Lepton number violation in Little Higgs model

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In this note we examine the constraints imposed by muon anomalous magnetic moment $((g-2)_\mu)$ and $\mu^- \to e^+ e^- e^-$ on lepton number violating (LNV) couplings of the triplet Higgs in Little Higgs (LH) model.

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The Standard Model (SM) has been remarkably successful in explaining experimental data upto the highest energies available at present. The precision electroweak data suggests that the Higgs boson remains light [1], $m_H < 219$ GeV at 95% CL, upto the Planck's scale. In SM, the Higgs boson however, gets quadratically divergent contribution to its mass and requires fine tuning of parameters which are sensitive to new physics that may be present at scales much higher than electroweak scale. Fine tuning and naturalness requires this new physics to be at the TeV scale. Supersymmetry (SUSY) provides a particularly elegant solution to the Hierarchy problem where quadratic divergences in Higgs mass are canceled between contributions of SM particles and their superpartners. This has the desired effect of stabilizing the electro-weak scale. In Technicolor theories, the hierarchy problem is deferred by introducing new dynamics at a scale not too much above electroweak scale. Theories of large extra dimensions resolve the hierarchy problem by lowering the Planck's scale and modifying quantum gravity at the TeV scale. Phenomenological consequences of these theories have been studied in the literature and constraints obtained [2].

Recently there has been a proposal to consider Higgs fields as pseudo-Nambu-Goldstone boson of a Global symmetry which is spontaneously broken at some high scale [3]. The Higgs fields acquire mass through electroweak symmetry breaking triggered by radiative corrections leading to Coleman-Weinberg type of potential. Since the Higgs is protected by approximate global symmetry, it remains light and the quadratic divergent contributions to its mass are canceled by the contributions of heavy gauge bosons and a heavy Fermionic state that are introduced in the model. The Littlest Higgs (LH) [3, 4, 5] model is a minimal model of this class which accomplishes this task to one loop order within a minimal matter content. The LH model consists of an SU(5) non-linear sigma model which is spontaneously broken to its subgroup SO(5) by vacuum expectation value (VEV) of order f. The gauged group $[SU(2) \times U(1)]^2$ is broken at the same time to its diagonal electroweak SM sub-

group $SU(2) \times U(1)$. The new heavy states in this model consists of heavy gauge bosons (W_H, Z_H, A_H), a triplet Higgs Φ and a vector like 'top quark' which cancels the quadratic divergences coming from the SM top quark. All these particles have masses of the order f and are typically in the TeV range. The effect of these heavy states on electro-weak precision measurements in colliders [4, 5, 6] and some of the low energy processes [7, 8, 9] have been studied earlier in literature.

In the LH model, existence of complex triplet Higgs provides an opportunity to introduce lepton number violation (LNV) and generation of neutrino mass in the theory. Lately there have been studies [9, 10, 11] which explore these possibilities. In the present work we have studied the effects of such LNV couplings in Little Higgs model and have tried to constrain such couplings from $(g-2)_\mu$ and $\mu^- \to e^+ e^- e^-$ data.

In the notation of [3] the LNV interaction, which is invariant under the full gauge group can be written as :

$$\mathcal{L}_{LNV} = -\frac{1}{2} Y_{ab} \left(L_a^T\right)_i \Sigma_{ij}^* C^{-1} \left(L_b^T\right)_j + h.c. \quad (1)$$

where a, b are generation indicies, $i, j = 1, 2$ and $L = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L$ and $Y's$ are coupling constants. This interaction generates a neutrino mass matrix after electro-weak symmetry breaking and because of non-linear nature of Σ_{ij}^*, it has the form ;

$$M_{ab} = Y_{ab} \left(v' + \frac{v^2}{4f}\right) \quad (2)$$

which involve the vacuum expectation values v and v' of Higgs doublet and triplet respectively. No stringent limits on v', the vev of triplet Higgs, exists from the study of electroweak precision tests in LH model except for the bound $\frac{v'^2}{v^2} < \frac{v^2}{12f^2}$ obtained by demanding positive definite mass for the triplet Higgs. Thus in principle it is possible to put $v' = 0$, but as has been argued in [11], it is not a natural choice. The current bounds [12, 13] on neutrino mass from neutrino oscillation, cosmological (WMAP) data and from neutrino-less double β-decay then require Yukawa coupling to be $Y_{ab} \sim 10^{-11}$, which is indeed unnaturally small.

One can however write a LNV interaction using only the complex Higgs triplet Φ which is invariant only under

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the electro-weak gauge symmetry and not under the full gauge symmetry of the LH model [10, 11] ;

$$
\begin{aligned}
\mathcal{L}_{LNV} &= iY_{ab}\left(L_a^T\right)_i \Phi_{ij} C^{-1}\left(L_b^T\right)_j + h.c. \\
&= iY_{ab}\Bigg[\ell_{La}^T C^{-1}\ell_{Lb}\Phi^{++} + \frac{1}{\sqrt{2}}\left(\nu_{La}^T C^{-1}\ell_{Lb}\right. \\
&\quad \left. +\ell_{La}^T C^{-1}\nu_{Lb}\right)\Phi^+ + \nu_{La}^T C^{-1}\nu_{Lb}\Phi^0\Bigg] + h.c.
\end{aligned} \tag{3}
$$

The interaction generates a neutrino mass matrix $M_{ab} = Y_{ab}v'$, after electroweak symmetry breaking.

In this scenario we can have the Yukawa coupling Y_{ab} to be of natural order one provided the triplet VEV v' is restricted to be extremely small. This can be achieved by tuning the parameters such that the coupling of the standard doublet Higgs with triplet Higgs is very small. In this formulation the attractive feature of LH model namely, the cancellation of quadratic divergences in Higgs mass remains unchanged and the interaction is renormalizable.

Neutrino mass bounds require

$$
Y_{ab}v' \sim 10^{-10} GeV \tag{4}
$$

Branching ratios of triplet Higgs scalars in the region of parameter space eqn(4) have been calculated in [10] to search for signals of LNV interactions in collider environment. Bounds on coupling for LNV processes like neutrinoless double β-decay and $K^+ \rightarrow \pi^-\mu^+\mu^-$ decay independent of vev v' have been given in [11].

In LH model the contributions to $(g-2)_\mu$ coming from the exchange of heavy vector bosons, Higgs bosons and heavy vector 'top quark' exchanges has been calculated [1] in [6, 8]. In the presence of LNV interactions given in eqn(3) we have additional Feynman diagrams as given in Figure (1) where the photon is hooked to either the Higgs or to the internal charged lepton line. The contribution
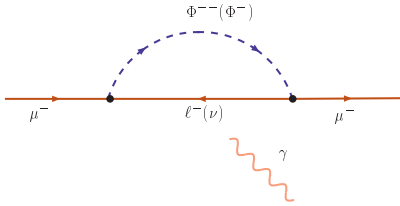

FIG. 1: Feynman diagrams contributing to $(g-2)_\mu$. The photon is to be hooked to all possible internal charged lines

to $(g-2)_\mu$ when photon is hooked to charged Higgs line

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[1] the contributions of heavy particles was found to be negligible and the dominant contribution came from corrections to SM Z & W couplings in LH model

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is [2];

$$
[a_\mu]_1 = q_\Phi \frac{Y_{\mu i}Y_{\mu i}^*}{4\pi^2}m_\mu^2 \int_0^1 dx \frac{x(1-x)^2}{D_1} \tag{5}
$$

where q_Φ is the charge (in the units of μ charge) of Φ and $D_1 = \left[(1-x)m_\Phi^2 + xm_\ell^2 - x(1-x)m_\mu^2\right]$. The contribution from the diagram when photon is hooked to internal lepton line is :

$$
[a_\mu]_2 = q_\ell \frac{Y_{\mu i}Y_{\mu i}^*}{4\pi^2}m_\mu^2 \int_0^1 dx \frac{x(1-x)^2}{D_2} \tag{6}
$$

where q_ℓ is the charge of the internal lepton line (in the units of μ charge) and $D_2 = \left[xm_\Phi^2 + x(1-x)m_\ell^2 + (1-x)m_\mu^2\right]$.

There is another diagram similar to the diagram where photon is emitted from the Higgs. In this diagram the lepton line is replaced by a neutrino line and the doubly charged Higgs is replaced by a singly charged Higgs. The contribution of this diagram is given by :

$$
[a_\mu]_3 = \frac{1}{2}[a_\mu]_1 \tag{7}
$$

where in eqn(7), the q_Φ and m_i are the charges of Φ^- and neutrino mass respectively.

The total contribution in the limit of neglecting lepton masses in comparison to triplet Higgs mass ($m_\Phi >> m_i$) is :

$$
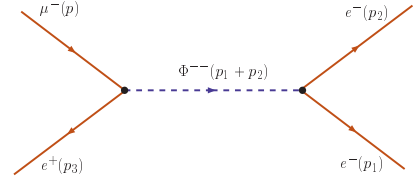[a_\mu]_{tot} = \sum_{i=e,\mu,\tau} \frac{3}{16\pi^2}\frac{m_\mu^2}{m_\Phi^2}|Y_{\mu i}|^2 \tag{8}
$$

FIG. 2: Feynman diagrams contributing to $\mu^- \rightarrow e^+e^-e^-$.

The lepton number violating $\mu^- \rightarrow e^+e^-e^-$ decay is possible through the exchange of doubly charged triplet Higgs as given in Figure 2.

The matrix element for the diagram 2 responsible for the process $\mu^- \rightarrow e^+e^-e^-$ can be written as :

$$
\begin{aligned}
\mathcal{M} &= 4Y_{\mu e}Y_{ee}^* e_L^T(p_3)C^{-1}\mu_L(p)\frac{1}{(p_1+p_2)^2 - m_\Phi^2}\{L(p_1,p_2) \\
&\quad -L(p_2,p_1)\}
\end{aligned} \tag{9}
$$

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[2] where $a_\mu = \frac{g-2}{2}$

where $L(p_1, p_2) = \bar{e}_L(p_1)\bar{e}_L^T(p_2)$. The decay rate now can be calculated from the above matrix element. Neglecting the electron mass we can get the analytical result :

$$\Gamma(\mu^- \rightarrow e^+ e^- e^-) = \frac{|Y_{\mu e}Y_{ee}^*|^2}{48\pi^2}\frac{m_\mu^5}{m_\Phi^4} \qquad (10)$$

The total contributions to $(g-2)_\mu$ due to the new set of diagrams given in fig. (1) would be :

$$\triangle a_\mu = [a_\mu]_1 + [a_\mu]_2 + [a_\mu]_3 \qquad (11)$$

where various $[a_\mu]_i$ are given in eqns(5),(6) and (7).

On comparing the theoretical predictions of SM for $(g-2)_\mu$ with the experimental results we get [14] :

$$\triangle a_\mu(E821 - SM) = a_\mu(E821) - a_\mu(SM)$$
$$= (25.2 \; to \; 26.0 \pm 9.4) \times 10^{-10} \qquad (12)$$

From above result we can see that the discrepancy in SM (from experimental data) is a 2.7σ effect. In our numerical results we have shown the constraints imposed on the LH parameter space if we consider 1σ and 2σ deviations of the above results (eqn 12). In Figure (3) we have shown the contour plots of various values of $\triangle a_\mu$ (as given by eqn 11) in m_Φ and Y_{ab} plane. In the plots the shaded portion corresponds to allowed range of LH parameter space corresponding to 1σ and 2σ deviations. In the next set of plots (Figure 4) we have plotted $\triangle a_\mu$

as a function of triplet Higgs mass for various values of Y_{ab}. As we can see from the plots there is a region in the parameter space where the deviations in $(g-2)_\mu$ can be explained. The allowed region indicates that Y should be of order one.

Now we discuss the constraints imposed by lepton number violating $\mu^- \rightarrow e^+ e^- e^-$. The expression of decayrate for this process is given in eqn(10). The present experimental bound on this process is [1]:

$$\Gamma(\mu^- \rightarrow e^- e^+ e^-)/\Gamma < 1 \times 10^{-12} \qquad (13)$$

This process will not be able to constrain Y independently but would be able to constrain the combination $|Y_{\mu e}Y_{ee}|$. For this purpose we have given two plots in Figure (5). In first of these figures we have given the contour plots of the branching ratio in $|Y_{\mu e}Y_{ee}|$, m_Φ plane. In the second plot we have shown the variation of the branching fraction of $\mu^- \rightarrow e^+ e^- e^-$ as a function of the Higgs mass (m_Φ).

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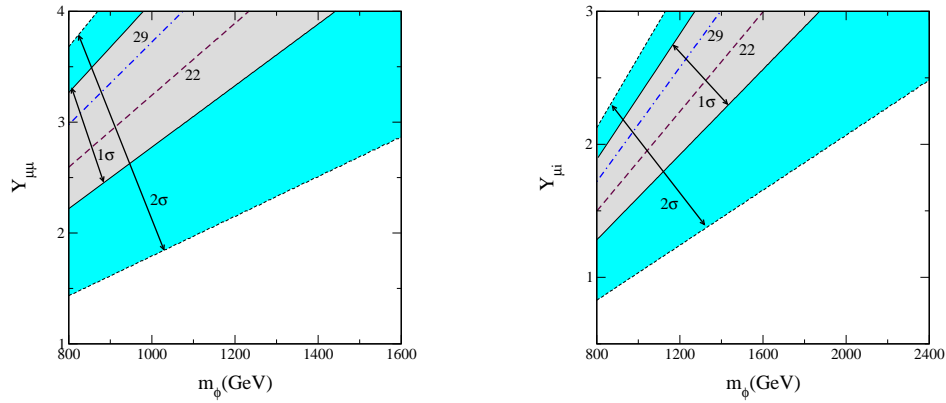[10] T. Han, H. E. Logan, B. Mukhopadhyaya and

FIG. 3: Contour plots in m_ϕ and Y plane. In left panel we have assumed $Y_{\mu i} = 0$ with $i = e, \mu, \tau$ and $i \neq \mu$ and right panel we have assumed $Y_{\mu i} = Y_{\mu\mu}$. Shaded area indicates region of 1σ and 2σ deviations.
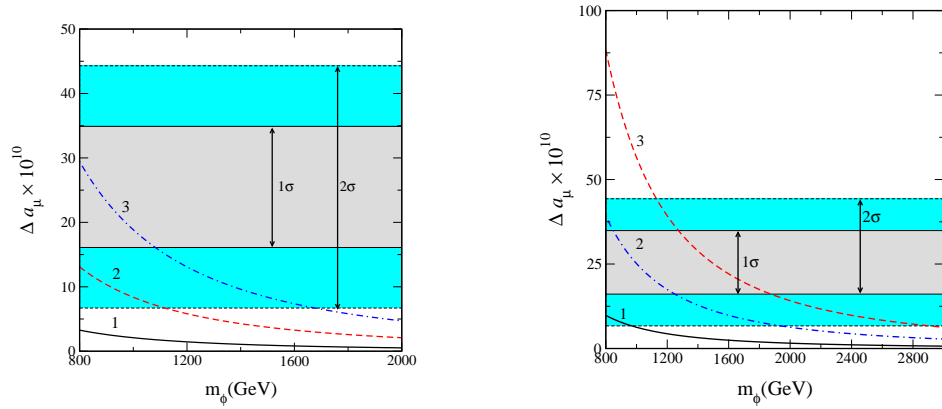

FIG. 4: Plot of $\triangle a_\mu$ as a function of Higgs mass for various values of Y. In left panel we have assumed $Y_{\mu i} = 0$ with $i = e, \mu, \tau$ and $i \neq \mu$ and right panel we have assumed $Y_{\mu i} = Y_{\mu\mu}$. Shaded area indicates 1σ and 2σ deviations.

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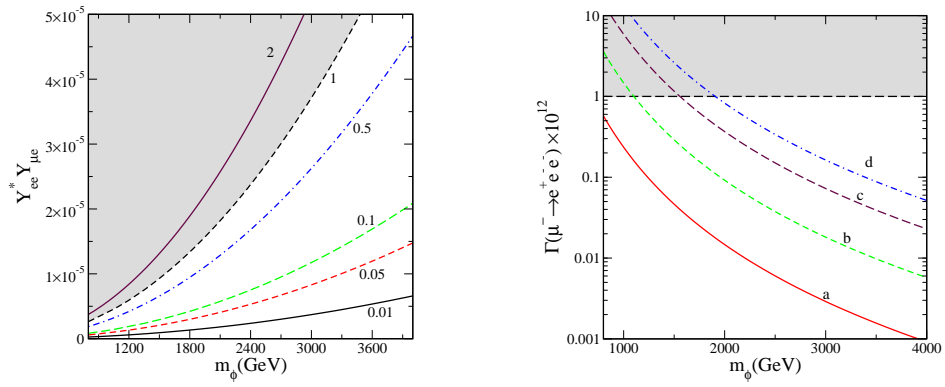[14] B. L. Roberts [Muon g-2 Collaboration], arXiv:hep-ex/0501012.

FIG. 5: Contour plot of branching ratio of $\mu^- \to e^+e^-e^-$ in m_Φ- $Y_{ee}^*Y_{\mu e}$ plane (left). Plot of Branching ratio of $\mu^- \to e^+e^-e^-$ as a function of Higgs mass for various Yukawa couplings (right). In right panel the legends a, b, c, d corresponds to $Y_{ee}^*Y_{\mu e}$ values of $(1.5, 1, 0.5, 0.2) \times 10^{-5}$ respectively. Shaded area indicates region ruled out by experimental data.