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Decay and Decoupling of heavy Right-handed Majorana Neutrinos in the L-R model

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Abstract

Heavy right-handed neutrinos are of current interest. The interactions and decay of such neutrinos determine their decoupling epoch during the evolution of the universe. This in turn affects various observable features like the energy density, nucleosynthesis, CMBR spectrum, galaxy formation, and baryogenesis. Here, we consider reduction of right-handed electron-type Majorana neutrinos, in the left-right symmetric model, by the $W_R^+ W_R^-$ channel and the channel originating from an anomaly, involving the $SU(2)_R$ gauge group, as well as decay of such neutrinos. We study the reduction of these neutrinos for different ranges of left-right model parameters, and find that, if the neutrino mass exceeds the right-handed gauge boson mass, then the neutrinos never decouple for realistic values of the parameters, but, rather, decay in equilibrium. Because there is no out-of-equilibrium decay, no mass bounds can be set for the neutrinos.

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1 Introduction

If a see-saw mechanism [1] is to account for left-handed electron neutrino ν masses sufficiently small to be consistent with current ideas on neutrino oscillations, right-handed N neutrinos with mass, M, in the TeV scale come into the picture [2, 3]. High N masses in the range 1-20 TeV, and even higher masses, have been considered in studies of leptogenesis – baryogenesis [2, 4, 5] and e^-e^- collisions [3]. While some studies [5] consider M smaller than M_W, the right-handed W_R boson mass, others specifically use N masses greater than the W_R mass [3, 6]. As cosmological and laboratory lower bounds for Z', W_R masses are of the order of 0.5 TeV, 1.6-3.2 TeV, respectively [7], there is no reason not to consider N masses greater than Z', W_R masses. Cosmological mass bounds for M, with $M \gg M_W$, have been considered in [8]-[12].

In a recent work [12], $(B+L)$–violation from an anomaly, involving the $SU(2)_R$ gauge group [2, 13, 14], was considered as a generation/reduction channel for N neutrinos satisfying $M > M_W$. It was found that this anomalous channel played a role, in the decoupling of such neutrinos, at least as important as the $N\bar{N} \to W_R^+ W_R^-$ channel (each of these channels was found more important than the $N\bar{N} \to F\bar{F}$ channel, F representing a relevant fermion). Matrix elements for $N\bar{N} \to F\bar{F}$ and $N\bar{N} \to W_R^+ W_R^-$ were calculated in [12] from the Left-Right symmetric extension [15, 16] of the standard model, as an illustration.

In the above work [12], the N neutrinos were assumed to be stable, for simplicity. If, however, the Left-Right symmetric model (L-R model, hereafter) is to be taken as a serious working basis, N neutrinos cannot be considered to be stable. Decays involving $\nu - N$ mixing, $W_L - W_R$ mixing, and generation mixing, and CP-violating decays have been extensively studied. The last scenario has been widely used in generating lepton and, hence, baryon number from decay of Majorana-type N neutrinos [4, 5, 17, 18, 19]. If the N mass is considered to be greater than the W_R mass, then these decay channels will be marginalised by the channel $N \to W_R^+ + e^-$. Such a fast decay should have important effects on decoupling. It is this which is studied in detail in the present paper. The effect on leptogenesis has already received considerable attention [20].

The effect of the decay of massive neutrinos on energy density, nucleosynthesis, the Cosmic Microwave Background Radiation (CMBR), galaxy formation, and stellar evolution has been well-studied [21, 22]. In these studies, the decay time was taken to be large (typically, larger than 100-200 seconds), and the effect of decay at the crucial epochs followed up. The decay time was chosen greater than the decoupling time, *i.e.*, it was assumed that the neutrinos first decoupled, and, then, their cosmological and astrophysical effects were felt, as a result of subsequent out-of-equilibrium decay [22]. If a fast decay like $N \to W_R^+ + e^-$ is considered, the relationship of decoupling and decay may not be like this, and the two have been considered, in the present paper, together, in one Boltzmann equation.

The plan of the paper is as follows. Section 1 is the Introduction. In section 2, the thermally averaged $N\bar{N} \to W_R^+ W_R^-$ annihilation cross-section times relative velocity and the thermally averaged decay rate are calculated in the L-R model, assuming pure Majorana neutrinos. The anomalous rate per unit volume, used in ref.[12], is slightly modified to accommodate pure Majorana neutrinos. In section 3, the Boltzmann equation is writ-

ten down and treated approximately to obtain a decoupling criterion. The possibility of decoupling is investigated numerically. Section 4 discusses the conclusions.

2 Annihilation, Decay, and Anomalous Reduction of Right-handed neutrinos: Thermal averages in L-R model

2.1 Summary of relevant features of the L-R model

In the L-R model, there are two doublets (ν, e_L) and (N, e_R) belonging to the representations $(\frac{1}{2}, 0, -1)$ and $(0, \frac{1}{2}, -1)$, respectively, of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where the quantum numbers refer to the values of T_L, T_R, and $B-L$ respectively. To simplify the issues, only one generation is considered (the lightest), and $N-\nu$, $W_L - W_R$, and $Z - Z'$ mixings are neglected.

The symmetry-breaking from $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(2)_L \times U(1)_Y$ is achieved by means of a scalar triplet ($T_L = 0, T_R = 1$ and $B - L = 2$) $\Delta \equiv (\Delta^{++}, \Delta^{+}, \Delta^{0})$, putting $< \Delta^0 >= v_R/\sqrt{2}$ (chosen real). The right-handed gauge boson becomes massive due to the piece of the Lagrangian

$$L_{\Delta W^+ W^-} = D^\mu \Delta^\dagger D_\mu \Delta \tag{1}$$

with $D_\mu = \partial_\mu + ig\vec{T}.\vec{W}_\mu + i(g'/2)B_\mu$. From now on, the subscript R is dropped except when essential. The T_i form a 3×3 representation of the $SU(2)$ generators in a spherical basis. Z'_μ has the definition

$$Z'_\mu = (\sqrt{\cos 2\theta}/\cos\theta) W_3^\mu - \tan\theta B^\mu \tag{2}$$

(1) gives a W mass $M_W^2 = \frac{1}{2} g^2 v_R^2$, and an interaction

$$L_{HW^+W^-} = \sqrt{2} g M_W W_\mu^+ W^{\mu -} H \tag{3}$$

where $H(x)/\sqrt{2} = Re(\Delta^0(x)) - v_R/\sqrt{2}$, the displaced neutral Higgs field.
Neglecting $Z - Z'$ mixing, one gets the Z' neutral current piece

$$L_{N\bar{N}}^{Z'} = (g/2\sqrt{\cos 2\theta}) \cos\theta \bar{N} \gamma^\mu P_R N Z'_\mu. \tag{4}$$

θ is the weak mixing angle. The required charged current piece of the Lagrangian is

$$L_{Ne}^W = (g/\sqrt{2})(\bar{N}\gamma^\mu P_R e W_\mu + h.c.). \tag{5}$$

The Z', W^+, W^- interaction piece is

$$\begin{aligned} L_{W^+W^-}^{Z'} &= -ig(\sqrt{\cos 2\theta}/\cos\theta)[Z'^\mu(\partial_\mu W_\nu^+ W^{\nu -} - \partial_\nu W_\mu^+ W^{\nu -} - \partial_\mu W_\nu^- W^{\nu +} \\ &\quad + \partial_\nu W_\mu^- W^{\nu +}) + \frac{1}{2}(\partial_\mu Z'_\nu - \partial_\nu Z'_\mu)(W^{\mu +} W^{\nu -} - W^{\mu -} W^{\nu +})] \end{aligned} \tag{6}$$

The Majorana mass of N is thought to arise from the piece

$$L_{R\bar{R}\Delta} = f R^T C \epsilon (\vec{\tau}/\sqrt{2}).\vec{\Delta} R + h.c. \tag{7}$$

where R^T is the $SU(2)_R$ doublet (N, e_R), C is the charge conjugation matrix, and $\epsilon = i\tau_2$. $(\vec{\tau}/\sqrt{2}).\vec{\Delta}$ is the matrix

$$\begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix},$$

and $\epsilon(\vec{\tau}/\sqrt{2}).\vec{\Delta}$ is symmetric. This term gives a Majorana mass for the N neutrinos, $M = fv_R/\sqrt{2}$, so that $g/f = M_W/M = r_W$, and an interaction

$$L_{N\bar{N}H} = (g/\sqrt{2}r_W)\bar{N}^C H N \tag{8}$$

N^C is the field conjugate to N. It has been pointed out [19, 23] that the effective one-loop mass matrix for multi-generation unstable Majorana neutrinos is not Hermitian and, strictly speaking, $N \neq N^C$. But, here, at the level of tree-order calculations for a single generation, $N = N^C$ will be assumed.

2.2 $< \sigma|v| >$ for $N\bar{N} \to W_R^+ W_R^-$

Three amplitudes $iM_{Z'}, iM_e, iM_H$ have been considered for $N\bar{N} \to W^+W^-$, these arising, respectively, from $N\bar{N} \xrightarrow{Z'} W^+W^-$ in the s-channel, $N\bar{N} \xrightarrow{e} W^+W^-$ in the t-channel, and $N\bar{N} \xrightarrow{H} W^+W^-$ in the s-channel. Earlier calculations, with $r_W \ll 1$ [9], considered a heavy charged lepton exchange in the t-channel. In this paper, as required by the L-R model, an ordinary electron is considered to be exchanged.

$iM_{Z'}$ is calculated from (4) and (6), iM_e from (5), and iM_H from (3) and (8). One finds that, in the limit $M \to 0$ and $s \gg M_W^2$, iM_e and $iM_{Z'}$ cancel in tree-order, as expected. With a massive $M \gg M_W$, an extra M-dependent term remains in $iM_{Z'} + iM_e$. On calculating $\sum |\mathcal{M}|^2$, it is found that the interference terms between iM_H, and $iM_{Z'} + iM_e$ cancel, and for $r_W \ll 1$, in the CM frame,

$$\sum |\mathcal{M}|^2 = \frac{2g^4}{r_W^4}\left[\frac{1}{t^2}E^2\mathbf{k}^2\sin^2\theta_{CM} + \frac{16}{M^2(s-M_H^2)^2}E^6\right]. \tag{9}$$

$(E, \vec{\mathbf{k}})$ is the 4-momentum of the N neutrino in the CM frame, and θ_{CM}, the angle of scattering. $\mathbf{k}$ has been written for $|\vec{\mathbf{k}}|$.

The thermally averaged cross-section times relative velocity is

$$\begin{aligned} < \sigma|v| > \;\; &= \;\; (1/n_{eq}^2)\int d\pi_N d\pi_{\bar{N}} d\pi_{W^+} d\pi_{W^-}(2\pi^4)\delta^4(p_N + p_{\bar{N}} - p_{W^+} - p_{W^-}) \\ &\quad \times \sum |\bar{\mathcal{M}}|^2 e^{-E_N/T} e^{-E_{\bar{N}}/T}. \end{aligned} \tag{10}$$

n_{eq} is the equilibrium value of the number density n of the N neutrinos.

$$n_{eq} = g_N\left[\frac{MT}{(2\pi)}\right]^{\frac{3}{2}} e^{-M/T}. \tag{11}$$

$g_N = 2$ for Majorana neutrinos. The measure $d\pi_i = g_i d^3p_i/[(2\pi)^3 2E_i]$. $\sum |\bar{\mathcal{M}}|^2$ is the spin-averaged matrix element squared, with symmetry factor 1/2!, arising from the identification $N = N^C$.

The invariant integral $\int d\pi_{W^+} d\pi_{W^-} (2\pi)^4 \delta^4(p_N + p_{\bar{N}} - p_{W^+} - p_{W^-}) \sum |\bar{\mathcal{M}}|^2$ is calculated in the CM frame, then transformed to the comoving "lab" frame, in which $N, \bar{N}$ have energies $E_N, E_{\bar{N}}$, respectively, and thermally averaged according to (10). In all calculations, N neutrinos are non-relativistic, *viz.* $E_N = M + \mathbf{k}_N^2/(2M)$, $\mathbf{k}_N^2 \ll M^2$. This is because the interesting region for decoupling studies of a massive particle has $T \ll M$.

The result is

$$< \sigma|v| >= \frac{1}{2!} \frac{g^4}{64\pi r_W^4} \left[\frac{T}{M^3} + \frac{16}{M^2(4 - r_H^2)^2} \left\{ 1 - \frac{3T(4 + r_H^2)}{2M(4 - r_H^2)} \right\} \right]. \tag{12}$$

$r_H = M_H/M$.
The first term arises from Z'- and e-exchange, and apart from the 1/2! factor, agrees with the result of [11], where the results of [9] have been considered in the limit $s \to 4M^2$. The second term originates from H-exchange. In the calculation of this term, a further approximation has been made, *viz.* $4\mathbf{k}^2 \ll |4M^2 - M_H^2|$, *i.e.*, this calculation is reliable provided M_H is not very near in value to $2M$. In [9], the H-exchange contribution was found to be negligible as $s \to 4M^2$, because of a factor $(s - 4M^2)$ which arises for Dirac neutrinos. For Majorana neutrinos, this factor is absent, and this term cannot be neglected. As noted earlier, there is no interference between the H-exchange amplitude and those arising from Z'- and e-exchange. In [12], only the first term in (12) was considered, which is the approximation $M_H \gg M$.

The processes $N\bar{N} \to F\bar{F}$, where F is a relevant fermion, have not been considered here. For $M \gg M_W$, the contribution of these processes is small compared to that of $N\bar{N} \to W^+W^-$ [9], and their effect on decoupling is overshadowed by the effects of $N\bar{N} \to W^+W^-$ and the anomalous reduction of N neutrinos [12].

2.3 Thermally averaged decay width of N neutrinos

The decay width for $N \to W^+ + e^-$ can be calculated from (5). Calculation gives the spin-averaged matrix element squared in the neutrino rest frame

$$\sum |\bar{\mathcal{M}}|^2 = g^2 \left[\frac{1}{2} M\mathbf{p} + \frac{M(M - \mathbf{p})M\mathbf{p}}{M_W^2} \right],$$

with $\mathbf{p} = \frac{1}{2}M(1 - r_W^2)$ being the momentum of the decay products, resulting in the width

$$\Gamma_e = \frac{g^2 M}{32\pi} (1 - r_W^2)^2 \left(1 + \frac{1}{2r_W^2} \right). \tag{13}$$

In the frame in which the neutrino has energy E_N, the width becomes

$$\Gamma_e^E = (M/E_N)\Gamma_e. \tag{14}$$

The thermally averaged decay width [24, 25] is defined as

$$\begin{aligned}\bar{\Gamma}_e &= (1/n_{eq})\int d\pi_N d\pi_{W^+} d\pi_{W^-}(2\pi)^4\delta^4(p_N - p_{W^+} - p_{W^-})\sum|\bar{\mathcal{M}}|^2 e^{-E_N/T} \\ &= (1/n_{eq})\int (g_N d^3p_N/(2\pi)^3)\Gamma_e^E e^{-E_N/T}.\end{aligned}$$

Using (13) and (14), one gets

$$\bar{\Gamma}_e = \frac{g^2 M}{32\pi}(1-r_W^2)^2\left(1+\frac{1}{2r_W^2}\right)\left(1-\frac{3T}{2M}\right). \tag{15}$$

The calculation has been done in the approximation $T \ll M, \mathbf{k}_N^2 \ll M^2$. For $M \gg M_W^2$,

$$\bar{\Gamma}_e = \frac{g^2 M}{64\pi r_W^2}\left(1-\frac{3T}{2M}\right). \tag{16}$$

If the mass of the N neutrino is greater than the Δ^+ mass, (7) allows the decay $N \to \Delta^+ + e^-$. The corresponding

$$\sum|\bar{\mathcal{M}}|^2 = (1/2)f^2(M^2 - M_+^2),$$

where M_+ is the Δ^+ mass. In the rest frame of the neutrino, this decay width comes out as

$$\Gamma_+ = (g^2/64\pi)(M^2/M_W^2)M(1-r_+^2)^2, \tag{17}$$

where $r_+ = M_+/M$. There is another Yukawa piece of the Lagrangian

$$L_{LR\Phi} = \sum_{i,j}(h_{ij}\bar{L}_i\Phi R_j + h'_{ij}\bar{L}_i\tau_2\Phi^\star\tau_2 R_j) + h.c. \tag{18}$$

The scalar field Φ transforms under the gauge group as $(\frac{1}{2},\frac{1}{2},0)$ and is represented by the matrix

$$\Phi = \begin{pmatrix} \phi^0 & \phi'^+ \\ \phi^- & \phi'^0 \end{pmatrix}.$$

Such pieces have been used in different models to break CP and induce baryogenesis through leptogenesis [4, 5, 17, 18, 19, 20]. If one generation is considered, the coupling constant h contributes to the electron mass and must be very small. To fit the observed baryon asymmetry, with N mass in the TeV range, $|h_{ij}|^2$ values of the order of 10^{-10} to 10^{-13} have been considered [4, 5]. In this situation, as CP breaking has not been considered here, and only one generation taken into account, it has not been thought useful to consider N-decays arising from (18). In any case, it will be found that any enhancement of the decay width given in (15) and (16) will only strengthen the main conclusion of the paper.

2.4 Anomalous generation of Majorana neutrinos

The one generation, sphaleron-mediated, fermion number violating transition rate per unit volume, with $|\Delta L| = 1$, $|\Delta B| = 1$, for the quantum anomaly, involving the $SU(2)_R$ gauge group, was written, in [12], by extrapolation from the $SU(2)_L$ case [26], as

$$\begin{aligned} A_R &= (1.4 \times 10^6) \left(\frac{bM_W^7}{g^6 T^3}\right) \left[1 - \left(\frac{T}{zM_W}\right)^2\right]^{7/2} \\ &\quad \times \exp\left\{-\frac{16\pi M_W}{g^2 T}\left[1 - \left(\frac{T}{zM_W}\right)^2\right]^{1/2}\right\}. \end{aligned} \tag{19}$$

M_W is the zero temperature W_R mass. $z = T_R/M_W$, where T_R is the critical temperature associated with the breaking of the $SU(2)_R$ gauge symmetry. So, z is essentially a quantity which reflects the uncertainty in the values of the L-R model parameters, while b captures the uncertainties involved in the extrapolation from the $SU(2)_L$ to the $SU(2)_R$ case in addition to those in the estimation of the prefactor of the anomaly driven transition [27].

In [12], the anomalous rate of reduction of N neutrinos was considered, maintaining a distinction between N and N^C. Here, $N = N^C$ neutrinos are considered, to maintain uniformity with the $N\bar{N} \to W^+W^-$ calculations. This entails an extra factor of 2, as seen below.

For an anomalous process l, with $\Delta_L = +1$, such that

$$l : i + j + \cdots \to N + a + b + \cdots,$$

one writes [12],

$$\begin{aligned} A_l &= \int d\pi_N d\pi_a d\pi_b \cdots d\pi_i d\pi_j \cdots |\mathcal{M}_l|^2 \\ &\quad \times (2\pi)^4 \delta^4(p_N + p_a + p_b + \cdots - p_i - p_j - \cdots) f_N^{eq} f_a f_b \cdots \\ &= I_l n_{eq}, \end{aligned} \tag{20}$$

where I_l contains the result of the phase space integrations, apart from n_{eq} [24, 25], and $i, j, \cdots a, b, \cdots$ are all supposed to be in equilibrium.

Taking the view that leptogenesis and baryogenesis are effects of a smaller order, CP-symmetry is assumed. Then [12], for each process l, there is a $\Delta L = -1$ process

$$\bar{l}' : N + a + b + \cdots \to i + j + \cdots,$$

with the same $|\mathcal{M}_l|^2$. For this process,

$$\begin{aligned} A_{\bar{l}'} &= \int d\pi_N d\pi_a d\pi_b \cdots d\pi_i d\pi_j \cdots |\mathcal{M}_l|^2 \\ &\quad \times (2\pi)^4 \delta^4(p_N + p_a + p_b + \cdots - p_i - p_j - \cdots) f_N f_a f_b \cdots \\ &= I_l n \end{aligned} \tag{21}$$

The $\Delta L = +1$ processes, generating N neutrinos, add up to

$$\sum_l A_l = n_{eq} \sum_l I_l = \frac{1}{2} A_R \exp(-\beta \mu_L/2),$$

from [26]. $\mu_L = \mu_N$ is the chemical potential ($\mu_e = 0$, as electrons are in equilibrium). The 1/2 factor arises from the assumption [12], that, to a first approximation, the rate of generation of one member of a lepton doublet may be taken to be the same as that of the other, near equilibrium (not a bad condition at decoupling when the neutrinos are just falling out of equilibrium).
Similarly,

$$\sum_{\bar{l}'} A_{\bar{l}'} = n \sum_l I_l = \frac{1}{2} A_R \exp(+\beta \mu_L/2).$$

So, for small μ_N [26],

$$n_{eq} \textstyle\sum_l I_l \approx \frac{1}{2} A_R (1 - \beta \mu_N/2),$$
$$n \textstyle\sum_l I_l \approx \frac{1}{2} A_R (1 + \beta \mu_N/2).$$

One gets

$$\sum_l I_l = A_R/(n + n_{eq}),$$

and the anomalous rate of reduction of N neutrinos per unit volume

$$A_N = \sum_{\bar{l}'} A_{\bar{l}'} - \sum_l A_l = \frac{n - n_{eq}}{n + n_{eq}} A_R. \tag{22}$$

This has an extra factor of 2, compared to [12], because anti-particle processes, which had to be considered separately there, do not appear here, because of the assumption $N = N^C$.

3 Effect of decay on decoupling

3.1 The Boltzmann equation for N neutrinos

Using the results of the last section, one can write the Boltzmann equation

$$\frac{dn}{dt} + 3Hn = -2 < \sigma |v| > (n^2 - n_{eq}^2) - 2\bar{\Gamma}_e (n - n_{eq}) - \frac{n - n_{eq}}{n + n_{eq}} A_R, \tag{23}$$

where the second term on the left gives the effect of expansion, and the three terms on the right are to be taken from (12), (16), and (22), respectively.
The expression for the Hubble parameter H is

$$H = 1.66 g^{\star \frac{1}{2}} T^2 / M_{Pl}. \tag{24}$$

$g^\star$ is taken ≈ 100, and $M_{Pl} = 1.22 \times 10^{19}$ GeV. The 2 factor with $< \sigma |v| >$ appears because two neutrinos are disappearing in $N\bar{N} \to W^+ W^-$, considering $N = N^C$. The 2 factor with

$\bar{\Gamma}_e$ appears because of the two decay channels $N \to W^+ + e^-$ and $N \to W^- + e^+$.Writing $x = M/T$ and $Y = n/s$, where $s = g^{\star S}(2\pi^2/45)T^3$, with $g^{\star S} \approx 100$, (23) becomes

$$\frac{dY}{dx} = -f(x)(Y^2 - Y_{eq}^2) - d(x)(Y - Y_{eq}) - g(x)\frac{Y - Y_{eq}}{Y + Y_{eq}}. \tag{25}$$

In (25),

$$f(x) = \left(\frac{1.41 \times 10^{16}\ \text{GeV}}{M_W}\right)\left(\frac{a}{x}\right)^3 \left[\frac{2x}{(1 - r_H^2/4)^2} - h(r_H)\right], \tag{26}$$

where $a = 1/r_W = M/M_W$ and

$$h(y) = \frac{1}{(1 - y^2/4)^3}\left[1 + y^2\left\{(1 - y^2/8)(1 - y^2/4) + 5/4\right\}\right], \tag{27}$$

$$d(x) = \left(\frac{3.08 \times 10^{15}\ \text{GeV}}{M_W}\right) ax \left[1 - \frac{3}{2x}\right], \tag{28}$$

$$\begin{aligned} g(x) \quad &= \quad \left(\frac{3.06 \times 10^{23}\ \text{GeV}}{M_W}\right)\frac{b}{a}\left[\frac{1}{u^2} - \frac{1}{z^2}\right]^{\frac{7}{2}} \\ &\quad \times \exp\left\{-118.98\left[\frac{1}{u^2} - \frac{1}{z^2}\right]^{\frac{1}{2}}\right\}, \end{aligned} \tag{29}$$

with $u = a/x = T/M_W$.

Writing $Y = Y_{eq} + \Delta$, it is noted that, before decoupling, Y is close to Y_{eq}, and Δ' may be put equal to zero. Then, (25) may be put in the form

$$\Delta = \frac{-Y'_{eq}}{f(x)(2Y_{eq} + \Delta) + d(x) + \frac{g(x)}{2Y_{eq} + \Delta}}.$$

From n_{eq} in (11), one gets

$$Y_{eq} = 2.89 \times 10^{-3} x^{\frac{3}{2}} e^{-x}, \quad \text{and} \quad Y'_{eq} \approx -Y_{eq},$$

at decoupling, when it is expected that $x = x_d \gg 1$. The criterion for decoupling may be taken as $\Delta = c' Y_{eq}$, where c' is of order 1. As in [12], c' is chosen to be 1.
Then, the condition for decoupling is

$$3f(x_d)Y_{eq}(x_d) + d(x_d) + \frac{g(x_d)}{3Y_{eq}(x_d)} = 1 \tag{30}$$

This is the key condition in our analysis of decoupling. Each factor on the lhs of (30) is positive [28]. So, there is no scope for cancellation between different terms. We will show in the following subsections that, in fact, the condition can never be satisfied. The argument proceeds as follows. First, we consider (30), excluding the second (decay) term in the lhs. We show that there is a value x_a, which we obtain numerically below, such that, for $x < x_a$ or $x > x_a$, the lhs is, respectively, greater or less than unity in the absence of decay. We then check that for $x > x_a$ the decay term is much larger than unity so that there is no value of x $(= M/T)$ for which eq. (30) is satisfied.

3.2 Decoupling in the absence of decay

First, $d(x_d)$ is omitted, and the decoupling condition

$$l(x) = 3f(x)Y_{eq}(x) + \frac{g(x)}{3Y_{eq}(x)} = 1 \tag{31}$$

is solved to give $x = x_a$. x_a represents the value of M/T for which decoupling would occur in the absence of decay.

Y_{eq} gives a factor e^{-x}, and, so, the term with $f(x)$ increases as x decreases. $g(x)$ has an exponential factor of the form $e^{-E_{sp}/T} = e^{-Kx/a}$ [12, 14, 26, 29, 30], where E_{sp} is the energy of the sphaleron mode which decays to cause anomalous L generation. The kinematic constraint on N production, $E_{sp} > M$, gives $K/a > 1$ [12], and so $g(x)/Y_{eq} \sim e^{-(\frac{K}{a}-1)x}$, and, again, increases as x decreases. This means that $l(x) > 1$ if $x < x_a$. So, $x < x_a$ will not satisfy the decoupling condition (30). Therefore, on the whole, it may be said approximately [24] that, for $x > x_a$, the annihilation rate plus the anomalous reduction rate is less than H.

This programme is followed numerically. First, for definiteness, we fix b and z, the two parameters in the expression for $g(x)$ – see eq. (29) – at the values 1 and 4, respectively [12]. a is varied from 2-100, for r_H[4] = 0, 1, 3, 10 and x_a found by solving (31) numerically.

r_H=1		r_H=3		r_H=10	
a	x	a	x	a	x
2	25.79	2	25.06	2	22.30
5	28.49	5	27.74	5	24.89
10	30.54	10	29.77	10	26.86
20	32.60	20	31.81	20	28.83
50	35.32	50	34.52	50	32.18
75	73.77	75	73.77	75	73.77
100	238.83	100	238.83	100	238.83

Table I: $x = M/T$ at decoupling for different choices of $a = M/M_W$ and $r_H = M_H/M$ without the inclusion of the decay contribution in the Boltzmann equation. M_W has been chosen to be 4000 GeV

The results for $M_W = 4000$ GeV are shown in Table I for r_H=1,3,10. There is a clear trend, showing an increase in x_a for an increase in a (constant r_H), and a decrease in x_a for an increase in r_H (constant a). The values for r_H=100 and r_H=1000, we have checked, do not differ at all.It is safe to say that $x_a > 20$ for the parameter ranges considered.

We now address the uncertainty regarding the anomalous rate [27]. The uncertainty is embodied in the parameters b and z appearing in the expression for this rate. The parameter $z = T_R/M_W$ is of order unity ($z = 3.8$ in the $SU(2)_L$ case [26]). To gauge the

[4] $h(r_H)$ assumes its asymptotic value of –2 for $r_H > 5$.

a=2		a=50	
b	x	b	x
10^3	22.30	10^3	36.79
10^2	22.30	10^2	35.11
10	22.30	10	33.50
1	22.30	1	32.18
10^{-1}	22.30	10^{-1}	31.60
10^{-2}	22.30	10^{-2}	31.47
10^{-3}	22.30	10^{-3}	31.46

Table II: Effect of uncertainty in anomalous rate on the results of Table I

sensitivity of the results on z, a calculation was made with a=50 (to give good weightage to the anomalous transition factor $g(x)$) and r_H=1. x_a, obtained by solving (31), was 42 for $z = 2$, 35.6 for $z = 3$, and reached an asymptotic value of 35.3 for $z \geq 4$. For r_H=10, x_a was 42 for $z = 2$, 35 for $z = 3$, and had an asymptotic value of 31.5. The conclusion $x_a > 20$, of the last paragraph is not disturbed. In [12], the uncertainty in b was taken into account by varying it through six orders of magnitude about $b = 1$. We do a similar calculation for $z = 4$, $r_H = 10$, $M_W = 4000$ GeV and present the results in Table II. It is clear that even this large variation of b through six orders of magnitude does not affect the conclusion of the previous paragraph.

For comparison, the results for $M_W = 2000$ GeV, for the same values of the other parameters, are shown in Table III.

r_H=1		r_H=3		r_H=10	
a	x	a	x	a	x
2	26.47	2	25.73	2	22.95
5	29.18	5	28.41	5	25.54
10	31.23	10	30.45	10	27.51
20	33.28	20	32.49	20	29.49
50	36.00	50	35.20	50	32.75
75	75.01	75	75.01	75	75.01
100	242.83	100	242.83	100	242.83

Table III: Same as in Table I but for $M_W = 2000$ GeV

The expectation that $l(x)$ – see eq. (31) – decreases below 1, as x increases through x_a, was verified numerically for $M_W = 4000$ GeV, and a = 10,20,50,75, for each of r_H=1,10.

Table IV shows the results for r_H = 1,10, and a = 10,50.

r_H=1				r_H=10			
a=10		a=50		a=10		a=50	
x	$l(x)$	x	$l(x)$	x	$l(x)$	x	$l(x)$
20	45.23×10^3	30	227.87	20	14.61×10^2	25	13.13×10^3
35	1.09×10^{-2}	40	8.70×10^{-3}	30	3.67×10^{-2}	40	1.51×10^{-4}

Table IV: Behavior of $l(x)$ (31) around x_a in the absence of decay. M_W = 4000 GeV for this Table.

3.3 Effect of decay

From the numerical results of the previous subsection it would be safe to say that for $x < 20, l(x) > 1$. Now, if one looks at $d(x)$ (28), it is clear that, for $x > 20, d(x) \gg 1$, and, moreover, as x increases further, $d(x)$ increases. So, there is no possibility of the decoupling condition (30) being satisfied.

$d(x)$, of course, has a simple physical meaning. In the lhs of (23),

$$\frac{dn}{dt} + 3Hn = Hsx\frac{dY}{dx}.$$

Comparing (23) and (25), $d(x) = (2\bar{\Gamma}_e/Hx)$. If $d(x) \gg 1$ for $x = x_a$, with $x_a \sim 20$, this means that $\bar{\Gamma}_e \gg H$ at this point. Considering the physical meaning of x_a, it may be concluded that, although, at temperatures lower than $T_a = M/x_a$, the annihilation rate plus anomalous reduction rate falls below the expansion rate, the decay rate remains much faster than the expansion, and this prevents decoupling.

In the absence of decoupling, the fast decay constrains the N neutrino number density to follow the equilibrium density $\sim e^{-M/T}$. There is no out-of-equilibrium decay.

One may check that this conclusion is not an artefact of the approximation $M^2 \gg M_W^2$ in the calculations. Without this approximation, (15) gives the decay rate

$$\bar{\Gamma}_e = \frac{g^2 M(a^2-1)^2(a^2+2)(1-(3/2x))}{64\pi a^4} \tag{32}$$

so that

$$d(x) = \left(\frac{3.08 \times 10^{15}\ \text{GeV}}{M_W}\right)\frac{(a^2-1)^2(a^2+2)x}{a^5}\left[1-\frac{3}{2x}\right]. \tag{33}$$

Simple calculation shows that if $d(x)$ is to be < 1, for $x \sim 20$, then one must have $a \sim 1$, to 1 part in 10^7 (for $M_W = 4000$ GeV). There is no reason for such fine tuning between N and W masses.

If the N decay width is augmented by the Δ^+, e^- channel, *i.e.*, if $M > M_+$,

$$d(x) = \left(\frac{3.08 \times 10^{15}\ \text{GeV}}{M_W}\right)\left[\frac{(a^2-1)^2(a^2+2)}{a^5} + a(1-r_+^2)^2\right] x\left[1 - \frac{3}{2x}\right]. \qquad (34)$$

In this case, if $d(x)$ is to be < 1 for $x \sim 20$, with $M_W = 4000$ GeV, not only must $r_W \sim 1$, but, also, $r_+ \sim 1$, each to 1 part in 10^7, an unacceptable situation.

Clearly, if the decay width is further augmented by introducing other channels, this will change Γ_e, but not the x-dependence of $\bar{\Gamma}_e$ or $d(x)$, so that $d(x) < 1$ for $x \sim 20$ will be an even remoter possibllity.

$d(x)$ can be decreased by increasing M_W. Taking $r_H = 1$, a little calculation shows that $d(x) = 0.999$ and $l(x) = 0.001$ for $x = 4.25$ and $M_W = 1.7 \times 10^{16}$ GeV if $a = 2$, and for $x = 6.6$ and $M_W = 1.6 \times 10^{17}$ GeV if $a = 10$.

This scale of M_W agrees with [20] where the decoupling condition was, however, chosen simply as $\bar{\Gamma}_e < H$ at $T = M$, $\bar{\Gamma}_e$ having been assigned the value $g^2T/8\pi$. In this paper, we have calculated $\Gamma_e = \frac{g^2(M^2-M_W^2)^2(1+M^2/2M_W^2)}{32\pi M^3}$, and taken the thermal average of $(M/E_N)\Gamma_e$ for $T < M$. We have used a decoupling criterion which includes the effect of annihilations and anomalous reduction, for a wide range of the parameters $r_H = M_H/M$ and $r_W = M_W/M$. Also, our method of approximate solution of the Boltzmann equation keeps the temperature of decoupling open and calculable, *e.g.*, in the previous paragraph it was found that decoupling occurs for $M > 10^{16}, (10^{17})$ GeV at $T = M/4$ $(M/6)$, and not at $T = M$.

So, the main point which emerges is that for there to be decoupling of massive neutrinos in the L-R model, it is necessary to consider right-handed gauge boson mass values far above the physically expected L-R mass scale. Further as $M = aM_W$, the see-saw mechanism will, then, give values of the ν mass, which will be unacceptably small, when compared to neutrino oscillation values of Δm^2.

4 Conclusions

It is to be concluded that, in the L-R model, with right-handed neutrino mass greater than the W_R mass, even when the annihilation plus anomalous reduction rate for these neutrinos has fallen below the expansion rate, the decay remains faster than expansion, and becomes increasingly faster. So, massive right-handed neutrinos will decay while remaining in equilibrium.

We find that this is true for a wide range of values of M_W/M and M_H/M, and also for a wide allowance of uncertainty in the anomalous rate.

As the equilibrium number density varies as $e^{-M/T}$, right-handed neutrinos rapidly dwindle in number once the temperature falls below their mass. The decay products, with much lower masses, equilibriate immediately. Hence, there is no question of influencing the present density of the universe, CMBR, and nucleosynthesis or later events, and no mass bound can be set for right-handed neutrinos in the L-R model, if their mass is greater than the W_R boson mass (apart from an upper bound from unitarity [10, 11]).

Because there is no out-of-equilibrium decay, the lepton (and baryon) number generation scenarios, utilising the decay of massive Majorana neutrinos will not work in the L-R model, if the neutrino mass is greater than the W_R boson mass, a result also previously noted in [20].

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