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Open-Closed Duality at Tree Level

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Abstract

We study decay of unstable D-branes in string theory in the presence of electric field, and show that the classical open string theory results for various properties of the final state agree with the properties of closed string states into which the system is expected to decay. This suggests a duality between tree level open string theory on unstable D-branes and closed strings at high density.

Recent studies involving decay of unstable D-branes in string theory show that at open string tree level such systems evolve to a configuration of zero pressure but non-zero energy density localized in the plane of the brane[1]. Study of closed string emission from this system indicates that once this process is taken into account the system decays into a set of highly non-relativistic, very massive closed strings[2, 3]. If we regard each of these closed strings as a *point-like* massive object without any internal structure, such a system of closed strings will have small ratio of pressure to energy density, and furthermore since the strings move very slowly away from the plane of the brane, the stress tensor of this system of closed strings will be localized close to the plane of the brane. This agrees with what is seen in the tree level open string analysis, and suggests that the tree level open string theory already knows about the fate of the closed strings that the system decays into[4].[1] There is, however, an intrinsic spread of the energy density away from the plane of the brane due to the internal oscillation of the final state closed strings. This effect does not seem to be captured by the tree level open string results. This may not necessarily be a contradiction, since defining the detailed profile of the energy-momentum tensor $T_{\mu\nu}$ requires us to make an ad hoc choice of the off-shell continuation of the graviton field. In contrast, the integrated $T_{\mu\nu}$ in the asymptotic future is the source of a (near) on-shell zero momentum graviton[6], and for this there is perfect agreement between the open and the closed string results.

In this paper we study the decay of unstable D-branes in string theory in the presence of electric field. Such a system carries fundamental string winding charge along the direction of the electric field. The time evolution of the stress tensor and the winding charge density was determined in [7, 8] at open string tree level. In appropriate region in the parameter space we can estimate the stress tensor and winding charge density of the final state closed strings to which this system is expected to decay. We compare these with the asymptotic values of the answers obtained in tree level open string theory and again find agreement between the two sets of quantities. Although we shall not discuss this explicitly, the dilaton coupling computed from open and closed string descriptions also agree, both being zero. This analysis therefore supports the conjectured duality between the results in tree level open string theory on an unstable D-brane and the properties of the high density closed string states to which such D-branes decay. This is different

[1] Of course, the precise final state will depend on the details of the quantum state of the initial D-brane[4, 5], but this is presumably reflected in an ambiguity in the quantum corrections to the tree level open string results.

from the usual open closed duality, in which the open string *loop amplitudes* contain information about the closed string poles[9].

Our analysis will be valid for D-branes in bosonic string theory, as well as non-BPS D-branes or brane-antibrane systems in type IIA or IIB string theory. We shall assume, for technical reasons, that the coordinates along the D-brane world-volume are compact so that the total energy and other charges of the system are finite. We shall denote by x^1 the coordinate along which the electric field on the non-BPS D-p-brane points, by x^i $(2 \le i \le p)$ the rest of the spatial world-volume coordinates on the D-p-brane, and by $\vec{x}_\perp$ the coordinates transverse to the D-p-brane. The classical solution describing the spatially homogeneous decay of the D-p-brane is parametrized by two parameters, the electric field e, and a constant $\widetilde{\lambda}$ labelling the initial position from which the tachyon starts rolling. In the asymptotic future the components of the stress tensor $T^{\mu\nu}$ and the source $S^{\mu\nu}$ for the antisymmetric tensor field $B_{\mu\nu}$ approach the form[7, 8]

$$
\begin{aligned}
& T^{00} = \Pi\, e^{-1}\, \delta(\vec{x}_\perp)\,, \quad T^{11} = -\Pi\, e\, \delta(\vec{x}_\perp)\,, \quad S^{01} = \Pi\, \delta(\vec{x}_\perp)\,, \\
& T^{\mu\nu} = S^{\mu\nu} = 0 \quad \text{for other } \mu,\, \nu\,,
\end{aligned} \tag{1}
$$

where

$$
\Pi \equiv e\,(1-e^2)^{-1/2} \widetilde{\mathcal{T}}_p\, \cos^2(\widetilde{\lambda}\pi)\,. \tag{2}
$$

$\widetilde{\mathcal{T}}_p$ denotes the (sum of the) tension(s) of the original D-branes. In the limit

$$
e \to 1, \qquad \widetilde{\lambda} \to \frac{1}{2}, \qquad \Pi \quad \text{fixed}\,, \tag{3}
$$

we get[7]

$$
T^{00} = \Pi\, \delta(\vec{x}_\perp)\,, \qquad T^{11} = -\Pi\, \delta(\vec{x}_\perp)\,, \qquad S^{01} = \Pi\, \delta(\vec{x}_\perp)\,. \tag{4}
$$

Since $|T^{00}| = |S^{01}|$, we see that the total energy available to the system is saturated by the BPS bound from the closed string winding charge. This in turn implies that the only possible final closed string states into which the system can decay in this limit is a set of zero velocity closed strings wound along the direction x^1 without any oscillator excitations. The stress-tensor and winding charge density associated with such a system of closed strings agree precisely with (4)[7]. Thus we see that in this case the classical open string theory results (4) reproduce the expected property of the system of closed strings to which the system decays. In fact, the identification of these solutions with closed strings was first proposed in the context of effective field theory[10] before exact classical solutions were found using the techniques of boundary conformal field theory.

In what follows, we shall carry out a similar analysis without taking the limit (3). Since $\widetilde{\mathcal{T}}_p \sim g^{-1}$, g being the closed string coupling constant, we see from (2) that Π is of order g^{-1} for generic e and $\widetilde{\lambda}$ of order one. We shall work in the weak coupling limit so that $|\Pi|$ is large, and assume, without any loss of generality, that Π is positive. Our goal will be to analyze the final closed string state produced in the decay of this D-brane and check if the stress tensor and winding number density associated with this state agree with the tree level open string answers (1). We note first of all that due to winding charge and energy conservation, the final state must carry a net winding charge and energy equal to that given by S^{01} and T^{00} in (1). Thus the final state must have (a set of) closed strings wound along x^1. From (1) we also see that the system has an excess energy density $(T^{00} - S^{01}) \propto \Pi(e^{-1} - 1)$ beyond what is given by the BPS bound. In order to calculate properties of the final closed string state, we need to understand where this excess energy density resides in the final state. We can consider two extreme cases:

1. All the excess energy density resides in the oscillation modes of 'macroscopic' closed strings for which the (winding charge / energy) ratio is finite.

2. The decay product contains 'microscopic' closed strings for which the winding charge is an insignificant fraction of their energy, and all the excess energy density resides in these microscopic closed string states.

Of course we also have the more general possibility where some part of the excess energy resides in the macroscopic strings and the rest resides in the microscopic strings. We shall now present several (not totally independent) arguments showing that for generic e and $\widetilde{\lambda}$ possibility 1 is realized, *i.e.* the excees energy density almost fully resides in the macroscopic strings. Furthermore in the $g \to 0$ limit the transverse velocity of these strings vanish, and the (energy / winding charge) ratio of each of these strings is given by e^{-1}.

1. In [2] it was shown that as the tachyon on a non-BPS D-brane rolls down towards the vacuum in the absence of any electric field, the total energy carried by the closed strings produced during the decay is formally infinite. This in turn indicates that all the energy of the initial D-brane is carried away by these closed strings. In this

analysis there was a delicate cancellation between the exponentially growing density of states of the final state closed strings and the exponentially suppressed amplitude for production of these strings. If we repeat the analysis for decay of a non-BPS D-brane with electric field, we get a finite answer (measured in $\alpha' = 1$ unit) for the total energy carried by the microscopic closed strings produced during this process. This is simply a consequence of the fact that the presence of the electric field slows down the decay process[7] and hence gives rise to a larger exponential suppression for the production amplitude, whereas the density of states remains the same. This indicates that the total fraction of energy of the initial D-brane that is carried away by the microscopic closed strings during the rolling of the tachyon is negligible in the weak coupling limit. Thus the energy must be somewhere else.

2. Consider a fundamental string wound (possibly multiple times) along x^1 carrying total winding charge W. If we excite such a string to oscillator level N, the energy of the state in $\alpha' = 1$ unit is given by:

$$E = \sqrt{W^2 + 4N + \vec{p}_\perp^{\,2}}\,, \tag{5}$$

where $\vec{p}_\perp$ is the transverse momentum. First we note that since the density of states grow exponentially with $\sqrt{N}$ but only as a power of $|\vec{p}_\perp|$, for a given energy it is 'entropically favourable' to have $N >> |\vec{p}_\perp|^2$. In particular, by following the argument of [2] one can show that we can maximize the density of states by taking $|\vec{p}_\perp|^2 \sim \sqrt{N}$. This allows us to set $\vec{p}_\perp = 0$ in (5) if we are willing to ignore terms of order unity in the expression for E. Then the density of closed string states grow as[2]

$$D(E, W) \sim \exp(2\beta_H \sqrt{N}) \sim \exp\left(\beta_H \sqrt{E^2 - W^2}\right) \tag{6}$$

where β_H is the inverse Hagedorn temperature. In (6) we have ignored multiplicative factor involving powers of $(E^2 - W^2)$.

Now we note that if the same amount of energy and winding charge are distributed among two strings, – one with energy E_1 and winding charge W_1, and the other with energy $E - E_1$ and winding charge $W - W_1$, then up to power law corrections the density of states of this system will be of order

$$D(E_1, W_1) D(E - E_1, W - W_1)\,, \tag{7}$$

[2]The analysis of [2] indicates that the actual number of closed string states available for the decay is only square root of the number given in (6), but the argument given below (6) still holds.

where $D(E,W)$ has been defined in (6). For fixed but large W_1, E and W, this has a sharp maximum at $E_1/W_1 = E/W$. Thus if the matrix elements for the decay of the D-brane into these states are comparable for different values of E_1, W_1, then the D-brane will decay predominantly into configurations of closed strings where the ratio of energy to the winding charge of each closed string is equal to the ratio of total energy to the total winding charge, *i.e.* e^{-1}. This, in particular, shows that microscopic strings carry a negligible fraction of the final state energy.

3. Consider the decay of a static D0-brane. According to Ref.[2] the final decay products are very massive closed strings with mass of order g^{-1} and velocity of order $g^{1/2}$ where g is the closed string coupling constant. Now consider boosting the system by a velocity e in the x^1 direction. If the x^1 direction is non-compact, then due to the Lorentz invariance of the theory we would expect that in this case the final state closed strings will be moving with a velocity e along the x^1 direction, with a small spread of order $g^{1/2}$ around e due to the initial velocity of the decay products. Now suppose the direction x^1 is compact so that the D0-brane is moving along a circle. Since boost along a circle direction is not a symmetry of the theory, we cannot strictly conclude that the final state closed strings will still be moving with a velocity e along x^1, but let us assume that this is the case and proceed. In this case, the final state closed strings have energy/momentum ratio e^{-1}. Now making a T-duality transformation along x^1 converts the initial moving D0-brane into a D1-brane with electric field e along it, and converts the final closed string states moving along S^1 into fundamental strings wound along x^1 with the energy/winding charge ratio e^{-1}. Thus if we are allowed to use boost along a compact direction to study the system, we would conclude that most of the energy of the initial D1-brane is carried by final state closed strings which are wound along x^1, each with T^{00}/S^{01} ratio e^{-1}. This argument can be generalized to D-p-branes by making further T-duality transformation along the other directions.[3]

Thus we arrive at the conclusion that the final state closed strings in the decay of a non-BPS D-brane with electric field along x^1 are predominantly closed strings wound along x^1, each carrying T^{00}/S^{01} ratio e^{-1}. We shall now compute the stress tensor and

[3]Indeed, T-duality transformation and the assumption of boost invariance along a compact direction can be used to derive the results (11) from the corresponding results in the absence of winding charge.

winding charge associated with such a system of closed strings and show that the result agrees with (1). For simplicity we shall carry out the calculation for the bosonic string, but it can be easily extended to include the fermionic terms in the action. We work in the covariant gauge where the degrees of freedom of the string are represented by D free bosonic fields $X^\mu(\sigma,\tau)$, D being the total dimension of space-time (26 for bosonic string theory). The stress tensor $T^{\mu\nu}$ and the winding charge density $S^{\mu\nu}$ are given by:

$$
\begin{aligned}
T^{\mu\nu}(y) &= C\int d\sigma d\tau \prod_{\alpha=0}^{D-1} \delta(y^\alpha - X^\alpha(\sigma,\tau))\left(\partial_\tau X^\mu \partial_\tau X^\nu - \partial_\sigma X^\mu \partial_\sigma X^\nu\right), \\
S^{\mu\nu}(y) &= C\int d\sigma d\tau \prod_{\alpha=0}^{D-1} \delta(y^\alpha - X^\alpha(\sigma,\tau))\left(\partial_\tau X^\mu \partial_\sigma X^\nu - \partial_\tau X^\nu \partial_\sigma X^\mu\right),
\end{aligned} \tag{8}
$$

for some constant C.

The fields X^μ have expansion

$$
X^\mu = x^\mu + p^\mu\tau + w^\mu\sigma + X^\mu_{osc}\,, \qquad X^\mu_{osc} \equiv \sum_{n\neq 0}\frac{1}{n}\left(\alpha^\mu_n e^{in(\tau-\sigma)} + \bar\alpha^\mu_n e^{in(\tau+\sigma)}\right) \tag{9}
$$

where p^μ and w^μ denote respectively the momentum and winding along x^μ, and X^μ_{osc} is the oscillator contribution. We shall focus on the case where the only non-zero components of p^μ and w^μ are p^0 and w^1 respectively, and also set the center of mass coordinates x^μ to zero. We note first of all that due to the oscillation of the fundamental string in the transverse direction (the X^i_{osc} part for $2\le i\le (D-1)$) the various charge densities clearly have spread in the transverse direction, and hence we shall not get strict localization of these charges on the plane of the brane. However, as argued earlier, this does not necessarily lead to a contradiction. Given this, we might as well look at the expression for the various sources after integrating them over the transverse directions y^i for $2\le i\le (D-1)$. This removes the transverse δ-functions from (8). We can now use the remaining two delta functions to fix τ and σ, but we shall proceed in a different manner. Since we are interested in calculating the average property of the final state closed strings, we shall also integrate over the coordinates y^0 and y^1, and divide by the volume V of space-time in the y^0-y^1 plane. This removes the remaining two delta functions and give:

$$
\begin{aligned}
\langle T^{\mu\nu}\rangle &= C\,V^{-1}\int d\sigma d\tau\left(\partial_\tau X^\mu \partial_\tau X^\nu - \partial_\sigma X^\mu \partial_\sigma X^\nu\right), \\
\langle S^{\mu\nu}\rangle &= C\,V^{-1}\int d\sigma d\tau\left(\partial_\tau X^\mu \partial_\sigma X^\nu - \partial_\tau X^\nu \partial_\sigma X^\mu\right),
\end{aligned} \tag{10}
$$

where $\langle\ \rangle$ denotes integration over the transverse directions and averaging over the y^0, y^1 directions. We now note that the integral over σ and τ make the contribution from the oscillator parts vanish. Thus we finally have:

$$
\begin{aligned}
&\langle T^{00}\rangle = C\,V^{-1}(p^0)^2 \int d\sigma d\tau, \qquad \langle T^{11}\rangle = -C\,V^{-1}(w^1)^2 \int d\sigma d\tau, \\
&\langle S^{01}\rangle = C\,V^{-1}p^0 w^1 \int d\sigma d\tau, \\
&\langle T^{\mu\nu}\rangle = \langle S^{\mu\nu}\rangle = 0 \quad \text{for other } \mu,\, \nu\,.
\end{aligned} \tag{11}
$$

This form agrees with the classical open string theory result (1) provided we identify w^1/p^0 as e. It is also clear that if we have a collection of such long strings (or multiply wound strings) with the same w^1/p^0 ratio, then the total stress tensor and winding charge density carried by the system, obtained by summing over (11) with different values of p^0 and $w^1 = ep^0$, will also have the form given by (1).

This finishes our analysis showing that for the decay of D-branes in the presence of an electric field, the tree level open string theory reproduces the properties of the closed strings produced in the decay of the D-brane. While the origin of this new duality is not completely clear, it could be related to the fact that there are no open string excitations around the tachyon vacuum. Thus if Ehrenfest theorem, – which states that the classical results in a theory can be interpreted as quantum expectation values, – has to work in this case, it must be that the tree level open string theory contain informtion about the properties of high density closed string states produced in the decay of the D-brane. This, in turn, suggests that upon quantization this open string theory might contain closed strings in its spectrum, and could lead to a non-perturbative formulation of string theory. Recent observation[11, 5] that in the matrix model description of two dimensional string theory the matrix eigenvalue can be related to the open string tachyon on an unstable D0-brane provides strong evidence for this proposal.

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