutrinos will get an induced mass. We can now write down the mass matrix in the basis $(\,\nu_L \quad {\nu^c}_L \quad S_L\,)$,

$$M_\nu = \begin{pmatrix} 0 & 0 & m_L \\ 0 & 0 & m_R \\ m_L & m_R & M_S \end{pmatrix}. \tag{12}$$

Diagonalization of this matrix will give two heavy states S_L and ${\nu^c}_L$ with eigenvalues M_S and m_R^2/M_S, but the left-handed neutrinos will remain massless. One loop contributions do not solve this problem since they are also proportional to the effective contributions one can get after integrating out the heavy singlet field. As a result the left-handed neutrinos still remain massless. One may try to extend the theory with two or more singlets, but even then the determinant of the mass matrix vanishes and the lightest left-handed neutrino cannot get any mass. Apparently the left-right $\mathcal{D}$ parity symmetry makes the determinant of the neutrino mass matrix to vanish as we shall discuss next.

The effective neutrino Dirac mass term comes from the operator of equation 8 and including the Majorana masses the effective operator can be written as

$$\nu_L\;\nu_L\;{\chi^c}_R\;{\chi^c}_R + {\nu^c}_L\;{\nu^c}_L\;\chi_R\;\chi_R + {\nu^c}_L\;\nu_L\;\chi_R\;{\chi^c}_R \subset \psi_L\;\psi_L\;\Gamma^\dagger\;\Gamma^\dagger + H.c.. \quad (13)$$

Thus the effective neutrino mass term in the basis $\begin{pmatrix}\nu_L & {\nu^c}_L\end{pmatrix}$ takes the form (with β taken to be some effective coupling constant)

$$M_\nu = \frac{\beta}{M_S}\begin{pmatrix} m_L^2 & m_L m_R \\ m_L m_R & m_R^2 \end{pmatrix}$$

whose determinant vanishes because the coupling constants are the same for ν_L and ν_L^c, implying a massless left-handed neutrino, since $u_R \gg u_L$. It should be noted that this is also related to the $D-$parity. Thus if we can include the $D-$parity odd singlet Φ in this effective neutrino mass operator, then this problem may be solved. We do this by combining the effective operator of equation 13 with the effective operator

$$\psi_L\;\psi_L\;\Gamma^\dagger\;\Gamma^\dagger\;\Phi + H.c \supset -\nu_L\;\nu_L\;{\chi^c}_R\;{\chi^c}_R\;\eta + {\nu^c}_L\;{\nu^c}_L\;\chi_R\;\chi_R\;\eta. \quad (14)$$

Since the field η is odd under parity and the first term goes to the Hermitian conjugate of the second term, there is a $(-ve)$ sign in the first term. In addition, the Dirac mass now disappears since it is even under D-parity while η is odd.

The effective operators of equation 13 and equation 14 could come from the figures 3(a) and 3(b) respectively. Including both these contributions we can now write down the neutrino mass matrix as,

$$M_\nu = \begin{pmatrix} (1-\alpha)\frac{m_L^2}{M_S} & \frac{m_L m_R}{M_S} \\ \frac{m_L m_R}{M_S} & (1+\alpha)\frac{m_R^2}{M_S} \end{pmatrix}. \quad (15)$$

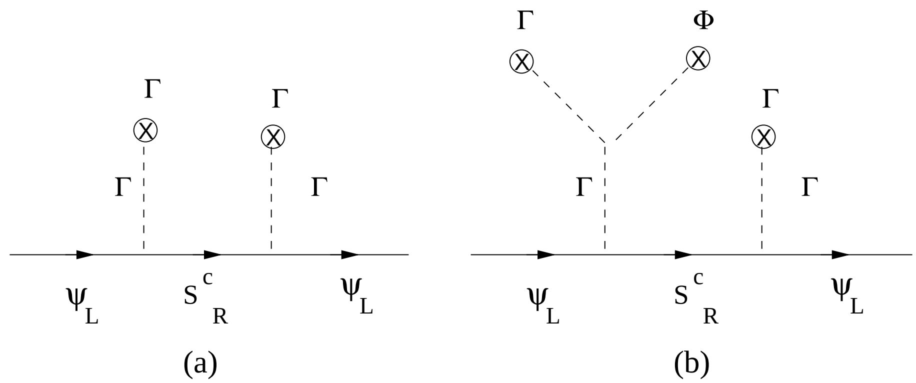

Figure 3: Tree level diagrams contributing to the neutrino masses.

where, $\alpha = M_D \langle\Phi\rangle / m_\Phi^2$. In the limit $\alpha > -1$ and $m_L \ll m_R$, diagonalization of this matrix gives a heavy neutrino ${\nu^c}_L$ with mass of the order of m_R^2/M_S and the left-handed neutrino remains light with mass

$$m_\nu = \frac{\alpha^2}{(1+\alpha)} \frac{m_L^2}{M_S}. \tag{16}$$

With m_L to be of the order of electroweak symmetry breaking scale and M_S to be of the order of the GUT scale, this gives the required neutrino mass to be fraction of an eV. It is to be noted that all fermion masses are of see-saw type. Since the Dirac masses are of the form, $m_L m_R/M_S$, they can be as heavy as top quark masses, while the Majorana mass for the left-handed neutrinos is of the form m_L^2/M_S and hence remain very light and of the order of fraction of eV.

Since the neutrino masses now depend on the couplings with the singlets, there is no stringent restriction coming from the up quark masses. As a result, it may be possible to get large neutrino mixing angles. The right-handed neutrinos and the new singlet fermions can now decay into light leptons. The Majorana masses of the left-handed and right-handed singlets violate lepton numbers, which in turn can generate enough lepton asymmetry. Before the electroweak phase transition this asymmetry can then generate a baryon asymmetry of the universe [20]. Since there is no supersymmetry, the gravitino bounds are not present. The out-of-equilibrium condition can be satisfied near the GUT scale since the couplings are large to get the required neutrino mass with large see-saw scale. In this model there is another interesting feature that the singlets combine with the right-handed neutrinos to form pseudo-Dirac particles and hence resonant leptogenesis is also possible [21, 22]. We shall present all these details in a forthcoming article [23].

In conclusion, we constructed an $SO(10)$ GUT without any Higgs bi-doublets. All the symmetry breaking could be achieved by only two Higgs scalars, a **210** and a **16**. By including a massive singlet fermion we break chiral symmetry which can then give masses to all the fermions radiatively without introducing any new scalar fields. All fermion masses have the same see-saw form. The spontaneous parity breaking plays a crucial role in breaking the left-handed and right-handed $SU(2)$ groups at two widely different scales and also giving masses to the left-handed neutrinos in this scenario. Large neutrino mixing and leptogenesis is also possible in this scenario.

Acknowledgement We thank Prof E. Ma and Dr M. Frigerio for discussions. One of us (US) would like to thank Prof W. Buchmuller for inviting him to visit DESY and acknowledge hospitality at DESY. GR acknowledges the support of the DAE-BRNS Senior Scientist Scheme and the hospitality of the Physics Department,University of California, Riverside. BRD acknowledges the support in part by the U.S. Department of Energy under Grant No. DE-FG03-94ER40837.

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