

## Universal theory of weak interactions in the paracharge scheme and quark-lepton analogy

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**Abstract.** A universal theory of weak interaction is constructed by exploiting an analogy inherent between the four leptons and the four quarks of the paracharge scheme proposed recently to deal with the  $\psi$ -particles. The leptons ( $\nu_e, \nu_\mu, e_L, \mu_L$ ) are assigned to the representation  $(\frac{1}{2}, \frac{1}{2})$  and the quarks  $(p, n_w)_L$  and  $(\lambda, \lambda_w)_L$  to the representations  $(\frac{1}{2}, 0)$  and  $(0, \frac{1}{2})$ , respectively, of the group  $O_4$  ( $L$  stands for the left-handed projections and  $W$  for the Cabibbo rotated orthogonal combinations of  $n$  and  $\lambda$ ). Universality is ensured by embedding the above (weak)  $O_4$  into the simple group  $O_5$  and gauging the latter. In the final effective weak interaction, besides the conventional  $V-A$  charged-current part, a  $(V-A)$  neutral current interaction (consistent with the present data) is naturally present. The neutral current has a  $\bar{\nu}_\mu \nu_\mu$  term but no  $\bar{\nu}_e \nu_e$  term, thus providing a crucial test of the theory.

**Keywords.** Paracharge;  $O_5$ -gauge symmetry; universality; weak-interaction; neutral-current; quark-lepton analogy.

### 1. Introduction

We have recently proposed (Das *et al* 1975 *a*, 1975 *b*)\* the existence of a new additive quantum number, the paracharge  $Z$ , conserved strictly in strong interactions, to deal with the properties of the new unusually narrow  $\psi$ -particles. The older strong interaction symmetry group  $SU_3$  was thus extended to  $SU_4$ . The basic representations of this larger group are provided by the quartet  $\xi = \text{column}(p, n, \lambda, \chi)$  and its adjoint  $\bar{\xi} = \text{row}(\bar{p}, \bar{n}, \bar{\lambda}, \bar{\chi})$ . The new  $SU_3$ -singlet quark  $\chi$  is distinguished from the older  $SU_3$  triplet of quarks  $(p, n, \lambda)$  by the values  $Z = 1$  and  $Z = 0$ , respectively. The four quarks have† the electric charge  $Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$ , the third component of isospin  $I_3 = \text{diag}(+\frac{1}{2}, -\frac{1}{2}, 0, 0)$ , the hypercharge  $Y \equiv (2Q - 2I_3) = \text{diag}(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3})$ , the baryon number  $B = \text{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ , and the strangeness  $S \equiv Y - B = \text{diag}(0, 0, -1, -1)$ . To account for the production and decays of the  $\psi$ -particles we were led to introduce a new  $Z$ -changing Pauli type "non-minimal" electromagnetic interaction for the hadrons. In the work so far (*see I*) no real attention was paid to the weak interactions in our scheme. The present work is devoted entirely to the description of a *universal* theory of weak interactions in the paracharge scheme.

In attempting the construction of a theory of weak interactions a very interesting possibility comes to hand with the introduction of the paracharged quark  $\chi$ . We

\* We call this work as I (*a, b*) in the present paper.

† The usual modifications such as assigning "colour" or parastatistics to the quarks can be easily made, but, for simplicity, will not be gone into here.

have a quartet of left-handed leptons ( $\nu_e, \nu_\mu, e_L, \mu_L$ ), where

$$e_L \equiv \frac{1 + \gamma_5}{2} e, \quad \mu_L \equiv \frac{1 + \gamma_5}{2} \mu,$$

on the one hand and a quartet of quarks on the other. The muonic lepton number ( $L_\mu$ ) makes a distinction between the leptons that does not depend on the electric charge. In the same way  $Z$ , which does not contribute to the electric charge of the quarks, also makes a distinction between the quarks. This suggests that perhaps for purposes of weak interactions

$$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} P \\ n_w \end{pmatrix}_L$$

may act as doublets of a weak-isospin  $I(W1)$ , while

$$\begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix}, \quad \begin{pmatrix} \mu_L \\ e_L \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} X \\ \lambda_w \end{pmatrix}_L$$

may act as doublets of *another independent* weak isospin  $I(W2)$ . The subscript L stands for the left-handed projections and  $n_w$  and  $\lambda_w$  denote the two orthogonal combinations of  $n$  and  $\lambda$ :

$$n_w \equiv n \cos \theta + \lambda \sin \theta, \quad \lambda_w \equiv -n \sin \theta + \lambda \cos \theta,$$

in terms of the Cabibbo angle  $\theta$ . In other words, we may be dealing with the weak (broken) symmetry group  $O_4(W) = O_3(W1) \otimes O_3(W2)$  with the representations  $(\frac{1}{2}, 0)$  and  $(0, \frac{1}{2})$  for the quark doublets

$$\begin{pmatrix} P \\ n_w \end{pmatrix}_L \quad \text{and} \quad \begin{pmatrix} X \\ \lambda_w \end{pmatrix}_L,$$

respectively, and the representation  $(\frac{1}{2}, \frac{1}{2})$  for the leptons ( $\nu_e, \nu_\mu, \mu_L, e_L$ ).

To achieve a *universal* theory of weak interactions we adopt the above picture and embed  $O_4(W)$  in the *simple* group  $O_5 \equiv C_2$ . The latter has two fundamental representations\*, one  $D^{(4)}$  of four dimensions, and the other  $D^{(5)}$  of five dimensions. The above (weak) quartet of quarks is taken to belong to  $D^{(4)}$  and the above lepton quartet, along with another hypothetical lepton  $\mathcal{L}_L$ , is taken to belong to  $D^{(5)}$  (see figure 1). The subtle *quark-lepton analogy* is thus apparent at the  $O_4 \subset O_5$  level (the “*plus-cross*” analogy). It will be shown below that having performed a useful role, the  $\mathcal{L}$  disappears from our final effective weak interactions on account of the requirement of the electric-charge superselection rule. To obtain a theory with built-in universality we construct the  $O_5$ -gauge invariant weak interaction. This ensures us a universal interaction of the 10 gauge-boson vector fields (belonging to the 10-dimensional adjoint representation of  $O_5$ ) with the corresponding currents involving both the leptons and the quarks *simultaneously*, in terms of one single coupling constant  $g$  (since  $O_5$  is simple). The quark-lepton analogy thus succeeds in ensuring that the “charges” of both the weak leptonic and hadronic currents satisfy the same  $O_5$  algebra. The effective Lagrangian emerges when the intermediate vector bosons (IVB's) are given non-zero masses. These masses are presumed to arise naturally in the physically stable solution displaying the mass spectrum of particles observed in nature. This solution

\* All the relevant information on the group  $O_5 \equiv C_2$  can be found, e.g. in Behrends et al (1962).

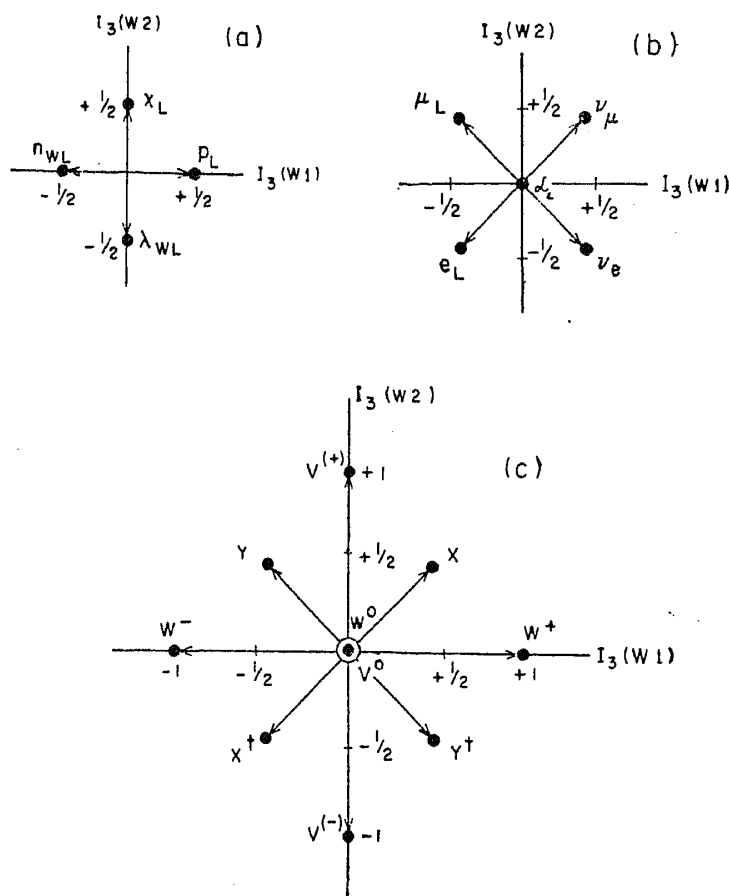


Figure 1. Weight diagrams of  $O_5$  for (a) the left-handed (weak) quarks ( $\sim 4$ ); (b) the left-handed leptons ( $\sim 5$ ); and (c) the IVB's ( $\sim 10$ ). Weight vectors are denoted by solid lines with arrows.

will represent a "spontaneous" breaking of the  $O_5$ -gauge symmetry. Since at present no way of arriving at an exact solution is known, the masses of the intermediate vector bosons must, as is usual, be assigned through phenomenological considerations. Thus the emergence of the mass spectrum as well as the Cabibbo angle  $\theta$  remain profound problems for the future.

In our scheme the dominant effective weak interactions have the following noteworthy features: (i) The range of the weak interactions is strictly non-zero. (ii) The chiral behaviour of the basic fermions of  $O_5$  leads only to the  $V-A$  structure for the weak currents. (iii) The conventional charge-changing currents with CVC and Cabibbo suppression are incorporated quite simply. (iv) Most importantly, we have the natural emergence of a strangeness-preserving *neutral current* (with the associated gauge vector boson  $W^0$ ) with  $V-A$  structure, with the hadronic part having both  $I = 1$  and  $I = 0$  and belonging to the 15-representation of the paracharge  $SU_4$  group. The  $I = 0$  piece has only the  $\lambda$  and the  $\chi$  quarks contributing to it. (v) This neutral current has a  $\bar{\nu}_\mu \nu_\mu$  term but *no*  $\bar{\nu}_e \nu_e$  term. This provides a *crucial test* of our theory since neutral current events are possible here only through  $\nu_\mu$  or  $\bar{\nu}_\mu$  absorption but not through  $\nu_e$  or  $\bar{\nu}_e$  absorption. (vi) In deep inelastic neutrino scattering we expect scale invariance asymptotically

in contrast with the  $e$  or  $\mu$  scattering, where eventually the (hadronic) electromagnetic Pauli couplings become dominant. (vii) The mass  $m(W^0)$  of the neutral intermediate vector boson is expected to be *less* than that of the charged one [ $m(W^\pm) \simeq 1.5 m(W^0)$ ]. (viii) The reality of the representations of the weak  $O_5$  group ensures the absence of the so-called axial-current anomalies. (ix) The relative strengths of all the currents are fixed by the equal-time  $O_5$ -algebra of the "charges".

We present in section 2 the  $O_5$ -gauge invariant theory needed for describing the weak interactions. In section 3 we discuss the physical solution of the broken  $O_5$ -gauge symmetry and the phenomenology of the effective weak interactions. Some useful group theoretic results are collected in Appendix A.

## 2. The $O_5$ -gauge invariant weak interaction

The Lie group  $O_5 \equiv C_2$  is a compact simple ten parameter group with rank two. We can thus take its ten generators  $G_a$ , ( $a = 1, 2, \dots, 10$ ), to be hermitian, and satisfying commutation relations specified by completely antisymmetric real structure constants  $C_{abc}$  (see Appendix A):

$$[G_a, G_b] = iC_{abc}G_c. \quad (2.1)$$

The quartet

$$\xi_{\text{WL}} \equiv \begin{pmatrix} p \\ n_{\text{w}} \\ \lambda_{\text{w}} \\ \chi \end{pmatrix}_{\text{L}}; \quad \left( \xi_{\text{WL}} \equiv \frac{1 + \gamma_5}{2} \xi_{\text{w}} \right) \quad (2.2)$$

involving the Cabibbo angle  $\theta$  through

$$\begin{aligned} n_{\text{w}} &\equiv n \cos \theta + \lambda \sin \theta, & \lambda_{\text{w}} &\equiv -n \sin \theta + \lambda \cos \theta \\ (\bar{n}_{\text{w}} n_{\text{w}} + \bar{\lambda}_{\text{w}} \lambda_{\text{w}}) &= \bar{n} n + \bar{\lambda} \lambda \end{aligned} \quad (2.3)$$

is taken to provide the 4-dimensional fundamental representation  $D^{(4)}$  of our  $O_5$  group (see figure 1a). The right-handed projections (subscript R), e.g.

$$p_{\text{R}} \equiv \frac{1 - \gamma_5}{2} p,$$

of the quarks are taken as  $O_5$  singlets. The other fundamental representations  $D^{(5)}$  of 5 dimensions of our  $O_5$  is taken to be given by the left-handed lepton 5-plet (see figure 1b):

$$l_{\text{L}} \equiv \begin{pmatrix} \nu_{\text{e}} \\ \nu_{\mu} \\ \mu_{\text{L}} \\ e_{\text{L}} \\ \mathcal{L}_{\text{L}} \end{pmatrix} \quad (2.4)$$

where  $\mathcal{L}$  is a fifth hypothetical lepton that will be seen in the end not to appear in the effective weak interactions on account of the deep reason of the charge superselection rule (see section 3). The right-handed leptons  $e_{\text{R}}, \mu_{\text{R}}, \mathcal{L}_{\text{R}}$  are taken as singlets of  $O_5$ . It is *assumed* that there are no right-handed neutrinos.

It is important to note that the electric charge operator  $Q$  cannot be expressed as a linear combination of the generators of  $O_5$  and is thus outside the weak group.

In  $D^{(4)}$  we have the hermitian  $4 \times 4$  matrix representation of the generators in terms of the matrices  $\rho_a$ ,  $G_a = \frac{1}{2} \rho_a$ ; and in  $D^{(5)}$  the  $5 \times 5$  matrix representation in terms of matrices  $\delta_a$ ,  $G_a = \frac{1}{2} \delta_a$ , satisfying the commutation and normalization relations (see Appendix A):

$$[\rho_a, \rho_b] = 2iC_{abc}\rho_c, \quad [\delta_a, \delta_b] = 2iC_{abc}\delta_c, \quad (2.5)$$

$$\text{Tr}(\rho_a\rho_b) = 2\delta_{ab}, \quad \text{Tr}(\delta_a\delta_b) = 4\delta_{ab}. \quad (2.6)$$

Now we start in the standard manner to generate the  $O_5$ -gauge invariant interaction†. We start from the free fermion Lagrangian (mass-less):

$$L_0 = \bar{\xi}_{\text{WL}}\gamma_\mu\partial_\mu\xi_{\text{WL}} + \bar{l}_L\gamma_\mu\partial_\mu l_L + \text{terms for the R-components}. \quad (2.7)$$

Demanding invariance under the  $O_5$ -gauge transformations (repeated index  $a$  summed over 1 to 10):

$$\xi_{\text{WL}}(x) \rightarrow e^{i\epsilon_a(x)\rho_a/2} \xi_{\text{WL}}(x), \quad l_L(x) \rightarrow e^{i\epsilon_a(x)\delta_a/2} l_L(x) \quad (2.8)$$

the R-fields remaining unchanged, necessitates the introduction of ten hermitean vector gauge boson fields  $u_{\mu a}(x)$  with the transformation property [for infinitesimal  $\epsilon_a(x)$ ]:

$$u_{\mu a}(x) \rightarrow u_{\mu a}(x) - C_{abc}\epsilon_b(x)u_{\mu c}(x) - 1/g\partial_\mu\epsilon_a(x), \quad (2.9)$$

and the replacements to gauge covariant derivatives

$$\begin{aligned} \partial_\mu\xi_{\text{WL}} &\rightarrow (\partial_\mu + igu_{\mu a}\rho_a/2)\xi_{\text{WL}}, \\ \partial_\mu l_L &\rightarrow (\partial_\mu + igu_{\mu a}\delta_a/2)l_L. \end{aligned} \quad (2.10)$$

Then the  $O_5$ -gauge invariant Lagrangian is

$$\begin{aligned} L = & -\frac{1}{4}G_{\mu\nu a}G_{\mu\nu a} + \bar{\xi}_{\text{WL}}\gamma_\mu(\partial_\mu + igu_{\mu a}\rho_a/2)\xi_{\text{WL}} \\ & + \bar{l}_L\gamma_\mu(\partial_\mu + igu_{\mu a}\delta_a/2)l_L \\ & + \bar{p}_R\gamma_\mu\partial_\mu p_R + \bar{n}_{\text{WR}}\gamma_\mu\partial_\mu n_{\text{WR}} + \bar{\lambda}_{\text{WR}}\gamma_\mu\partial_\mu\lambda_{\text{WR}} \\ & + \bar{\chi}_R\gamma_\mu\partial_\mu\chi_R + \bar{e}_R\gamma_\mu\partial_\mu e_R + \bar{\mu}_R\gamma_\mu\partial_\mu\mu_R + \bar{\mathcal{L}}_R\gamma_\mu\partial_\mu\mathcal{L}_R, \end{aligned} \quad (2.11)$$

where

$$G_{\mu\nu a} \equiv \partial_\mu u_{\nu a} - \partial_\nu u_{\mu a} - gC_{abc}u_{\mu b}u_{\nu c}. \quad (2.12)$$

The interaction term for the gauge fields is

$$L_{\text{int}} = gu_{\mu a}J_{\mu a}, \quad (2.13)$$

where the ten conserved currents  $J_{\mu a}$  are given by

$$J_{\mu a} = i\bar{\xi}_{\text{WL}}\gamma_\mu\frac{\rho_a}{2}\xi_{\text{WL}} + i\bar{l}_L\gamma_\mu\frac{\delta_a}{2}l_L - C_{abc}G_{\mu\nu b}u_{\nu c}. \quad (2.14)$$

† For a review of gauge theories and spontaneous gauge symmetry breaking see Tuli Dass (1973). For notations used for the Dirac matrices, space-time metric, and a review of the conventional weak interaction theory, see Marshak *et al* (1969).

Thus our quark-lepton analogy has forced the simultaneous appearance of the leptonic and the hadronic terms in the weak currents.

It should be noted that since  $\bar{\xi}_{\text{WL}}\sigma_{\mu\nu}\xi_{\text{WL}} \equiv \bar{l}_{\text{L}}\sigma_{\mu\nu}l_{\text{L}} \equiv 0$  (because  $\gamma_5$  commutes with  $\sigma_{\mu\nu}$ ) no Pauli tensor interaction can be written consistent with  $O_5$ -gauge invariance. This is in contrast with the case of the electromagnetic interaction of the quarks. This will also lead to *asymptotic* scale invariance in weak (but not in electromagnetic) deep inelastic lepton-scattering.

For further discussion it is useful to define the following intermediate vector boson (IVB) fields:

$$\begin{aligned}
W_{\mu}^{-} &\equiv \frac{1}{\sqrt{2}}(u_{\mu 1} + iu_{\mu 2}), & W_{\mu}^{+} &\equiv \frac{1}{\sqrt{2}}(u_{\mu 1} - iu_{\mu 2}); \\
V_{\mu}^{(-)} &\equiv \frac{1}{\sqrt{2}}(u_{\mu 4} + iu_{\mu 5}), & V_{\mu}^{(+)} &\equiv \frac{1}{\sqrt{2}}(u_{\mu 4} - iu_{\mu 5}); \\
W_{\mu}^0 &\equiv \frac{1}{\sqrt{2}}(u_{\mu 3} + u_{\mu 6}), & V_{\mu}^0 &\equiv \frac{1}{\sqrt{2}}(u_{\mu 3} - u_{\mu 6}); \\
X_{\mu} &\equiv \frac{1}{\sqrt{2}}(u_{\mu 7} + iu_{\mu 8}), & X_{\mu}^{\dagger} &\equiv \frac{1}{\sqrt{2}}(u_{\mu 7} - iu_{\mu 8}); \\
Y_{\mu} &\equiv \frac{1}{\sqrt{2}}(u_{\mu 9} + iu_{\mu 10}), & Y_{\mu}^{\dagger} &\equiv \frac{1}{\sqrt{2}}(u_{\mu 9} - iu_{\mu 10}).
\end{aligned} \tag{2.15}$$

The superscripts ( $\pm$ ) on  $V_{\mu}$  refer to  $Z + L_{\mu}$ ; the  $V$ 's do not carry any electric charge.

The IVB's are displayed in the weight diagram shown in figure 1c, appropriate to the adjoint representation of  $O_5$  (to which the generators also belong). From the diagrams of figure 1 it is clear that  $O_5$  admits a *discrete* operation  $R$  that reflects the weight diagrams through the line  $I_3(W1) = I_3(W2)$ . Our interaction eq. (2.13) is invariant under  $R$ . In eq. (2.15) we should note that the strictly neutral bosons  $W^0$  and  $V^0$  are eigenstates of  $R$  with  $R = +1$  and  $R = -1$ , respectively. This separation is useful in discussing the effective weak interaction in section 3.

The interaction can now be written as follows:

$$\begin{aligned}
L_{\text{int}} = &\left\{ g \frac{1}{\sqrt{2}} W_{\mu}^{-} i [\cos \theta (\bar{n}_{\text{L}}\gamma_{\mu}p_{\text{L}}) + \sin \theta (\bar{\lambda}_{\text{L}}\gamma_{\mu}p_{\text{L}}) + \right. \\
&+ (\bar{e}_{\text{L}}\gamma_{\mu}v_{\text{e}}) + (\bar{\mu}_{\text{L}}\gamma_{\mu}v_{\mu}) + i(C_{1bc} - iC_{2bc}) G_{\mu\nu b}u_{\nu c}] + \text{h.c.} \left. \right\} \\
&+ g \frac{1}{2\sqrt{2}} W_{\mu}^0 i [(\bar{p}_{\text{L}}\gamma_{\mu}p_{\text{L}}) + (\bar{\chi}_{\text{L}}\gamma_{\mu}\chi_{\text{L}}) - (\bar{n}_{\text{L}}\gamma_{\mu}n_{\text{L}}) - (\bar{\lambda}_{\text{L}}\gamma_{\mu}\lambda_{\text{L}}) + \\
&+ 2(\bar{v}_{\mu}\gamma_{\mu}v_{\mu}) - 2(\bar{e}_{\text{L}}\gamma_{\mu}e_{\text{L}}) + 2i(C_{3bc} + C_{6bc}) G_{\mu\nu b}u_{\nu c}] \\
&+ g \frac{1}{2\sqrt{2}} V_{\mu}^0 i [(\bar{p}_{\text{L}}\gamma_{\mu}p_{\text{L}}) - (\bar{\chi}_{\text{L}}\gamma_{\mu}\chi_{\text{L}}) + \cos 2\theta (\bar{\lambda}_{\text{L}}\gamma_{\mu}\lambda_{\text{L}})
\end{aligned}$$

$$\begin{aligned}
 & -\cos 2\theta (\bar{n}_L \gamma_\mu n_L) - \sin 2\theta (\bar{\lambda}_L \gamma_\mu n_L) - \sin 2\theta (\bar{n}_L \gamma_\mu \lambda_L) + 2(\bar{\nu}_e \gamma_\mu \nu_e) \\
 & - 2(\bar{\mu}_L \gamma_\mu \mu_L) + 2i(C_{2bc} - C_{6bc}) G_{\mu\nu b} u_{\nu c}] \\
 & + \left\{ g \frac{1}{\sqrt{2}} V_\mu^{(-)} i [\sin \theta (\bar{n}_L \gamma_\mu \chi_L) - \cos \theta (\bar{\lambda}_L \gamma_\mu \chi_L) + (\bar{\nu}_e \gamma_\mu \nu_e) \right. \\
 & \left. + (\bar{e}_L \gamma_\mu \mu_L) + i(C_{4bc} - iC_{5bc}) G_{\mu\nu b} u_{\nu c}] + \text{h.c.} \right\} \\
 & + \left\{ g \frac{1}{2} X_\mu i [(\bar{\lambda}_{wL} \gamma_\mu p_L) - (\bar{n}_{wL} \gamma_\mu \chi_L) + \sqrt{2} (\mathcal{L}_L \gamma_\mu \nu_\mu) \right. \\
 & \left. - \sqrt{2} (\bar{e}_L \gamma_\mu \mathcal{L}_L) + \sqrt{2} i(C_{7bc} - iC_{8bc}) G_{\mu\nu b} u_{\nu c}] + \text{h.c.} \right\} \\
 & + \left\{ g \frac{1}{2} Y_\mu i [(\bar{p}_L \gamma_\mu \chi_L) + (\bar{\lambda}_{wL} \gamma_\mu n_{wL}) + \sqrt{2} (\bar{\mathcal{L}}_L \gamma_\mu \mu_L) \right. \\
 & \left. + \sqrt{2} (\bar{\nu}_e \gamma_\mu \mathcal{L}_L) + \sqrt{2} i(C_{9bc} - iC_{10bc}) G_{\mu\nu b} u_{\nu c}] + \text{h.c.} \right\}. \quad (2.16)
 \end{aligned}$$

It is important to note that\* our theory does not have the so-called axial-current anomalies, since the group  $O_5$  has only real representations.

### 3. The effective weak interaction and broken $O_5$ -gauge symmetry

In the highly symmetric  $O_5$ -gauge invariant theory presented above, all the IVB's have zero bare mass. If the physical solution of the theory is presumed to describe the observed weak interaction, it is clear that the physical masses of the IVB's must be very large indeed to account for the extremely short ranges of the effective weak interaction. The physical solution must thus display a drastic breaking† of the  $O_5$ -gauge symmetry. In the absence of a method for obtaining the exact solution of our field theory, we shall simply *assume* that the desired mass spectrum does emerge from the theory. We assign the values of the masses on the basis of phenomenological considerations. This is similar to the introduction of the Cabibbo angle  $\theta$  phenomenologically. The emergence of both the desired physical mass spectrum as well as the angle‡  $\theta$  in the stable physical solution from such a theory, if at all possible, will remain profound (and possibly connected) problems for the future.

It is clear that for low energy weak processes the effective coupling constant for a weak interaction mediated by a vector boson  $B$  will be simply proportional to  $g^2/m^2(B)$ . Comparison with the conventional Fermi-coupling constant  $G$  thus leads to

$$G = \frac{g^2}{4\sqrt{2} m^2 (W^\pm)}, \quad G = 1.024 \times 10^{-5} \times m_p^{-2}. \quad (3.1)$$

This fixes the relative scale of comparison for the different masses, which are obtained as follows:

\* For review and references, see Tuli Dass (1973).

† For a review of gauge theories and spontaneous gauge symmetry breaking, see Tuli Dass (1973). For notations used for the Dirac matrices, space time metric, and a review of the conventional weak interaction theory, see Murshak *et al.*

‡ For examples of attempts at understanding  $\theta$ , cf. Michel and Radicati (1968); Gatto, Sartori and Tonin (1968); Cabibbo (1969); Pais (1972).

(i) From eq. (2.16) it is clear that there are no real linear combinations of  $J_{\mu a}$ ,  $a = 7, 8, 9, 10$ , such that every term satisfies the same electric-charge selection rule. Consequently, no physical boson can couple to these currents. In other words, the condition of the electric-charge *superselection rule* on the effective interaction forces the result:

$$\frac{m(W^\pm)}{m(X)} = \frac{m(W^\pm)}{m(Y)} = 0. \quad (3.2)$$

Hence, the effective couplings mediated by the  $X$ ,  $Y$ ,  $X^\dagger$ ,  $Y^\dagger$  are all strictly zero. We note that such a use of electric charge superselection rule has been possible here because the group  $O_5$  does not include electric charge  $Q$  among its ten generators.

(ii) We find from the  $L_{int}$ , given by eq. (2.16), that only the vector boson  $V_\mu^0 \equiv 1/\sqrt{2}(u_{\mu 3} - u_{\mu 6})$  combination couples to a strangeness changing neutral current; the other neutral vector boson  $W_\mu^0 \equiv 1/\sqrt{2}(u_{\mu 3} + u_{\mu 6})$  does not do so. We shall regard  $V_\mu^0$  and  $W_\mu^0$  to be the physical neutral vector bosons and eigenstates of the mass. From the extreme smallness of the rates for the strangeness-changing neutral current processes, e.g., from the branching ratio (Kleinknecht 1974)

$$\text{Br}(K_L^0 \rightarrow \mu^+\mu^-) = (12_{-4}^{+8}) \times 10^{-9}, \quad (3.3)$$

we obtain (see figure 2 a):

$$\frac{m^2(W^\pm)}{m^2(V^0)} \lesssim 10^{-4}. \quad (3.4)$$

Thus the effective coupling constant of the  $V^0$  mediated interaction is nearly 10,000 times smaller than  $G$ .

(iii) From the limit§

$$\text{Br}(\mu \rightarrow ee\bar{e}) < 6 \times 10^{-11}, \quad (3.5)$$

and from the consideration that this decay can proceed (see, e.g. figure 2b) through the  $V^\pm$  interaction along with the electromagnetic  $Z$ -changing Pauli coupling

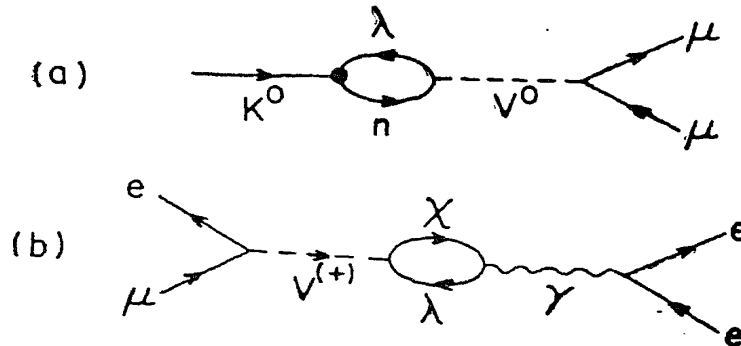


Figure 2. (a) Mechanism for the  $K^0 \rightarrow \mu\bar{\mu}$  decay via the  $V^0$ . (b) Mechanism for the decay  $\mu \rightarrow ee\bar{e}$  via the  $V^{(+)}$  and the anomalous electromagnetic interaction.

§ The lowest allowed order amplitude for  $\mu \rightarrow e\gamma$  is accidentally zero in our scheme for a real photon on account of the tensor structure of the relevant  $Z$ -changing electromagnetic interaction.



[Appendix A, eqs (15) to (18)] we obtain

$$\frac{m^2(W^\pm)}{m^2(V^{(\pm)})} \lesssim 3 \times 10^{-3} \quad (3.6)$$

Thus the effective coupling constant of the  $V^{(\pm)}$  mediated interaction is around at least a thousand times smaller than  $G$ . This then also automatically suppresses the contribution of the  $V^{(\pm)}$  interaction to the  $\mu$ -decay ( $\mu \rightarrow e\bar{\nu}_e\nu_\mu$ ) to less than the radiative correction to the normal dominant contribution of the  $W^\pm$  interaction.

We are thus effectively left with the dominant weak interactions mediated by the  $W^\pm$  and the  $W^0$ . The former is the conventional well established charged-current interaction. The latter is the *strangeness preserving neutral-current* interaction (experimentally discovered recently) and is seen to emerge quite naturally in our theory.

In terms of the  $SU_4$  tensor operators:

$$\mathcal{F}_\mu^i \equiv i\bar{\xi} \frac{\lambda_i}{2} \gamma_\mu \xi, \quad \mathcal{F}_{5\mu}^i \equiv i\bar{\xi} \frac{\lambda_i}{2} \gamma_\mu \gamma_5 \xi, \quad i = 1, 2, \dots, 15, \quad (3.7)$$

we may write now the *dominant* effective weak interaction by dropping all the IVB's except the  $W^\pm$  and the  $W^0$ :

$$\begin{aligned} L_{\text{eff}} = & \frac{g}{2\sqrt{2}} W_\lambda^- [\cos \theta \{(\mathcal{F}_\lambda^1 - i\mathcal{F}_\lambda^2) + (\mathcal{F}_{5\lambda}^1 - i\mathcal{F}_{5\lambda}^2)\} \\ & + \sin \theta \{(\mathcal{F}_\lambda^4 - i\mathcal{F}_\lambda^5) + (\mathcal{F}_{5\lambda}^4 - i\mathcal{F}_{5\lambda}^5)\} \\ & + (\bar{e}\gamma_\lambda (1 + \gamma_5) \nu_e) + (\bar{\mu}\gamma_\lambda (1 + \gamma_5) \nu_\mu) \\ & - 2i \{W_\nu^0 G_{\lambda\nu} (W^+) - W_\nu^+ G_{\lambda\nu} (W^0)\} + \text{h.c.} \\ & + \frac{g}{4\sqrt{2}} W_\lambda^0 \left[ \left( \mathcal{F}_\lambda^3 + \frac{1}{\sqrt{3}} \mathcal{F}_\lambda^8 - \sqrt{\frac{2}{3}} \mathcal{F}_\lambda^{15} \right) + \left( \mathcal{F}_{5\lambda}^3 + \frac{1}{\sqrt{3}} \mathcal{F}_{5\lambda}^8 \right. \right. \\ & \left. \left. - \sqrt{\frac{2}{3}} \mathcal{F}_{5\lambda}^{15} \right) + 2 (\bar{\nu}_\mu \gamma_\lambda (1 + \gamma_5) \nu_\mu) - 2 (\bar{e}\gamma_\lambda (1 + \gamma_5) e) \right. \\ & \left. - 4i \{G_{\lambda\nu} (W^-) W_\nu^+ - G_{\lambda\nu} (W^+) W_\nu^-\} \right], \quad (3.8) \end{aligned}$$

where

$$G_{\lambda\nu} (W^0) \equiv \partial_\lambda W_\nu^0 - \partial_\nu W_\lambda^0 + \frac{i}{\sqrt{2}} g (W_\lambda^+ W_\nu^- - W_\lambda^- W_\nu^+);$$

$$G_{\lambda\nu} (W^\pm) \equiv \partial_\lambda W_\nu^\pm - \partial_\nu W_\lambda^\pm \pm ig (W_\lambda^0 W_\nu^\pm - W_\nu^0 W_\lambda^\pm).$$

The neutral current has the following features:

(i) A pure  $V$ - $A$  structure at the quark (parton) level.

(ii) It has both the isospin  $I = 1$  as well as  $I = 0$  hadronic pieces and they all belong to the adjoint representation 15 of the strong interaction symmetry group  $SU_4$ . It is to be noted that the  $I = 0$  piece does not involve the  $p$  and the  $n$  quarks

(iii) The leptonic part of the neutral current has only  $(\bar{\nu}_\mu \nu_\mu)$  and  $(\bar{e} e)$  terms but *no*  $(\bar{\nu}_e \nu_e)$  or  $(\bar{\mu} \mu)$  terms.

In comparing with results of the neutral current experiments, we shall use the simple parton picture. This is justified from the similarity of the space-time structures of the neutral and the charged currents, and by the observation of scaling behaviour at the lower energies in the charged current  $\nu_\mu$  ( $\bar{\nu}_\mu$ ) interactions. Then in terms of the experimental parameters

$$r \equiv \frac{\sigma_N(\nu_\mu \mathcal{N})}{\sigma_C(\nu_\mu \mathcal{N})}; \quad \bar{r} \equiv \frac{\sigma_N(\bar{\nu}_\mu \mathcal{N})}{\sigma_C(\bar{\nu}_\mu \mathcal{N})}, \quad (3.10)$$

where the subscripts  $N$  and  $C$  refer (Rajasekaran and Sarma 1974)\* to the neutral and the charged currents, respectively, and  $\mathcal{N}$  denotes the  $I$ -spin averaged nucleon. We have from the  $V-A$  structure the results (assuming that only the  $p$  and the  $n$  quarks are effective at lower energies):

$$\sigma_C(\bar{\nu}_\mu \mathcal{N}) = \frac{1}{3} \sigma_C(\nu_\mu \mathcal{N}), \quad (3.11)$$

$$r = \bar{r}, \quad (3.12)$$

$$r = \frac{1}{8} \frac{m^2(W^\pm)}{m^2(W^0)}. \quad (3.13)$$

At higher energies (where the  $\lambda$  and  $\chi$  quarks play a role) the result  $r = \bar{r}$  would be somewhat modified. From the lower energy experimental result (Hasegawa et al 1974) (CERN-corrected for cuts)

$$r = 0.23 \pm 0.04,$$

$$\bar{r} = 0.33 \pm 0.07, \quad (3.14)$$

taking  $r \simeq \bar{r} \simeq \frac{1}{4}$ , we obtain

$$m(W^0) \simeq \frac{1}{\sqrt{2}} m(W^\pm). \quad (3.15)$$

Thus, the neutral IVB,  $W^0$ , is expected in our theory to be around  $\simeq 1.5$  times less massive than the charge IVB's  $W^\pm$ . Concerning the isospin structure some experimental information is available. For example, from the result (Barish et al 1974)

$$\frac{\sigma(\nu_\mu p \rightarrow (\nu_\mu) p \pi^0)}{\sigma(\nu_\mu p \rightarrow (\nu_\mu) n \pi^+)} = 3.1 \pm 2.1, \quad (3.16)$$

pure  $\Delta I = 0$  appears to be ruled out (as the expected value for the ratio would be  $\frac{1}{2}$ ). Similarly for pure  $\Delta I = 1$  (or pure  $\Delta I = 0$ ) we expect

$$\frac{\sigma(\nu_\mu d \rightarrow (\nu_\mu) \pi^+ nn)}{\sigma(\nu_\mu d \rightarrow (\nu_\mu) \pi^- pp)} = 1. \quad (3.17)$$

As pointed out earlier, the  $I = 0$  neutral current does not have the  $p$  or  $n$  quark terms, but has the  $\lambda$  and  $\chi$  quark terms. Thus, at higher  $\nu_\mu$  ( $\bar{\nu}_\mu$ ) energies, where the  $\lambda$  and the  $\chi$  quarks play a role, the effective isospin of the neutral current should change from a nearly pure  $I = 1$  to a mixture of  $I = 1$  and  $I = 0$ . Thus the  $I$ -behaviour of the current will show some energy dependence in deep inelastic  $\nu_\mu$  and  $\bar{\nu}_\mu$  scattering off nucleons.

\* The preliminary conclusion of this paper, that  $(V \pm A)$ -structure of the neutral current is not favoured, is to be modified by the more recent experiments. We thank Drs G Rajasekaran and K V L Sarma for a discussion on this.

The absence of any  $(\bar{\nu}_e \nu_e)$  term in the neutral current implies that (denoting by subscript  $C$  the charged-current contribution)

$$\frac{\sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e)}{\sigma_c(\bar{\nu}_e e \rightarrow \bar{\nu}_e e)} = 1. \quad (3.17)$$

Experimentally (Gurr *et al* 1972)  $\sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) < 1.9 [\sigma_c(\bar{\nu}_e e \rightarrow \bar{\nu}_e e)]_{V-A}$ . Further there should be *no* observable reaction of the type  $\bar{\nu}_e + d \rightarrow \bar{\nu}_e + n + p$ . Experimentally (Gurr *et al* 1974), for energy between 2.2 and 5 MeV, this is seen to be satisfied. Furthermore, our leptonic neutral current allows for the scattering

$$\nu_\mu(\bar{\nu}_\mu) + e \rightarrow \nu_\mu(\bar{\nu}_\mu) + e, \quad (3.18)$$

with pure  $V-A$  structure. This is quite consistent with the data (Hasert *et al* 1973) so far available.

The absence of a  $(\bar{\mu}\mu)$  term in the neutral current implies that no observable weak modification of the electrodynamic process (McDonald 1974)

$$e^+e^- \rightarrow \mu^+\mu^- \quad (3.19)$$

should be present. Weak effects should, however, be present in the process (McDonald 1974)

$$e^+e^- \rightarrow \text{hadron} + \text{anything}. \quad (3.20)$$

The neutral current is thus seen to violate  $\mu$ - $e$  universality in a well-specified manner.

In both the  $W^\pm$  mediated, as well as the  $W^0$ -mediated, weak inelastic scattering off nuclei at high energies  $\mu^+\mu^-$  (uncorrelated) pairs can be produced by processes involving a virtual photon. We shall await more definitive higher statistics experiments on the observation of "dimuon events" (Benvenuti *et al* 1975) before making quantitative comments based on our theory.

The structure of our weak currents (both charged and neutral) would lead to scaling behaviour in neutrino inelastic reactions at the extreme asymptotic energies after the scaling-violations due to new particle production thresholds have died down. This is in contrast with the non-scaling behaviour expected in electromagnetic  $e$  and  $\mu$  reactions at asymptotic energies when (hadronic) Pauli couplings become the dominant ones.

## Appendix A: Some useful group theoretic results

Most of the theory of Lie groups relevant to our work can be found in the article by Behrends *et al* (1962). For convenience we list below the representation matrices used by us.

1. *The matrix representation of the generators of the fundamental representations of  $O_5$*

We denote the 10 hermitian generators by  $G_a$ ,  $a = 1, 2, \dots, 10$ . They satisfy the commutation relations:

$$[G_a, G_b] = iC_{ab0}G_0, \quad (A 1)$$

where  $C_{ab0}$  are the completely antisymmetric and real structure constants. The hermitian generators of  $O_3(W1) \otimes O_3(W2) = O_4(W) \subset O_5$  are denoted by

$I_i(W1)$ ,  $I_i(W2)$ ,  $i = 1, 2, 3$ , and are identified as:

$$\begin{aligned} G_1 &= I_1(W1), \quad G_2 = I_2(W1), \quad G_3 = I_3(W1); \quad G_4 = I_1(W2), \\ G_5 &= I_2(W2), \quad G_6 = I_3(W2). \end{aligned} \quad (\text{A } 2)$$

In the fundamental representation  $D^{(4)}$  the generators are given in terms of the ten  $4 \times 4$  hermitian matrices  $\rho_a$ :

$$G_a^{(4)} = \frac{1}{2} \rho_a, \quad [\rho_a, \rho_b] = 2iC_{ab0}\rho_c, \quad \text{Tr}(\rho_a\rho_b) = 2\delta_{ab}. \quad (\text{A } 3)$$

In the orthonormal basis  $|\xi_1\rangle = |p_L\rangle$ ,  $|\xi_2\rangle = |x_L\rangle$ ,  $|\xi_3\rangle = |n_{WL}\rangle$ ,  $|\xi_4\rangle = |\lambda_{WL}\rangle$ , we have the  $\rho$ -matrices in the "operator form":

$$\begin{aligned} \rho_1 &= |\xi_1\rangle\langle\xi_3| + |\xi_3\rangle\langle\xi_1|, \\ \rho_2 &= -i(|\xi_1\rangle\langle\xi_3| - |\xi_3\rangle\langle\xi_1|), \\ \rho_3 &= |\xi_1\rangle\langle\xi_1| - |\xi_3\rangle\langle\xi_3|; \\ \rho_4 &= -(|\xi_2\rangle\langle\xi_4| + |\xi_4\rangle\langle\xi_2|), \\ \rho_5 &= i(|\xi_2\rangle\langle\xi_4| - |\xi_4\rangle\langle\xi_2|), \\ \rho_6 &= |\xi_2\rangle\langle\xi_2| - |\xi_4\rangle\langle\xi_4|; \\ \rho_7 &= \frac{1}{\sqrt{2}}(|\xi_1\rangle\langle\xi_4| - |\xi_2\rangle\langle\xi_3| + |\xi_4\rangle\langle\xi_1| - |\xi_3\rangle\langle\xi_2|), \\ \rho_8 &= \frac{-i}{\sqrt{2}}(|\xi_1\rangle\langle\xi_4| - |\xi_2\rangle\langle\xi_3| - |\xi_4\rangle\langle\xi_1| + |\xi_3\rangle\langle\xi_2|); \\ \rho_9 &= \frac{1}{\sqrt{2}}(|\xi_2\rangle\langle\xi_1| + |\xi_3\rangle\langle\xi_4| + |\xi_1\rangle\langle\xi_2| + |\xi_4\rangle\langle\xi_3|), \\ \rho_{10} &= \frac{-i}{\sqrt{2}}(|\xi_2\rangle\langle\xi_1| + |\xi_3\rangle\langle\xi_4| - |\xi_1\rangle\langle\xi_2| - |\xi_4\rangle\langle\xi_3|). \end{aligned} \quad (\text{A } 4)$$

In the fundamental representation  $D^{(5)}$  the generators are given in terms of the ten  $5 \times 5$  hermitian matrices  $\delta_a$ :

$$G_a^{(5)} = \frac{1}{2} \delta_a, \quad [\delta_a, \delta_b] = 2iC_{abc}\delta_c, \quad \text{Tr}(\delta_a\delta_b) = 4\delta_{ab} \quad (\text{A } 5)$$

In the orthonormal basis  $|l_1\rangle = |v_e\rangle$ ,  $|l_2\rangle = |v_\mu\rangle$ ,  $|l_3\rangle = |\mu_L\rangle$ ,  $|l_4\rangle = |e_L\rangle$ ,  $|l_5\rangle = |\mathcal{L}_L\rangle$ , we have the  $\delta$ -matrices in the "operator form":

$$\begin{aligned} \delta_1 &= |l_1\rangle\langle l_4| + |l_2\rangle\langle l_3| + |l_4\rangle\langle l_1| + |l_3\rangle\langle l_2|, \\ \delta_2 &= -i(|l_1\rangle\langle l_4| + |l_3\rangle\langle l_3| - |l_4\rangle\langle l_1| - |l_3\rangle\langle l_2|), \\ \delta_3 &= |l_1\rangle\langle l_1| + |l_2\rangle\langle l_2| - |l_3\rangle\langle l_3| - |l_4\rangle\langle l_4|; \\ \delta_4 &= |l_2\rangle\langle l_1| + |l_3\rangle\langle l_4| + |l_1\rangle\langle l_2| + |l_4\rangle\langle l_3|, \\ \delta_5 &= -i(|l_2\rangle\langle l_1| + |l_3\rangle\langle l_4| - |l_1\rangle\langle l_2| - |l_4\rangle\langle l_3|), \\ \delta_6 &= -|l_1\rangle\langle l_1| + |l_2\rangle\langle l_2| + |l_3\rangle\langle l_3| - |l_4\rangle\langle l_4|; \\ \delta_7 &= |l_2\rangle\langle l_5| - |l_5\rangle\langle l_4| + |l_5\rangle\langle l_2| - |l_4\rangle\langle l_5|, \\ \delta_8 &= -i(|l_2\rangle\langle l_5| - |l_5\rangle\langle l_4| - |l_5\rangle\langle l_2| + |l_4\rangle\langle l_5|); \end{aligned}$$

$$\begin{aligned}\delta_9 &= |l_3\rangle\langle l_5| + |l_5\rangle\langle l_1| + |l_5\rangle\langle l_3| + |l_1\rangle\langle l_5|, \\ \delta_{10} &= -i(|l_3\rangle\langle l_5| + |l_5\rangle\langle l_1| - |l_5\rangle\langle l_3| - |l_1\rangle\langle l_5|).\end{aligned}\quad (\text{A } 6)$$

## 2. The $\lambda$ -matrices of the fundamental representation 4 of $SU_4$

These have been listed in Appendix-1 of paper I (b). We repeat them here in the alternate "operator form" for convenience of the reader.

The 15 hermitian generators  $F_i$ ,  $i = 1, 2, \dots, 15$ , of  $SU_4$  satisfy the commutation relations

$$[F_i, F_j] = if_{ijk}F_k, \quad (\text{A } 7)$$

with completely antisymmetric and real structure constants  $f_{ijk}$ . In the basic representation 4 given by  $\xi = \text{column}(p, n, \lambda, \chi)$  the generators are given in terms of 15 hermitean  $4 \times 4$  matrices  $\lambda_i$ ,  $i = 1, \dots, 15$ ,

$$F_i^{(4)} = \frac{1}{2} \lambda_i, \quad [\lambda_i, \lambda_j] = 2if_{ijk}\lambda_k, \quad \text{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}. \quad (\text{A } 8)$$

Note that the normalizations of the  $F_i^{(4)}$  here are different from those given in paper I. Confusion should be avoided on this. In the orthonormal basis  $|1\rangle = |p\rangle$ ,  $|2\rangle = |n\rangle$ ,  $|3\rangle = |\lambda\rangle$ ,  $|4\rangle = |\chi\rangle$ , the  $\lambda$ -matrices in the "operator form" are given by:

$$\begin{aligned}\lambda_1 &= |1\rangle\langle 2| + |2\rangle\langle 1|, & \lambda_2 &= -i(|1\rangle\langle 2| - |2\rangle\langle 1|), \\ \lambda_3 &= |1\rangle\langle 1| - |2\rangle\langle 2|; & \lambda_4 &= |1\rangle\langle 3| + |3\rangle\langle 1|, \\ \lambda_5 &= -i(|1\rangle\langle 3| - |3\rangle\langle 1|); & \lambda_6 &= |2\rangle\langle 3| + |3\rangle\langle 2|, \\ \lambda_7 &= -i(|2\rangle\langle 3| - |3\rangle\langle 2|); \\ \lambda_8 &= \frac{1}{\sqrt{3}}(|1\rangle\langle 1| + |2\rangle\langle 2| - 2|3\rangle\langle 3|); \\ \lambda_9 &= |1\rangle\langle 4| + |4\rangle\langle 1|, & \lambda_{10} &= -i(|1\rangle\langle 4| - |4\rangle\langle 1|); \\ \lambda_{11} &= |2\rangle\langle 4| + |4\rangle\langle 2|, & \lambda_{12} &= -i(|2\rangle\langle 4| - |4\rangle\langle 2|); \\ \lambda_{13} &= |3\rangle\langle 4| + |4\rangle\langle 3|, & \lambda_{14} &= -i(|3\rangle\langle 4| - |4\rangle\langle 3|); \\ \lambda_{15} &= \frac{1}{\sqrt{6}}(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3| - 3|4\rangle\langle 4|).\end{aligned}\quad (\text{A } 9)$$

We also introduce the matrix

$$\lambda_0 = \frac{1}{\sqrt{2}}(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3| + |4\rangle\langle 4|) \quad (\text{A } 10)$$

If  $i, j, k$  take values  $0, 1, \dots, 15$ , we may write

$$[\lambda_i, \lambda_j] = 2if_{ijk}\lambda_k, \quad (\text{A } 11)$$

where  $f_{ijk} = 0$  if any index is 0. We also define the completely symmetric symbol  $d_{ijk}$  through the anti-commutation relation ( $i, j, k = 0, 1, \dots, 15$ ):

$$[\lambda_i, \lambda_j]_+ = 2d_{ijk}\lambda_k, \quad (\text{A } 12)$$

$$\text{Trace}(\lambda_i \lambda_j) = 2\delta_{ij}, \quad \text{Trace}(\lambda_k [\lambda_i, \lambda_j]) = 4if_{ijk},$$

$$\text{Trace}(\lambda_k [\lambda_i, \lambda_j]_+) = 4d_{ijk}, \quad (\text{A } 13)$$

The charge matrix for the  $\xi$ -quartet is

$$Q = \frac{1}{2} \left( \lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 + \frac{1}{\sqrt{6}} \lambda_{15} - \frac{1}{3\sqrt{2}} \lambda_0 \right), \quad (\text{A } 14)$$

so that the  $u_1(Q)$ -gauge invariant electromagnetic interaction for the quarks is

$$L_{\text{em}} = eA_\mu(x)j_\mu(x) + \frac{e}{2M}F_{\mu\nu}(x)J_{\mu\nu}(x) \quad (\text{A } 15)$$

where

$$j_\mu(x) = i\bar{\xi}Q\gamma_\mu\xi = \mathcal{F}_\mu^3 + \frac{1}{\sqrt{3}}\mathcal{F}_\mu^8 + \frac{1}{\sqrt{6}}\mathcal{F}_\mu^{15} - \frac{1}{3\sqrt{2}}\mathcal{F}_\mu^0; \quad (\text{A } 16)$$

and

$$\begin{aligned} J_{\mu\nu} &= \kappa\bar{\xi}Q\sigma_{\mu\nu}\xi + \eta\bar{\xi}\frac{\lambda_{13}}{2}\sigma_{\mu\nu}\xi \\ &= \kappa\left[\mathcal{F}_{\mu\nu}^3 + \frac{1}{\sqrt{3}}\mathcal{F}_{\mu\nu}^8 + \frac{1}{\sqrt{6}}\mathcal{F}_{\mu\nu}^{15} - \frac{1}{3\sqrt{2}}\mathcal{F}_{\mu\nu}^0\right] + \eta\mathcal{F}_{\mu\nu}^{13} \end{aligned} \quad (\text{A } 17)$$

where

$$\mathcal{F}_{\mu\nu}^i \equiv \bar{\xi}\frac{\lambda_i}{2}\sigma_{\mu\nu}\xi, \quad (\text{A } 18)$$

and  $\kappa$  and  $\eta$  are the strength parameters for the Z-preserving and the Z-changing parts, respectively.

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