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Measurement of the τ electric dipole moment using longitudinal polarization of e^+e^- beams

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Abstract

Certain CP-odd momentum correlations in the production and subsequent decay of τ pairs in e^+e^- collisions are enhanced significantly when the e^+ and e^- beams are longitudinally polarized. These may be used to probe the real and imaginary parts of d_τ^γ, the electric dipole moment of the τ. Closed-form expressions for these "vector correlations" and the standard deviation of the operators defining them due to standard model interactions are presented for the two-body final states of τ decays. If 42% average polarization of each beam is achieved, as proposed for the tau-charm factories, with equal integrated luminosities for each sign of polarization and a total yield of $2\cdot 10^7$ $\tau^+\tau^-$ pairs, it is possible to attain sensitivities for $|\delta \mathrm{Re} d_\tau^\gamma|$ of $8\cdot 10^{-19}$, $1\cdot 10^{-19}$, $1\cdot 10^{-19}$ e cm respectively and for $|\delta \mathrm{Im} d_\tau^\gamma|$ of $4\cdot 10^{-14}$, $6\cdot 10^{-15}$, $5\cdot 10^{-16}$ e cm respectively at the three operating center-of-mass energies of 3.67, 4.25 and 10.58 GeV. These bounds emerge when the effects of a posible weak dipole form factor d_τ^Z are negligible as is the case when it is of the same order of magnitude as d_τ^γ. Furthermore, in such a polarization experiment where different polarizations are possible, a model-independent disentangling of their individual effects is possible, and a technique to achieve this is described. A strong longitudinal polarization physics programme at the tau-charm factory appears warranted.

I. Introduction

Leptonic CP violation would signal interactions not described in the framework of the standard model since it arises there only at the multi-loop level and is way below any measurable level [1]. The presence of a non-zero and large electric dipole moment (edm) of any elementary particle is a signature of CP-violating interactions [2]. Whereas the edm of the electron is constrained to be $\lesssim 10^{-26}$ e cm and that of the muon is $\lesssim 10^{-19}$ e cm [3], the constraint on the edm of the τ lepton [4] is less stringent, viz., $\lesssim 5 \cdot 10^{-17}$ e cm [3]. Thus an important experimental challenge is to measure the τ electric dipole moment far more accurately than at present. The analogous coupling of the τ to the Z boson, the weak dipole form factor (wdff), is better constrained from LEP data be $\lesssim 3.7 \cdot 10^{-17}$ e cm at the Z resonance [4]. It has recently been proposed [5] that the availability of large polarization at SLC might improve this measurement some more. The purpose of this note is to demonstrate that the availability of large polarization will go a long way in improving the measurement of the τ edm. Further, the discussion presented here may also be easily extended to other physiucal situations which include the measurement of CP-violating form factors in W^+W^- or $t\bar{t}$ production.

The approach proposed consists measuring CP-odd correlations [6,7] amongst the momenta of the final state particles in the reaction $e^+e^- \to \tau^+\tau^- \to X^+\bar{\nu}_\tau X^-\nu_\tau$. In particular, one may construct scalar, vector and tensor correlations [8] from the momenta $\mathbf{q}_+$ and $\mathbf{q}_-$ of the decay products of the τ^+ and τ^-. One such tensor has been used to constrain the real part of the τ wdff from LEP data [9] where the Z contribution dominates the cross-section. Indeed, other tensor correlations have been found to be sensitive to the imaginary part of the wdff as well [10] and may be used at LEP to constrain it in the event of the absence of a significant non-zero measurement of such correlations. In [5] it has been shown that the presence of large longitudinal polarization renders certain simple vector correlations sensitive to the real as well as to the imaginary parts of the wdff at the Z factory SLC.

Here we investigate the sensitivity of these correlations to the real and imaginary parts of the edm when the production of $\tau^+\tau^-$ is no longer dominated by Z exchange and instead by photon exchange as is typically the case when $\sqrt{s} \ll m_z$. In particular, we will present much of our numerical results for the proposed tau-charm factories (τcF) [11] where there exists an ample opportunity to have substantial polarization of the e^+ and e^- beams [12]. The prospects for the measurement of the edm at the tau-charm factory with unpolarized beams has already been considered [10] by measuring tensor correlations amongst the momenta of final state particles in the τ decays. Algebraically our approach proves simpler since the vector correlations (more correctly their scalar product with the e^+ beam direction) we consider can be expressed in closed form and the standard deviation of the operators defining the correlations due to the standard model interactions can also be so expressed for the two-body final states of the τ decays. In practice the expressions of Ref.[5] valid at the Z peak are now generalized to include the pure γ^* as well as the $\gamma^* - Z^*$ interference terms, using in addition to SM, the CP-violating terms in the effective Hamiltonian for the reaction

$$e^+e^- \rightarrow \gamma^*, Z^* \rightarrow \tau^+\tau^- \rightarrow \overline{B}A\overline{\nu}_\tau\nu_\tau \tag{1}$$

given in Ref.[10]. (Note that the expressions obtained here are also valid at much larger center of mass energies where contributions from γ^* and Z^* are significant, and can also be easily modified for W^+W^- and $t\bar{t}$ production [13] where it may be possible to probe CP violation). For comparable magnitudes of the edm and wdff, at the τcF energies, the CP-odd correlations obtain their most significant contribution from the edm [14].

The CP-odd momentum correlations we consider here are associated with the c.m. momenta $\mathbf{p}$ of e^+, $\mathbf{q}_{\overline{B}}$ of $\overline{B}$ and $\mathbf{q}_A$ of A, where the $\overline{B}$ and A arise in the decays $\tau^+ \to \overline{B} + \overline{\nu}_\tau$ and $\tau^- \to A + \nu_\tau$, and where A, B run over π, ρ, A_1, etc. In the case when A and B are different, one has to consider also the decays with A and B interchanged, so as to construct correlations which are explicitly CP-odd. The calculations include two-body decay modes of the τ in general and is applied specifically to the case of $\tau \to \pi + \nu_\tau$ and $\tau \to \rho + \nu_\tau$ due to the fact that these modes possess a good resolving power of the τ polarization, parametrized in terms of the constant α which takes the value 1 for the π channel (with branching fraction of about 11%) and 0.46 for the ρ channel [9] (with branching fraction of about 22%). It may be noted that with these final states the substantive fraction of the channels that are sensitive to such correlations are accounted for; three-body leptonic final states must also be included; they are characterized by a somewhat smaller $\alpha = -0.33$ (with branching fraction of about 35%). Thus with the channels studied here, one more or less reaches the limits of discovery in such experiments. (It would also be possible to apply this to the decay $\tau \to A_1 + \nu$; α_{A_1} is however too small to be of any experimental relevance.) Further, we also present closed-form expressions for the variance of the correlations considered due to standard model interactions. These, because of finite statistics, provide a measure of the CP-invariant background to the determination of the CP-odd contributions to the correlations. In case of a negative result, the limit on the CP-violating interactions is obtained using the value of the variance and the size of the data sample.

It must be noted that correlations which are CP violating in the absence of initial beam polarization are not strictly CP odd for arbitrary e^+ and e^- polarizations, since the initial state is then not necessarily CP even. We argue, however, that this is true to a high degree of accuracy in the case at hand. Besides, for our numerical results, we restrict ourselves to the case where the e^+ and e^- polarizations are equal and opposite, thus making up a CP-even initial state.

We follow a slightly different notation notation from Bernreuther *et al.* [10] and use the symbols B_i and B_j to denote the intermediate vector bosons, the photon and the Z. In the mean as well as in the variances and in the cross-sections the contributions would eventually have to be summed over $i,\ j$. Our main result is that the contribution to certain CP-odd correlations, which are relatively small in the absence of polarization, since they come with a factor $r_{ij} = (V_e^i A_e^j + V_e^j A_e^i)/(V_e^i V_e^j + A_e^i A_e^j)$ and get enhanced in the presence of polarization, now being proportional to $(r_{ij} - P)$, with the corresponding contribution to the cross-section being multiplied by $(1 - r_{ij}P)$. Here V_e^i, A_e^i are the vector and axial vector couplings of e^- to B_i, and P is the effective polarization defined by

$$P = \frac{P_e - P_{\overline{e}}}{1 - P_e P_{\overline{e}}},$$

where P_e ($P_{\overline{e}}$) is the polarization of the electron (positron) and is positive for right-circular polarization for each particle in our convention.

The correlations which have this property are those which have an odd number of factors of the e^+ c.m. momentum $\mathbf{p}$, since this would need P and C violation at the electron vertex. Furthermore, we suggest a procedure for obtaining these correlations from the difference in the event distributions for a certain polarization P and the sign-flipped polarization $-P$. With this procedure, the correlations are further enhanced, leading to increased sensitivity. The inclusion of the ρ channel leads to a considerable improvement in the sensitivity that can be reached in the measurement of Im d_τ while improving the measurement of Re d_τ less spectacularly.

More specifically, we have considered the observables $O_1 \equiv \frac{1}{2}[\hat{\mathbf{p}}\cdot(\mathbf{q}_{\overline{B}}\times\mathbf{q}_A) + \hat{\mathbf{p}}\cdot(\mathbf{q}_{\overline{A}}\times\mathbf{q}_B)]$ and $O_2 \equiv \frac{1}{2}[\hat{\mathbf{p}}\cdot(\mathbf{q}_A+\mathbf{q}_{\overline{B}}) + \hat{\mathbf{p}}\cdot(\mathbf{q}_{\overline{A}}+\mathbf{q}_B)]$ (the caret denoting a unit vector) and obtained analytic expressions for their mean values and standard deviations in the presence of longitudinal polarization. O_2, being CPT-odd, measures $\mathrm{Im}\, d_\tau^i$, whereas O_1 measures $\mathrm{Re}\, d_\tau^i$. Inclusion of other exclusive τ decay modes (not studied here) would improve the sensitivity further.

As a result, we find it possible to define 1 s.d. sensitivities $\mid \delta \mathrm{Re} d_\tau^\gamma \mid$ and $\mid \delta \mathrm{Im} d_\tau^\gamma \mid$ from the two-body decay modes when we make the reasonable assumption that the edm and wdff are of comparable magnitudes. To facilitate comparison with Ref.[10] we assume center of mass energies of 3.67, 4.25 and 10.58 GeV.

In order to answer what makes our correlations viable, we now discuss what prospects exist for longitudinal polarization at the τcF [15]. One proposal [12] is that the e^+ and e^- beams be polarized in separate rings to achieve an average degree of polarization of each beam as large as 42% before being injected in tho the main ring. (It is also important to note that this would not lead to a large loss in luminosity, in contrast to the situation at linear colliders where the large polarization is accompanied by modest luminosities as, for instance, in the case of SLC). This proposal also envisages all four possiblities in the combinations of the polarizations. In particlular, as an effective polarization P can be as large as 0.71 and of either sign in the e^+e^- collisions at the τcF. We show that with equal integrated luminosities with either sign, $\int \mathcal{L}(P)dt = \int \mathcal{L}(-P)dt$ and a total yield $N_{\tau^+\tau^-}$ of $2\cdot 10^7$ $\tau^+\tau^-$ pairs, we can probe the real part of the edm of the τ to the remarkable 1 s.d. precision of $\sim 10^{-19}$ e cm. The imaginary part however is not probed to such a spectacular degree. We finally describe a technique whereby the reasonable assumption of the comparability of magnitudes of the edm and wdff can be avoided in such a polarization experiment.

These considerations enable us to build a very strong case for introducing longitudinal polarization at the τcF [16, 17].

II. Notation and Formalism

Although much of this section has already been described in our previous papers [5] we will repeat it for the sake of completeness and to make the generalization to the inclusion of γ and Z (we drop the asterisk in what follows since no confusion is bound to arise) more transparent.

The process we consider is

$$e^-(p_-) + e^+(p_+) \rightarrow \tau^-(k_-) + \tau^+(k_+), \tag{2}$$

with the subsequent decays

$$\tau^-(k_-) \rightarrow A(q_A) + \nu_\tau,\ \tau^+(k_+) \rightarrow \overline{B}(q_{\overline{B}}) + \overline{\nu}_\tau, \tag{3}$$

together with decays corresponding to A and B interchanged in (2).

Under CP, the various three-momenta transform as

$$\mathbf{p}_- \leftrightarrow -\mathbf{p}_+,\ \mathbf{k}_- \leftrightarrow -\mathbf{k}_+,\ \mathbf{q}_{A,B} \leftrightarrow -\mathbf{q}_{\overline{A},\overline{B}}. \tag{4}$$

We choose for our analysis the two CP-odd observables $O_1 \equiv \frac{1}{2}[\hat{\mathbf{p}} \cdot (\mathbf{q}_{\overline{B}} \times \mathbf{q}_A) + \hat{\mathbf{p}} \cdot (\mathbf{q}_{\overline{A}} \times \mathbf{q}_B)]$ and $O_2 \equiv \frac{1}{2}[\hat{\mathbf{p}} \cdot (\mathbf{q}_A + \mathbf{q}_{\overline{B}}) + \hat{\mathbf{p}} \cdot (\mathbf{q}_{\overline{A}} + \mathbf{q}_B)]$, which have an odd number of factors of $\hat{\mathbf{p}}$, the unit vector along $\mathbf{p}_+$. As mentioned before, they are expected to get enhanced in the presence of polarization.

Though these observables are CP odd, their observation with polarized e^+ and e^- beams is not necessarily an indication of CP violation, unless the e^+ and e^-

longitudinal polarizations are equal and opposite, so that the initial state is described by a CP-even density matrix. The case when only the e^- is polarized, has already been discussed [5]. Though our expressions for correlations will be valid for arbitrary polarizations, our results will be only for equal and opposite electron and positron polarizations, so that the correlations are strictly CP odd.

Of O_1 and O_2, O_1 is even under the combined CPT transformation, and O_2 is CPT-odd. A CPT-odd observable can only have a non-zero value in the presence of an absorptive part of the amplitude. It is therefore expected that $\langle O_2 \rangle$ will be proportional to the imaginary part of the dipole form factors $\mathrm{Im}\, d_\tau^i$, since final-state interaction, which could give rise to an absorptive part, is negligible in the weak τ decays. Since $\langle O_1 \rangle$ and mean values of other CPT-even quantities will be proportional to $\mathrm{Re}\, d_\tau^i$, phase information on d_τ^i can only be obtained if $\langle O_2 \rangle$ (or some other CPT-odd quantity) is also measured.

We assume SM couplings for all particles except τ, for which an additional edm and wdff interaction is assumed, viz.,

$$\mathcal{L}_{CPV} = -\frac{i}{2} d_\tau^Z \overline{\tau} \sigma^{\mu\nu} \gamma_5 \tau \left(\partial_\mu Z_\nu - \partial_\nu Z_\mu \right) - \frac{i}{2} d_\tau^\gamma \overline{\tau} \sigma^{\mu\nu} \gamma_5 \tau \left(\partial_\mu A_\nu - \partial_\nu A_\mu \right), \tag{5}$$

Using (5), we now proceed to calculate $\langle O_1 \rangle$ and $\langle O_2 \rangle$ in the presence of an effective longitudinal polarization P.

We can anticipate the effect of P in general for the process (1). We can write the matrix element squared for the process in the leading order in perturbation theory, neglecting the electron mass, as

$$|M|^2 = \sum_{i,j} L_{\mu\nu}^{ij}(e) L^{ij\mu\nu *}(\tau) \frac{1}{s - M_i^2} \frac{1}{s - M_j^2}, \tag{6}$$

where the summation is over the gauge bosons $(\gamma, Z, \ldots)$ exchanged in the s channel, and $L^{ij}_{\mu\nu}(e,\tau)$ represent the tensors arising at the e and τ vertices:

$$L^{ij}_{\mu\nu} = V^i_\mu V^{j*}_\nu. \tag{7}$$

For the electron vertex, with only the SM vector and axial-vector couplings,

$$V^i_\mu(e) = \overline{v}(p_+,s_+)\gamma_\mu\left(V^i_e - \gamma_5 A^i_e\right)u(p_-,s_-), \tag{8}$$

We have the definitions

$$V^\gamma_{e/\tau} = -e,\ A^\gamma_{e/\tau} = 0; \tag{9}$$

$$V^Z_{e/\tau} = (-\frac{1}{2} + 2\sin^2\theta_W)\frac{e}{2\sin\theta_w\cos\theta_w},\ A^Z_{e/\tau} = (-\frac{1}{2})\frac{e}{2\sin\theta_w\cos\theta_w}. \tag{10}$$

It is easy to check, by putting in helicity projection operators, that

$$\begin{aligned} L^{ij}_{\mu\nu}(e) = \\ & \Big\{\Big[(1-P_eP_{\overline{e}})\left(V^i_eV^j_e + A^i_eA^j_e\right) - (P_e - P_{\overline{e}})\left(V^i_eA^j_e + A^i_eV^j_e\right)\Big]\, Tr(\not{p}_-\gamma_\mu\not{p}_+\gamma_\nu) \\ & + \Big[(P_e - P_{\overline{e}})\left(V^i_eV^j_e + A^i_eA^j_e\right) - (1-P_eP_{\overline{e}})\left(V^i_eA^j_e + A^i_eV^j_e\right)\Big]\, Tr\left(\gamma_5\not{p}_-\gamma_\mu\not{p}_+\gamma_\nu\right)\Big\} \end{aligned} \tag{11}$$

in the limit of vanishing electron mass, where P_e ($P_{\overline{e}}$) is the degree of the e^- (e^+) longitudinal polarization. Eq.(11) gives a simple way of incorporating the effect of the longitudinal polarization.

$$\begin{array}{rcl} V^i_eV^j_e + A^i_eA^j_e & \to & V^i_eV^j_e + A^i_eA^j_e \quad - \quad P\left(A^i_eV^j_e + A^j_eV^i_e\right), \\ (A^i_eV^j_e + A^j_eV^i_e) & \to & (A^i_eV^j_e + A^j_eV^i_e) \quad - \quad P\left(V^i_eV^j_e + A^i_eA^j_e\right), \end{array} \tag{12}$$

where P is as defined earlier.

To calculate correlations of O_1 and O_2, we need the differential cross section for (1) followed by (2) arising from SM γ and Z couplings of e and τ, together dipole couplings of τ arising from eq.(4). The calculation may be conveniently done, following ref.[10], in steps, by first determining the production matrix χ for $\tau^+\tau^-$ in spin space, and then taking its trace with the decay matrices $\mathcal{D}^{\pm}$ for $\tau^{\pm}$ decays into single charged particle in addition to the invisible neutrino.

The differential cross section for (1) is given by

$$\frac{d\sigma}{d\Omega_k d\Omega_-^* d\Omega_+^* dE_-^* dE_+^*} = \frac{k}{8\pi s} \frac{1}{(4\pi)^3} \chi^{\beta\beta',\alpha\alpha'} \mathcal{D}^-_{\alpha'\alpha} \mathcal{D}^+_{\beta'\beta}, \tag{13}$$

where $d\Omega_k$ is the solid angle element for $\mathbf{k}_+$ in the overall c.m. frame, $k = |\mathbf{k}_+|$, and $d\Omega_\pm^*$ are the solid angle elements for $\mathbf{q}^*_{\overline{B},A}$, the $\overline{B}$ and A momenta in the $\tau^{\pm}$ rest frame. The $\mathcal{D}$ matrices are given by

$$\begin{aligned} \mathcal{D}^+ &= \delta\left(E_B^* - E_{0B}\right)\left[1 - \alpha_B \sigma_+ \cdot \hat{\mathbf{q}}_B^*\right] \\ \mathcal{D}^- &= \delta\left(E_A^* - E_{0A}\right)\left[1 + \alpha_A \sigma_- \cdot \hat{\mathbf{q}}_A^*\right], \end{aligned} \tag{14}$$

where $\sigma_\pm$ are the Pauli matrices corresponding to the $\tau^{\pm}$ spin, $E_\pm^*$ are the charged particle energies in the $\tau^{\pm}$ rest frame, and

$$E_{0A,B} = \frac{1}{2} m_\tau (1 + p_{A,B}); \; p_{A,B} = m_{A,B}^2 / m_\tau^2. \tag{15}$$

The expressions for χ arising from SM as well as the CP-violating form factor couplings of τ are rather long, and we refer the reader to ref.[10] for these expressions in the absence of polarization. It is straightforward to incorporate polarization using (12).

III. Results

Using eqns. (13)-(15) above, as well as the expression for the $\tau^+\tau^-$ production matrix χ from [10], we can obtain expressions for $\langle O_1 \rangle$ and $\langle O_2 \rangle$ by writing O_1 and O_2 in terms of the τ rest frame variables and performing the integrals over them analytically. The expressions for the correlations $\langle O_1 \rangle$ and $\langle O_2 \rangle$ obtained are, neglecting

$\sum_{i,j} d_\tau^i d_\tau^j$,

$$\langle O_1\rangle = -\tfrac{1}{36x\sigma}\sum_{i,j} K_{ij} s^{3/2} m_\tau^2 (1-x^2)\left(\tfrac{r_{ij}-P}{1-r_{ij}P}\right)$$

$$[(A_\tau^i \mathrm{Re}\ d_\tau^j + A_\tau^j \mathrm{Re}\ d_\tau^i)\alpha_A\alpha_B(1-p_A)(1-p_B)-$$

$$\tfrac{3}{2}(V_\tau^i \mathrm{Re}\ d_\tau^j + V_\tau^j \mathrm{Re}\ d_\tau^i)[\alpha_A(1-p_A)(1+p_B)+\alpha_B(1-p_B)(1+p_A)], \tag{16}$$

and

$$\langle O_2\rangle = \tfrac{1}{3\sigma}\sum_{i,j} K_{ij} s^{3/2} m_\tau \left(\tfrac{r_{ij}-P}{1-r_{ij}P}\right)$$

$$\tfrac{1}{4}(A_\tau^i \mathrm{Im}\ d_\tau^j + A_\tau^j \mathrm{Im}\ d_\tau^i)(1-x^2)(\alpha_A(1-p_A)+\alpha_B(1-p_B)), \tag{17}$$

where $x = 2m_\tau/\sqrt{s}$ and σ, which is the cross-section apart from a normalization factor, is given by:

$$\sigma = \sum_{i,j} K_{ij} s[V_\tau^i V_\tau^j (1+\frac{x^2}{2}) + A_\tau^i A_\tau^j (1-x^2)], \tag{18}$$

and

$$K_{ij} = \frac{s(V_e^i V_e^j + A_e^i A_e^j)(1-r_{ij}P)}{(s-M_i^2)(s-M_j^2)}. \tag{19}$$

Here we neglect the width of the Z since we work now at $\sqrt{s} << m_Z$. However at the Z peak we neglect γ and treat the system in the narrow-width approximation.

We have also obtained analytic expressions for the variance $\langle O^2\rangle - \langle O\rangle^2 \approx \langle O^2\rangle$ in each case, arising from the CP-invariant SM part of the interaction:

$$\langle O_1^2\rangle = \tfrac{1}{720x^2\sigma}\sum_{i,j} K_{ij} s m_\tau^4 \Big((1-p_A)^2(1-p_B)^2$$

$$[V_\tau^i V_\tau^j (6+8x^2+x^4) + A_\tau^i A_\tau^j (6-2x^2-4x^4)]$$

$$+(1-x^2)\left([(1+p_A)^2(1-p_B)^2 + (1+p_B)^2(1-p_A)^2]\right.$$

$$[3V_\tau^i V_\tau^j (3+2x^2) + 9A_\tau^i A_\tau^j (1-x^2)]$$

$$+4\alpha_A\alpha_B(1-p_B^2)(1-p_A^2)(1-x^2)[V_\tau^i V_\tau^j - A_\tau^i A_\tau^j])$$

$$-6(1-p_A)(1-p_B)(V_\tau^i A_\tau^j + V_\tau^j A_\tau^i)(1-x^2)(1-\tfrac{x^2}{6})$$

$$[\alpha_A(1+p_A)(1-p_B)+\alpha_B(1+p_B)(1-p_A)]\Big), \tag{20}$$

$$\langle O_2^2\rangle = \tfrac{1}{360x^2\sigma}\textstyle\sum_{i,j} K_{ij} s m_\tau^2$$

$$\Big[\Big(3[(1-p_A)^2+(1-p_B)^2][V_\tau^i V_\tau^j(4+7x^2+4x^4)+A_\tau^i A_\tau^j 2(1-x^2)(2+3x^2)]$$

$$-2\alpha_A\alpha_B(1-p_A)(1-p_B)[V_\tau^i V_\tau^j(4+7x^2+4x^4)+A_\tau^i A_\tau^j 4(1-x^2)^2]\Big)$$

$$+6\Big(6(1-x^2)(p_A-p_B)^2[V_\tau^i V_\tau^j(1+\tfrac{x^2}{4})+A_\tau^i A_\tau^j(1-x^2)]$$

$$-(V_\tau^i A_\tau^j + V_\tau^j A_\tau^i)(1-x^2)(4+x^2)(p_A-p_B)[\alpha_A(1-p_A)-\alpha_B(1-p_B)]\Big)\Big]. \tag{21}$$

The results for the significant two-body decay channels are presented in the tables. In Tables 1-6 we have presented, for three typical values of $\sqrt{s}$ at which the τcF is expect to run, the values of c_{AB} for O_1 and O_2 respectively, defined as the correlation for a value of $\mathrm{Re}\, d_\tau^\gamma$ or $\mathrm{Im}\, d_\tau^\gamma$ (as the case may be) equal to $e/\sqrt{s}$, for some values of P chosen to correspond to average beam polarizations of 0, 35%, 42% and 100%. We have also presented the value of $\sqrt{\langle O_a^2\rangle}$, $(a=1,2)$. This 1 s.d. limit is the value of d_τ^γ which gives a mean value of O_a equal to the s.d. $\sqrt{\langle O_a^2\rangle/N_{AB}}$ in each case:

$$c_{AB}^{1(2)}\delta\mathrm{Re(Im)}d_\tau^\gamma = \frac{e}{\sqrt{s}}\frac{1}{\sqrt{N_{AB}}}\sqrt{\langle O_{1(2)}^2\rangle}. \tag{22}$$

Here N_{AB} is the number of events in the channel $A\overline{B}$ (or $\overline{A}B$), and is given by

$$N_{AB} = N_{\tau^+\tau^-}B(\tau^-\to A\nu_\tau)B(\tau^+\to\overline{B}\overline{\nu}_\tau), \tag{23}$$

where we take $N_{\tau^+\tau^-}(P)=10^7$.

These limits can be improved by looking at correlations of the same observables, but in a sample obtained by counting the difference between the number of events for a certain polarization, and for the corresponding sign-flipped polarization. If the partial cross section for the process for a polarization P is given by

$$d\sigma(P) = \sum_{i,j} \left\{ (X_{ij} + r_{ij} Y_{ij}) - P(r_{ij} X_{ij} + Y_{ij}) \right\}, \tag{24}$$

we can define a polarization asymmetrized distribution

$$|d\sigma(P) - d\sigma(-P)| = 2|P \sum_{i,j} (r_{ij} X_{ij} + Y_{ij})|. \tag{25}$$

We can then compute the mean and standard deviation for the correlations over this distribution and these are tabulated in Tables 7-9. The correlations get contributions from the $\pm 2P \sum Y_{ij}$ term in eq.(25) as compared to the $\sum r_{ij} Y_{ij}$ and is therefore enhanced, since $|r_{ij}| < 1$. However the sensitivities are now computed for smaller event samples whose size is given by $|P \sum_{i,j} r_{ij} N_{ij}|$ where $\sum_{i,j} N_{ij}$ stands for the total number of $\tau^+\tau^-$ pairs including both polarizations P and $-P$. The standard deviations are only slightly affected. The net result is an increase in the sensitivity. For the different values of $\sqrt{s}$ we tabulate the associated quantity, $\frac{|\sum_{i,j} r_{ij} N_{ij}|}{\sum_{i,j} N_{ij}}$, the effective polarization asymmetry in Table 10. Indeed, the improvement in sensitivity is seen to be by an order of magnitude.

We can combine the sensitivities from the different τ channels in inverse quadrature, to get the improved numbers for $|\delta \mathrm{Re} d_\tau^\gamma|$ of $8 \cdot 10^{-19}$, $1 \cdot 10^{-19}$, $1 \cdot 10^{-19}$ e cm respectively and for $|\delta \mathrm{Im} d_\tau^\gamma|$ of $4 \cdot 10^{-14}$, $6 \cdot 10^{-15}$, $5 \cdot 10^{-16}$ e cm respectively at the three center of mass energies of 3.67, 4.25 and 10.58 GeV.

Thus far and for the purposes of Tables 1-9, we have made the altogether reasonable assumption that the contribution of the wdff is negligible which is justified so long as the edm and wdff are of comparable magnitude. However, no such assumption is really necessary in polarization experiments such as these where the ability to run the experiment at different polarizations allows one to disentangle their individual contributions to the correlations considered here. Indeed, it has been pointed out in the context of CP violation in the $t\bar{t}$ system that varying the polarization allows a model independent determination of the separate contributions of the edm and wdff to the correlations of the type considered here [13]. The principle is that at a given polarization, a certain linear combination of the two form factors alone can be measured. Performing the experiment at two different polarizations enables us to disentangle the two form factors. Similarly, the 1 s.d. limits also can only be placed on such a linear combination. Indeed, such 1 s.d. limits would be defined by straight lines given by equations such as

$$\delta \mathrm{Re} d_\tau^\gamma / a + \delta \mathrm{Re} d_\tau^Z / b = \pm 1 \tag{26}$$

for the limits arising from O_1 and by

$$\delta \mathrm{Im} d_\tau^\gamma / c + \delta \mathrm{Im} d_\tau^Z / d = \pm 1 \tag{27}$$

for the limits arising from O_2 where the numbers $a,\ b,\ c$ and d can be explicitly computed for a given P and N. This is also presented for the polarization asymmetrized distribution for which we have set $P = 1$ (with the understanding that this would have to be scaled by $\sqrt{P}$ if P is the polarization realized in a certain experiment). In particular, a (c) is the sensitivity of the real (imaginary) part of the edm in inverse quadrature when the real (imaginary) part of the wdff is set to zero and b (d) is the sensitivity of the real (imaginary) part of the wdff in inverse quadrature when the real (imaginary) part of the edm is set to zero. We tabulate these quantities for the three different c.m. energies and for different polarizations for the parent distributions in Table 11 and for the asymmetrized distributions in Table 12. Note that one can read from the columns for a and c in Table 12 the bounds cited in the abstract and in the preceding paragraph (scaled by $\sqrt{0.71}$). These implicitly assumes that the magnitudes of the edm and wdff are comparable and therefore the latter may be ignored for such considerations.

We now discuss how the results of Table 11 may be used in order to essentially define regions in the Re (Im) d_τ^γ– d_τ^Z planes due to finite statistics, say at the 1 s. d. level. By performing the experiment at two values of P, say P_1 and P_2, one obtains two sets of straight lines defined above. The vertices of the interesection of these 4 lines defines the parallellogram in each of these planes which cannot be ruled out due to the finite statistics. The best results may be obtained by taking the largest value of polarization realizable P_{max} and taking $P_1 = -P_2 = P_{max}$. In Table 13 we tabulate for the three different values of P_{max} two pairs (A, B) for the Real and (C, D) for the Imaginary planes, which give the coordinates of two vertices in the Real and Imaginary d_τ^γ– d_τ^Z planes respectively with $(-A, -B)$ and $(-C, -D)$ giving the remaining pairs. Thus the availability of polarization and of either sign provides for a model independent scheme for constraining regions of the parameter space spanned by the CP-violating form factors. It must be noted that the price to be paid for such a model-independent bound on each of the form factors is large. In particular, from Table 13, the most stringent such bound on the magnitude of Re(Im)d_τ^γ is only the larger of the $|A|(|C|)$.

IV. Conclusions

We have presented closed-form expressions for the correlations of O_1 and O_2 parametrized by the real and imaginary parts of the edm and wdff and for their standard deviations due to standard model interactions. We have tabulated for unit values of these parameters (in units of $e/\sqrt{s}$) the values of the correlation and standard deviations for a variety of energies at whichthe τCF is expected to operate and have computed the 1 s.d. sensitivities for a modest sample of 10^7 $\tau^+\tau^-$ pairs. A polarization asymmetry we define is a useful tool to improve this sensitivity. We have described a technique to implement a model independent analysis by varying the polarization which does not require us to neglect the contributions of a possible wdff that is justified when the edm and wdff are of comparable magnitudes. For e^+ and e^- longitudinal beam polarizations of 42% achievable at the τcF the sensitivities can be as excellent as (few)$\cdot 10^{-19} e$ cm for the real part and (few)$\cdot 10^{-16} e$ cm for the imaginary part. We demonstrate that the absence of an axial vector coupling of the electron to the photon is not necessarily a detriment to the use of polarization in probing CP violation. An improvement by at least an order of magnitude over the sensitivity for the real part of the edm in the unpolarized case (Table 2 of ref. [10]) is noted.

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[15] And yet, it is worthy of note that these correlations and their standard deviations yield an important normalization tool for Monte Carlo simulations that have to be constructed in order to compute three (and larger) body final states and tensor correlations, making them *of more than academic interest* even in the event no polarization experiment is performed.

[16] An alternative proposal is to polarize the electron beam alone [J. Kirkby, private communication] with the possibility of a degree of polarization of nearly 100%, as for example at SLC where an 80% polarization has now been achieved [B. Schumm, Talk given at Third Workshop on Tau Lepton Physics, Montreux, Switzerland, September 1994]. However, a hard collinear photon bremmstrahlung process would induce a CP-even background to the correlation $\langle O_2 \rangle$ at $O(\alpha)$. Such a background would have to computed and subtracted. It is possible to do this in the formalism applicable to helicity-flip bremmstrahlung following B. Falk and L. M. Sehgal, Phys. Lett. B 325, 509 (1994). Note that at SLC the cross-section of this process is suppressed by a large factor compared to the production cross-section (in fact vanishing in the narrow width approximation) but is not necessarily so at the τcF energies.

[17] After the completion of this work we received a paper by Y-S. Tsai, SLAC report, SLAC-PUB-6685, hep-ph/9410265, wherein the use of longitudinally polarized beams to probe CP violation in τ-decays is discussed.

Table Captions

1. (a) c_{AB}, standard deviation and $|\delta \mathrm{Re}\, d_\tau^\gamma|$ computed for 10^7 $\tau^+\tau^-$ pairs for $\pi\pi$ channel and $\sqrt{s} = 3.67$ GeV for operator O_1 for different P.
(b) Same as above for $\pi\rho$ channel.
(c) Same as above for $\rho\rho$ channel.

2. (a) c_{AB}, standard deviation and $|\delta \mathrm{Im}\, d_\tau^\gamma|$ computed for 10^7 $\tau^+\tau^-$ pairs for $\pi\pi$ channel and $\sqrt{s} = 3.67$ GeV for operator O_2 for different P.
(b) Same as above for $\pi\rho$ channel.
(c) Same as above for $\rho\rho$ channel.

3. Same as (1) for $\sqrt{s} = 4.25$ GeV.

4. Same as (2) for $\sqrt{s} = 4.25$ GeV.

5. Same as (1) for $\sqrt{s} = 10.58$ GeV.

6. Same as (2) for $\sqrt{s} = 10.58$ GeV.

7. (a) c_{AB}, standard deviation and $|\delta \mathrm{Re}\, d_\tau^\gamma|$ computed for $\int \mathcal{L}(P)dt = \int \mathcal{L}(-P)dt$ and $\sum_{i,j} N_{ij} = 2 \cdot 10^7$ $\tau^+\tau^-$ pairs from O_1 for $\pi\pi$, $\pi\rho$ and $\rho\rho$ channels for $\sqrt{s} = 3.67$ GeV from polarization asymmetrized distribution.
(b) c_{AB}, standard deviation and $|\delta \mathrm{Im}\, d_\tau^\gamma|$ computed for $\int \mathcal{L}(P)dt = \int \mathcal{L}(-P)dt$ and $\sum_{i,j} N_{ij} = 2 \cdot 10^7$ $\tau^+\tau^-$ pairs from O_2 for $\pi\pi$, $\pi\rho$ and $\rho\rho$ channels for $\sqrt{s} = 3.67$ GeV from polarization asymmetrized distribution.

8. Same as (7) for $\sqrt{s} = 4.25$ GeV.

9. Same as (7) for $\sqrt{s} = 10.58$ GeV.

10. The effective polarization asymmetry $|\sum_{i,j} N_{ij} r_{ij}| / \left(\sum_{i,j} N_{ij}\right)$ for $\sqrt{s} = 3.67$, 4.25 and 10.58 GeV.

11. (a) The quantities $a,\ b,\ c$ and d in e cm defining the lines of sensitivity for different polarization with $N_{\tau^+\tau^-} = 10^7$ for $\sqrt{s} = 3.67$ GeV.
(b) As above for $\sqrt{s} = 4.25$ GeV.
(c) As above for $\sqrt{s} = 10.58$ GeV.

12. The quantities $a,\ b,\ c$ and d for the asymmetrized distributions with $P = 1$, $\int \mathcal{L}(P)dt = \int \mathcal{L}(-P)dt$ and $\sum_{ij} N_{ij} = 2 \cdot 10^7$ for the three different center of mass energies.

13. (a) A and B, and C and D, defining the parallelograms in the Red_τ^γ – Red_τ^Z and Imd_τ^γ – Imd_τ^Z planes respectively for various values of P_{max} for $a,\ b,\ c$ and d of Table 11 as described in the text at $\sqrt{s} = 3.67$ GeV.
(b) As above for $\sqrt{s} = 4.25$ GeV.
(c) As above for $\sqrt{s} = 10.58$ GeV.

P	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert\delta\,\mathrm{Re}\,d_\tau^\gamma\rvert$ e cm
0.00	-3.12×10^{-6}	0.399	1.88×10^{-12}
-0.62	-1.36×10^{-2}	0.399	4.32×10^{-16}
$+0.62$	1.35×10^{-2}	0.399	4.32×10^{-16}
-0.71	-1.55×10^{-2}	0.399	3.77×10^{-16}
$+0.71$	1.55×10^{-2}	0.399	3.77×10^{-16}
-1.00	-2.19×10^{-2}	0.399	2.68×10^{-16}
$+1.00$	2.19×10^{-2}	0.399	2.68×10^{-16}

(a)

P	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert\delta\,\mathrm{Re}\,d_\tau^\gamma\rvert$ e cm
0.00	-7.72×10^{-7}	0.336	4.64×10^{-12}
-0.62	-1.05×10^{-2}	0.336	3.39×10^{-16}
$+0.62$	1.05×10^{-2}	0.336	3.39×10^{-16}
-0.71	-1.21×10^{-2}	0.336	2.96×10^{-16}
$+0.71$	1.21×10^{-2}	0.336	2.96×10^{-16}
-1.00	-1.70×10^{-2}	0.336	2.10×10^{-16}
$+1.00$	1.70×10^{-2}	0.336	2.10×10^{-16}

(b)

P	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert\delta\,\mathrm{Re}\,d_\tau^\gamma\rvert$ e cm
0.00	-1.39×10^{-7}	0.282	1.57×10^{-11}
-0.62	-6.02×10^{-3}	0.282	3.63×10^{-16}
$+0.62$	6.02×10^{-3}	0.282	3.63×10^{-16}
-0.71	-6.89×10^{-3}	0.282	3.17×10^{-16}
$+0.71$	6.89×10^{-3}	0.282	3.17×10^{-16}
-1.00	-9.70×10^{-3}	0.282	2.25×10^{-16}
$+1.00$	9.70×10^{-3}	0.282	2.25×10^{-16}

(c)

Table 1

P	c_{AB} GeV	$\sqrt{\langle O_2^2 \rangle}$ GeV	$\lvert\delta\,\mathrm{Im}\,d_\tau^\gamma\rvert$ e cm
0.00	1.36×10^{-5}	0.596	6.44×10^{-13}
-0.62	1.42×10^{-5}	0.596	6.14×10^{-13}
$+0.62$	1.29×10^{-5}	0.596	6.78×10^{-13}
-0.71	1.43×10^{-5}	0.596	6.10×10^{-13}
$+0.71$	1.28×10^{-5}	0.596	6.83×10^{-13}
-1.00	1.46×10^{-5}	0.596	5.97×10^{-13}
$+1.00$	1.25×10^{-5}	0.596	7.01×10^{-13}

(a)

P	c_{AB} GeV	$\sqrt{\langle O_2^2 \rangle}$ GeV	$\lvert\delta\,\mathrm{Im}\,d_\tau^\gamma\rvert$ e cm
0.00	9.33×10^{-6}	0.615	7.02×10^{-13}
-0.62	9.79×10^{-6}	0.615	6.69×10^{-13}
$+0.62$	8.86×10^{-6}	0.615	7.39×10^{-13}
-0.71	9.86×10^{-6}	0.615	6.64×10^{-13}
$+0.71$	8.88×10^{-6}	0.615	7.44×10^{-13}
-1.00	1.01×10^{-5}	0.615	6.50×10^{-13}
$+1.00$	8.58×10^{-6}	0.615	7.63×10^{-13}

(b)

P	c_{AB} GeV	$\sqrt{\langle O_2^2 \rangle}$ GeV	$\lvert\delta\,\mathrm{Im}\,d_\tau^\gamma\rvert$ e cm
0.00	5.10×10^{-6}	0.575	8.72×10^{-13}
-0.62	5.35×10^{-6}	0.575	8.31×10^{-13}
$+0.62$	4.85×10^{-6}	0.575	9.18×10^{-13}
-0.71	5.39×10^{-6}	0.575	8.25×10^{-13}
$+0.71$	4.81×10^{-6}	0.575	9.25×10^{-13}
-1.00	5.51×10^{-6}	0.575	8.08×10^{-13}
$+1.00$	4.69×10^{-6}	0.575	9.48×10^{-13}

(c)

Table 2

P	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert \delta\, \mathrm{Re}\, d_\tau^\gamma \rvert$ e cm
0.00	-2.67×10^{-5}	0.543	2.58×10^{-13}
-0.62	-8.64×10^{-2}	0.543	7.96×10^{-17}
$+0.62$	8.63×10^{-2}	0.543	7.97×10^{-17}
-0.71	-9.89×10^{-2}	0.543	6.95×10^{-17}
$+0.71$	9.88×10^{-2}	0.543	6.96×10^{-17}
-1.00	-1.39×10^{-1}	0.543	4.94×10^{-17}
$+1.00$	1.39×10^{-1}	0.543	4.94×10^{-17}

(a)

P	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert \delta\, \mathrm{Re}\, d_\tau^\gamma \rvert$ e cm
0.00	-6.59×10^{-6}	0.488	6.80×10^{-13}
-0.62	-6.72×10^{-2}	0.488	6.68×10^{-17}
$+0.62$	6.72×10^{-2}	0.488	6.68×10^{-17}
-0.71	-7.69×10^{-2}	0.488	5.83×10^{-17}
$+0.71$	7.69×10^{-2}	0.488	5.83×10^{-17}
-1.00	-1.08×10^{-1}	0.488	4.14×10^{-17}
$+1.00$	1.08×10^{-1}	0.488	4.14×10^{-17}

(b)

P	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert \delta\, \mathrm{Re}\, d_\tau^\gamma \rvert$ e cm
0.00	-1.19×10^{-6}	0.432	2.43×10^{-12}
-0.62	-3.83×10^{-2}	0.432	7.53×10^{-17}
$+0.62$	3.83×10^{-2}	0.432	7.53×10^{-17}
-0.71	-4.39×10^{-2}	0.432	6.57×10^{-17}
$+0.71$	4.39×10^{-2}	0.432	6.57×10^{-17}
-1.00	-6.18×10^{-2}	0.432	4.67×10^{-17}
$+1.00$	6.18×10^{-2}	0.432	4.67×10^{-17}

(c)

Table 3

P	c_{AB} GeV	$\sqrt{\langle O_2^2 \rangle}$ GeV	$\lvert\delta\,\mathrm{Im}\,d_\tau^\gamma\rvert$ e cm
0.00	1.00×10^{-4}	0.632	8.00×10^{-14}
−0.62	1.05×10^{-4}	0.632	7.62×10^{-14}
+0.62	9.51×10^{-5}	0.632	8.41×10^{-14}
−0.71	1.06×10^{-4}	0.632	7.57×10^{-14}
+0.71	9.44×10^{-5}	0.632	8.48×10^{-14}
−1.00	1.08×10^{-4}	0.632	7.40×10^{-14}
+1.00	9.20×10^{-5}	0.632	8.69×10^{-14}

(a)

P	c_{AB} GeV	$\sqrt{\langle O_2^2 \rangle}$ GeV	$\lvert\delta\,\mathrm{Im}\,d_\tau^\gamma\rvert$ e cm
0.00	6.89×10^{-5}	0.655	8.74×10^{-14}
−0.62	7.23×10^{-5}	0.655	8.33×10^{-14}
+0.62	6.54×10^{-5}	0.655	9.20×10^{-14}
−0.71	7.28×10^{-5}	0.655	8.27×10^{-14}
+0.71	6.49×10^{-5}	0.655	9.27×10^{-14}
−1.00	7.44×10^{-5}	0.655	8.09×10^{-14}
+1.00	6.33×10^{-5}	0.655	9.50×10^{-14}

(b)

P	c_{AB} GeV	$\sqrt{\langle O_2^2 \rangle}$ GeV	$\lvert\delta\,\mathrm{Im}\,d_\tau^\gamma\rvert$ e cm
0.00	3.76×10^{-5}	0.610	1.08×10^{-13}
−0.62	3.95×10^{-5}	0.610	1.03×10^{-13}
+0.62	3.58×10^{-5}	0.610	1.14×10^{-13}
−0.71	3.98×10^{-5}	0.610	1.02×10^{-13}
+0.71	3.55×10^{-5}	0.610	1.15×10^{-13}
−1.00	4.06×10^{-5}	0.610	1.00×10^{-13}
+1.00	3.46×10^{-5}	0.610	1.18×10^{-13}

(c)

Table 4

P	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert\delta\,\mathrm{Re}\,d_\tau^\gamma\rvert$ e cm
0.00	-1.58×10^{-5}	1.78	5.71×10^{-15}
-0.62	-8.19×10^{-1}	1.78	1.10×10^{-17}
$+0.62$	8.15×10^{-1}	1.78	1.11×10^{-17}
-0.71	-9.37×10^{-1}	1.78	9.65×10^{-18}
$+0.71$	9.33×10^{-1}	1.78	9.67×10^{-18}
-1.00	-1.32	1.78	6.86×10^{-18}
$+1.00$	1.31	1.78	6.86×10^{-18}

(a)

P	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert\delta\,\mathrm{Re}\,d_\tau^\gamma\rvert$ e cm
0.00	-3.91×10^{-4}	1.66	1.57×10^{-14}
-0.62	-6.36×10^{-1}	1.66	9.63×10^{-18}
$+0.62$	6.35×10^{-1}	1.66	9.64×10^{-18}
-0.71	-7.29×10^{-1}	1.66	8.41×10^{-18}
$+0.71$	7.27×10^{-1}	1.66	8.42×10^{-18}
-1.00	-1.03	1.66	5.98×10^{-18}
$+1.00$	1.02	1.66	5.98×10^{-18}

(b)

P	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert\delta\,\mathrm{Re}\,d_\tau^\gamma\rvert$ e cm
0.00	-7.03×10^{-5}	1.51	5.76×10^{-14}
-0.62	-3.63×10^{-1}	1.51	1.12×10^{-17}
$+0.62$	3.62×10^{-1}	1.51	1.12×10^{-17}
-0.71	-4.15×10^{-1}	1.51	9.77×10^{-18}
$+0.71$	4.15×10^{-1}	1.51	9.77×10^{-18}
-1.00	-5.85×10^{-1}	1.51	6.93×10^{-18}
$+1.00$	5.85×10^{-1}	1.51	6.94×10^{-18}

(c)

Table 5

P	c_{AB} GeV	$\sqrt{\langle O_2^2\rangle}$ GeV	$\lvert\delta\,\mathrm{Im}\,d_\tau^\gamma\rvert$ e cm
0.00	2.83×10^{-3}	1.19	2.53×10^{-15}
-0.62	2.50×10^{-3}	1.19	2.41×10^{-15}
$+0.62$	2.26×10^{-3}	1.19	2.67×10^{-15}
-0.71	2.52×10^{-3}	1.19	2.40×10^{-15}
$+0.71$	2.25×10^{-3}	1.19	2.69×10^{-15}
-1.00	2.58×10^{-3}	1.19	2.34×10^{-15}
$+1.00$	2.19×10^{-3}	1.19	2.75×10^{-15}

(a)

P	c_{AB} GeV	$\sqrt{\langle O_2^2\rangle}$ GeV	$\lvert\delta\,\mathrm{Im}\,d_\tau^\gamma\rvert$ e cm
0.00	1.64×10^{-3}	1.26	2.83×10^{-15}
-0.62	1.72×10^{-3}	1.26	2.69×10^{-15}
$+0.62$	1.56×10^{-3}	1.26	2.98×10^{-15}
-0.71	1.73×10^{-3}	1.26	2.67×10^{-15}
$+0.71$	1.55×10^{-3}	1.26	3.00×10^{-15}
-1.00	1.77×10^{-3}	1.26	2.62×10^{-15}
$+1.00$	1.51×10^{-3}	1.26	3.08×10^{-15}

(b)

P	c_{AB} GeV	$\sqrt{\langle O_2^2\rangle}$ GeV	$\lvert\delta\,\mathrm{Im}\,d_\tau^\gamma\rvert$ e cm
0.00	8.96×10^{-4}	1.15	3.43×10^{-15}
-0.62	9.41×10^{-4}	1.15	3.26×10^{-15}
$+0.62$	8.51×10^{-4}	1.15	3.61×10^{-15}
-0.71	9.48×10^{-4}	1.15	3.24×10^{-15}
$+0.71$	8.45×10^{-4}	1.15	3.64×10^{-15}
-1.00	9.69×10^{-4}	1.15	3.17×10^{-15}
$+1.00$	8.24×10^{-4}	1.15	3.73×10^{-15}

(c)

Table 6

	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert\delta\, \mathrm{Re}\, d_\tau^\gamma\rvert$ e cm
$\pi\pi$	2.39×10^2	0.292	1.32×10^{-18}
$\pi\rho$	1.86×10^2	0.267	1.13×10^{-18}
$\rho\rho$	1.06×10^2	0.240	1.29×10^{-18}

(a)

	c_{AB} GeV	$\sqrt{\langle O_2^2 \rangle}$ GeV^2	$\lvert\delta\, \mathrm{Im}\, d_\tau^\gamma\rvert$ e cm
$\pi\pi$	1.18×10^{-2}	0.596	5.45×10^{-14}
$\pi\rho$	8.15×10^{-3}	0.615	5.94×10^{-14}
$\rho\rho$	4.45×10^{-3}	0.575	7.38×10^{-14}

(b)

Table 7

	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert\delta\, \mathrm{Re}\, d_\tau^\gamma\rvert$ e cm
$\pi\pi$	1.13×10^3	0.529	3.77×10^{-19}
$\pi\rho$	8.82×10^2	0.289	1.90×10^{-19}
$\rho\rho$	5.03×10^2	0.127	1.08×10^{-19}

(a)

	c_{AB} GeV	$\sqrt{\langle O_2^2 \rangle}$ GeV^2	$\lvert\delta\, \mathrm{Im}\, d_\tau^\gamma\rvert$ e cm
$\pi\pi$	6.52×10^{-2}	0.632	7.83×10^{-15}
$\pi\rho$	4.47×10^{-2}	0.657	8.59×10^{-15}
$\rho\rho$	2.45×10^{-2}	0.610	1.06×10^{-14}

(b)

Table 8

	c_{AB} GeV^2	$\sqrt{\langle O_1^2 \rangle}$ GeV^2	$\lvert\delta\,\mathrm{Re}\,d_\tau^\gamma\rvert$ e cm
$\pi\pi$	1.72×10^3	3.46	2.61×10^{-19}
$\pi\rho$	1.34×10^3	2.38	1.68×10^{-19}
$\rho\rho$	7.62×10^2	1.48	1.33×10^{-19}

(a)

	c_{AB} GeV	$\sqrt{\langle O_2^2 \rangle}$ GeV^2	$\lvert\delta\,\mathrm{Im}\,d_\tau^\gamma\rvert$ e cm
$\pi\pi$	2.49×10^{-1}	1.19	6.20×10^{-16}
$\pi\rho$	1.71×10^{-1}	1.28	7.03×10^{-16}
$\rho\rho$	9.35×10^{-2}	1.15	8.39×10^{-16}

(b)

Table 9

$\sqrt{s}$ GeV	$\lvert\sum N_{ij} r_{ij}\rvert / \sum N_{ij}$
3.67	9.2×10^{-5}
4.25	1.2×10^{-4}
10.58	7.7×10^{-4}

Table 10

P	a	b	c	d
0.00	1.73×10^{-12}	1.38×10^{-13}	4.17×10^{-13}	2.70×10^{-9}
-0.62	2.15×10^{-16}	1.32×10^{-13}	3.97×10^{-13}	5.52×10^{-10}
$+0.62$	2.15×10^{-16}	1.45×10^{-13}	4.39×10^{-13}	9.32×10^{-10}
-0.71	1.88×10^{-16}	1.31×10^{-13}	3.95×10^{-13}	4.94×10^{-10}
$+0.71$	1.88×10^{-16}	1.47×10^{-13}	4.42×10^{-13}	7.80×10^{-10}
-1.00	1.33×10^{-16}	1.28×10^{-13}	3.86×10^{-13}	3.71×10^{-10}
$+1.00$	1.33×10^{-16}	1.50×10^{-13}	4.53×10^{-13}	5.11×10^{-10}

(a)

P	a	b	c	d
0.00	2.39×10^{-13}	2.03×10^{-14}	5.18×10^{-14}	2.50×10^{-10}
-0.62	4.23×10^{-17}	1.93×10^{-14}	4.93×10^{-14}	5.11×10^{-11}
$+0.62$	4.23×10^{-17}	2.13×10^{-14}	5.45×10^{-14}	8.63×10^{-11}
-0.71	3.69×10^{-17}	1.92×10^{-14}	4.90×10^{-14}	4.58×10^{-10}
$+0.71$	3.70×10^{-17}	2.15×10^{-14}	5.49×10^{-14}	7.22×10^{-10}
-1.00	2.62×10^{-17}	1.88×10^{-14}	4.80×10^{-14}	3.43×10^{-11}
$+1.00$	2.62×10^{-17}	2.21×10^{-14}	5.63×10^{-14}	4.73×10^{-11}

(b)

P	a	b	c	d
0.00	5.34×10^{-15}	4.66×10^{-16}	1.65×10^{-15}	1.27×10^{-12}
-0.62	6.09×10^{-18}	4.44×10^{-16}	1.57×10^{-15}	2.60×10^{-13}
$+0.62$	6.09×10^{-18}	4.91×10^{-16}	1.74×10^{-15}	4.40×10^{-13}
-0.71	5.32×10^{-18}	4.41×10^{-16}	1.56×10^{-15}	2.33×10^{-13}
$+0.71$	5.32×10^{-18}	4.94×10^{-16}	1.75×10^{-15}	3.68×10^{-13}
-1.00	3.78×10^{-18}	4.31×10^{-16}	1.53×10^{-15}	1.75×10^{-13}
$+1.00$	3.78×10^{-18}	5.07×10^{-16}	1.80×10^{-15}	2.41×10^{-13}

(c)

Table 11

$\sqrt{s}$ GeV	a	b	c	d
3.67	7.14×10^{-19}	9.29×10^{-15}	3.52×10^{-14}	2.91×10^{-12}
4.25	9.11×10^{-20}	8.79×10^{-16}	5.07×10^{-15}	3.13×10^{-13}
10.58	9.68×10^{-20}	1.49×10^{-16}	4.06×10^{-16}	3.99×10^{-15}

Table 12

P_{max}	A	B	C	D
0.62	-4.34×10^{-15}	2.79×10^{-12}	5.17×10^{-13}	-1.66×10^{-10}
	2.16×10^{-16}	-4.55×10^{-16}	-2.02×10^{-12}	3.35×10^{-9}
0.71	-3.31×10^{-15}	2.44×10^{-12}	5.58×10^{-13}	-2.06×10^{-10}
	1.88×10^{-16}	-3.25×10^{-16}	-2.50×10^{-12}	3.62×10^{-9}
1.00	-1.66×10^{-15}	1.73×10^{-12}	8.40×10^{-13}	-4.36×10^{-10}
	1.33×10^{-16}	-1.94×10^{-16}	-5.28×10^{-12}	5.44×10^{-9}

(a)

P_{max}	A	B	C	D
0.62	-8.54×10^{-16}	4.10×10^{-13}	6.42×10^{-14}	-1.54×10^{-11}
	4.25×10^{-17}	-6.78×10^{-17}	-2.50×10^{-13}	3.10×10^{-10}
0.71	-6.51×10^{-16}	3.58×10^{-13}	6.94×10^{-14}	-1.91×10^{-11}
	3.70×10^{-17}	-4.83×10^{-17}	-3.10×10^{-13}	3.35×10^{-10}
1.00	-3.28×10^{-16}	2.54×10^{-13}	1.04×10^{-13}	-4.04×10^{-11}
	2.63×10^{-17}	-3.38×10^{-17}	-6.56×10^{-13}	5.04×10^{-10}

(b)

P_{max}	A	B	C	D
0.62	-1.24×10^{-16}	9.50×10^{-15}	2.05×10^{-15}	-7.91×10^{-14}
	6.17×10^{-18}	-5.61×10^{-18}	-7.99×10^{-15}	1.58×10^{-12}
0.71	-9.43×10^{-17}	8.26×10^{-15}	2.22×10^{-15}	-9.79×10^{-14}
	5.36×10^{-18}	-3.57×10^{-18}	-9.89×10^{-15}	1.71×10^{-12}
1.00	-4.72×10^{-17}	5.83×10^{-15}	3.34×10^{-15}	-2.07×10^{-13}
	3.78×10^{-18}	-3.86×10^{-19}	-2.09×10^{-14}	2.56×10^{-12}

(c)

Table 13